Granul. Comput. (2017) 2:187–197
DOI 10.1007/s41066-017-0038-5

ORIGINAL PAPER

# *GIFIHIA* operator and its application to the selection of cold chain logistics enterprises

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Received: 27 July 2016 / Accepted: 22 January 2017 / Published online: 19 April 2017


**Abstract** In this paper, we present some induced hybrid interaction averaging operators under intuitionistic fuzzy environments, including the induced hybrid interaction averaging operator and the generalized induced hybrid interaction averaging operator under intuitionistic fuzzy environments. The properties of these operators are investigated. The main advantages of these operators are that, (1) the interactions of different intuitionistic fuzzy values are taken into consideration, (2) the involved intuitionistic fuzzy values are reordered according to the induced values and then are aggregated into a collective one, (3) the attitudes of decision makers are considered by taking different values of parameter according to decision makers' preferences. We make comparisons between the results of this paper and the exsiting ones and apply the proposed operators to the selection of cold chain logistics enterprises under intuitionistic fuzzy environment. We also construct the intuitionistic fuzzy values with granularity and show the feasibility of the new approach with numerical examples.

**Keywords** Intuitionistic fuzzy set · Induced hybrid interaction averaging operator · Cold chain logistics enterprises

## 1 Introduction

As an important part of modern decision science, multiple attribute decision making has been widely used. Many operators focus on correctly aggregating information decision making problems have been developed on this issue (Zhao et al. 2010; Ye 2010; He et al. 2013; 2016; Beliakov et al. 2010; Merigó et al. 2011).

As a common form of decision information, Zadeh (1965) developed the fuzzy sets. To describe vague information more flexibly and practicably, Atanassov (1986) developed intuitionistic fuzzy sets (IFSs). Atanassov (1994) presented some basic operations on IFSs. Xu (2007) presented some aggregation operators under intutionistic fuzzy environments and obtained their detailed formulas with mathematical induction. Much more attention has been paid to Granular Computing and decision making problems (Xu and Yager 2006; Rodríguez et al. 2012, 2013; Chen 2014; Rodríguez et al. 2014; Pedrycz and Chen 2015; Livi and Sadeghian 2016; Chen and Chang 2015; He et al. 2015; Lorkowski and Kreinovich 2015; Chen et al. 2016a, b; Apolloni et al. 2016; Antonelli et al. 2016; Ciucci 2016; Kovalerchuk and Kreinovich 2017; Loia et al. 2016; Lingras et al. 2016; Liu and Cocea 2017; Liu et al. 2016; Maciel et al. 2016; Min and Xu 2016; Peters and Weber 2016; Skowron et al. 2016; Wilke and Portmann 2016; Xu and Wang 2016; Yao 2016; Sanchez et al. 2017; Song and Wang 2016; Wang et al. 2017; Zhou 2017). Xu and Xia (2011) dealt with financial decision making with induced generalized aggregation operators. Mendel (2016) synthesized the interval type-2 fuzzy set model for a word by comparing Hao-Mendel Approach, Interval Approach and Enhanced Interval Approach. Xu and Gou (2017) made an overview of interval-valued intuitionistic fuzzy information aggregations and applications. Wei and Zhao (2012) dealt with decision making by the induced correlated aggregating operators. Considering that distorted conclusions would be obtained if the decision makers don't take account the relationships

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among the evaluated values (Yager 2008), Wei (2012) developed some prioritized aggregation operators. Xu and Xia (2011) proposed some induced generalized intuitionistic fuzzy operators. Yager and Filev (1999) proposed the induced ordered weighted averaging operators. Chen et al. (2016a, b) proposed a multi-criteria decision making method based on the TOPSIS method under intuitionistic fuzzy environments. Apolloni et al. (2016) proposed a neurofuzzy algorithm for learning from complex granules. Dubois and Prade (2016) investigated the notion of extensional fuzzy set and highlighted its common features with the notion of formal concept in regard to a similarity relation. Das et al. (2017) developed a robust decision making method using intuitionistic fuzzy values. Chen and Tsai (2016) developed multiple attribute decision making method based on novel interval-valued intuitionistic fuzzy geometric averaging operators. Kreinovich (2016) solved systems of equations under uncertainty and explained how different practical problems lead to different mathematical and computational formulations.

Recently, some new operational laws on intuitionistic fuzzy values had been proposed in He et al. (2014a, b), which can be used in some special cases. As a good complement to the existing works, the new operational laws consider the interactions between membership function and non-membership function of different intuitionistic fuzzy values. However, when concerned with the decision making situations that the given intuitionistic fuzzy values and their ordered positions by the induced values should be considered with the interaction theory, very little work had been done. As a result, based on the works in He et al. (2014a, b), Xu (2007) and Xu and Yager (2006), we present some induced hybrid interaction aggregation operators on intuitionistic fuzzy values, such as the IFIHIA operator and the *G*IFIHIA operator. We investigate the properties of these new aggregation operators and apply them to the selection of cold chain logistics enterprises under intuitionistic fuzzy environment. The main advantage of these operators is concluded as follows, (1) the interactions of different intuitionistic fuzzy values are taken into consideration, (2) the involved intuitionistic fuzzy values are reordered according to the induced values and then are aggregated into a collective one, (3) the attitudes of decision makers are considered by taking different values of parameter according to decision makers' preferences.

The rest of the paper is organized as follows. Section 2 reviews some basic concepts. Section 3 presents someinduced hybrid interaction averaging operators under intuitionistic fuzzy environments and the corresponding properties are investigated. Section 4 investigates the selection of cold chain logistics enterprises based on the proposed operators under intuitionistic fuzzy environment. In Sect. 5, numerical examples show the feasibility and validity of the presented approach. Finally, Sect. 6 concludes the paper.

## 2 Preliminaries

**Definition 1** (*Atanassov* 1986). Suppose that $X$ is a fixed non-empty set.

$A = \left\{ \langle x, u_A(x), v_A(x), \pi_A(x) \rangle | x \in X \right\}$ indicates intuitionistic fuzzy sets (IFSs) in $X$, where $u_A(x), v_A(x) \in [0, 1,$ representing the membership degree and the non-membership degree respectively. $\pi_A(x) = 1 - u_A(x) - v_A(x)$, reflecting the hesitant degree of $x \in X$.
Xu (2007) and Xu and Yager (2006) called $A = \langle u, v \rangle$ intuitionistic fuzzy number (IFN) for computational convenience and all IFNs are denoted as IFNs(X) in this paper.

Atanassov (1994) and De et al. (2000) introduced some basic operations on IFNs, which have been widely used in multiple attribute decision making.

Let $A = <u_A, v_A>$ be an IFN, Chen and Tan (1994) described the suitable degree of an alternative meets the decision maker's demand with score function $S(A) = u_A - v_A$. Hong and Choi (2000) described the accurate degree of IFN $A$ with accuracy function $H(A) = u_A + v_A$.

Based on the score function and accuracy function, Xu (2007) and Xu and Yager (2006) defined the comparison law for IFNs as follows.

**Definition 2** Let $A = \langle u_A, v_A \rangle \in$ IFNs($X$) and $B = \langle u_B, v_B \rangle \in$ IFNs($X$). Then $A < B$ if and only if.

i. $S(A) < S(B)$ or
ii. $S(A) = S(B)$ and $H(A) < H(B)$.

Recently, the new addition operation, scalar multiplication operation, multiplication operation and power operation are developed in He et al. (2014a, b) as follows.

**Definition 3** Let $A = \langle u_A, v_A \rangle \in$ IFNs($X$) and $B = \langle u_B, v_B \rangle \in$ IFNs($X$).

$$A \hat{\oplus} B = \left\langle 1 - (1 - u_A) \cdot (1 - u_B), (1 - u_A) \cdot (1 - u_B) - (1 - (u_A + v_A)) \cdot (1 - (u_B + v_B)) \right\rangle \quad (1)$$

$$\lambda A = \left\langle 1 - (1 - u_A)^{\lambda}, (1 - u_A)^{\lambda} - (1 - (u_A + v_A))^{\lambda} \right\rangle, \lambda > 0 \quad (2)$$

$$A \hat{\otimes} B = \left\langle (1 - v_A)(1 - v_B) - (1 - (u_A + v_A))(1 - (u_B + v_B)), 1 - (1 - v_A)(1 - v_B) \right\rangle \quad (3)$$

$$A^{\lambda} = \left\langle (1 - v_A)^{\lambda} - (1 - (u_A + v_A))^{\lambda}, 1 - (1 - v_A)^{\lambda} \right\rangle, \lambda > 0 \quad (4)$$

## 3 Intuitionistic fuzzy induced hybrid interaction averaging (IFIHIA) operator

**Definition 4** Let $A_i = \langle u_{A_i}, v_{A_i} \rangle \in \text{IFNs}(X)$ $(i = 1, 2, \ldots, n)$. $p_i$ $(i = 1, 2, \ldots, n)$ is the induced value. The IFIHIA operator is defined as

$$\text{IFIHIA}_{\omega,w}(A_1, \ldots, A_n) = \bigoplus_{i=1}^{n} w_i \tilde{A}_{index(i)}, \tag{5}$$

where $\omega = (\omega_1, \ldots \omega_n)^T$ is the weight vector of $A_i$ $(i = 1, 2, \ldots, n)$, satisfying $\omega_i \in [0, 1$ and $\sum_{i=1}^{n} \omega_i = 1$, $\tilde{A}_i$ denotes $n\omega_i A_i$ $(i = 1, \ldots, n)$, $\tilde{A}_i$ is reordered according to $p_i$ as $\tilde{A}_{index(i)}$ $(i = 1, 2, \ldots, n)$, $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $\tilde{A}_{index(i)}$ $(i = 1, 2, \ldots, n)$, satisfying $w_i \in [0, 1$ and $\sum_{i=1}^{n} w_i = 1$.

**Theorem 1** *Let* $A_i = \langle u_{A_i}, v_{A_i} \rangle \in \text{IFNs}(X)$ $(i = 1, 2, \ldots, n)$. $p_i$ $(i = 1, 2, \ldots, n)$ *is the induced value. Then*

$$\text{IFIHIA}_{\omega,w}(A_1, \ldots, A_n) = \left\langle 1 - \prod_{i=1}^{n} (1 - u_{\tilde{A}_{index(i)}})^{w_i}, \prod_{i=1}^{n} (1 - u_{\tilde{A}_{index(i)}})^{w_i} - \prod_{i=1}^{n} \left(1 - (u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}})\right)^{w_i} \right\rangle \tag{6}$$

And, $\text{IFIHIA}_{\omega,w}(A_1, \ldots, A_n) \in \text{IFNs}(X)$.

**Theorem 2** (Idempotency) *Let* $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of IFNs, if all* $A_i$ *are equal, supposed as A, then* $IFIHIA_{\omega,w}(A_1, \ldots, A_n) = A$.

**Theorem 3** (Commutativity). *Let* $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of IFNs, if* $A_i'(i = 1, \ldots n)$ *is any permutation of* $A_i(i = 1, \ldots n)$, *then*

$$IFIHIA_{\omega,w}(A_1, \ldots, A_n) = IFIHIA_{\omega,w}(A_1', \ldots, A_n').$$

*Example 1* Let $A_1 = \langle 0.2, 0.7 \rangle A_2 = \langle 0.5, 0.3 \rangle$ $A_3 = \langle 0.4, 0.5 \rangle A_4 = \langle 0.3, 0.4 \rangle A_5 = \langle 0.6, 0 \rangle$ be five IFNs. $\omega = \{0.25, 0.20, 0.15, 0.18, 0.22\}^T$ is the weight vector of $A_i$ $(i = 1, 2, \ldots, 5)$.

By the operational law in Atanassov (1986), De et al. (2000) and Xu (2007), it has.

$\tilde{A}_1 = \langle 1 - (1 - 0.2)^{5\times0.25}, 0.7^{5\times0.25} \rangle = \langle 0.243, 0.640 \rangle$,
$\tilde{A}_2 = \langle 1 - (1 - 0.5)^{5\times0.20}, 0.3^{5\times0.20} \rangle = \langle 0.500, 0.300 \rangle$,
$\tilde{A}_3 = \langle 1 - (1 - 0.4)^{5\times0.15}, 0.5^{5\times0.15} \rangle = \langle 0.318, 0.595 \rangle$,
$\tilde{A}_4 = \langle 1 - (1 - 0.3)^{5\times0.18}, 0.4^{5\times0.18} \rangle = \langle 0.275, 0.438 \rangle$,
$\tilde{A}_5 = \langle 1 - (1 - 0.6)^{5\times0.22}, 0^{5\times0.22} \rangle = \langle 0.635, 0.00 \rangle$.

By Definition 2, we have

$$S(\tilde{A}_1) = -0.3969,\ S(\tilde{A}_2) = 0.2000,\ S(\tilde{A}_3) = -0.2763,\ S(\tilde{A}_4) = -0.1639,\ S(\tilde{A}_5) = 0.6350.$$

For a fair comparison, we suppose the score values of IFNs are the induced values, Obviously,

$$S(\tilde{A}_5) > S(\tilde{A}_2) > S(\tilde{A}_4) > S(\tilde{A}_3) > S(\tilde{A}_1).$$

So

$$\tilde{A}_{index(1)} = \langle 0.6350,\ 0.0000 \rangle,\ \tilde{A}_{index(2)} = \langle 0.5000,\ 0.3000 \rangle,\ \tilde{A}_{index(3)} = \langle 0.2745,\ 0.4384 \rangle,\ \tilde{A}_{index(4)} = \langle 0.3183,\ 0.5946 \rangle,\ \tilde{A}_{index(5)} = \langle 0.2434,\ 0.6403 \rangle.$$

Suppose that $w = (0.112, 0.236, 0.304, 0.236, 0.112)^T$. Then, by the intuitionistic fuzzy hybrid interaction averaging (IFHA) operator in Xu (2007), it follows that.

$$A = IFHA_{w,\omega}(A_1, A_2, A_3, A_4, A_5) = \left\langle 1 - \prod_{j=1}^{5} \left(1 - u_{\tilde{A}_{\sigma(j)}}\right)^{w_j}, \prod_{j=1}^{5} \left(v_{\tilde{A}_{\sigma(j)}}\right)^{w_i} \right\rangle = \langle 1 - (1 - 0.6350)^{0.112} \cdot (1 - 0.5)^{0.236} \cdot (1 - 0.2745)^{0.304} \cdot (1 - 0.3183)^{0.236} \cdot (1 - 0.2434)^{0.112}, 0.1703^{0.112} \cdot 0.3^{0.236} \cdot 0.4384^{0.304} \cdot 0.5946^{0.236} \cdot 0.0000^{0.112} \rangle = \langle 0.3910, 0.000 \rangle.$$

While according to Definition 3, we have

$\tilde{A}_1 = \langle 1 - (1 - 0.2)^{5\times0.25}, (1 - 0.2)^{5\times0.25} - (1 - 0.9)^{5\times0.25} \rangle = \langle 0.2434, 0.7004 \rangle$,
$\tilde{A}_2 = \langle 1 - (1 - 0.5)^{5\times0.20}, (1 - 0.5)^{5\times0.20} - (1 - 0.8)^{5\times0.20} \rangle = \langle 0.500, 0.300 \rangle$,
$\tilde{A}_3 = \langle 1 - (1 - 0.4)^{5\times0.15}, (1 - 0.4)^{5\times0.15} - (1 - 0.9)^{5\times0.15} \rangle = \langle 0.3183, 0.5039 \rangle$,
$\tilde{A}_4 = \langle 1 - (1 - 0.3)^{5\times0.18}, (1 - 0.3)^{5\times0.18} - (1 - 0.7)^{5\times0.18} \rangle = \langle 0.2745, 0.3870 \rangle$,
$\tilde{A}_5 = \langle 1 - (1 - 0.6)^{5\times0.22}, (1 - 0.6)^{5\times0.22} - (1 - 0.6)^{5\times0.22} \rangle = \langle 0.6350, 0.0000 \rangle$.

According to Definition 2, we obtain

$S(\tilde{A}_1) = -0.4570,\ S(\tilde{A}_2) = 0.2000,\ S(\tilde{A}_3) = -0.1856,$
$S(\tilde{A}_4) = -0.1125,\ S(\tilde{A}_5) = 0.6350.$

obviously,

$S(\tilde{A}_5) > S(\tilde{A}_2) > S(\tilde{A}_4) > S(\tilde{A}_3) > S(\tilde{A}_1).$

Thus,

$\tilde{A}_{index(1)} = \langle 0.6350, 0.0000\rangle,\ \tilde{A}_{index(2)} = \langle 0.5000, 0.3000\rangle,$
$\tilde{A}_{index(3)} = \langle 0.2745, 0.3870\rangle,\ \tilde{A}_{index(4)} = \langle 0.3183, 0.5039\rangle,$
$\tilde{A}_{index(5)} = \langle 0.2434, 0.7004\rangle.$

Suppose that $w = (0.112, 0.236, 0.304, 0.236, 0.112)^T$, which is determined by the normal distribution based method (Xu 2007). Then, by Theorem 1, it follows that

$$lA = IFIHIA_{w,\omega}(A_1, A_2, A_3, A_4, A_5) = \left\langle 1 - \prod_{j=1}^{5}(1 - u_{\tilde{A}_{index(j)}})^{w_j}, \prod_{j=1}^{5}\left[1 - u_{\tilde{A}_{index(j)}}\right]^{w_i} - \prod_{j=1}^{5}\left[1 - (u_{\tilde{A}_{index(j)}} + v_{\tilde{A}_{index(j)}})\right]^{w_i}\right\rangle$$
$$= \left\langle 1 - (1 - 0.6350)^{0.112}\cdot(1 - 0.5)^{0.236}\cdot(1 - 0.2750)^{0.304}\cdot(1 - 0.3183)^{0.236}\cdot(1 - 0.2434)^{0.112},\ 0.6090 - 0.1703^{0.112}\right.$$
$$\left.\cdot 0.2^{0.236}\cdot 0.338^{0.304}\cdot 0.1778^{0.236}\cdot 0.0562^{0.112}\right\rangle = \langle 0.3910, 0.3950\rangle.$$

It is obvious that $v_{\mathrm{IFIHA}_{\omega,w}(A_1,\ldots,A_n)} = 0.3950 \neq 0$. Thus, $v_{A_5}$ doesn't play a decisive role, which is the advantage of the proposed operator in this paper over that in Xu (2007).

The IFIHIA operator can be interpreted from three aspects as follows.

1. It considers not only the effects of membership function of different IFN*s* and effects of non-membership degree of different IFN*s*, but also the interactions of different IFN*s*.
2. The weighted IFNs $n\omega_i A_i$ $(i = 1, \ldots, n)$ are reordered according to the induced value $p_i$ $(i = 1, 2, \ldots, n)$. The weighted IFNs $n\omega_i A_i$ $(i = 1, \ldots, n)$ are obtained by multiplies $A_i$ $(i = 1, \ldots, n)$ by the corresponding weights $\omega = (\omega_1, \ldots \omega_n)^T$ and a balancing coefficient $n$.
3. In the process of all the weighted IFNs $w_i\tilde{A}_{index(i)}$ $(i = 1, 2, \ldots, n)$ are aggregated into a collective one, both the given IFNs and their induced values $p_i$ $(i = 1, 2, \ldots, n)$ are considered.

## 4 Generalized intuitionistic fuzzy induced hybrid interaction averaging (*GIFIHIA*) operator

**Definition 5** Let $A_i = \langle u_{A_i}, v_{A_i}\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IFNs, $\lambda > 0$, $p_i$ $(i = 1, 2, \ldots, n)$ be the induced value. The generalized intuitionistic fuzzy induced hybrid interaction averaging (*GIFIHIA*) operator of dimension $n$ is a mapping $\mathrm{GIFIHIA}_\lambda:\mathrm{IFN}^n \to \mathrm{IFN}$, which has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$, satisfying $w_i \in [0, 1$ and $\sum_{i=1}^{n} w_i = 1$ such that

$$GIFIHIA_\lambda(A_1, \ldots, A_n) = \left(\bigoplus_{i=1}^{n} w_i\tilde{A}^{\lambda}_{index(i)}\right)^{1/\lambda}, \tag{7}$$

where $\omega = (\omega_1, \ldots \omega_n)^T$ is the weight vector of $A_i$ $(i = 1, 2, \ldots, n)$, satisfying $\omega_i \in [0, 1$ and $\sum_{i=1}^{n}\omega_i = 1$, $\tilde{A}_i$ denotes $n\omega_i A_i$ $(i = 1, 2, \ldots, n)$, $n$ is the balancing coefficient, which plays a role of balance. $\tilde{A}_i$ is reordered according to $p_i$ as $\tilde{A}_{index(i)}$ $(i = 1, 2, \ldots, n)$.

**Lemma 1** *Let $A_i = \langle u_{A_i}, v_{A_i}\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of* IFN*s. Then*

$$\bigoplus_{i=1}^{n} w_i A_i{}^{\lambda} = \left\langle 1 - \prod_{i=1}^{n}\left(1 - (1 - v_{A_i})^{\lambda} + (1 - (u_{A_i} + v_{A_i}))^{\lambda}\right)^{w_i}, \right.$$
$$\prod_{i=1}^{n}\left(1 - (1 - v_{A_i})^{\lambda} + (1 - (u_{A_i} + v_{A_i}))^{\lambda}\right)^{w_i}$$
$$\left. - \prod_{i=1}^{n}\left(1 - (u_{A_i} + v_{A_i})\right)^{\lambda w_i}\right\rangle$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weighting vector of $A_i$, satisfying $w_i \in [0, 1$ and $\succ$.

*Proof* By using mathematical induction on $n$, we prove Lemma 1 as follows.

when $n = 1$, $w_1 = 1$, we obtain

$$w_1 A_1{}^{\lambda} = A_1{}^{\lambda} = \left\langle (1 - v_{A_1})^{\lambda} - (1 - (u_{A_1} + v_{A_1}))^{\lambda}, \right.$$
$$\left. 1 - (1 - v_{A_1})^{\lambda}\right\rangle = \left\langle 1 - \left(1 - (1 - v_{A_1})^{\lambda} + (1 - (u_{A_1} + v_{A_1}))^{\lambda}\right)^{1}, \right.$$
$$\left. \left(1 - (1 - v_{A_1})^{\lambda} + (1 - (u_{A_1} + v_{A_1}))^{\lambda}\right)^{1} - (1 - (u_{A_1} + v_{A_1}))^{1\cdot\lambda}\right\rangle$$
$$= \left\langle 1 - \left(1 - (1 - v_{A_1})^{\lambda} + (1 - (u_{A_1} + v_{A_1}))^{\lambda}\right)^{w_1}, \right.$$
$$\left. \left(1 - (1 - v_{A_1})^{\lambda} + (1 - (u_{A_1} + v_{A_1}))^{\lambda}\right)^{w_1} - (1 - (u_{A_1} + v_{A_1}))^{\lambda\cdot w_1}\right\rangle.$$

Thus, Lemma 1 is established for $n = 1$.

If Lemma 1 holds for $n=k$. Then, $n=k+1$, by inductive assumption and Eq. (1), we get

$$l \mathop{\oplus}_{i=1}^{k+1} w_i {A_i}^{\lambda} = \left( \mathop{\oplus}_{i=1}^{k} w_i {A_i}^{\lambda} \right) \oplus w_{k+1} {A_{k+1}}^{\lambda}$$
$$= \left\langle 1 - \prod_{i=1}^{k} \left(1 - (1 - v_{A_i})^{\lambda} + (1 - (u_{A_i} + v_{A_i}))^{\lambda}\right)^{w_i}, \prod_{i=1}^{k} \left(1 - (1 - v_{A_i})^{\lambda} + (1 - (u_{A_i} + v_{A_i}))^{\lambda}\right)^{w_i} - \prod_{i=1}^{k} (1 - (u_{A_i} + v_{A_i}))^{\lambda w_i} \right\rangle$$
$$\oplus \left\langle 1 - \left(1 - \left(1 - v_{A_{k+1}}\right)^{\lambda} + \left(1 - \left(u_{A_{k+1}} + v_{A_{k+1}}\right)\right)^{\lambda}\right)^{w_{k+1}}, \left(1 - \left(1 - v_{A_{k+1}}\right)^{\lambda} + \left(1 - \left(u_{A_{k+1}} + v_{A_{k+1}}\right)\right)^{\lambda}\right)^{w_{k+1}} - \left(1 - \left(u_{A_{k+1}} + v_{A_{k+1}}\right)\right)^{\lambda \cdot w_{k+1}} \right\rangle$$
$$= \left\langle 1 - \prod_{i=1}^{k+1} \left(1 - (1 - v_{A_i})^{\lambda} + (1 - (u_{A_i} + v_{A_i}))^{\lambda}\right)^{w_i}, \prod_{i=1}^{k+1} \left(1 - (1 - v_{A_i})^{\lambda} + (1 - (u_{A_i} + v_{A_i}))^{\lambda}\right)^{w_i} - \prod_{i=1}^{k+1} (1 - (u_{A_i} + v_{A_i}))^{\lambda w_i} \right\rangle$$

i.e. Lemma 1 is established for $n = k + 1$.

Thus, Lemma 1 holds for all $n$ with the mathematical induction on $n$.

**Theorem 4** *Let* $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of* IFNs. $\lambda > 0$. *Then* $\text{GIFIHIA}_{\lambda}(A_1, A_2, \ldots A_n) \in \text{IFNs}(X)$. *Furthermore,*

$$GIFIHIA_{\lambda}(A_1, A_2, \ldots A_n) = \left\langle \left( 1 - \prod_{i=1}^{n} \left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} + \prod_{i=1}^{n} \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda \cdot w_i} \right)^{1/\lambda} - \prod_{i=1}^{n} \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{w_i}, 1 - \left( 1 - \prod_{i=1}^{n} \left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} + \prod_{i=1}^{n} \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda w_i} \right)^{1/\lambda} \right\rangle. \tag{8}$$

*Proof* By Lemma 1 we get

$$\mathop{\oplus}_{i=1}^{n} w_i \tilde{A}_{index(i)}^{\lambda} = \left\langle 1 - \prod_{i=1}^{n} \left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i}, \prod_{i=1}^{n} \left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} - \prod_{i=1}^{n} \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda w_i} \right\rangle.$$

Then according to Eqs. (4) and (7), we have

$$\text{GIFIHIA}_{\lambda}(A_1, A_2, \ldots A_n) = \left( \mathop{\oplus}_{i=1}^{n} w_i (\tilde{A}_{index(i)})^{\lambda} \right)^{1/\lambda}$$
$$= \left\langle \left( 1 - \prod_{i=1}^{n} \left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} + \prod_{i=1}^{n} \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda \cdot w_i} \right)^{1/\lambda} - \prod_{i=1}^{n} \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{w_i}, 1 - \left( 1 - \prod_{i=1}^{n} \left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} + \prod_{i=1}^{n} \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda w_i} \right)^{1/\lambda} \right\rangle$$

Therefore, Eq. (8) is established.

Next, we prove the result that $\text{GIFIHIA}_{\lambda}(A_1, A_2, \ldots A_n) \in \text{IFNs}(X)$.

Let $\text{GIFIHIA}_{\lambda}(A_1, A_2, \ldots A_n) = E = \langle u_E, v_E \rangle$, By Eq. (8), we have

$$v_E = 1 - \left(1 - \prod_{i=1}^{n}\left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} + \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda w_i}\right)^{1/\lambda}, \tag{9}$$

$$u_E = \left(1 - \prod_{i=1}^{n}\left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} + \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda \cdot w_i}\right)^{1/\lambda} - \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{w_i} \tag{10}$$

According to Definition 1, we have $0 \leqslant 1 - \left(1 - v_{\tilde{A}_{\sigma(i)}}\right)^{\lambda} \leqslant 1$,then

$$v_E \geqslant 1 - \left(1 - \prod_{i=1}^{n}\left(0 + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} + \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda w_i}\right)^{1/\lambda} = 0. \tag{11}$$

And

$$1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda} = 1 - \left(\left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} - \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right) \in [0, 1]. \tag{12}$$

Thus,

$$v_E \leq 1 - \left(1 - \prod_{i=1}^{n}(1)^{w_i} + \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda w_i}\right)^{1/\lambda} = 1 - \left(1 - 1 + \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda w_i}\right)^{1/\lambda}$$
$$= 1 - \left(\prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda w_i}\right)^{1/\lambda} = 1 - \left(\prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{w_i}\right) \leq 1 \tag{13}$$

By Eqs. (11) and (13), we have

$$0 \leqslant v_E \leqslant 1. \tag{14}$$

By Eq. (12) and $0 \leqslant w_i \leqslant 1$, we obtain

$$\left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} \in [0, 1],$$

Thus,

$$1 - \prod_{i=1}^{n}\left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} \in [0, 1]. \tag{15}$$

By Eqs. (10) and (15), we have

$$u_E \geqslant \left(0 + \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda \cdot w_i}\right)^{1/\lambda} - \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{w_i} = 0. \tag{16}$$

And

$$\left(\prod_{i=1}^{n}\left(1 - \left(1 - v_{\tilde{A}_{index(i)}}\right)^{\lambda} + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} - \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda \cdot w_i}\right)$$
$$\geqslant \prod_{i=1}^{n}\left(0 + \left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda}\right)^{w_i} - \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{\lambda \cdot w_i} = 0, \tag{17}$$

By Eqs. (10) and (17), we have

$$u_E \leqslant (1 - 0)^{1/\lambda} - \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{w_i} = 1 - \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{w_i} \leqslant 1. \tag{18}$$

Then by Eqs. (16) and (18), we obtain

$$0 \leq u_E \leq 1. \tag{19}$$

According to Eqs. (9) and (10), we have

$$u_E + v_E = 1 - \prod_{i=1}^{n}\left(1 - \left(u_{\tilde{A}_{index(i)}} + v_{\tilde{A}_{index(i)}}\right)\right)^{w_i} \in [0, 1]. \tag{20}$$

Therefore, according to Eqs. (14), (19) and (20) and Definition 1, we have

$\mathrm{GIFIHIA}_{\lambda}(A_1, A_2, \ldots A_n) \in \mathrm{IFNs}(X)$.

**Theorem 5** *Let* $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of* IFNs, $\lambda > 0$. $\lambda \to 0$. *Then the GIFIHIA operator approaches the following limit.*

$$\underset{\lambda \to 0}{Lim}\, GIFIHIA_{\lambda}(A_1, \cdots A_n) = \left\langle e^{\sum_{i=1}^{n}\left(w_i\left(ln\left(1-v_{\tilde{A}_{index(i)}}\right)-ln\left(1-\left(u_{\tilde{A}_{index(i)}}+v_{\tilde{A}_{index(i)}}\right)\right)\right)\right)+ln\left(\prod_{i=1}^{n}\left(1-\left(u_{\tilde{A}_{index(i)}}+v_{\tilde{A}_{index(i)}}\right)\right)^{w_i}\right)} - \prod_{i=1}^{n}\left(1-\left(u_{\tilde{A}_{index(i)}}+v_{\tilde{A}_{index(i)}}\right)\right)^{w_i},\ 1 - e^{\sum_{i=1}^{n}\left(ln\left(1-v_{\tilde{A}_{index(i)}}\right)-ln\left(1-\left(u_{\tilde{A}_{index(i)}}+v_{\tilde{A}_{index(i)}}\right)\right)\right)+ln\prod_{i=1}^{n}\left(1-\left(u_{\tilde{A}_{index(i)}}+v_{\tilde{A}_{index(i)}}\right)\right)^{w_i}} \right\rangle. \quad (21)$$

*And* $\underset{\lambda \to 0}{Lim}\, GIFIHIA_{\lambda}(A_1, A_2, \ldots A_n) \in IFNs(X)$.

*Proof* Similar to He et al. (2014b) and omitted here.

**Theorem 6** (Idempotency) *Let* $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of* IFNs, $\lambda > 0. w = (w_1, w_2, \ldots, w_n)^T$ *is the weighting vector of* $A_i$, *satisfying* $w_i \in [0, 1$ *and* $>$. *If all* $A_i$ *are equal, denoted as* $A$, *and* $\omega = \left(\frac{1}{n}, \cdots \frac{1}{n}\right)$, *then* $\mathrm{GIFIHIA}_{\lambda}(A_1, A_2, \ldots A_n) = A$.

*Proof* Let $A_i = A = \langle u_A, v_A \rangle$ $(i = 1, 2, \ldots, n)$, if $\omega = \left(\frac{1}{n}, \cdots \frac{1}{n}\right)$, it has $n\omega A_i = A$. By Eq. (8), taking note of $\sum_{i=1}^{n} w_i = 1$, we obtain

$$\begin{aligned}
\mathrm{GIFIHIA}_{\lambda}(A_1, A_2, \ldots A_n) &= \left\langle \left(1 - \prod_{i=1}^{n}\left(1-(1-v_A)^{\lambda} + (1-(u_A+v_A))^{\lambda}\right)^{w_i} + \prod_{i=1}^{n}(1-(u_A+v_A))^{\lambda \cdot w_i}\right)^{1/\lambda} - \prod_{i=1}^{n}(1-(u_A+v_A))^{w_i}, \right. \\
&\quad \left. 1 - \left(1 - \prod_{i=1}^{n}\left(1-(1-v_A)^{\lambda} + (1-(u_A+v_A))^{\lambda}\right)^{w_i} + \prod_{i=1}^{n}(1-(u_A+v_A))^{\lambda w_i}\right)^{1/\lambda} \right\rangle \\
&= \left\langle \left(1 - \left(1-(1-v_A)^{\lambda} + (1-(u_A+v_A))^{\lambda}\right)^{\sum_{i=1}^{n} w_i} + (1-(u_A+v_A))^{\sum_{i=1}^{n} \lambda \cdot w_i}\right)^{1/\lambda} \right. \\
&\quad \left. -(1-(u_A+v_A))^{\sum_{i=1}^{n} w_i}\ 1 - \left(1 - \left(1-(1-v_A)^{\lambda} + (1-(u_A+v_A))^{\lambda}\right)^{\sum_{i=1}^{n} w_i} + (1-(u_A+v_A))^{\sum_{i=1}^{n} \lambda w_i}\right)^{1/\lambda} \right\rangle \\
&= \left\langle \left(1 - \left(1-(1-v_A)^{\lambda} + (1-(u_A+v_A))^{\lambda}\right) + (1-(u_A+v_A))^{\lambda}\right)^{1/\lambda} \right. \\
&\quad \left. -\ (1-(u_A+v_A)),\ 1 - \left(1 - \left(1-(1-v_A)^{\lambda} + (1-(u_A+v_A))^{\lambda}\right) + (1-(u_A+v_A))^{\lambda}\right)^{1/\lambda} \right\rangle \\
&= \left\langle \left((1-v_A)^{\lambda}\right)^{1/\lambda} - (1-(u_A+v_A)), 1 - \left((1-v_A)^{\lambda}\right)^{1/\lambda} \right\rangle = \langle u_A, v_A \rangle.
\end{aligned}$$

**Theorem 7** (Commutativity) *Let* $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of* IFNs. $\lambda > 0$. $w = (w_1, w_2, \ldots, w_n)^T$ *is the weighting vector of* $A_i$ *and* $w_i = \frac{1}{n}\,(i = 1, \ldots, n)$. *If* $(A'_1, A'_2, \ldots A'_n)$ *is any permutation of* $(A_1, A_2, \ldots A_n)$, *then* $GIFIHIA_{\lambda}(A_1, A_2, \ldots A_n) = GIFIHIA_{\lambda}(A'_1, A'_2, \ldots A'_n)$..

*Proof* By Eq. (8) and the condition that $(A'_1, A'_2, \ldots A'_n)$ is any permutation of $(A_1, A_2, \ldots A_n)$, we can get the result directly.

## 5 Selection of cold chain logistics enterprises under intuitionistic fuzzy environment

Intuitionistic fuzzy multiple attribute decision making (IFMADM) problems are the process of choosing the best alternative from all of the possible alternatives which are evaluated by several attributes. A cold chain is a temperature-controlled supply chain, which is used to help extend and ensure the shelf life of products. Suppose that $X = \{x_1, x_2, \ldots, x_n\}$ is a set of cold chain logistics enterprises, $G = \{G_1, G_2, \ldots, G_m\}$ is a set of attributes with the associated weighting vector $w = (w_1, w_2, \ldots, w_m)^T$, satisfying $w_i \in [0, 1$ and $\sum_{i=1}^{m} w_i = 1$.

Suppose that the evaluated values of the cold chain logistics enterprises $x_i$ $(i = 1, 2, \ldots, n)$ under the attribute $G_j$ are represented by IFNs $A_{ij} = \left\langle u_{A_{ij}}, v_{A_{ij}} \right\rangle$ $(i = 1, 2, \ldots, n j = 1, 2, \ldots, m)$, where $u_{A_{ij}}$ reflects the degree that the alternative $x_i$ $(i = 1, 2, \ldots, n)$ satisfies the attribute $G = \{G_1, G_2, \ldots, G_m\}$ $v_{A_{ij}}$ indicates the opposite meaning. The construction of IFNs can refer to Pedrycz and Chen (2015), Lorkowski and Kreinovich (2015) and Naim and Hagras (2015).

Then the selection of cold chain logistics enterprises under intuitionistic fuzzy environment based on the *GIFIHIA* operator can be listed as follows.

*Step 1* Determine the weights $w = (w_1, w_2, \ldots, w_n)^T$ and $\omega = (\omega_1, \ldots \omega_n)^T$, the meaning of $w$ and $\omega$ please refer to Definitions 4 and 5.

*Step 2* Based on the *GIFIHIA* operator in Deinition 5, we aggregate the decision information by experts into collective ones, and the *GIFIHIA* operator is listed as Eq. (22).

$$A_i = \mathrm{GIFIHIA}_\lambda(A_{i1}, A_{i2}, \ldots A_{in}) = \left\langle \left( \left( 1 - \prod_{j=1}^{n} \left( 1 - \left( 1 - v_{\tilde{A}_{index(ij)}} \right)^\lambda + \left( 1 - \left( u_{\tilde{A}_{index(ij)}} + v_{\tilde{A}_{index(ij)}} \right) \right)^\lambda \right)^{w_j} + \prod_{j=1}^{n} \left( 1 - \left( u_{\tilde{A}_{index(ij)}} + v_{\tilde{A}_{index(ij)}} \right) \right)^{\lambda \cdot w_j} \right)^{1/\lambda} - \prod_{j=1}^{n} \left( 1 - \left( u_{\tilde{A}_{index(ij)}} + v_{\tilde{A}_{index(ij)}} \right) \right)^{w_j}, \right.$$
$$\left. 1 - \left( 1 - \prod_{j=1}^{n} \left( 1 - \left( 1 - v_{\tilde{A}_{index(ij)}} \right)^\lambda + \left( 1 - \left( u_{\tilde{A}_{index(ij)}} + v_{\tilde{A}_{index(ij)}} \right) \right)^\lambda \right)^{w_j} + \prod_{j=1}^{n} \left( 1 - \left( u_{\tilde{A}_{index(ij)}} + v_{\tilde{A}_{index(ij)}} \right) \right)^{\lambda w_j} \right)^{1/\lambda} \right\rangle. \tag{22}$$

*Step 3* Rank the final IFNs $A_i$ $(i = 1, 2, \ldots, n)$ by the score the accuracy functions in Definition 2.

*Step 4* Rank the possible cold chain logistics enterprises $x_i$ $(i = 1, 2, \ldots, n)$ and choose the best ones.

*Step 5* Adjust the values of the parameter $\lambda$ according to decision makers' preference, and analyze the rakings of the cold chain logistics enterprises with different values of the parameter $\lambda$.

*Step 6* Draw the figure of Step 5 to illustrate the selections of the cold chain logistics enterprises further.

## 6 Numerical example

Suppose that a foodstuff general corporation plan to choose a cold chain logistics enterprise to store and transport its goods. After thinking the market environment, three possible cold chain logistics enterprises ($x_1$, $x_2$, $x_3$) are to be considered. For the sake of choosing the best cold chain logistics enterprise to cooperate, the foodstuff general corporation has brought a panel. After careful thinking of the alternatives and decision environment, five attributes $G = \{G_1, G_2, G_3, G_4, G_5\}$ are concluded to evaluate the ability of the candidate cold chain logistics enterprises.

- $G_1$: Storage ability.
- $G_2$: The levels of processing.
- $G_3$: Transportation capability.
- $G_4$: Logistics support capability
- $G_5$: Coordinating optimization ability of business operations.

The evaluated decision information of the three possible alternatives under the above five attributes are presented by IFNs (Table 1).

*Preliminary* According to the method of constructing IFNs with granularity in Sect. 5, for example, experts mark her confidence by left membership value 2 and right membership value 5 on a scale from 1 to 10 to indicate the degrees $x_1$ satisfy the property $G_1$, thus $A_{11}$ in the numerical example is $\left\langle \underline{u_{A_{11}}}, 1 - \overline{u_{A_{11}}} \right\rangle = \langle 2/10, 1 - 5/10 \rangle = < 0.2, 0.5 >$. Similarly, we have $A_{ij}$ $(i = 1, 2, 3;\ j = 1, \ldots, 5)$, and all evaluated IFNs are listed in Table 1.

*Step 1* The weights $w$ and $\omega$ are given by the experts as $w = (0.112, 0.236, 0.304, 0.236, 0.112)$ and $\omega = (0.2, \ldots 0.2)$.

*Step 2* Suppose the induced values of $\tilde{A}_{ij}(i = 1, 2, 3; j = 1, \ldots, 5)$given by the exsperts are $p_{i1} = 0.9$, $p_{i2} = 0.8$, $p_{i3} = 0.6$, $p_{i4} = 0.5$, $p_{i5} = 0.2$ $(i = 1, 2, 3)$, then by Eq. (22), and taking $\lambda = 0.5$, we obtain

$$A_1 = GIFIHIA_{0.5}(A_{11}, \ldots A_{15}) = \langle 0.4387, 0.3262 \rangle.$$

Similarly, $A_2 = \langle 0.4515, 0.3625 \rangle$, $A_3 = \langle 0.3772, 0.4413 \rangle$.

*Step 3* By the score function proposed by Chen and Tan (1994), we have

$$S(A_1) = 0.1125,\ S(A_2) = 0.0891,\ S(A_3) = -0.0640.$$

*Step 4* By Definition 2, we have $S(A_1) > S(A_2) > S(A_3)$ and $x_1 \succ x_2 \succ x_3$.

**Table 1** Intuitionistic fuzzy matrix $(A_{ij})_{3\times 5}$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_5$ |
|---|---|---|---|---|---|
| $x_1$ | ⟨0.2,0.5⟩ | ⟨0.4,0.2⟩ | ⟨0.5,0.4⟩ | ⟨0.3,0.3⟩ | ⟨0.7,0.1⟩ |
| $x_2$ | ⟨0.2,0.7⟩ | ⟨0.6,0.3⟩ | ⟨0.4,0.3⟩ | ⟨0.4,0.4⟩ | ⟨0.6,0.1⟩ |
| $x_3$ | ⟨0.2,0.7⟩ | ⟨0.4,0.3⟩ | ⟨0.4,0.5⟩ | ⟨0.3,0.4⟩ | ⟨0.6,0.2⟩ |

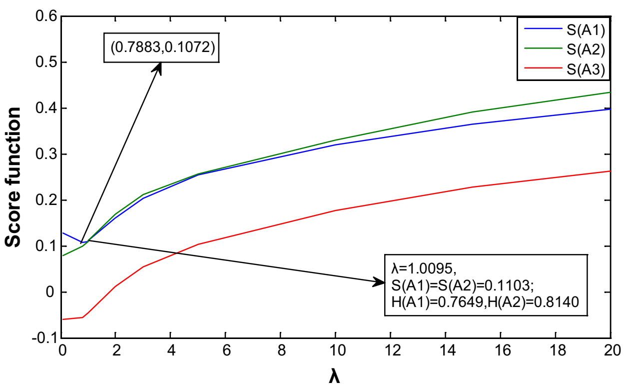


**Fig. 1** Scores of the alternatives with different values of parameter $\lambda$

*Step 5* Taking different values of the parameter $\lambda$ and $\lambda =$ 0.001, 0.002, 0.003 …, 20, here we just list the scores and the rankings of involved alternatives with $\lambda = 0.1,\ 1,\ 2,\ 3,\ 5,\ 10,\ 15,\ 20$ as in Table 2.

Step 6. The scores of invovled alternatives with different values of parameter $\lambda$ in Step 5 are shown in Fig. 1.

From Fig. 1, we find that $S(A_1)$ increases as $\lambda$ increases on (0,0.7883], and decreases as $\lambda$ increases on (0.7883, 20]. $S(A_2)$ and $S(A_3)$ increase as $\lambda$ increases on (0, 20]. Moreover, the ranking of the alternatives with different values of the parameter $\lambda$ are concluded as follows.

1. When $\lambda \in (0, 1.0095)$, the ranking of the three possible cold chain logistics enterprises is $x_1 \succ x_2 \succ x_3$. Therefore, $x_1$ is the best alternative.
2. When $\lambda \in (1.0095, 20)$, the ranking of the three possible cold chain logistics enterprises is $x_2 \succ x_1 \succ x_3$. Thus, $x_2$ is the best alternative.

If we use the aggregation operators in Zhao et al. (2010) to aggregate a set of IFNs, when there exists only one non-membership degree of IFN equals to zero, the non-membership degree of aggregation result of n IFNs is zero even if the non-membership degrees of $n-1$ IFNs are not zero, which is the weakness of the aggregation operators in Zhao et al. (2010). However, if we use the operators developed in this paper, the aggregation result can be explained reasonably. For example, Let $A_1 = \langle 0.3, 0.5\rangle$, $A_2 = \langle 0.4, 0.4\rangle$, $A_3 = \langle 0.3, 0.6\rangle$, $A_4 = \langle 0.4, 0.5\rangle$, $A_5 = \langle 0.5, 0\rangle$ be five IFNs, $w = \{0.25, 0.20, 0.15, 0.18, 0.22\}^T$ be the corresponding weight vector. By the generalized intuitionistic fuzzy weighted averaging (GIFWA) operator in Zhao et al. (2010), i.e.,

$$GIFWA_\lambda(A_1, A_2, \dots A_n) = \left\langle \left(1 - \prod_{i=1}^{n} \left(1 - {u_{A_i}}^{\lambda}\right)^{w_i}\right)^{1/\lambda}, 1 - \left(1 - \prod_{i=1}^{n} \left(1 - \left(1 - {v_{A_i}}^{\lambda}\right)\right)^{w_i}\right)^{1/\lambda} \right\rangle \quad (23)$$

Taking $\lambda = 0.5$, we get $GIFWA_{0.5}(A_1, A_2, \dots A_5) = \langle 0.3845, 0\rangle$.

If we use the generalized intuitionistic fuzzy induced hybrid interaction averaging (GIFIHIA) operator in this paper, taking $\lambda = 0.5$ and $\omega = (0.2, \dots 0.2)^T$ for the convenience of comparison, we obtain

$GIFIHIA_{0.5}(A_1, A_2, \dots A_5) = \langle 0.3758, 0.4295\rangle$.

Obviously $v_{\mathrm{GIFWA}_{0.5}(A_1, A_2, \dots A_5)} = 0$, while $v_{\mathrm{GIFIHIA}_{0.5}(A_1, A_2, \dots A_5)} = 0.4295 \neq 0$, which shows that $v_{A_5} = 0$ plays a decisive role by Eq. (23),While doesn't play a decisive role by Eq. (22). Therefore, the new generalized weighted operator developed by this paper is more practical from the averaging point of view.

The characteristics of *GIFIHIA* operator can be interpreted in the following five aspects.

1. It considers three relations of different IFN*s*: the interactions of membership function of different IFN*s*, the interactions of non-membership function of different IFN*s* and the interactions between membership function and non-membership function of different IFN*s*.
2. It weights the IFNs $A_i = \langle u_{A_i}, v_{A_i}\rangle$ $(i = 1, 2, \dots, n)$ by the associated weights $\omega = (\omega_1, \dots \omega_n)^T$and multiplies

**Table 2** Score function obtained by the *G*IFIHIA operator and the rank of the alternatives

| | $GIFIHIA_{0.1}$ | $GIFIHIA_1$ | $GIFIHIA_2$ | $GIFIHIA_3$ | $GIFIHIA_5$ | $GIFIHIA_{10}$ | $GIFIHIA_{15}$ | $GIFIHIA_{20}$ |
|---|---|---|---|---|---|---|---|---|
| $x_1$ | 0.1281 | 0.1100 | 0.1596 | 0.2036 | 0.2535 | 0.3187 | 0.3640 | 0.3977 |
| $x_2$ | 0.0786 | 0.1098 | 0.1692 | 0.2117 | 0.2568 | 0.3306 | 0.3902 | 0.4344 |
| $x_3$ | −0.0593 | −0.0466 | 0.0119 | 0.0536 | 0.1031 | 0.1775 | 0.2277 | 0.2631 |
| Ranking | $x_1{>}x_2{>}x_3$ | $x_1{>}x_2{>}x_3$ | $x_2{>}x_1{>}x_3$ | $x_1{>}x_2{>}x_3$ | $x_2{>}x_1{>}x_3$ | $x_2{>}x_1{>}x_3$ | $x_2{>}x_1{>}x_3$ | $x_2{>}x_1{>}x_3$ |

these numbers by a balancing coefficient $n$, and then gets the weighted IFNs $n\omega_i A_i$ $(i = 1, 2, \ldots, n)$.

3. It reorders the weighted IFNs $n\omega_i A_i$ $(i = 1, 2, \ldots, n)$ according to the induced value $p_i$ $(i = 1, 2, \cdots, n)$ as $\left(\tilde{A}_{index(1)}, \tilde{A}_{index(2)}, \ldots, \tilde{A}_{index(n)}\right)$.
4. Both the weighted IFNs $w_i\tilde{A}_{index(i)}$ $(i = 1, 2, \ldots, n)$ and their induced value $p_i$ $(i = 1, 2, \ldots, n)$ are considered, and all the IFNs $A_i$ $(i = 1, 2, \ldots, n)$ are aggregated into a collective one.
5. The attitude of decision makers are considered by taking different values of $\lambda$ according to decision makers' preferences.

## 7 Conclusions

In this paper, we present the IFIHIA operator and the *G*IFIHIA operator, taking the interactions of different IFNs into consideration, reordering the involved IFNs according to the induced values and then aggregate them into a collective one, considering the attitudes of decision makers by taking different values of parameter according to decision makers' preferences. We investigate the properties of these operators and apply them to the selection of cold chain logistics enterprises under intuitionistic fuzzy environment. Examples are illustrated to show the validity and feasibility of the new approach. We also give some comparisons between this paper and other papers.

In the succeeding work, we will develop a class of fuzzy numbers intuitionistic fuzzy hybrid interaction averaging operators based on the existing works and apply them to the selection of cold chain logistics enterprises, decision support, recommender systems and multiple attribute group decision making.

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