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ORIGINAL PAPER

Correlation coefficients for T-spherical fuzzy sets and their applications in pattern analysis and multi-attribute decision-making

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Abstract

T-spherical fuzzy set is an effective tool to deal with vagueness and uncertainty in real-life problems, especially in a situation when there are more than two circumstances, like in casting a ballot. The correlation coefficient of T-spherical fuzzy sets is a tool to calculate the association of two T-spherical fuzzy sets. It has several applications in various disciplines like science, management, and engineering. The noticeable applications incorporate pattern analysis, decision-making, medical diagnosis, and clustering. The aim of this article is to introduce some correlation coefficients for T-spherical fuzzy sets with their applications in pattern recognition and decision-making. The under discussion correlation coefficients are far more advantageous than the existing and many other tools used for T-spherical fuzzy sets, because they are used completely and demonstrate the nature as well as the limit of association between two T-spherical fuzzy sets. Further, an application of proposed correlation coefficients in pattern analysis is discussed here. In addition to it, the proposed correlation coefficients are applied to a multi-attribute decision-making problem, in which the selection of a suitable COVID-19 mask is presented. A comparative analysis has also been made to check the effectiveness of the proposed work with the existing correlation coefficients.

Keywords T-spherical fuzzy sets · Correlation coefficients · Pattern recognition · Multi-attribute decision-making

1 Introduction

In various parts of life, to deal with various problems like machine learning, decision-making (DM), and pattern recognition (PR), it is necessary to compare things from a different point of view. DM is important for the deciding science whose objective is to get the best choice from the group of similar choices. To solve the DM problems in daily life, there are many ways, but when the data are in fuzzy form, correlation coefficients (CCs) are found outstanding. The under consideration article, for the most part, is related to the T-spherical fuzzy (TSF) correlation coefficients (TSFCCs). As it is the direct extension of intuitionistic fuzzy sets (IFSs) and picture fuzzy sets (PFSs), therefore, it is appropriate to mention the pioneers and recent studies in terms of the development and applications of T-spherical fuzzy sets (TSFSs).

There exists plenty of unclear, ambiguous, and unstable data in real life. To handle such situations, Zadeh (1965) introduced fuzzy set (FS), in which each element of uncertainty is assigned with a membership grade (MG) denoted by t. The MG is taken in the FSs and one-minus gradation is taken as non-membership grade (NMG). Therefore, one is sure to find the NMG by considering the MG. Nonetheless, in practical life, one is not sure regarding the NMG owing to knowledge of MG. In such circumstances, it is recommended that there should be a free NMG function. To overcome the situation, Atanassov (1986) introduced the concept of IFS, in which each element is assigned to MG as well as a non-membership grade (NMG) denoted by f, with the condition $0 \leq t + f \leq 1$. Following are the references for further study on DM based on IFSs, Ye (2010a, b), Jamkhaneh (2018).

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An IFS is not able to deliver in some cases. For example, if a person is given $t = 0.6$ and $f = 0.5$, in such conditions, IFS will be unable to manage the situation, that is, $t + f = 0.6 + 0.5 = 1.1 \notin [0, 1]$. In the same way, some problems were faced in real life, where IFS was also deviated. Because of these limitations of the IFS, Yager (2013) initiated the system of Pythagorean FSs [PyFSs], containing both functions t and f with the condition $0 \leq t^2 + f^2 \leq 1$, extending the space of IFSs.

Various researchers used the PyFSs in various areas, keeping in view their forceful structure. However, in some conditions, PyFS is not kept in mind. For example, in a situation when there are more than two circumstances, like in casting a ballot (choice of inclusion, restraint, resistance, and exclusion), Understanding this, Cuong (2013, 2014) built up a new direction known as PFS, which describes the MG, hesitancy grade (HG) denoted by h and an NMG of an element or object in interval $[0, 1]$, with the condition $0 \leq t + h + f \leq 1$. As research hotspots of PFSs, CCs are needed to calculate the resemblance between two objects. Distance measures (DMs) and CCs are necessary and are used to show differences between two matters. Recently, the examination of DMs and CCs between PFSs has offered a lot of achievements. The CCs for PFSs are discussed by Singh (2015). The generalized CCs of the hesitant fuzzy sets are discussed by Singh (2018, 2019) with their application in DM and clustering. The CCs for interval valued IFSs are discussed by Park (2009). Further, the distance and correlation measures are discussed by Xu (2011); for further study, we may refer to Ganie (2020), Ullah (2020), Joshi (2021) and Jin (2019).

PFSs extend the model of FSs and IFSs, but there is a still limitation in the structure, i.e., the domain is limited For example, if a person is given $t = 0.6$, $h = 0.3$, and $f = 0.4$, in such situations, PFS will not be able to manage it, i.e., $t + h + f = 0.6 + 0.3 + 0.4 = 1.3 \notin [0, 1]$. In that condition, PFS is not kept in mind. In the same way, in real life, some problems were faced where PFS failed to handle the situation. To overcome the situation where PFS fails, Mahmood et al. (2018) proposed a new framework of spherical fuzzy set (SFS). Further, in generalization of FSs, IFSs, PyFSs, PFSs, and SFSs, a novel and most helpful tool in fuzziness was proposed by Mahmood et al. (2018) known as TSFS, which has no limitations at all. For some applicable work in this direction, we may refer to Ullah (2018, 2020), Wu (2020), Garg (2018), Munir (2020), Saad (2022), and Akram (2022).

From the above discussion, the drawbacks of the PF environment are clear. Keeping in view the drawbacks of PFSs, our aim is to develop the novel CCs, based on TSF information as it is the most generalized structure of the fuzzy sets with no limitations. The main advantages of the work we done in this article are that it can be used to solve the problems given in PF context also. Also, we have examined that the result obtained by applying proposed work is exactly the same as many other existing structures give. On the other hand, the existing structures in the PF environment are not able to solve the problems that are in the TSF context. This study includes the following ingredients:

1. The CCs based on TSFSs are developed in this article.
2. PR and multi-attribute decision-making (MADM) problems have been solved using proposed correlation measures.
3. The benefits of the proposed methods are discussed.

Further, the proposed article is organized as follows: The basic work that is necessary to study the proposed article is given in the next section. In Sect. 3, CCs based on the TSF environment has been proposed along with some related properties. Section 4 investigates the application of proposed work on pattern analysis and the MADM problem in the TSF context, and a comparative study has been made in Sect. 5. In Sect. 6, we have concluded the article.

2 Preliminaries

In this section, the notions discussed, provided a foundation for our work. Throughout the article, we have used t, h and f as an MG, HG, and, NMG, respectively. X acts as universal set and $PFS(X)$ be the family of PFSs on X. Further, $U, V \in PFS(X)$.

Definition 1 (Atanassov 1986) An IFS is of the form $F = \{\langle x, t(x_i), f(x_i)\rangle, such\ that\ x \in X\}$, where t and f are functions from X to an element in unit interval $[0, 1]$ with a restriction $0 \leq t + f \leq 1$ and $r = 1 - (t + f)$ is the refusal grade (RG) of x_i in F, where (t, f) is considered as an intuitionistic fuzzy number (IFN).

Because of the existing of some limitations in IFS, while allocating the MG and NMG, Yager (2013) initiated the system of PyFSs, by extending the space of IFSs.

Definition 2 (Yager 2013) A PyFS is of the form $P = \{\langle x, t(x_i), f(x_i)\rangle, such\ that\ x \in X\}$, where t and f are functions from X to an element in unit interval $[0, 1]$ with a restriction $0 \leq t^2 + f^2 \leq 1$ and $r = 1 - (t^2 + f^2)$ is the RG of x_i in P, where (t^2, f^2) is considered as Pythagorean fuzzy number (PyFN).

Definition 3 (Coung 2013) A PFS is of the form $T = \{\langle x, t(x_i), h(x_i), f(x_i)\rangle, such\ that\ x \in X\}$, where t, i, and f are functions from X to an element in unit interval $[0, 1]$ with a restriction $0 \leq t + h + f \leq 1$ and $r = 1 - (t + h + f)$

is the RG of x_i in T where (t, h, f) is considered as picture fuzzy number (PFN).

Definition 4 (Mahmood 2018) An SFS is of the form $M = \{\langle x, t(x_i), h(x_i), f(x_i)\rangle, \textit{such that } x \in X\}$, where t, h, and f are functions from X to an element in unit interval $[0, 1]$ with a restriction $0 \leq t^2 + h^2 + f^2 \leq 1$ and $r(x_i) = \sqrt{1 - (t^2 + h^2 + f^2)}$ is the RG of x_i in M, where (t, h, f) is considered as spherical fuzzy number (SFN).

Definition 5 (Mahmood 2018) A TSFS is of the form $I = \{\langle x, t(x_i), h(x_i), f(x_i)\rangle, \textit{such that } x \in X\}$ where t, h, and f are functions from X to an element in unit interval $[0, 1]$ with a restriction $0 \leq t^n + h^n + f^n \leq 1$ for $n \in \mathbb{Z}$ and $r(x_i) = \sqrt[n]{1 - (t^n + h^n + f^n)}$ is the RG of x_i in I, where (t, h, f) is considered as T-spherical fuzzy number (TSFN).

Some pre-existing CCs based on TSF information are given below. Later on, we have applied this related work for comparative study.

Definition 6 (Ullah 2020) For two TSFSs $U, V \in TSFS(X)$, the CC between U and V is defined as

$$\mathcal{K}^1_{TSFS}(U,V) = \frac{\sum_{i=1}^k \left\{\left(\left(t^n_U(x_i).t^n_V(x_i)\right) + \left(h^n_U(x_i).h^n_V(x_i)\right) + \left(f^n_U(x_i).f^n_V(x_i)\right) + \left(r^n_U(x_i).r^n_V(x_i)\right)\right)\right\}}{\sqrt{\begin{array}{c}\left(\sum_{i=1}^k\left(\left(t^2_U(x_i)\right)^n + \left(h^2_U(x_i)\right)^n + \left(f^2_U(x_i)\right)^n + \left(r^2_U(x_i)\right)^n\right)\right)\times \\ \left(\sum_{i=1}^k\left(\left(t^2_V(x_i)\right)^n + \left(h^2_V(x_i)\right)^n + \left(f^2_V(x_i)\right)^n + \left(r^2_V(x_i)\right)^n\right)\right)\end{array}}}. \tag{1}$$

Definition 7 (Ullah 2020) For two TSFSs $U, V \in TSFS(X)$, the CC between U and V is defined as

$$\mathcal{K}^2_{TSFS}(U,V) = \frac{\sum_{i=1}^k \left\{\left(\left(t^n_U(x_i).t^n_V(x_i)\right) + \left(h^n_U(x_i).h^n_V(x_i)\right) + \left(f^n_U(x_i).f^n_V(x_i)\right) + \left(r^n_U(x_i).r^n_V(x_i)\right)\right)\right\}}{max\left(\begin{array}{c}\left(\sum_{i=1}^k\left(\left(t^2_U(x_i)\right)^n + \left(h^2_U(x_i)\right)^n + \left(f^2_U(x_i)\right)^n + \left(r^2_U(x_i)\right)^n\right)\right)\times \\ \left(\sum_{i=1}^k\left(\left(t^2_V(x_i)\right)^n + \left(h^2_V(x_i)\right)^n + \left(f^2_V(x_i)\right)^n + \left(r^2_V(x_i)\right)^n\right)\right)\end{array}\right)}. \tag{2}$$

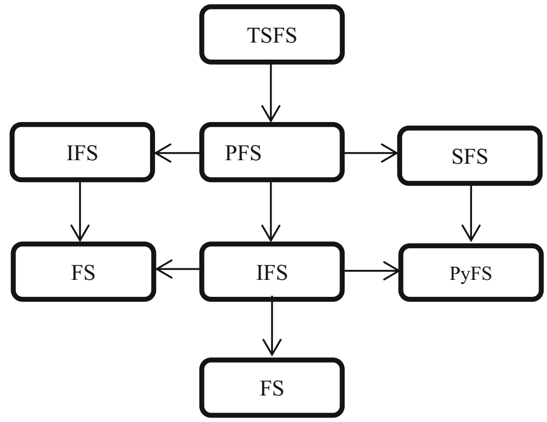

Fig. 1 T-spherical fuzzy sets

Definition 8 (Ullah 2020) For two TSFSs $U, V \in TSFS(X)$, the CC between U and V with the weight vector $w_i \in [0, 1]$ of each element of X, such that $\sum_{i=1}^k w_i = 1$ is defined as

$$WK^1_{TSFS}(U,V)=\frac{\sum_{i=1}^{k}w_i\left\{\left(\left(t^n_U(x_i).t^n_V(x_i)\right)+\left(h^n_U(x_i).h^n_V(x_i)\right)+\left(f^n_U(x_i).f^n_V(x_i)\right)+\left(r^n_U(x_i).r^n_V(x_i)\right)\right)\right\}}{\sqrt{\begin{array}{c}\left(\sum_{i=1}^{k}w_i\left(\left(t^2_U(x_i)\right)^n+\left(h^2_U(x_i)\right)^n+\left(f^2_U(x_i)\right)^n+\left(r^2_U(x_i)\right)^n\right)\right)\times\\ \left(\sum_{i=1}^{k}w_i\left(\left(t^2_V(x_i)\right)^n+\left(h^2_V(x_i)\right)^n+\left(f^2_V(x_i)\right)^n+\left(r^2_V(x_i)\right)^n\right)\right)\end{array}}}. \tag{3}$$

Definition 9 (Ullah 2020) For two TSFSs $U,V\in TSFS(X)$, the CC between U and V with the weight vector $w_i\in[0,1]$ of each element of X, such that $\sum_{i=1}^{k}w_i=1$ is defined as

$$WK^2_{TSFS}(U,V)=\frac{\sum_{i=1}^{k}w_i\left\{\left(\left(t^n_U(x_i).t^n_V(x_i)\right)+\left(h^n_U(x_i).h^n_V(x_i)\right)+\left(f^n_U(x_i).f^n_V(x_i)\right)+\left(r^n_U(x_i).r^n_V(x_i)\right)\right)\right\}}{max\left(\begin{array}{c}\left(\sum_{i=1}^{k}w_i\left(\left(t^2_U(x_i)\right)^n+\left(h^2_U(x_i)\right)^n+\left(f^2_U(x_i)\right)^n+\left(r^2_U(x_i)\right)^n\right)\right)\times\\ \left(\sum_{i=1}^{k}w_i\left(\left(t^2_V(x_i)\right)^n+\left(h^2_V(x_i)\right)^n+\left(f^2_V(x_i)\right)^n+\left(r^2_V(x_i)\right)^n\right)\right)\end{array}\right)}. \tag{4}$$

Figure 1 tells us that TSFS is the most generalized structure from the existing ones. Keeping in mind, the importance new CCs based on TSF information are proposed and further are used to solve the problems that the rest cannot.

3 A novel correlation coefficient based on TSFSs

As research hotspots of fuzzy theory, CCs are needed to calculate the resemblance between two objects. CCs are necessary and are used to show differences between two matters. Keeping in view, the importance and wide range of TSFSs, our aim is to develop novel CCs in the TSF environment as it is the most generalized structure of the fuzzy sets with no limitations.

Definition 10 For two TSFSs $U,V\in TSFS(X)$, the CC between U and V is defined as

$$C_{TSF}(U,V)=\frac{\sum_{i=1}^{k}\left\{\begin{array}{c}\left(\left(t^n_U(x_i)-\overline{t^n_U(x_i)}\right)+\left(h^n_U(x_i)-\overline{h^n_U(x_i)}\right)+\left(f^n_U(x_i)-\overline{f^n_U(x_i)}\right)\right)\times\\ \left(\left(t^n_V(x_i)-\overline{t^n_V(x_i)}\right)+\left(h^n_V(x_i)-\overline{h^n_V(x_i)}\right)+\left(f^n_V(x_i)-\overline{f^n_V(x_i)}\right)\right)\end{array}\right\}}{\sqrt{\begin{array}{c}\left(\sum_{i=1}^{k}\left(\left(t^n_U(x_i)-\overline{t^n_U(x_i)}\right)+\left(h^n_U(x_i)-\overline{h^n_U(x_i)}\right)+\left(f^n_U(x_i)-\overline{f^n_U(x_i)}\right)\right)^2\right)\times\\ \left(\sum_{i=1}^{k}\left(\left(t^n_V(x_i)-\overline{t^n_V(x_i)}\right)+\left(h^n_V(x_i)-\overline{h^n_V(x_i)}\right)+\left(f^n_V(x_i)-\overline{f^n_V(x_i)}\right)\right)^2\right)\end{array}}}, \tag{5}$$

where, $\overline{t^n_U}=\frac{1}{k}\sum_{i=1}^{k}t^n_U(x_i)$, $\overline{h^n_U}=\frac{1}{k}\sum_{i=1}^{k}h^n_U(x_i)$, $\overline{f^n_U}=\frac{1}{k}\sum_{i=1}^{k}f^n_U(x_i)$, $\overline{t^n_V}=\frac{1}{k}\sum_{i=1}^{k}t^n_V(x_i)$, $\overline{h^n_V}=\frac{1}{k}\sum_{i=1}^{k}h^n_V(x_i)$, and $\overline{f^n_V}=\frac{1}{k}\sum_{i=1}^{k}f^n_V(x_i)$.

Example 1 For two TSFSs, U and V given as: $U=\{(y_1,0.5,0.5,0.3),(y_2,0.5,0.4,0.4),(y_3,0.5,0.33,0.3)\}$ and

$V=\{(y_1,0.4,0.5,0.3),(y_2,0.4,0.2,0.3),(y_3,0.4,0.3,0.5)\}$; clearly, $U\neq V$. Then, the CC between U and V can be calculated as: $C_{TSF}(U,V)=0.405606$.

Further, Theorem 1 develops some features of the TSFCC $C_{TSF}(U,V)$, which are as follows.

Theorem 1 *For two TSFSs $U,V\in TSFS(X)$, then*

1 $C_{TSF}(U,V)=C_{TSF}(V,U)$;
2 $C_{TSF}(U,V)=1 if U=V$;
3 $|C_{TSF}(V,U)|\leq 1$;
4 If $U=\alpha V$, then $C_{TSF}(U,V)=1$ for $\alpha>0$ and $C_{TSF}(U,V)=-1$ for $\alpha<0$.

Proof:

1. By applying the definition, we have

$$C_{TSF}(U,V)=\frac{\sum_{i=1}^{k}\left\{\begin{array}{c}\left(\left(t_U^n(x_i)-\overline{t_U^n(x_i)}\right)+\left(h_U^n(x_i)-\overline{h_U^n(x_i)}\right)+\left(f_U^n(x_i)-\overline{f_U^n(x_i)}\right)\right)\times\\ \left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)\end{array}\right\}}{\sqrt{\begin{array}{c}\left(\sum_{i=1}^{k}\left(\left(t_U^n(x_i)-\overline{t_U^n(x_i)}\right)+\left(h_U^n(x_i)-\overline{h_U^n(x_i)}\right)+\left(f_U^n(x_i)-\overline{f_U^n(x_i)}\right)\right)^2\right)\times\\ \left(\sum_{i=1}^{k}\left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)^2\right)\end{array}}}$$

$$=\frac{\sum_{i=1}^{k}\left\{\begin{array}{c}\left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)\times\\ \left(\left(t_U^n(x_i)-\overline{t_U^n(x_i)}\right)+\left(h_U^n(x_i)-\overline{h_U^n(x_i)}\right)+\left(f_U^n(x_i)-\overline{f_U^n(x_i)}\right)\right)\end{array}\right\}}{\sqrt{\begin{array}{c}\left(\sum_{i=1}^{k}\left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)^2\right)\times\\ \left(\sum_{i=1}^{k}\left(\left(t_U^n(x_i)-\overline{t_U^n(x_i)}\right)+\left(h_U^n(x_i)-\overline{h_U^n(x_i)}\right)+\left(f_U^n(x_i)-\overline{f_U^n(x_i)}\right)\right)^2\right).\end{array}}}$$

$$=C_{TSF}(V,U)$$

2. Taking $U=V$, i.e., $t_U^n(x_i)=t_V^n(x_i)$, $h_U^n(x_i)=h_V^n(x_i)$ and $f_U^n(x_i)=f_V^n(x_i),\forall i$. Then

$$C_{TSF}(U,V)=\frac{\sum_{i=1}^{k}\left\{\begin{array}{c}\left(\left(t_U^n(x_i)-\overline{t_U^n(x_i)}\right)+\left(h_U^n(x_i)-\overline{h_U^n(x_i)}\right)+\left(f_U^n(x_i)-\overline{f_U^n(x_i)}\right)\right)\times\\ \left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)\end{array}\right\}}{\sqrt{\begin{array}{c}\left(\sum_{i=1}^{k}\left(\left(t_U^n(x_i)-\overline{t_U^n(x_i)}\right)+\left(h_U^n(x_i)-\overline{h_U^n(x_i)}\right)+\left(f_U^n(x_i)-\overline{f_U^n(x_i)}\right)\right)^2\right)\times\\ \left(\sum_{i=1}^{k}\left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)^2\right)\end{array}}}$$

$$C_{TSF}(U,V)=\frac{\sum_{i=1}^{k}\left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)^2}{\left(\sqrt{\sum_{i=1}^{k}\left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)^2}\right)^2}$$

$$=1.$$

3. By Cauchy–Schwarz inequality

$$(C_{TSF}(U,V))^2=\left(\frac{\sum_{i=1}^{k}\left\{\begin{array}{c}\left(\left(t_U^n(x_i)-\overline{t_U^n(x_i)}\right)+\left(h_U^n(x_i)-\overline{h_U^n(x_i)}\right)+\left(f_U^n(x_i)-\overline{f_U^n(x_i)}\right)\right)\times\\ \left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)\end{array}\right\}}{\sqrt{\begin{array}{c}\left(\sum_{i=1}^{k}\left(\left(t_U^n(x_i)-\overline{t_U^n(x_i)}\right)+\left(h_U^n(x_i)-\overline{h_U^n(x_i)}\right)+\left(f_U^n(x_i)-\overline{f_U^n(x_i)}\right)\right)^2\right)\times\\ \left(\sum_{i=1}^{k}\left(\left(t_V^n(x_i)-\overline{t_V^n(x_i)}\right)+\left(h_V^n(x_i)-\overline{h_V^n(x_i)}\right)+\left(f_V^n(x_i)-\overline{f_V^n(x_i)}\right)\right)^2\right)\end{array}}}\right)^2$$

$$\leq \frac{\begin{pmatrix} \sum_{i=1}^{k}\left(\left(t_U^n(x_i) - \overline{t_U^n(x_i)}\right) + \left(h_U^n(x_i) - \overline{h_U^n(x_i)}\right) + \left(f_U^n(x_i) - \overline{f_U^n(x_i)}\right)\right)^2 \times \\ \sum_{i=1}^{k}\left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right)^2 \end{pmatrix}}{\left(\sqrt{\begin{array}{c} \left(\sum_{i=1}^{k}\left(\left(t_U^n(x_i) - \overline{t_U^n(x_i)}\right) + \left(h_U^n(x_i) - \overline{h_U^n(x_i)}\right) + \left(f_U^n(x_i) - \overline{f_U^n(x_i)}\right)\right)^2\right) \times \\ \left(\sum_{i=1}^{k}\left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right)^2\right) \end{array}}\right)^2} = 1$$

$|C_{TSF}(V,U)| \leq 1.$

4. Consider $U = \alpha V$, we have

$$C_{TSF}(U,V) = \frac{\sum_{i=1}^{k}\left\{\begin{array}{c} \left(\left(t_U^n(x_i) - \overline{t_U^n(x_i)}\right) + \left(h_U^n(x_i) - \overline{h_U^n(x_i)}\right) + \left(f_U^n(x_i) - \overline{f_U^n(x_i)}\right)\right) \times \\ \left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right) \end{array}\right\}}{\sqrt{\begin{array}{c} \left(\sum_{i=1}^{k}\left(\left(t_U^n(x_i) - \overline{t_U^n(x_i)}\right) + \left(h_U^n(x_i) - \overline{h_U^n(x_i)}\right) + \left(f_U^n(x_i) - \overline{f_U^n(x_i)}\right)\right)^2\right) \times \\ \left(\sum_{i=1}^{k}\left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right)^2\right) \end{array}}}$$

$$= \frac{\sum_{i=1}^{k}\left\{\begin{array}{c} \left(\left(\alpha t_V^n(x_i) - \overline{\alpha t_V^n(x_i)}\right) + \left(\alpha h_V^n(x_i) - \overline{\alpha h_V^n(x_i)}\right) + \left(\alpha f_V^n(x_i) - \overline{\alpha f_V^n(x_i)}\right)\right) \times \\ \left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right) \end{array}\right\}}{\sqrt{\begin{array}{c} \left(\sum_{i=1}^{k}\left(\left(\alpha t_V^n(x_i) - \overline{\alpha t_V^n(x_i)}\right) + \left(\alpha h_V^n(x_i) - \overline{\alpha h_V^n(x_i)}\right) + \left(\alpha f_V^n(x_i) - \overline{\alpha f_V^n(x_i)}\right)\right)^2\right) \times \\ \left(\sum_{i=1}^{k}\left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right)^2\right) \end{array}}}$$

$$= \frac{\alpha \sum_{i=1}^{k}\left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right)^2}{\left(\sqrt{\alpha^2\left(\sum_{i=1}^{k}\left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right)^2\right)^2}\right)}$$

$$= \begin{cases} -1 & if \quad \alpha < 0 \\ 1 & if \quad \alpha > 0 \end{cases}$$

Definition 11 For two TSFSs $U, V \in TSFS(X)$, the CC between U and V with the weight vector $w_i \in [0,1]$ of each element of X, such that $\sum_{i=1}^{k} w_i = 1$ is defined as

Table 1 Calculated TSF-correlation measure

Correlation	(P_1, Q)	(P_2, Q)	(P_3, Q)	Result
C_{TSF} (proposed)	0.126312	0.405606	0.103192	P_2
WC_{TSF} (proposed)	0.070464	0.236554	0.057925	P_2
$\mathcal{K}^3_{TSFS}$	0.797456	0.813975	0.780799	P_2
$\mathcal{K}^4_{TSFS}$	0.630155	0.652791	0.633758	P_2
$W\mathcal{K}^3_{TSFS}$	0.872724	0.939003	0.892172	P_2
$W\mathcal{K}^4_{TSFS}$	0.238748	0.231241	0.233969	P_1

Table 2 T-spherical fuzzy representation of masks

Masks	A_1	A_2	A_3	A_4
$\boldsymbol{M}_1$	$(0.4, 0.5, 0.3)$	$(0.7, 0.6, 0.3)$	$(0.3, 0.5, 0.3)$	$(0.7, 0.4, 0.3)$
$\boldsymbol{M}_2$	$(0.2, 0.5, 0.4)$	$(0.7, 0.6, 0.3)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.6, 0.3)$
$\boldsymbol{M}_3$	$(0.8, 0.2, 0.3)$	$(0.5, 0.6, 0.3)$	$(0.6, 0.6, 0.3)$	$(0.4, 0.6, 0.3)$
$\boldsymbol{M}_4$	$(0.4, 0.5, 0.3)$	$(0.3, 0.6, 0.5)$	$(0.7, 0.5, 0.3)$	$(0.3, 0.4, 0.3)$
$\boldsymbol{M}_5$	$(0.4, 0.7, 0.3)$	$(0.6, 0.4, 0.3)$	$(0.4, 0.4, 0.3)$	$(0.5, 0.4, 0.4)$
$\boldsymbol{M}_6$	$(0.5, 0.6, 0.3)$	$(0.5, 0.5, 0.3)$	$(0.4, 0.5, 0.3)$	$(0.4, 0.5, 0.5)$

Table 3 T-spherical fuzzy representation of masks with positive and negative ideal

Masks	A_1	A_2	A_3	A_4
M_1	$(0.4, 0.5, 0.3)$	$(0.7, 0.6, 0.3))$	$(0.3, 0.5, 0.3)$	$(0.7, 0.4, 0.3)$
M_2	$(0.2, 0.5, 0.4)$	$(0.7, 0.6, 0.3)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.6, 0.3)$
M_3	$(0.8, 0.2, 0.3)$	$(0.5, 0.6, 0.3)$	$(0.6, 0.6, 0.3)$	$(0.4, 0.6, 0.3)$
M_4	$(0.4, 0.5, 0.3)$	$(0.3, 0.6, 0.5)$	$(0.7, 0.5, 0.3)$	$(0.3, 0.4, 0.3)$
M_5	$(0.4, 0.7, 0.3)$	$(0.6, 0.4, 0.3)$	$(0.4, 0.4, 0.3)$	$(0.5, 0.4, 0.4)$
M_6	$(0.5, 0.6, 0.3)$	$(0.5, 0.5, 0.3)$	$(0.4, 0.5, 0.3)$	$(0.4, 0.5, 0.5)$
M^+	$(0.8, 0.2, 0.3)$	$(0.7, 0.4, 0.3)$	$(0.7, 0.4, 0.3)$	$(0.7, 0.4, 0.3)$
M^-	$(0.2, 0.2, 0.4)$	$(0.3, 0.4, 0.5)$	$(0.4, 0.4, 0.3)$	$(0.3, 0.4, 0.5)$

Table 4 Correlation of each mask with positive and negative ideal

Correlation	M_1	M_2	M_3	M_4	M_5	M_6
$C_{TSF}(M_i, M^+)$	0.03200	0.03414	0.04757	0.03784	0.05266	0.09586
$C_{TSF}(M_i, M^-)$	0.013080	0.01396	0.01945	0.01547	0.02153	0.03919
$WC_{TSF}(M_i, M^+)$	0.330796	0.317381	0.351506	0.458819	0.416565	0.796299
$WC_{TSF}(M_i, M^-)$	−0.19642	−0.18845	−0.20872	−0.27243	−0.24735	−0.47282
$\mathcal{K}^3_{TSFS}(M_i, M^+)$	0.851731	0.837082	0.874696	0.840206	0.839129	0.843937
$\mathcal{K}^3_{TSFS}(M_i, M^-)$	0.906375	0.895418	0.887774	0.92685	0.921431	0.934138
$\mathcal{K}^4_{TSFS}(M_i, M^+)$	0.772661	0.735621	0.787811	0.767884	0.772627	0.768963
$\mathcal{K}^4_{TSFS}(M_i, M^-)$	0.822232	0.786886	0.79959	0.847071	0.848407	0.851151
$W\mathcal{K}^3_{TSFS}(M_i, M^+)$	0.840257	0.819278	0.8846	0.831168	0.821156	0.837513
$W\mathcal{K}^3_{TSFS}(M_i, M^-)$	0.91822	0.906735	0.872932	0.934488	0.917434	0.934963
$W\mathcal{K}^3_{TSFS}(M_i, M^+)$	0.228103	0.220867	0.226318	0.229448	0.22907	0.226945
$W\mathcal{K}^4_{TSFS}(M_i, M^-)$	0.208125	0.195129	0.204879	0.210586	0.209892	0.206016

Table 5 Index of correlation

Correlation	M_1	M_2	M_3	M_4	M_5	M_6
C_{TSF} (proposed)	0.50463	0.50493	0.50680	0.50545	0.50750	0.51327
WC_TSF (proposed)	0.623505	0.6188	0.630722	0.667229	0.65303	0.773108
$\mathcal{K}^3_{TSFS}$ (Ullah, 2020)	0.49273	0.492185	0.498262	0.4885	0.489057	0.499363
$\mathcal{K}^4_{TSFS}$ (Ullah 2020)	0.493105	0.492723	0.498358	0.489047	0.489536	0.498648
$W\mathcal{K}^3_{TSFS}$ (Ullah 2020)	0.489628	0.488264	0.501553	0.486281	0.487124	0.508708
$W\mathcal{K}^4_{TSFS}$ (Ullah 2020)	0.5041	0.505327	0.504409	0.503865	0.503932	0.506301

Table 6 Ranking of masks

Correlation	Ranking
C_{TSF} (proposed)	$M_6 > M_5 > M_3 > M_4 > M_2 > M_1$
WC_{TSF}(proposed)	$M_6 > M_4 > M_5 > M_3 > M_1 > M_2$
$\mathcal{K}^3_{TSFS}$ (Ullah 2020)	$M_6 > M_3 > M_1 > M_2 > M_5 > M_4$
$\mathcal{K}^4_{TSFS}$ (Ullah 2020)	$M_6 > M_3 > M_1 > M_2 > M_5 > M_4$
$W\mathcal{K}^3_{TSFS}$(Ullah 2020)	$M_6 > M_3 > M_1 > M_2 > M_5 > M_4$
$W\mathcal{K}^4_{TSFS}$ (Ullah 2020)	$M_6 > M_2 > M_3 > M_1 > M_5 > M_4$

4 *If* $U = \alpha V$, *then* $WC_{TSF}(U, V) = 1$ *for* $\alpha > 0$ *and* $WC_{TSF}(U, V) = -1$ *for* $\alpha < 0$.

Remark 1: If we set $n = 2$ in Definition 10, then it becomes the CC of SFSs.

Remark 2: If we set $n = 2$ in Definition 11, then it becomes the CC of SFSs.

$$WC_{TSF}(U,V) = \frac{\sum_{i=1}^{k} w_i \left\{ \begin{array}{c} \left(\left(t_U^n(x_i) - \overline{t_U^n(x_i)}\right) + \left(h_U^n(x_i) - \overline{h_U^n(x_i)}\right) + \left(f_U^n(x_i) - \overline{f_U^n(x_i)}\right)\right) \times \\ \left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right) \end{array} \right\}}{\sqrt{\begin{array}{c} \left(\sum_{i=1}^{k} w_i \left(\left(t_U^n(x_i) - \overline{t_U^n(x_i)}\right) + \left(h_U^n(x_i) - \overline{h_U^n(x_i)}\right) + \left(f_U^n(x_i) - \overline{f_U^n(x_i)}\right)\right)^2 \right) \times \\ \left(\sum_{i=1}^{k} w_i \left(\left(t_V^n(x_i) - \overline{t_V^n(x_i)}\right) + \left(h_V^n(x_i) - \overline{h_V^n(x_i)}\right) + \left(f_V^n(x_i) - \overline{f_V^n(x_i)}\right)\right)^2 \right) \end{array}}}. \tag{6}$$

Theorem 2 *For two TSFSs* $U, V \in TSFS(X)$, *then*

1 $WC_{TSF}(U, V) = WC_{TSF}(V, U)$;
2 $WC_{TSF}(U, V) = 1 if U = V$;
3 $|WC_{TSF}(V, U)| \leq 1$;

4 Applications

This section demonstrates the solution of pattern recognition and decision-making problems based on TSF information using the proposed methods. Further, the comparative analysis has also been made to compare the

Table 7 Decision matrix of alternatives

Firms	A_1	A_2	A_3	A_4
M_1	(0.3, 0.5, 0.4)	(0.3, 0.6, 0.7)	(0.3, 0.5, 0.3)	(0.3, 0.4, 0.7)
M_2	(0.6, 0.7, 0.2)	(0.4, 0.6, 0.6)	(0.8, 0.5, 0.5)	(0.7, 0.2, 0.4)
M_3	(0.8, 0.4, 0.3)	(0.7, 0.2, 0.6)	(0.6, 0.6, 0.5)	(0.5, 0.4, 0.7)
M_4	(0.4, 0.7, 0.6)	(0.6, 0.5, 0.3)	(0.7, 0.6, 0.5)	(0.5, 0.4, 0.5)

Table 8 Normalize decision matrix of alternatives

Firms	A_1	A_2	A_3	A_4
$\boldsymbol{M}_1$	(0.4, 0.5, 0.3)	(0.7, 0.6, 0.3)	(0.3, 0.5, 0.3)	(0.7, 0.4, 0.3)
$\boldsymbol{M}_2$	(0.2, 0.7, 0.6)	(0.6, 0.6, 0.4)	(0.5, 0.5, 0.8)	(0.4, 0.2, 0.7)
$\boldsymbol{M}_3$	(0.3, 0.4, 0.8)	(0.6, 0.2, 0.7)	(0.5, 0.6, 0.6)	(0.7, 0.4, 0.5)
$\boldsymbol{M}_4$	(0.6, 0.7, 0.4)	(0.3, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.5, 0.4, 0.5)

Table 9 Correlation degrees

Correlation	$\boldsymbol{M}_1$	$\boldsymbol{M}_2$	$\boldsymbol{M}_3$	$\boldsymbol{M}_4$
$\mathcal{K}^3_{\boldsymbol{TSFS}}(\boldsymbol{M_i}, \boldsymbol{M})$	0.214128	0.119901	0.206141	0.143596
$\mathcal{K}^4_{\boldsymbol{TSFS}}(\boldsymbol{M_i}, \boldsymbol{M})$	0.194250	0.103250	0.177750	0.123250
$\boldsymbol{W}\mathcal{K}^3_{\boldsymbol{TSFS}}(\boldsymbol{M_i}, \boldsymbol{M})$	0.285348	0.119648	0.267953	0.137375
$\boldsymbol{W}\mathcal{K}^4_{\boldsymbol{TSFS}}(\boldsymbol{M_i}, \boldsymbol{M})$	0.223937	0.218348	0.205167	0.207098
$\boldsymbol{C_{TSF}}(\boldsymbol{M_i}, \boldsymbol{M})$	0.875630	0.352410	0.846960	0.476900

Table 10 Ranking

Correlation	Ranking
$\mathcal{K}^3_{\boldsymbol{TSFS}}(\boldsymbol{M_i}, \boldsymbol{M})$	$M_1 > M_3 > M_4 > M_1$
$\mathcal{K}^4_{\boldsymbol{TSFS}}(\boldsymbol{M_i}, \boldsymbol{M})$	$M_1 > M_3 > M_4 > M_1$
$\boldsymbol{W}\mathcal{K}^3_{\boldsymbol{TSFS}}(\boldsymbol{M_i}, \boldsymbol{M})$	$M_1 > M_3 > M_4 > M_1$
$\boldsymbol{W}\mathcal{K}^4_{\boldsymbol{TSFS}}(\boldsymbol{M_i}, \boldsymbol{M})$	$M_1 > M_2 > M_4 > M_3$
$\boldsymbol{C_{TSF}}(\boldsymbol{M_i}, \boldsymbol{M})$	$M_1 > M_3 > M_4 > M_1$

existing CCs with our newly developed CCs, along with advantages. First, we have solved a problem related to the pattern recognition in the next section; second, we have compared the results with the existing fuzzy structures with their merits. In the next part, the decision-making problem has been solved and its benefits are discussed.

4.1 Pattern recognition

In this section, a problem related to pattern recognition based on TSF information has been solved. The main purpose of pattern identification is to determine how similar an unknown pattern is to a previously identified pattern. The proposed CCs are used to accomplish this. The data are given in TSF form.

Scenario: We have been provided an unknown pattern along with some known patterns. Suppose $P_1, P_2, P_3, \ldots, P_k$ denote the known patterns, while Q denotes an unknown pattern and expressed in X as

$$P_j = \left\{ \left(x_i, t^n_{P_j}(x_i), h^n_{P_j}(x_i), f^n_{P_j}(x_i) \right) such\ that\ x_i \in X_i \right\}$$

$$Q = \left\{ \left(x_i, t^n_Q(x_i), h^n_Q(x_i), f^n_Q(x_i) \right) such\ that\ x_i \in X_i \right\},$$ for all $j = 1, 2, 3, \ldots, k$.

4.1.1 Aim

To find the unknown pattern Q into one of the known patterns P_j for all $j = 1, 2, 3, \ldots, k$.

4.1.2 Recognition principle

The unknown pattern Q can be symbolized to known pattern P_j with which it has more correlation.

After finding the known pattern with which the unknown pattern relates, next the results are compared with the existing methods.

Example 2 Consider, P_1, P_2 and P_3 are three known with the weight vector $(0.4, 0.3, 0.3)$ and Q is unknown pattern in the TSF context as

$$P_1 = \{(y_1, 0.7, 0.5, 0.3), (y_2, 0.5, 0.4, 0.3), (y_3, 0.6, 0.3, 0.3)\}$$

$$P_2 = \{(y_1, 0.5, 0.5, 0.3), (y_2, 0.5, 0.4, 0.4), (y_3, 0.5, 0.33, 0.3)\}$$

$$P_3 = \{(y_1, 0.8, 0.3, 0.2), (y_2, 0.4, 0.4, 0.4), (y_3, 0.6, 0.3, 0.4)\}$$

$$\mathrm{Q} = \{(y_1, 0.4, 0.5, 0.3), (y_2, 0.4, 0.2, 0.3), (y_3, 0.4, 0.3, 0.5)\}.$$

Our aim is to check which unknown pattern corresponds to the known pattern. For the sake, the above algorithm is applied on the given data. Put $n = 3$ in our proposed CC for pattern analysis.

The resultant calculations that drawn using the above algorithm between $P_i, i = 1, 2, 3$ and Q are given in Table 1.

From Table 1, it is seen that the value of P_2 is the highest, so P_2 is most closely related to Q. Which leads to the conclusion that the unknown pattern Q is actually is of the type P_2.

4.2 Multi-attribute decision-making

The global pandemic, COVID-19, is a dangerous virus that causes flu and various respiratory diseases. COVID-19 is a family of several types of viruses that, in addition to common cold, cause serious illnesses, such as SARS and Middle East respiratory syndrome. This is the first time that this type of COVID-19 has been detected in humans. People infected with the COVID-19 may experience symptoms, such as headache, cold, cough, and fatigue. It can also cause serious symptoms, such as difficulty in breathing, pneumonia, and lung failure, which can be fatal. Prevention measures include hand washing, covering the mouth when coughing and sneezing, and staying away from people with symptom of virus. At the same time, it is important to use well cooked meat, eggs, and animal foods. Remember a healthy immune system is your most important weapon in protecting against the COVID-19. We can also avoid this virus by wearing mask. Using a good mask can prevent this virus to a great extent. Many people have done a mathematical study of this virus in [34]. In this article, we have identified the best masks used to prevent COVID-19 form the availed masks in market.

In this part of article, the MADM problem is solved using proposed CCs. For this purpose, a problem related to mask selecting is discussed here. Our aim is to select best mask among the masks available in the market.

Example 3 Six masks that are easily found in the market are given. The feedback in TSF context is taken from the customers. The six masks are: N95 (M_1), gas mask (M_2), ordinary non-medical mask (M_3), medical protective mask (M_4), surgical mask (M_5), and disposable mask (M_6). The customer usually would like to wear a mask with four features in mind: used material A_1, reusability A_2, leakage rate A_3, and filtration capability A_4. All this information is taken in the form of TSFS which is given in Table 2.

The negative ideal TSF M^- and the positive ideal TSF M^+ are constructed as

$M^- = \left\{\left(t^n_{M^-}(x_i), h^n_{M^-}(x_i), f^n_{M^-}(x_i)\right) such\ that\ x_i \in X_i\right\}.,$
$M^+ = \left\{\left(t^n_{M^+}(x_i), h^n_{M^+}(x_i), f^n_{M^+}(x_i)\right) such\ that\ x_i \in X_i\right\}.$
Where, $t^n_{M^-}(x_i) = min_j t^n_{A_j}(x_i)$, $h^n_{M^-}(x_i) = min_j h^n_{A_j}(x_i)$, $f^n_{M^-}(x_i) = max_j f^n_{A_j}(x_i)$, $t^n_{M^+}(x_i) = max_j t^n_{A_j}(x_i)$, $h^n_{M^+}(x_i) = min_j h^n_{A_j}(x_i)$ and $f^n_{M^+}(x_i) = min_j f^n_{A_j}(x_i)$.

Extract the values of M^- and M^+ from Table 2 as

$$M^- = \{(0.2, 0.2, 0.4), (0.3, 0.4, 0.5), (0.4, 0.4, 0.3), (0.3, 0.4, 0.5)\},$$

$$M^+ = \{(0.8, 0.2, 0.3), (0.7, 0.4, 0.3), (0.7, 0.4, 0.3), (0.7, 0.4, 0.3)\}.$$

Next, the correlation of each $M_i, i = 1, 2, 3, 4, 5, 6$ given in Table 3 is extracted with M^+ and M^- by applying the proposed formula with the weight vector $(0.4, 0.2, 0.2, 0.2)$. The findings are demonstrated in Table 4. Then, we sorted the results by descending order, so that we could choose the best mask. We used the index of correlation given below to draw the conclusions. The results of the index of correlation are given Table 5. Further, the ranking of the masks is given in Table 6

$$\xi_i = \frac{1 + C_{TSF}(M_i, M^+)}{2 + C_{TSF}(M_i, M^+) + C_{TSF}(M_i, M^-)}, i = 1, 2, 3, 4, 5, 6. \quad (7)$$

From Table 6, it is seen that that the value of P_6 is the highest, so P_6 is possibly the best available mask in the market. This leads to the conclusion that the disposable mask is the best one.

5 Comparative study

In this part of article, advantages of proposed work are given and a comparative study has been made in which proposed CCs are compared with pre-existing. For comparative study, some remarks are given that shows the novelty and generalization of the proposed work. Further, an example is given for the sake of comparative study.

Remarks The following remarks show the importance of the proposed work.

1. If we set $n = 1$ in Definitions 10 and 11, then it becomes the CCs of PFSs proposed in (Singh, 2021).
2. If we put $RG = 0$ that is $h = 0$ in Definitions 10 and 11, then it becomes the CC of PyFSs.
3. If we put $n = 1$ and $RG = 0$, that is, $h = 0$ in Definitions 10 and 11, then it becomes the CC of IFSs.
4. If we put $n = 1$, $RG = 0$, that is, $h = 0$ and $NMG = 0$, that is, $f = 0$ in Definitions 10 and 11, then it becomes the CC of FSs.

Example 4 Let us consider an example of selecting a company based on different attributes by a supplier who wants to work with a best construction company. After screening and testing different companies on different companies, the selected four companies are M_1, M_2, M_3, and M_4. The selection of best company is based on the criteria of four attributes; A_1: Cost, A_2: previous work, A_3: time taken, and A_4: quality assurance. The weight vector for the attributes is given as $w = (0.2, 0.3, 0.1, 0.4)^T$. n The data regarding companies are given in TSF form. The decision-making process is done step by step.

Step 1: The data regarding companies are given in Table 7 in TSF form.

Step 2: By making all the attributes of benefits type, the normalized matrix is obtained in Table 8.

Step 3: The correlation with some existing CCs and newly proposed CC is provided in Table 9. The $M = (1, 0, 0)$ is used as an ideal value to compute the correlation.

Now, the results of calculations of each TSFN using the CCs proposed in (Ullah, 2020) and newly proposed one are given in Table 9.

Step 4: The ranking of all firms is given in Table 10.

Form Table 10, it can be seen that the results of the correlation degree of the CCs proposed in (Ullah, 2020) and our proposed CC are same that is the M_1 is the best one. It also can be seen that the proposed CC has many advantages. The advantages of the work we have done in this article are that it can be used to solve IF, Pythagorean fuzzy, Spherical, and TSF problems. From Example 6, it can be seen that the results obtained from this are exactly the same as the result which are obtained by applying the CCs proposed in (Ullah, 2020).

Further, the main advantages of the proposed work are as follows:

1. The proposed CCs can solve the problems that contain the data in intuitionistic fuzzy, Pythagorean fuzzy, picture fuzzy, and TSF information.
2. From examples, it is seen that the results obtained from the proposed work are exactly the same as the results obtained by applying the CCs proposed in (Ullah, 2020).
3. The data given in Example 3 cannot be solved by the CCs proposed in (Ullah, 2020), which show the superiority of the proposed CCs.
4. Some current CCs only show the degree of correlation between two TSFNs but do not show anything about the type of negative or positive correlation. Also, some CCs express both positive and negative correlation but do not take into account the degree of hesitancy, which in most cases leads to intuitive results. Further, many existing CCs consider all four degrees of membership, often producing incorrect results or failing to measure the correlation between two TSFNs. And many of them fail to calculate the results. Keeping in view the above discussion, it can be seen from the comparative study that our proposed CC is able to overcome the problems. Therefore, it is concluded that our proposed work is most generalized.

6 Conclusion

This article outlines the background of the FSs, IFSs, PyFSs, SFSs, and PFSs in their entirety and the shortcomings of the work done earlier. Some existing CCs have been discussed in the literature review. This article also states that SFSs and TSFSs are the most generalized structures in fuzzy theory which have no flaws. T-spherical fuzzy set is an effective tool to deal with vagueness and uncertainty in real-life problems, especially in a situation when there are more than two circumstances, like in casting a ballot. The correlation coefficient of T-spherical fuzzy sets is a tool to calculate the association of two T-spherical fuzzy sets. It has several applications in various disciplines like science, management, and engineering. The noticeable applications incorporate pattern analysis, decision-making, medical diagnosis, and clustering. Keeping in view the importance of TSFSs, some novel CCs are developed in this article along with their related properties. The under discussion correlation coefficients are far more advantageous than the existing and many other tools used for T- spherical fuzzy sets, because they are used completely and demonstrate the nature as well as the limit of association between two T- spherical fuzzy sets. The key finding of the present study are as follows:

1. Some novel CCs based on TSFSs are introduced. The proposed TSFCCs may compute both the stages of association and the nature of correlation. The proposed CCs are the same as the conventional CCs which are employed in statistics.
2. By implying computation of correlation degree, the rationality of the proposed TSFCCs is set up.
3. An algorithm for pattern recognition problems based on proposed CCs has been developed which is further demonstrated a numerical example.
4. An algorithm for multi-attribute decision-making problems is introduced which is explained by an example in which selection of a suitable COVID-19 mask has been addressed.

Furthermore, the fusion of such types of CCs in nature-inspired algorithms also seems to make use of positive solutions in optimization problems. We will also make use of the proposed TSFCCs for the real data regarding COVID-19.

Author contributions All the data analyzed during this study are included in this article.

Availability of data and materials This was not an interventional study and no new patient data were created or analyzed. Data sharing is not applicable to this article.

Declarations

Conflict of interest The authors announce that there are no conflicts of interest.

Ethical approval This article is not currently being considered elsewhere for possible publication.

Informed consent Informed consent was obtained from all authors.

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