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RESEARCH

A new three-dimensional rock strength criterion based on shape function in deviatoric plane

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Abstract Rock strength criteria are the theoretical grounding of geotechnical design and stability estimation, the Mohr–Coulomb (MC) and Hoek–Brown (HB) criteria are the widely accepted criteria at present, due to their reasonability and unambiguous concept, however they overlook the effect of intermediate principal stress, and contain six singular corners in π plane. Aimed at overcoming those limitations, the MC and normal parabolic criterion (NPC) were improved to their 3D versions that lead to smooth and convex for a wide range of strength parameters. The extended 3D strength criteria coincide with corresponding original forms in the triaxial compression and triaxial extension states, which not only take intermediate principal stress into account, but also provide great convenient in numerical calculation. Multigroup of poly-axial strength datasets gathered from the references are used to check the prediction accuracy of the proposed 3D criteria by the least absolute deviation method. Research proved that the 3D NPC criterion has a relatively larger deviation on poly-axial strength data prediction, but the proposed 3D MC criterion can describe peak strength with low misfit for soft or hard rocks. Peak strength σ_1 increases first and then decreases with the increase of σ_2, whether increasing or decreasing σ_2, both will result in rock failure. Moreover, the 3D MC can fit the poly-axial strength data well for lower or higher values of σ_3, which strongly suggests the proposed 3D MC criterion is adequate. Applicability of the proposed strength criterion will be discussed in further research.

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Article Highlights

- The normal parabolic criterion and Mohr-Coulomb criterion are modified to their 3D versions.
- The established criteria are checked for poly-axial data using the least absolute deviation method.
- The 3D MC can provide reliable predictions on poly-axial strength for various rock types.
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Keywords Strength theory · Shape functions · Lode angle · Poly-axial · Deviatoric plane · Normal parabolic criterion

List of symbols

σ_1, σ_2, σ_3	Maximum principal stress, intermediate principal stress, minimum principal stress
σ_n, τ_n	Normal stress, principal shear stress
c_o, φ	Cohesive, internal frictional angle
σ_c	Uniaxial compressive strength
ξ, r, θ	Hydrostatic stress axis, radius of strength envelope to hydrostatic stress axis, Lode angle
I_1, J_2, J_3	First stress invariant, second stress deviator invariant, third stress deviator invariant
σ_i^t, σ_i^p	The tested rock strength, predicted rock strength
r_i^t, r_i^p	The tested and criterion predictive diameter vector to the original point in π plane
$E\left[\sigma_i^t\right]$	The expected or statistical mean value of rock strength
f	The least absolute deviation
R^2	The regression square value
AAREP	The average absolute relative error percentage

1 Introduction

Rocks yield and failure are increasingly prominent as the underground mine gradually steeped into deep mining stage, which caused instability of wellbore during fossil energy extraction (Zhang et al. 2020, 2021a, b, 2020b; Fan et al. 2020a). Rock burst, water inrush, and a series of severe accidents occurred frequently and even had an anabatic trend. Meanwhile, the effect of intermediate principal stress and nonlinear characteristics of rock strength under deep in-situ state could not be described and characterized accurately by traditional rock strength criteria (Kim and Lade 1984; Al-Ajmi 2006).

Subsurface rocks are under true triaxial stress conditions ($\sigma_1 > \sigma_2 > \sigma_3$) as tectonic stresses exist. At present, the study of rock deformation and failure process is based on the conventional triaxial stress state ($\sigma_1 > \sigma_2 = \sigma_3$), but in the true triaxial stress environment, the deformation and failure characteristics of subsurface rock are different from the conventional triaxial stress environment (Zhang et al. 2020; Lee et al. 2012b; Boswell and Chen 1987; Carter et al. 1991). Therefore, the study of rock deformation and failure process under poly-axial stress has important reference significance for the geotechnical engineering construction. Rock mechanical scientists had done a great deal of works on the strength theories depicting the failure and yield of rocks under false or true triaxial stress condition. The MC theory and HB theory are the two mostly used and profound criteria. MC criterion has been proposed by Mohr in 1900, since its introduction, although it has irreplaceable status, the controversy of neglecting intermediate principal stress has never stopped (Warnke and Warnke 1975; Sriapai et al. 2013; Colmenares and Zoback 2002; Gudehus 1973; Xu and Geng 1985; Zhang 2016). Furthermore, failure of rocks has a nonlinear character in meridian plane, but the MC criterion gives a linear relationship between rock strength and confining pressure. This limitation was overcome by HB criterion, which has been developed by Hoek and Brown based on large amounts of experimental data. However, it also ignores intermediate principal stress and has six sharp corners in the deviatoric plane. Therefore, many efforts had been devoted to extending the 2D HB criteria to 3D, for example, Pan and Hudson (1988), Zan et al. (2002, 2004), Zan and Yu (2013), Priest (2005), Zhang and Zhu (2013), Cai and Zhu (2021), and so on had proposed a series of 3D HB criteria, which also named modified 3D HB criteria. Unsmooth limit trace of strength criteria in deviatoric plane will result in singularities and numerical instability, the Drucker–Prager (DP) criterion owning a circular limit trace in π plane is the most widely used criterion in numerical software, while its failure envelope yields an identical value for triaxial compression and extension regimes. When the yield curve of DP criterion is the circumcircle of MC criterion, it will overestimate rock strength, which is very dangerous in engineering (Kim and Lade 1984; Li et al. 2021; Lade 1975).

Colmenares and Zoback (2002) examined seven strength criteria by fitting them to published poly-axial datasets. The poly-axial criteria especially the Modified Wiebols and Cook (MWC) criterion achieved a good fit to the rocks with a highly σ_2 dependent failure behavior, but the triaxial failure criteria fitted the rocks that were less σ_2 dependently. Benz et al. (2008) evaluated the six rock failure criteria, results indicated Hoek–Brown–Matsuoka–Nakai (Matsuoka and Nakai 1974) gave the overall least misfit. You (2010a, b, 2011) pointed that the least square method would lead to the deviation of the overall fitting curve from most normal data because of the individual abnormal data, the least absolute deviation is suggested to obtain the undetermined parameters. Mogi (1967, 1971a, b, 2006) did a lot of work on the effect of intermediate principal stress on rock failure, the amount of increasing strength at failure is proportional to and smaller than the confining pressure. Also, the angle between the failure plane and maximum principal stress is reduced significantly with the increasing of second principal stress. The strength theory charactering smooth and convex in the deviate stress plane are preferred in a theoretical point of view, but the exponential or linear Mogi criterion is neither smooth nor convex and hence violates Drucker's convexity postulate (Drucker 1956). Singh et al. (2011) modified the Mohr–Coulomb criterion by employing the concept of Barton's critical state for rocks. In the further research, the critical confining pressure is equal to the UCS of the intact rock. Authors extended the criterion to jointed rocks, which are anisotropic in nature, so the effect of minor and intermediate principal stress on the strength of jointed rock mass can be accessed (Benz and Schwab 2008; Haimson and Chang 2000; Singh and Singh 2012; Yang et al. 2018; You 2009, 2013; Zhou et al. 2008; Zhao 2021; Zhang et al. 2017). Wang et al. (2021) proposed a new strength criterion for soft rocks, including two independent parameters, the UCS and the parameter characterizing the rock mass quality. Comparative analysis showed that this criterion balances accuracy and simplicity.

So far, the topic of rock strength is still active, over the years, scientists devoted to establishing a unified strength criterion (Argyris et al. 1974; Shi and Yang 1987). Voigt (Timoshenko 1983) indicated that the strength problem is so complex that it is impossible to provide a single theory, Timoshenko (1983) reiterated Voigt's conclusion. In the theoretical studying of rock strength, except for the maximum shear stress in plane, the influence of shear stress on yield in other planes has not been included, besides, various criteria have not been unified into a failure criterion. The unified strength criterion established by Yu on basis of Drucker's postulation (Drucker 1956) and the concept of twin shear theory is a great progress on this conundrum, which contained and linearly approximated various existing strength theories in a simple mathematical expression(Yu et al. 2017). However, this criterion owns singular feature in the π plane, which causes numerical singularity in the strength calculation. On account of this, Yu (2002, 2007, 2014, 2017) and Zan (2002, 2004, 2013) further investigated the series nonlinear limit surfaces of strength criteria in the π plane, established some nonlinear unified strength theories. The accuracy of laboratory rock strength experiments can't distinguish singular corner or smooth curve at different Lode angle, but the smooth will be convenient for numerical application (Lee et al. 2012b). The objective of this work is also aimed at modifying the limit trace of polygons to the smooth closed cures and eliminating singularity, based on shape function in π plane, the MC and NPC are improved to their 3D versions, their undetermined strength parameters are obtained by the minimum deviation absolute value. To check the applicability of the proposed strength criteria, they are applied to experimental data comprising of twelve sets of poly-axial strength data for hard and soft rocks, and the predictions are compared with the other popular criteria.

2 The new strength criterion

2.1 Reviews of rock strength criteria

Coulomb (1776) proposed that materials shear failure took place along the plane which had a maximum shear stress that can overcome the cohesive force and frictional force along the failure plane, Mohr (1900) systematically elaborated Coulomb criterion, and showed it by Mohr's circle, the Coulomb criterion, or linear Mohr–Coulomb criterion is referred as Mohr–Coulomb criterion (Al-Ajmi 2006),

$$\tau = c_0 + \sigma_n \tan \varphi \tag{1}$$

In the principal stress form, the MC criterion can be written as,

$$\sigma_1 = \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3 + \frac{2c_0 \cos \varphi}{1 - \sin \varphi} \tag{2}$$

You (2010c) studied the mathematical equation and parameter determination of strength criteria for rock, during the determination parameters of Fairhurst criterion. You (2010c, 2011) proposed the normal parabolic criterion (NPC), which has a more accuracy prediction result than MC criterion, is equal to or better than HB criterion, even if they contain two strength parameters. In Mohr stress space, the NPC is,

$$\tau_n^2 = \sigma_n \sigma_c \tag{3}$$

In the principal stress space, the Eq. (3) can be rewritten as,

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 = \frac{\sigma_1 + \sigma_3}{2}\sigma_c \tag{4}$$

The strength criteria can be formulated into different expressions in different stress spaces (Zienkiewicz and Pande 1977). For instance, a failure surface can be geometrically defined as $F(\sigma_1, \sigma_2, \sigma_3) = 0$ in the principal stress space, $F(\tau_n, \sigma_n) = 0$ in Mohr space, and $F(r, \xi, \theta) = 0$, when the ξ remain constant, it represents failure envelopes in the π plane, when the θ is fixed, it represents failure envelopes om the meridian plane.

Figure 1 depicts how the stress state of point P i.e., $(\sigma_1, \sigma_2, \sigma_3)$ in principal stress space can be characterized in the deviatoric plane. ON is the hydrostatic axis $(\sigma_1 = \sigma_2 = \sigma_3)$, which has an identical angular separation with three principal stress axes. The plane passing through the point P and perpendicular to ON is named deviatoric plane or π plane, it has a distance ξ from the origin, which is represented by OQ in Fig. 1. $\sigma_1^*, \sigma_2^*, \sigma_3^*$ are the projection of $\sigma_1, \sigma_2, \sigma_3$ on the π plane. And the Lode angle is defined as the departure of the stress state from axis of σ_1^*, which varies in the range of 0° and 60°. In Fig. 1, the angle between QP and the σ_1^*-axis is the Lode angle. The variates in the π plane can be calculated in terms of principal stresses (Lee et al. 2012; Yu et al. 2017; Cai et al. 2021),

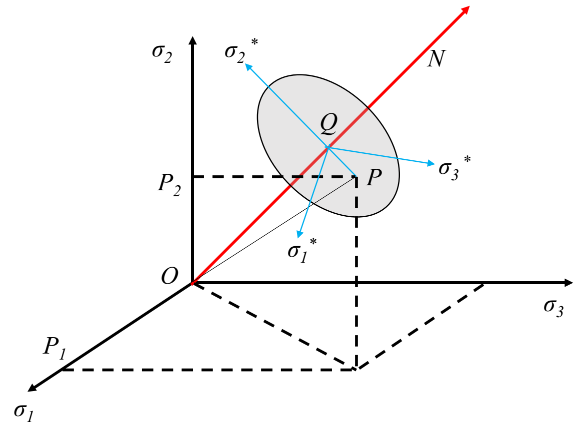


Fig. 1 Representation of the stress state in principal stress space and the deviatoric plane or π plane. (Yu et al. 2017)

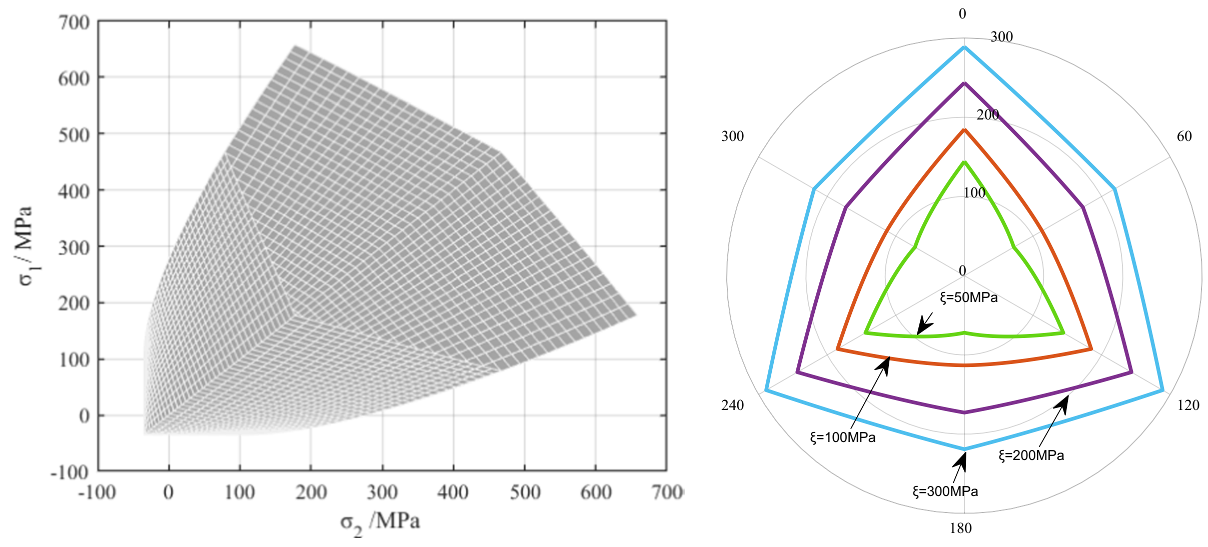
$$\xi = \frac{I_1}{\sqrt{3}}, \quad r = \sqrt{2J_2}, \quad \theta = \arctan\left(\frac{\sqrt{3}(\sigma_2 - \sigma_3)}{2\sigma_1 - \sigma_2 - \sigma_3}\right) \tag{5}$$

Inversely, the principal stress can be written in terms of (r, ξ, θ) as (Lee et al. 2012; Yu et al. 2017; Cai et al. 2021),

$$\begin{cases} \sigma_1 = \sqrt{\frac{2}{3}} r \cos\theta + \frac{\xi}{\sqrt{3}} \\ \sigma_2 = \sqrt{\frac{2}{3}} r \cos\left(\theta - \frac{2\pi}{3}\right) + \frac{\xi}{\sqrt{3}} \\ \sigma_3 = \sqrt{\frac{2}{3}} r \cos\left(\theta + \frac{2\pi}{3}\right) + \frac{\xi}{\sqrt{3}} \end{cases} \tag{6}$$

Substitution of Eq. (6) into MC and NPC, after some mathematical operations, the failure criteria in terms of variants in π plane can be expressed as Eqs. (7) and (8) respectively,

$$\frac{\xi}{\sqrt{3}} \sin\varphi + \frac{r}{\sqrt{2}} \sin\left(\frac{\pi}{3} + \theta\right) + \frac{r}{2\sqrt{6}} \sin\varphi \cos\left(\frac{\pi}{3} + \theta\right) = c_0 \cos\varphi \tag{7}$$



(*a*) in 3D principal stress space (*b*) in deviatoric plane

Fig. 2 Failure surface of normal parabolic criterion

$$r - \frac{\sqrt{\frac{3}{2}\sigma_c^2\left(\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)\right)^2+2\sqrt{3}\sigma_c\xi\left[\cos\theta-\cos\left(\theta+\frac{2\pi}{3}\right)\right]^2}}{\left[\cos\theta-\cos\left(\theta+\frac{2\pi}{3}\right)\right]^2} - \frac{\sqrt{\frac{3}{2}}\sigma_c\left[\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)\right]}{\left[\cos\theta-\cos\left(\theta+\frac{2\pi}{3}\right)\right]^2} = 0 \tag{8}$$

In order to have a better understanding of peak strength criteria in practice, they must be assessed in 3-dimensional stress invariants space. The limit trace of the material in the π plane has triaxial symmetry, and the shape of the yield surface in the range of 360° can be obtained from lode angle in the range of 0°–60°. Setting the strength parameter $\sigma_c = 138.6$MPa for normal parabolic criterion, their failure envelopes in 3D principal stress space and deviatoric plane are shown in Fig. 2. When the Lode angle is 0°, namely the triaxial compression state ($\sigma_1 > \sigma_2 = \sigma_3$), the distance of arbitrary point in the envelope line to the original point is the limit vector length r_c in the π plane. While θ is π/3, it corresponds to triaxial extension state ($\sigma_1 = \sigma_2 > \sigma_3$). The characteristics of NPC have never been investigated so far. As shown in Fig. 2, NPC has a nonlinear feature in meridian plane, and the shape of its limit trace in π plane changes with hydrostatic stress, with the increasing of ξ, the cross-section turns into a hexagonal curve from a curvilinear triangular curve.

2.2 Smooth approximation

In order to eliminate singularity, make the failure criteria possessing a smooth and convex failure envelope in π plane, rock mechanical masters proposed shape functions. Rock failure and yield function in terms of (r, ξ, θ) can be defined as follows (Lee et al. 2012),

$$F = r - g(\theta)r_c = 0 \tag{9}$$

where $r(\theta)$ is the vector length to the origin point in π plane for the Lode angle θ. In this work, θ = 0°

Table 1 The existing shape functions and their characteristics

Authors (year)	Equations	Characteristics	Weakness
Gudehus (1973) and Argyris et al. (1974)	$g(\theta) = \frac{2k}{(1+k)+(1-k)\cos 3\theta}$	It violates Drucker's postulation if $k \le 7/9$	Yes
Lin and Bažant (1986)	$g(\theta) = \frac{2k\left(C_1+C_2\cos 3\theta\right)}{\left(C_3+k\right)+\left(C_3-k\right)\cos 3\theta} \quad \left(C_1 - C_2 = 1,\ C_1 + C_2 = C_3\right)$	The convexity can be assured for entire range $0.5 \le k \le 1$, and is dependent on value of C_2	Yes
Warnke and Warnke (1975)	$g(\theta) = \frac{2\left(1-k^2\right)\cos(\theta-(\pi/3))+(2k-1)\sqrt{4\left(1-k^2\right)\cos^2(\theta-(\pi/3))+5k^2-4k}}{4\left(1-k^2\right)\cos^2(\theta-(\pi/3))+(2k-1)^2}$	Developed from an elliptical approximation, but has a complicated expression	No
Boswell and Chen (1987)	$g(\theta) = \frac{\sqrt{3}k}{\sqrt{3-4\left(1-k^2\right)\sin^2\theta}}$	It's convex for $0.5 \le k \le 1$, but not smooth as g '$(60°) \ne 0$	Yes
Jiang and Pietruszczak (1988)	$g_1(\theta) = \frac{\left(\sqrt{1+f}-\sqrt{1-f}\right)k}{k\sqrt{1+f}-\sqrt{1-f}+(1-k)\sqrt{1-f\cos 3\theta}}$	It satisfies smooth and convex when $0.56 \le k \le 1$ for f Infinitely close to1, but violates the requirement of convexity when ξ is small or negative	Yes
Jiang and Pietruszczak (1988)	$\begin{cases} g_2(\theta) = (1-k)B + \sqrt{(1-k)^2B^2 + 4k - 3} & g \ge \sqrt{4k^2 - 6k + 3} \\ g_2(\theta) = k/\cos\theta & g \le \sqrt{4k^2 - 6k + 3} \end{cases}$ $B = \cos\theta + \sqrt{3}\sin\theta$	It's smooth and convex in the entire range $0.5 < k \le 1$, but has a complicated expression	No
Shi and Yang (1987)	$g_1(\theta) = \frac{(7+2\mathrm{k})-2(1-k)\sin 3\theta}{9}$ $g_2(\theta) = \frac{\left[(1+\mathrm{k})+1.125(1-k)^2\right]-\left[(1-k)-1.125(1-k)^2\right]\sin 3\theta}{2}$	They can be smooth and convex when $k > 5/9$, but their deviatoric cross-section is only consistent with triaxial extension stress state	Yes
Lade (1975) and Matsuoka (1974)	$\begin{cases} g(\theta) = \frac{\cos\left(\frac{\pi}{3}-\frac{1}{3}\cos^{-1}A\right)}{\cos\left(\frac{1}{3}\cos^{-1}\left(A\cos 3\left(\theta-\frac{\pi}{3}\right)\right)\right)} & \mathit{for}\ \cos 3\left(\theta-\frac{\pi}{3}\right) \ge 0 \\ g(\theta) = \frac{\cos\left(\frac{\pi}{3}-\frac{1}{3}\cos^{-1}A\right)}{\cos\left(\frac{\pi}{3}-\frac{1}{3}\cos^{-1}\left(-A\cos 3\left(\theta-\frac{\pi}{3}\right)\right)\right)} & \mathit{for}\ \cos 3\left(\theta-\frac{\pi}{3}\right) \le 0 \end{cases}$ $\mathit{with}\quad A_{Lade} = \sqrt{k_1 - 27}/k_1, \quad k_1 \ge 27$ $A_{Matsuoka} = \sqrt{k_1^2\left(k_1 - 9\right)/\left(k_1 - 3\right)^3},\ k_1 \ge 9$	It's smooth and convex for $0.5 \le k \le 1$, but has a complicated expression	No
Yu and Liu (1990)	$g_1(\theta) = \frac{2k\left(1-k^2\right)\cos(\theta-\pi/3)+k^2\sqrt{12\left(1-k^2\right)\cos^2(\theta-\pi/3)+12k^2-3}}{(2k+1)\left[4\left(1-k^2\right)\cos^2(\theta-\pi/3)+(k-2)^2 3k^2\right]}$ $g_2(\theta) = \frac{2\left(1-k^2\right)-\sqrt{12\left(k^2-1\right)+3\left(4-k^2\right)\sec^2(\theta-2\pi/3)}}{4\left(1-k^2\right)-3\sec^2(\theta-2\pi/3)}\frac{\sqrt{6}k}{2+k}\sec\left(\theta-\frac{\pi}{3}\right)$	It is smooth and convex in the range of $0.5 < k \le 1$, and approximate to shape function proposed by William and Warnke	No

represents the triaxial compression condition, $\theta = 60°$ represents the triaxial extension condition. Therefore, it should be noted that (Lee et al. 2012),

$$r(0°) = r_c, \quad r(60°) = r_t, \quad g(0°) = 1, \quad g(60°) = r_c/r_t = k \tag{10}$$

There are many forms of shape function g(θ) that have been proposed so far. The convexity of frequently used forms of g(θ) are listed in Table 1. To avoid numerical singularity and iterative non-convergence in the numerical simulation, namely, the failure surface satisfies smooth and convex, the shape function requires (Jiang et al. 1988; Kim and Lade 1984; Lee et al. 2012; Lin et al. 1986),

$$\begin{cases} g'(\theta = 0°) = g'(\theta = 60°) = 0 \\ g^2 + 2g'^2 - gg'' = 0 \end{cases} \tag{11}$$

The smooth and convexity of the existing shape functions had been examined by Rock mechanics scientists based on geometrical or theoretical explanation. The shape functions established by Warnke and Warnke (1975), Jiang and Pietruszczak (1988), Lade (1975) and Matsuoka (1974) conform to Drucker's postulation and smooth in entire range of $0.5 \le k \le 1$, but most of them have a complicated expression. Yu (2017) compared their hyperbolic shape function with MC, TSS, Gudehus and Argyris's formulation, Warnke and Warnke's formulation, results indicated that William and Warnke's formulation was close to hyperbolic formulation, which is able to match with six experimental corners in π plane simultaneously. In this study, the hardly used shape function presented by Jiang and Pietruszczak (1988) is adopted which is also always convex and smooth,

$$\begin{cases} g(\theta) = (1-k)B + \sqrt{(1-k)^2B^2 + 4k - 3} & g \ge \sqrt{4k^2 - 6k + 3} \\ g(\theta) = k/\cos\theta & g \le \sqrt{4k^2 - 6k + 3} \end{cases} \tag{12}$$

In which,

$$B = \cos\theta + \sqrt{3}\sin\theta \tag{13}$$

According to Eq. 9, various approximations to strength criteria can be obtained by replacing g(θ) by a suitable shape function satisfying the constraint Eqs. 10 and 11. For this purpose, MC and NPC should rewritten as the general form of Eq. 9. Substituting θ = 0° and θ = 60° into Eq. 7, the vector length of MC criterion in π plane for triaxial compressional state and triaxial extension state, and the ratio of r_c to r_t can be obtained,

$$r_{c\_MC} = \frac{2\sqrt{2}\sin\varphi}{3-\sin\varphi}\xi + \frac{2\sqrt{6}c_0\cos\varphi}{3-\sin\varphi} \tag{14}$$

$$r_{t\_MC} = \frac{2\sqrt{2}\sin\varphi}{3+\sin\varphi}\xi + \frac{2\sqrt{6}c_0\cos\varphi}{3+\sin\varphi} \tag{15}$$

$$k_{\_MC} = \frac{r_t}{r_c} = \frac{3-\sin\varphi}{3+\sin\varphi} \tag{16}$$

Similarly, the normal parabolic criterion diameter vector to the origin point in π plane for triaxial compressional state and triaxial extension state, and their ratio can be derived,

$$r_{c\_NPC} = \frac{\sqrt{6}}{9}\sigma_c + \sqrt{\frac{2\sigma_c^2}{27} + \frac{8\sqrt{3}}{9}\sigma_c\xi} \tag{17}$$

$$r_{t\_NPC} = -\frac{\sqrt{6}}{9}\sigma_c + \sqrt{\frac{2\sigma_c^2}{27} + \frac{8\sqrt{3}}{9}\sigma_c\xi} \tag{18}$$

$$k_{\_NPC}(\xi) = \frac{-\sigma_c + \sqrt{\sigma_c^2 + 6\sqrt{2}\sigma_c\xi}}{\sigma_c + \sqrt{\sigma_c^2 + 6\sqrt{2}\sigma_c\xi}} \tag{19}$$

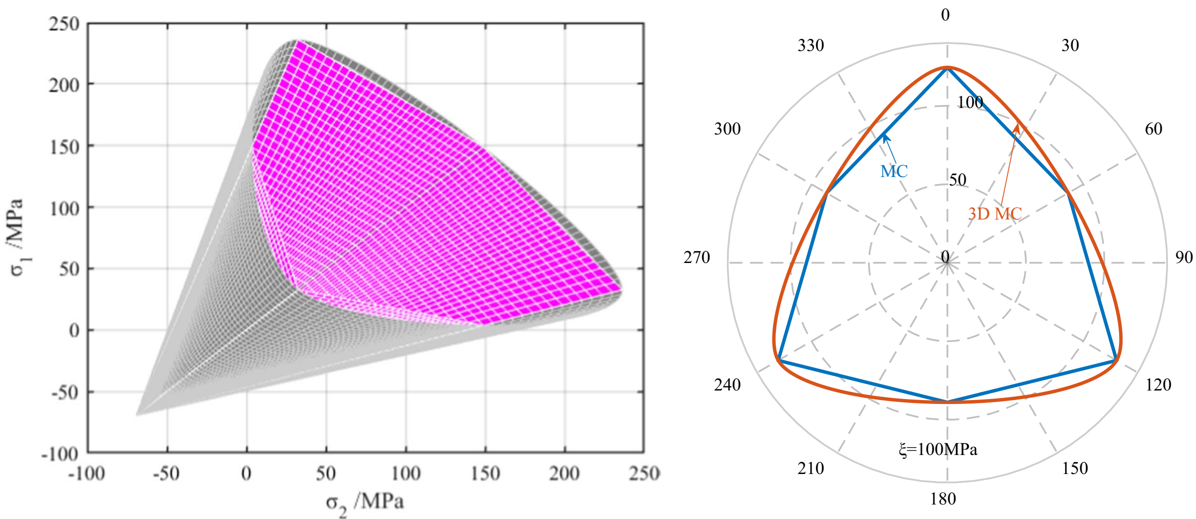
Combining the ratio of triaxial extension diameter to triaxial compression diameter with Jiang and Pietruszczak's formulation, substituting the result and r_c into Eq. 9, strength criteria can be extended to their three-dimensional version, as well as the intermediate principal stress is considered.

Setting the strength parameters of $c_0 = 40\text{MPa}$, $\varphi = 30°$ for MC, the principal stress space and deviatoric cross-section views of MC together with its extending 3D version and NPC together with its extending 3D are illustrated in Figs. 3 and 4. The figures intuitively show that 3D MC (3D NPC) is smooth and convex, and circumscribes MC surface (NPC surface). Moreover, the extended 3D strength criteria coincide with corresponding original forms in both the triaxial compression and triaxial extension states, which not only take intermediate principal stress into account, but also provide great convenient in numerical calculation. In the next section, three sets of conventional triaxial strength data and twelve sets of poly-axial strength data collected from references are used to evaluate the performance of MC, NPC and their extended 3D version.

3 Rock strength dataset and method

3.1 Data source

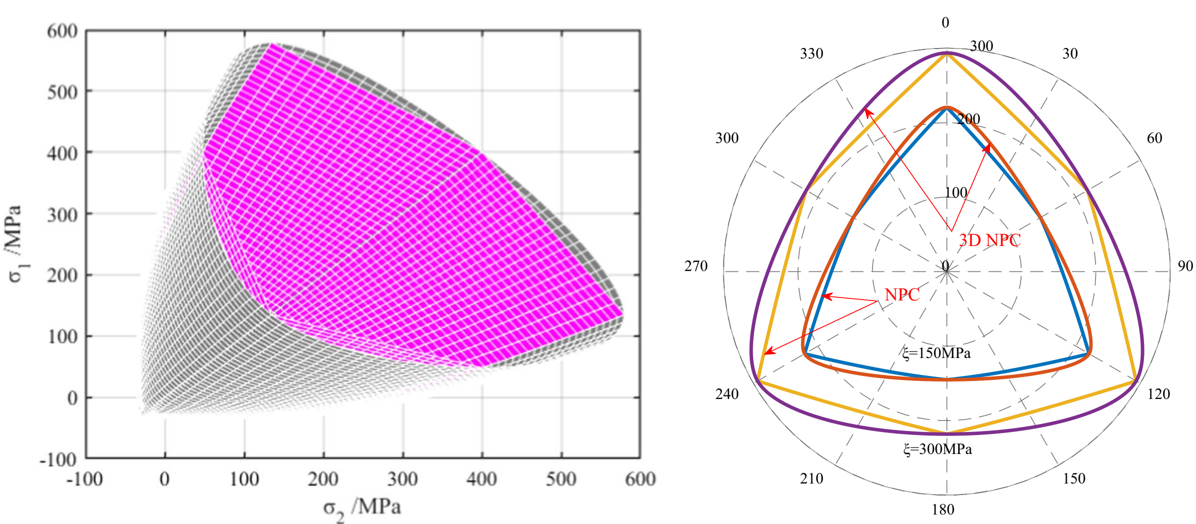
The strength datasets of Berea sandstone, Vosges sandstone and Indian limestone were collected from



(*a*) in 3D principal stress space (*b*) in deviatoric plane

Fig. 3 Failure envelope of MC corresponding to its 3D version

the reference published by Al-Ajmi (2006). One set of poly-axial strength data for Sandstone was obtained from Rukhaiyar and Samadhiya (2017), five sets of poly-axial strength data for Dunham dolomite, Solenhofen limestone, Yuubari shale, Shirahama sandstone and coarse grained dense marble were collected from Al-Ajmi (2006), another four sets data for Westerly Granite, Mizuho Trahchyte, KTB amphibolite, and Manazuru andesite were collected from Mogi (2006), two sets of poly-axial strength data for Limestone and Sandstone were collected from Yin (1987), one set of poly-axial strength data for Sandstone was collected from Zhang (1979), one set of poly-axial strength data for Sandstone was collected from Gao (1993). Furthermore, on set of poly-axial strength data for



(*a*) in 3D principal stress space (*b*) in deviatoric plane

Fig. 4 Failure envelope of NPC corresponding to its 3D version

Table 2 The best fitting strength parameters and evaluating index of 3D MC and 3D NPC for each rock type

Strength data	3D MC					3D NPC			
	c_0/MPa	φ/°	f/MPa	R^2	AAREP/%	σ_c/MPa	f/MPa	R^2	AAREP/%
Shirahama sandstone	19.70	35.90	2.82	0.98	2.531	60.9	8.73	0.876	9.124
Westerly Granite	49.90	50.30	9.54	1.00	2.27	284.88	70.38	0.81	20.01
Mizuho Trahchyte	52.70	27.20	7.57	0.94	4.09	98.36	9.26	0.90	4.98
KTB amphibolite	57.50	44.90	18.07	0.99	4.46	304.67	65.86	0.86	21.01
Manazuru andesite	64.20	43.30	5.76	0.99	1.41	231.22	28.02	0.83	7.25
Dunham dolomite	107.10	30.30	8.43	0.98	2.08	222.60	18.22	0.91	4.43
Solenhofen limestone	97.10	28.20	6.45	0.94	1.80	167.65	7.38	0.92	1.97
Yuubari shale	29.30	27.20	2.75	0.97	2.021	55.27	5.00	0.884	3.647
Marble	10.90	45.90	8.35	0.95	7.034	51.39	19.17	0.710	16.553
Limestone-Yin	17.50	43.20	4.53	0.99	3.270	89.67	16.76	0.879	18.219
Sandstone-Yin	19.80	40.00	4.06	0.99	3.958	74.46	13.64	0.888	14.605
Sandstone-Zhang	4.70	50.20	1.08	0.99	4.628	17.64	4.38	0.782	15.395
Sandstone-Rukhaiyar	6.23	50.09	1.25	0.99	1.98	32.69	7.76	0.76	13.40
Sandstone-Gao	11.00	48.60	1.68	0.99	2.107	44.39	7.00	0.853	10.820
Maha Sarakham salt	7.30	44.80	3.05	0.87	6.046	27.04	4.57	0.844	12.217

soft rock, i.e. Maha Sarakham salt was collected from Sriapai et al. (2013).

3.2 Fitting and evaluating method

The classical regression tool, least squares method is sensitive to abnormal data, as a result, few abnormal strength data may lead to a great deviation when this method is applied. Therefore, the least absolute deviation method is applied to determine the unknow strength parameters, the objective function used to fit false triaxial strength data is expressed as (Cai et al. 2021),

$$f = \min\left\{\sum_{i=1}^{N}\left|\sigma_i^t - \sigma_i^p\right|\right\} \tag{20}$$

The regression square value R^2 and the average absolute relative error percentage (AAREP), which are dimensionless, are used to access the performance of the strength criteria. Their definitions, respectively are written as (Wang et al. 2021),

$$R^2 = 1 - \sum_{i=1}^{N}\left(\sigma_i^t - \sigma_i^p\right)^2 / \sum_{i=1}^{N}\left(\sigma_i^t - E\left[\sigma_i^t\right]\right)^2 \tag{21}$$

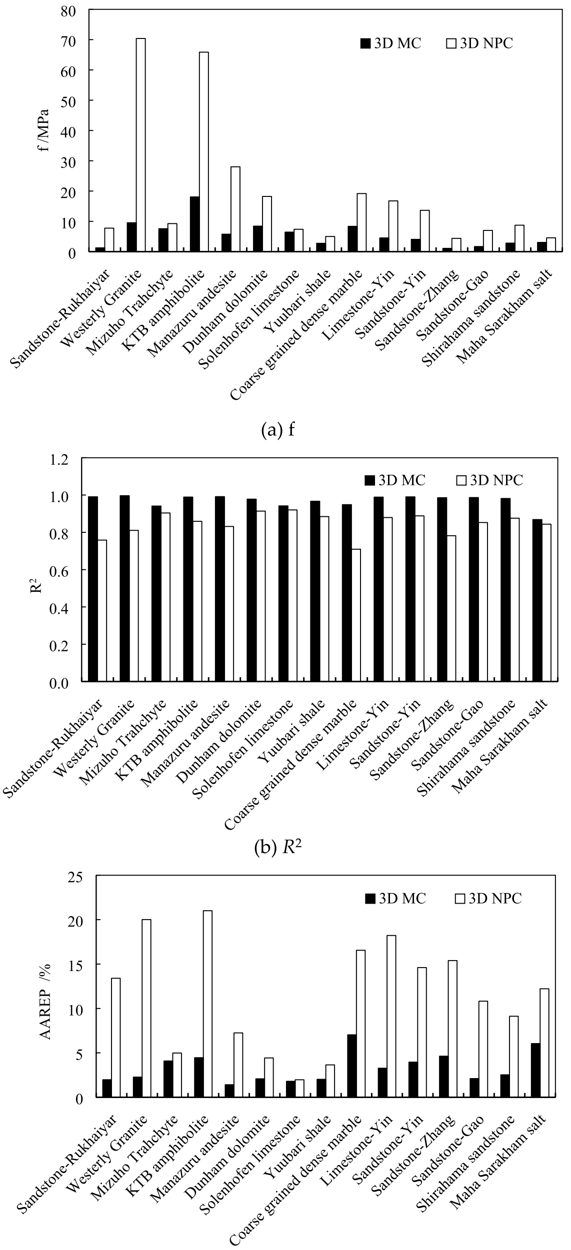
$$AAREP = \sum_{i=1}^{N}\left|\frac{\sigma_i^t - \sigma_i^p}{\sigma_i^t}\right| / N \times 100\% \tag{22}$$

Replacing the σ_i^t, σ_i^p by r_i^m, r_i^p, the objective function i.e., Eq. 20 can be used to fit poly-axial strength data and calculate the undetermined parameters in 3D strength criteria. And the Eqs. 21 and 22 can be used to evaluate the performance of the 3D strength criteria.

4 Results and discussion

4.1 Fitting index of the new criteria

The minimum absolute deviation method, namely Eq. 20 is applied to fitting the proposed 3D MC and 3D NPC to poly-axial strength data, which may not be exposed with huge misfit by a few abnormal data. As a result, it is better than the grid search method used by Colmenares and Zoback (2002), and the minimum squares method employed by Lee et al. (2012b). Once the best fitting strength parameters are determined, the rock failure strength predicted by the failure criteria are compared with the experimental data, meanwhile the Eqs. 21 and 22



(a) f

(b) R^2

(c) AAREP

Fig. 5 Evaluating index of failure criteria performance for different rock lithologies

are employed to evaluate the failure criteria performance. The best fitting strength parameters for 3D MC and 3D NPC are listed in Table 2.

Table 1 showed clearly although the normal parabolic criterion containing single strength parameter may be able to fit conventional triaxial strength data

Fig. 6 The best fitting results of 3D MC with poly-axial strength data for hard rocks ▶

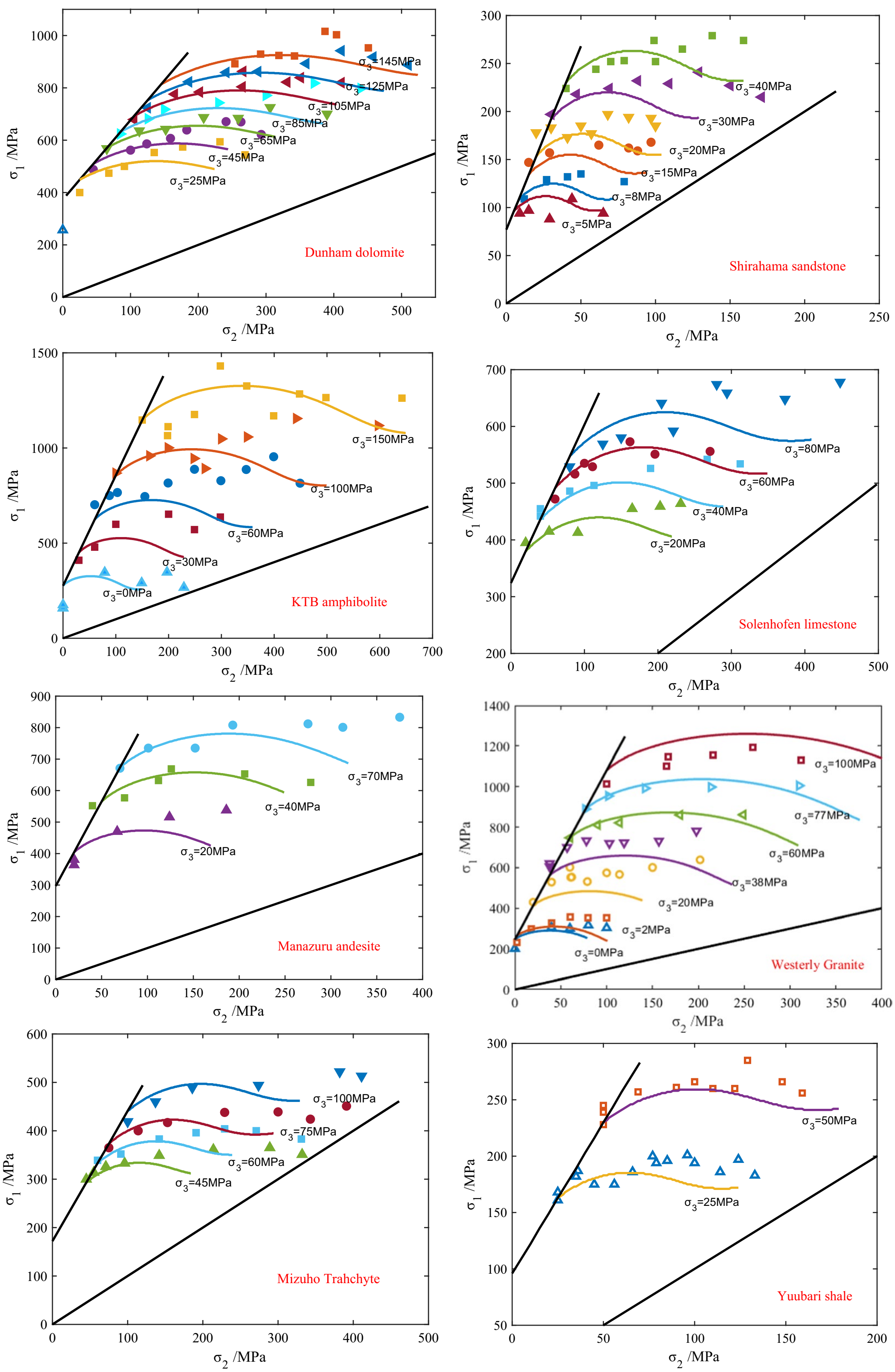
quite well, its extended 3D form has a relative larger deviation on poly-axial strength data prediction. While the 3D MC can provide reliable predictions on poly-axial strength for various rock types. The histograms of the least absolute deviation method, the regression square value and the average absolute relative error percentage are plotted in Fig. 5a, b and c, respectively.

As indicated in Fig. 5, f or AAREP associated with using the 3D NPC for poly-axial strength data of Westerly Granite and KTB amphibolite are several times of those using 3D MC, R^2 also indicates the 3D MC has a better performance than 3D NPC in poly-axial strength data fitting. But R^2 for 3D MC and 3D NPC have little difference, its values of 3D MC for different rock types are close to 1, meanwhile, R^2 of 3D NPC for most rock types are larger than 0.8, which may be not able to distinguish the fitting accuracy of the propose failure criteria obviously. The evaluating index f and AAREP have huge difference as displayed in Fig. 5a and c, which is able to estimate data fitting effect.

In order to observe the effect of intermediate principal stress on rock failure, the rock strength of different rock types predicted by 3D MC are compared with the experimental data in the σ_1-σ_2 plane. As displayed in Fig. 6, which illustrates rock failure under relative high confining pressure, Fig. 7 plots rock failure under low confining pressure, which presents that all rock types possess intermediate principal stress effect. Peak strength σ_1 increases firstly then decreases with the increase of σ_2. It should be noted that whether increasing or decreasing σ_2, both will result in rock failure. Moreover, the 3D MC can fit the poly-axial strength data well for lower or higher values of σ_3, which strongly suggests the proposed 3D MC criterion is adequate.

4.2 Comparison of fitting precision of existing criteria and the new criteria

In order to verify the fitting precision of the new proposed criteria, the fitting indexes of four existing criteria should be obtained and compared with those predicted by the new proposed criteria (Bahrehdar and Lakirouhani 2022). Except for the MC and MWC


Dunham dolomite
σ3=145MPa
σ3=125MPa
σ3=105MPa
σ3=85MPa
σ3=65MPa
σ3=45MPa
σ3=25MPa
Shirahama sandstone
σ3=40MPa
σ3=30MPa
σ3=20MPa
σ3=15MPa
σ3=8MPa
σ3=5MPa
KTB amphibolite
σ3=150MPa
σ3=100MPa
σ3=60MPa
σ3=30MPa
σ3=0MPa
Solenhofen limestone
σ3=80MPa
σ3=60MPa
σ3=40MPa
σ3=20MPa
Manazuru andesite
σ3=70MPa
σ3=40MPa
σ3=20MPa
Westerly Granite
σ3=100MPa
σ3=77MPa
σ3=60MPa
σ3=38MPa
σ3=20MPa
σ3=2MPa
σ3=0MPa
Mizuho Trahchyte
σ3=100MPa
σ3=75MPa
σ3=60MPa
σ3=45MPa
Yuubari shale
σ3=50MPa
σ3=25MPa
σ1 /MPa
σ2 /MPa

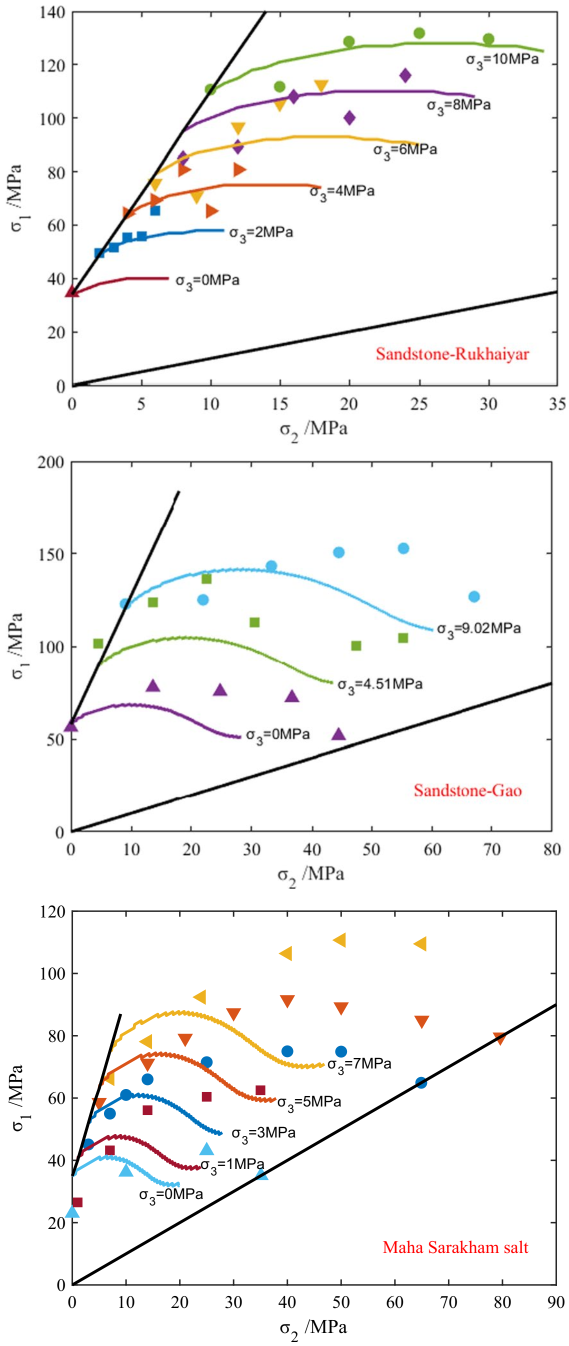


Fig. 7 The best fitting results of 3D MC with poly-axial strength data for soft rocks

criterion, the Mogi–Coulomb criterion (MGC) and Modified Lade criterion (ML) are also widely used in rock failure analysis, so these four existing strength criteria are used to test the fitting accuracy of the new criterion, the evaluated parameters of these four existing criteria are shown in Table 3.

The evaluating parameters of fitting precision determined by the exiting criteria and new criteria are compared by radar map, as shown in Figs. 8 and 9 respectively. It can be concluded that the 3D MC criterion owns the minimum absolute deviations *f* for different strength datasets, which indicate it is better than the mostly used existing criteria. Furthermore, the 3D NPC, ML, MWC and MWC have similar prediction accuracy, the MC criterion gets the maximum value of *f* and has a maximum prediction error for true triaxial compression strength. Meanwhile, as shown in Fig. 9, the regression squares R^2 obtained by fitting the 3D MC criterion to experimental data is mainly above 0.9, which proved that this newly established criterion has high fitting accuracy to true triaxial strength experimental data. The fitting accuracy of 3D NPC criterion is close to the existing rock strength criteria for the same strength tested data, the minimum regression squares R^2 obtained by MC criterion also indicated it has a large error in predicting rock strength under real stress environment, due to the ignoring of intermediate principal stress.

5 Conclusions

Rock strength criteria are the theoretical grounding of geotechnical design and stability estimation, the widely used MC and HB criteria not only ignore the effect of intermediate principal stress but also have six singular corners in π plane. In order to solve the above problems, one of the proposed shape functions in π plane, which is smooth and convex in the entire range of $0.5 < k \leq 1$, was selected for modifying conventional failure criteria, i.e., MC and NPC, to accommodate the requirements of both convexity and smoothness, meanwhile the effect of intermediate principal stress on failure can be considered. The extended 3D strength criteria coincide with corresponding original forms in both the triaxial compression and triaxial extension states, which not only take intermediate principal stress into account, but also provide great convenient in numerical calculation. The proposed 3D MC and 3D NPC share identical meridian sections and strength parameters with their original forms, their deviatoric sections are modified to be convex and smooth for numerical

Table 3 The best fitting indexes of four existing criteria

Strength data	MC		MGC		ML		MWC	
	f/MPa	R^2	f/MPa	R^2	f/MPa	R^2	f/MPa	R^2
Shirahama sandstone	12.05	0.90	10.93	0.94	11.68	0.92	13.50	0.91
Westerly Granite	52.75	0.95	62.28	0.93	70.29	0.92	52.01	0.95
Mizuho Trahchyte	25.33	0.87	16.22	0.92	13.89	0.95	19.50	0.90
KTB amphibolite	92.46	0.89	79.23	0.92	81.63	0.91	74.87	0.93
Manazuru andesite	47.74	0.83	40.36	0.89	43.21	0.88	39.72	0.91
Dunham dolomite	46.54	0.86	22.12	0.96	23.49	0.96	27.66	0.95
Solenhofen limestone	33.69	0.74	16.24	0.94	19.21	0.92	22.83	0.88
Yuubari shale	9.69	0.88	8.93	0.91	10.77	0.88	14.23	0.79
Marble	41.12	0.56	22.75	0.84	20.76	0.87	22.34	0.86
Limestone-Yin	22.49	0.89	18.70	0.93	20.31	0.93	19.82	0.92
Sandstone-Yin	18.89	0.92	11.63	0.96	14.14	0.95	12.23	0.95
Sandstone-Zhang	4.87	0.79	7.28	0.55	7.93	0.49	7.55	0.55
Sandstone-Rukhaiyar	8.97	0.84	5.41	0.93	4.85	0.88	5.10	0.94
Sandstone-Gao	10.81	0.81	16.97	0.67	16.42	0.57	14.92	0.69
Maha Sarakham salt	13.22	0.51	6.49	0.82	4.85	0.88	6.37	0.81

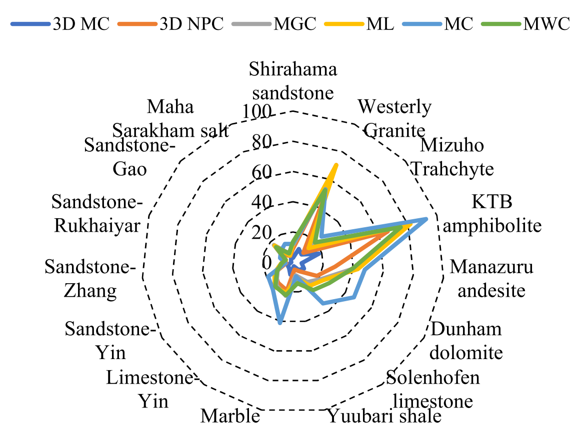


Fig. 8 Comparison of f determined by existing criteria and new criteria

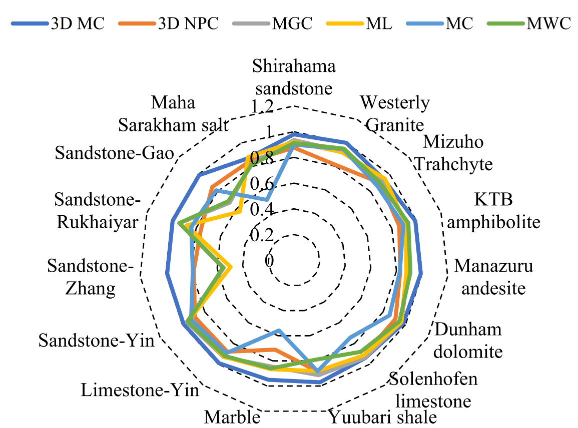


Fig. 9 Comparison of R^2 determined by existing criteria and new criteria

simulation. The applicability of the proposed strength criterion was verified by applying them to fifteen sets of poly-axial experimental data from literatures. Fitting results indicate that 3D NPC has a relative larger deviation on poly-axial strength data prediction, while the 3D MC can provide reliable predictions on poly-axial strength for various rock types under low or high confining pressure, which strongly suggests the proposed 3D MC criterion is adequate. The comparison of fitting precision indicates that the 3D MC criterion has a better fitting ability than the existing criteria, and the 3D NPC is close to the fitting accuracy of the existing criteria. Moreover, the peak strength σ_1 increases first and decreases with the increase of σ_2. It should be worth that whether increasing or decreasing σ_2, both will result in rock failure.

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Data availability The data and materials used in this study are available upon request. Please contact Mingming Zhang (zhangmm60862.sripe@sinopec.com) to inquire about the availability of the data and materials, including any restrictions that may apply due to privacy or confidentiality concerns.

Declarations

Competing interests The authors declare no competing interests.

Ethics approval and consent to participate All research activities were conducted in accordance with the ethical guidelines and principles outlined by the Committee on Publication Ethics.

Consent for publication All individuals involved in this study have provided their consent for the publication of the study findings. Any personal or identifying information that could potentially compromise privacy has been carefully removed or anonymized.

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