



Investigation of the relationship between dynamic and static deformation moduli of rocks

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Abstract The determination of deformation parameters of rock material is an essential part of any design in rock mechanics. The goal of this paper is to show, that there is a relationship between static and dynamic modulus of elasticity (E), modulus of rigidity (G) and bulk modulus (K). For this purpose, different data on igneous, sedimentary and metamorphic rocks, all of which are widely used as construction materials, were collected and analyzed from literature. New linear and nonlinear relationships have been proposed and results confirmed a strong correlation between static and dynamic moduli of rock species. According to rock types, for igneous rocks, the best correlation between static and dynamic modulus of elasticity (E) were

nonlinear logarithmic and power ones; for sedimentary rocks were linear and for metamorphic rocks were nonlinear logarithmic and power correlation. Moreover, with respect to different published linear correlations between static modulus of elasticity (E_{stat}) and dynamic modulus of elasticity (E_{dyn}), an interesting correlation for rock material constants was established. It was found that the static modulus of elasticity depends on the dynamic modulus only with one parameter formula.

Keywords Modulus of elasticity (E) · Modulus of rigidity (G) · Bulk modulus (K) · Rock types · Linear and non-linear correlations

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1 Introduction

An accurate estimate of geomechanical properties of rocks is crucial for almost any form of design and analysis in geomechanical projects. The strength and deformation behavior of rocks have been studied by many authors Xiong et al. (2019), Yang et al. (2016), Zhao et al. (2017), Ranjith et al. (2004), Rahimi and Nygaard (2018), Davarpanah et al. (2019) and Berzovski and Ván (2017). Among these properties modulus of elasticity (E), modulus of rigidity (G) and bulk modulus (K) are the basic parameters used in rock engineering. There are two common

methods to calculate the moduli: destructive and non-destructive procedures. In the destructive one, moduli are calculated from the stress–strain curves of the rock material. This is characteristic of the modulus of elasticity. For the non-destructive one, the most common method is an ultrasonic test, measuring both longitudinal and shear wave velocities.

Measuring the longitudinal and shear wave velocities, the dynamic elastic material parameters for isotropic and ideal elastic rocks are calculated with the help of the following formula (Martinez-Martinez et al. 2012):

$$E_{dyn} = \frac{\rho V_s^2 (3V_p^2 - 4V_s^2)}{V_p^2 - V_s^2} \quad (1)$$

$$\vartheta_{dyn} = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)} \quad (2)$$

$$G_{dyn} = \frac{E_{dyn}}{2(1 + \vartheta_{dyn})} \quad (3)$$

$$K_{dyn} = \frac{E_{dyn}}{3(1 - 2\vartheta_{dyn})} \quad (4)$$

where E_{dyn} = dynamic modulus of elasticity, GPa, G_{dyn} = dynamic modulus of rigidity, GPa, K_{dyn} = dynamic bulk modulus, GPa, $J_{dynamic}$ = dynamic Poisson's ratio,

ρ = density, $\frac{g}{cm^3}$, V_s = shear velocity, km/s,

V_p = longitudinal infinite medium velocity, km/s.

The Young's modulus obtained from the compression (destructive) test is called the static elastic modulus (E_{stat}). The International Society for Rock Mechanics (ISRM) suggests three standard methods for its determination (Ulusay and Hudson 2007). They are as followings:

- Tangent Young's modulus E_{tan} —at fixed percentage of ultimate stress. This is defined as the slope of a line tangent to the stress–strain curve at a fixed percentage of the ultimate strength (Fig. 1a);
- Average Young's modulus E_{av} —of the straight-line part of a curve. The elastic modulus is defined as the slope of the straight-line part of the stress–strain curve for the given test (Fig. 1b);
- Secant Young's modulus E_{sec} —at a fixed percentage of ultimate stress. It is defined as the slope of the line from the origin (usually point (0; 0)) to some fixed percentage of ultimate strength, usually

50% (Fig. 1c). In this paper, the secant static modulus has been calculated following ASTM D 3148-69 (1996).

The difference between dynamic (E_{dyn}) and static (E_{stat}) Young's Modulus for rocks has been addressed widely in rock engineering (van Heerden 1977; Lama and Vutukuri 1978; Barton 2006). Ratios for E_{dyn}/E_{stat} are typically in the range from 1 to 2 (Eissa and Kazi 1988).

Generally, the dynamic modulus of elasticity is slightly higher than the static value (Zhang 2006; Stacey et al. 1987; Al-Shayea 2004; Ide 1936; Kolesnikov 2009; Vanheerden 1987). The discrepancies between the dynamic and static elastic moduli have been widely attributed to microcracks and pores in the rocks. Figure 2 shows the ratio of dynamic elastic modulus to static elastic modulus compiled by Stacey et al. (1987). The ratio varies between about 1 and 3.

In other research (Martinez-Martinez et al. 2012) conducted a laboratory experiment on ten different carbonate rocks quarried in Spain and received the following diagram (Fig. 3). As shown, for the majority of cases dynamic (E_{dyn}) Young's Modulus is higher than static (E_{stat}) Young's Modulus.

The rocks consist of homogeneous limestones, low anisotropic travertines, limestones and dolostones with abundant stylolites, veins and fissures. Ultrasonic waves were measured using non-polarised Panametric transducers (1 MHz), a precise ultrasonic device consisting of signal emitting–receiving equipment and an oscilloscope (TDS 3012B Tektronix) (Martinez-Martinez et al. 2012) (Fig. 4).

Types of rocks are denoted below the picture and can be described as following. Blanco Alconera (BA): white crystalline limestone. Piedra de Colmenar (PdC): grey and white lacustrine fossiliferous limestone (99% calcite). Travertino Amarillo (TA): porous layered limestone. Travertino Rojo (TR): porous layered limestone. Gris Macael (GM): grey calcite marble. Blanco Tranco (BT): white homogeneous calcite marble. Amarillo Triana (AT): yellow dolomite marble. Crema Valencia (CV): cream micritic limestone (99% calcite). Rojo Cehégín (RC): micritic limestone. Marrón Emperador (ME): brown brecciated dolostone.

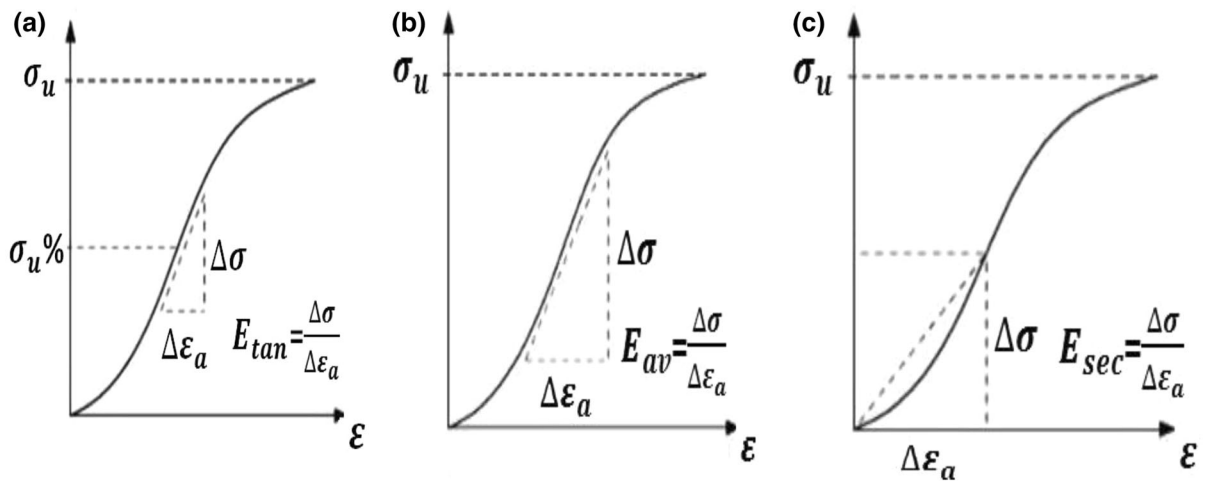
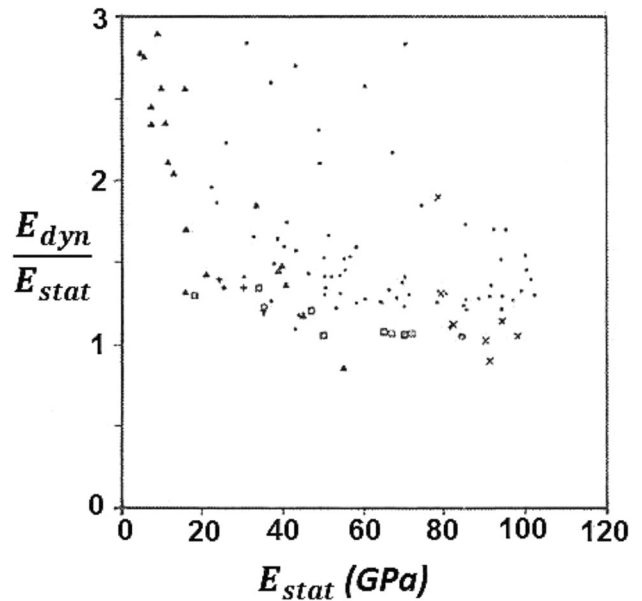


Fig. 1 a Tangent Young's modulus E_{tan} , b Average Young's modulus E_{av} , c Secant Young's modulus E_{sec}

Fig. 2 Comparison of static and dynamic elastic modulus (Stacey et al. 1987)



2 Empirical relationships between dynamic and static Young's modulus

There are several relationships between the static and dynamic Young's moduli (Belikov et al. 1970; King 1983; Eissa and Kazi 1988; McCann and Entwisle 1992; Eissa and Kazi 1988; Christaras et al. 1994; Nur and Wang 1999; Brotons et al. 2014, 2016; Małkowska et al. 2018). These equations can be divided into the following groups:

- Linear

- Non-linear: Logarithmic (power-law) and polynomial

2.1 Linear relationships

Up to now, several linear relationships have been established between the dynamic and the static Young's modulus. The following form was used:

$$E_{stat} = aE_{dyn} - b \tag{5}$$

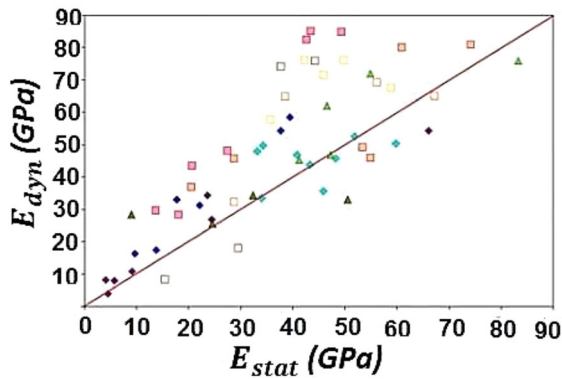


Fig. 3 Relationship between E_{dyn} and E_{stat} in the studied samples. The straight red line corresponds to the ideal ratio $\frac{E_{dyn}}{E_{stat}} = 1$. (Martinez–Martinez et al. 2012)

where a and b are material parameters. These values are summarized in Table 1.

2.2 Non-linear relationships

Some published papers use a logarithmic relationship. Analyzing the measured data, they suggest the following formula between the dynamic and the static Young’s modulus:

$$\log_{10} E_{stat} = c \log_{10} (\rho_{bulk} E_{dyn}) - d \tag{6}$$

The values of E_{stat} , and E_{dyn} are expressed in GPa while that of ρ_{bulk} in g/cm^3 . Here c and d are material constants (the published values are summarized in Table 2).

These equations can be rewritten to the following form:

$$E_{stat} = \alpha E_{dyn}^\beta \tag{7}$$

where the parameters are presented in Table 3.

The power-law relation (7) was suggested in other researches, too (see Table 4). E.g. according to the data of Ohen (2003), the dynamic Young’s modulus is about 18 times the static Young’s modulus (Peng and Zhang 2007).

From ultrasonic test data of 600 core samples in the Gulf of Mexico, Lacy (1997) obtained the following polynomial correlation for sandstones (Peng and Zhang 2007):

$$E_{stat} = eE_{dyn}^2 + fE_{dyn} \tag{8}$$

A similar connection exists for shales and sedimentary rocks (Horsrud 2001; see Table 5).

$$E_{stat} = 0.0428E_{dyn}^2 + 0.2334E_{dyn} \tag{9}$$

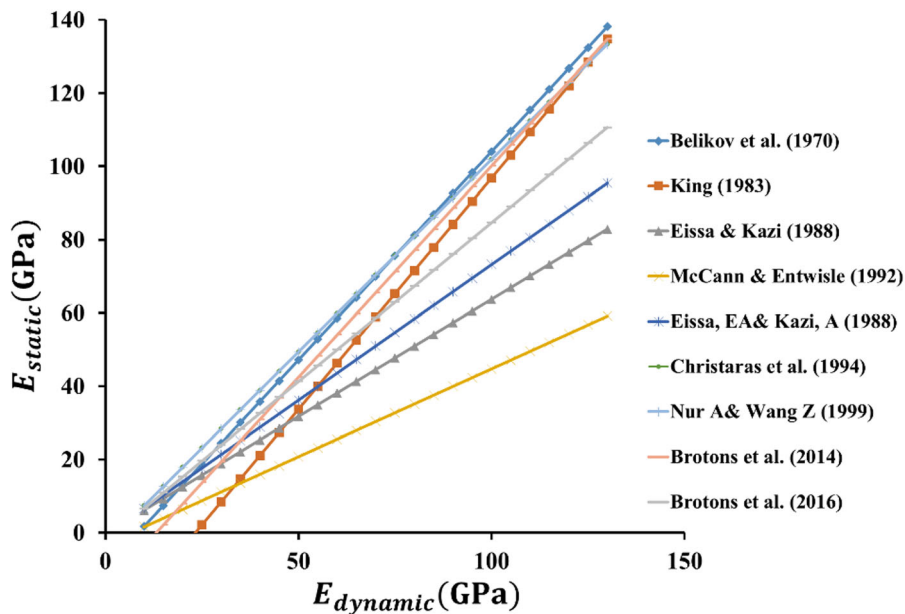


Fig. 4 Plot of the relationship between the measured static and dynamic modulus of elasticity ($E_{stat} = a E_{dyn} - b$)

Table 1 Linear relationship between static (E_{stat}) and dynamic (E_{dyn}) modulus ($E_{stat} = a E_{dyn} - b$)

A	B	Rock type	Refs.
1.137	9.68	Granite	Belikov et al. (1970)
1.263	29.5	Igneous and metamorphic rocks	King (1983)
0.64	0.32	All types	Eissa and Kazi (1988)
0.48	3.26	Crystalline rocks	McCann and Entwisle (1992)
0.74	0.82	All types	Eissa and Kazi (1988)
1.05	3.16	All types	Christaras et al. (1994)
1.153	15.2	All types	Nur and Wang (1999)
0.86	2.085	Calcarenite	Brotos et al. (2014)
0.932	3.42	All types	Brotos et al. (2016)

Table 2 Relationship between static (E_{stat}) and dynamic (E_{dyn}) Young’s modulus $\log_{10} E_{stat} = c \log_{10}(\rho_{bulk} E_{dyn}) - d$

c	d	Rock type	Refs.
0.77	- 0.02	All types	Eissa and Kazi (1988)
1.28	4.71	Calcarenite	Brotos et al. (2014)
0.96	3.306	All types	Brotos et al. (2016)

According to Fig. 6, there is a linear regression between static and dynamic modulus of elasticity for all studied rocks, giving $R^2 = 0.89$. More precisely, Fig. 7 shows the relationship between static and dynamic modulus of elasticity based on rock types. As it is shown, for igneous rocks there is a linear correlation that gives the value of $R^2 = 0.95$, for sedimentary rocks the value is $R^2 = 0.90$ and for

3 Data analysis

For this study, 40 samples of different types of rocks from the literature were analyzed (Lama and Vutukuri 1978). Data were classified according to rock types and the relationship between dynamic and static constants was investigated in Table 6. Also, the histogram of investigated parameters is shown in Fig. 5.

Table 5 Relationship between static (E_{stat}) and dynamic (E_{dyn}) Young’s modulus $E_{stat} = eE_{dyn}^2 + fE_{dyn}$

E	F	Rock type	Refs.
0.018	0.422	Sedimentary	Lacy (1997)
0.0293	0.4533	Sandstones	Lacy (1997)
0.0428	0.2334	shales	Lacy (1997)

Table 3 Relationship between static (E_{stat}) and dynamic (E_{dyn}) Young’s modulus $E_{stat} = \alpha E_{dyn}^\beta$

α	β	Rock type	Refs.
0.6019 $\rho_{bulk} + 0.42$	0.77	All types	Eissa and Kazi (1988)
1.66 $\rho_{bulk} + 4.01$	1.28	Calcarenite	Brotos et al. (2014)
1.24 $\rho_{bulk} + 2.78$	0.96	All types	Brotos et al. (2016)

Table 4 Relationship between static (E_{stat}) and dynamic (E_{dyn}) Young’s modulus

α	β	Rock type	Refs.
0.097–0.152	1.38–1.48	Sandstone-granite	Vanheerden (1987)
0.014	1.96	Limestone	Najibi et al. (2015)
0,0158	2.74	Various	Ohen (2003)

Table 6 Static and dynamic deformation constants of studied rocks (Lama and Vutukuri 1978)

Rock name	$V_P \left(\frac{\text{km}}{\text{s}} \right)$	$V_S \left(\frac{\text{km}}{\text{s}} \right)$	ν_{Stat}	ν_{Dyn}	$E_{Stat}(\text{GPa})$	$E_{Dyn}(\text{GPa})$	$G_{Stat}(\text{GPa})$	$G_{Dyn}(\text{GPa})$	$K_{Stat}(\text{GPa})$	$K_{Dyn}(\text{GPa})$
<i>Sedimentary rocks</i>										
Chalcedonic limestone	5	2.73	0.18	0.25	55.16	46.85	23.37	18.74	28.73	31.23
Limestone	5.39	3.32	0.25	0.28	66.88	70.96	26.75	27.72	44.59	53.76
Oolitic limestone	5.45	2.98	0.18	0.21	45.51	53.74	19.28	22.21	23.70	30.89
Quartzose shale	–	–	0.08	–	16.5	22.04	7.64	11.02	6.55	7.35
Stylolitic limestone	5.1	2.98	0.11	0.27	38.61	56.49	17.39	22.24	16.50	40.93
Limestone	4.5	2.16	0.18	0.2	16.55	28.24	7.01	11.77	8.62	15.69
Limestone	4.7	2.72	0.17	0.31	33.78	52.36	14.44	19.98	17.06	45.93
Siltstone	5	2.23	0.05	0.08	13.1	26.87	6.24	12.44	4.85	10.66
Subgraywacke	4.5	2.09	0.03	0.19	12.41	26.18	6.02	11.00	4.40	14.08
Sericite schist	–	–	0.02	0.44	7.58	17.91	3.87	6.22	2.43	49.75
Subgraywacke	5	2.22	0.02	0.06	11.03	26.18	5.41	12.35	3.83	9.92
Calcareous shale	–	–	0.02	–	15.86	24.80	7.77	12.40	5.51	8.27
Subgraywacke	4	1.95	0.02	0.29	9.65	24.80	4.73	9.61	3.35	19.68
Subgraywacke	4.95	2.2	0.05	0.08	8.96	26.18	4.27	12.12	3.32	10.39
Leuders limestone	4.66	2.3	0.21	0.22	24.13	33.37	9.97	13.68	13.87	19.86
Leuders limestone	4.65	2.3	0.21	0.22	24.82	33.37	10.26	13.68	14.26	19.86
Green River shale	4.92	2.56	0.18	0.22	29.65	40.06	12.56	16.42	15.44	23.85
Green River shale	4.69	2.3	0.17	0.27	35.16	42.54	15.03	16.75	17.76	30.83
Sandstone with chalcedonic	–	–	–	–	71.58	76.29	35.79	38.15	23.86	25.43
Equigranular dolomite	–	–	–	–	49.52	52.06	24.76	26.03	16.51	17.35
Limestone	–	–	–	–	18.43	23.73	9.22	11.87	6.14	7.91
Calcareous dolomite	–	–	–	–	34.22	46.28	17.11	23.14	11.41	15.43
Fine-grained detrital limestone	–	–	–	–	46.77	55.99	23.39	28.00	15.59	31.23
<i>Igneous rocks</i>										
Granite	–	–	–	–	64.71	69.62	32.36	34.81	21.57	18.66
Gabbro	–	–	–	–	69.62	73.54	34.81	36.77	23.21	23.21
Dunite	–	–	–	–	149.1	160.81	74.55	80.41	49.70	24.51
Granite (slightly altered)	4.19	1.65	0.04	0.1	5.52	15.15	2.65	6.89	2.00	53.60
Monzonite porphyry	5.53	3.05	0.18	0.21	41.37	56.49	17.53	23.34	21.55	6.31
Quartz diorite	4.69	2.25	0.05	0.19	21.37	30.31	10.18	12.74	7.91	32.47

Table 6 continued

Rock name	$V_P \left(\frac{\text{km}}{\text{s}}\right)$	$V_S \left(\frac{\text{km}}{\text{s}}\right)$	ν_{Stat}	ν_{Dyn}	$E_{Stat}(\text{GPa})$	$E_{Dyn}(\text{GPa})$	$G_{Stat}(\text{GPa})$	$G_{Dyn}(\text{GPa})$	$K_{Stat}(\text{GPa})$	$K_{Dyn}(\text{GPa})$
Uralite basalt	6.57	3.66	0.15	0.28	78.5	104.7	34.13	40.90	37.38	16.30
Dolerite	6.37	3.44	0.13	0.29	82	91.9	36.28	35.62	36.94	79.32
Uralite diabase	6.13	3.13	0.25	0.32	91	82	36.40	31.06	60.67	72.94
Dolerite	6.48	3.73	0.2	0.25	93.9	109.3	39.13	43.72	52.17	75.93
Syenite	–	–	–	–	72.56	79.42	36.28	39.71	24.19	72.87
<i>Metamorphic rocks</i>										
Quartzose phyllite	–	–	– 0.03	–	7.58	18.60	3.91	9.30	2.38	6.20
Graphitic phyllite	–	–	–	–	9.65	26.87	4.83	13.44	3.22	8.96
Tremolite schist	6.32	3.46	0.11	0.29	89.6	92.7	40.36	35.93	38.29	73.57
Hornblende schist	6.3	3.92	0.28	0.29	98.2	104.2	38.36	40.39	74.39	82.70
Actinolite schist	–	–	0.29	0.26	77.9	148.6	30.19	58.97	61.83	103.19

metamorphic rocks, the value is $R^2 = 0.70$. Similarly, Fig. 8 illustrates the correlation between static and dynamic modulus of rigidity for all samples, giving $R^2 = 0.89$. Figure 9 demonstrates the relationship between static and dynamic modulus of rigidity based on rock types.

As it is clear, for igneous rocks the value of $R^2 = 0.96$, for sedimentary rocks the value of $R^2 = 0.91$ and for metamorphic rocks the value of $R^2 = 0.63$. Figure 10 exhibits the linear correlation between static and dynamic bulk modulus for all studied rocks, giving $R^2 = 0.77$. Figure 11 shows the relationship between static and dynamic bulk modulus with respect to rock types. As can be seen, for igneous rock the value of $R^2 = 0.77$, for sedimentary rocks the value of $R^2 = 0.38$ and for metamorphic rocks, the value of $R^2 = 0.87$. The results are summarised in Tables 7, 8, 9.

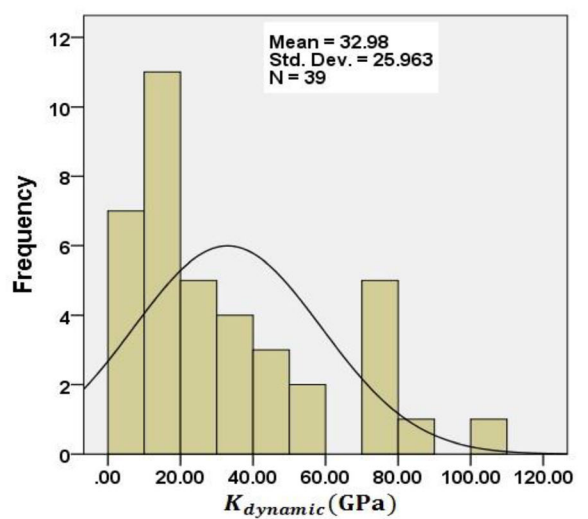
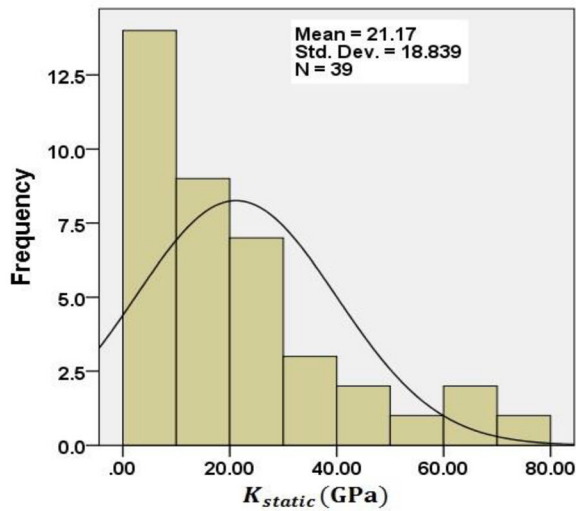
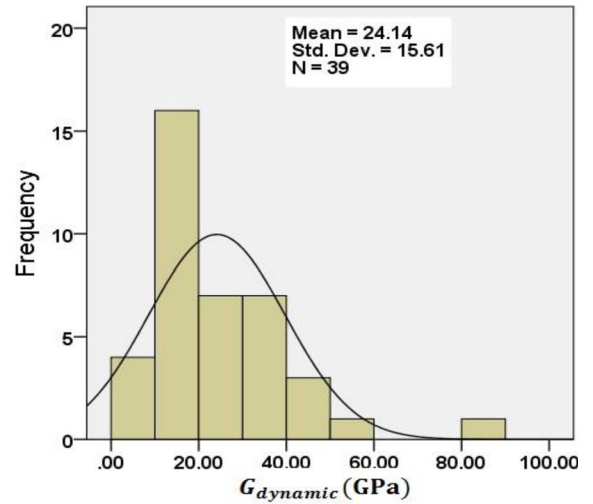
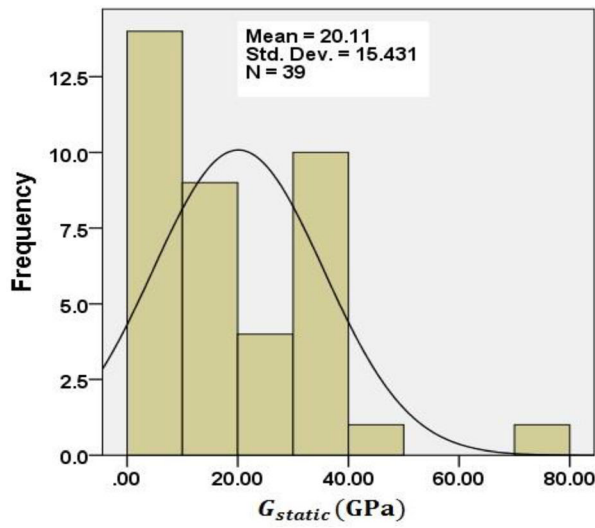
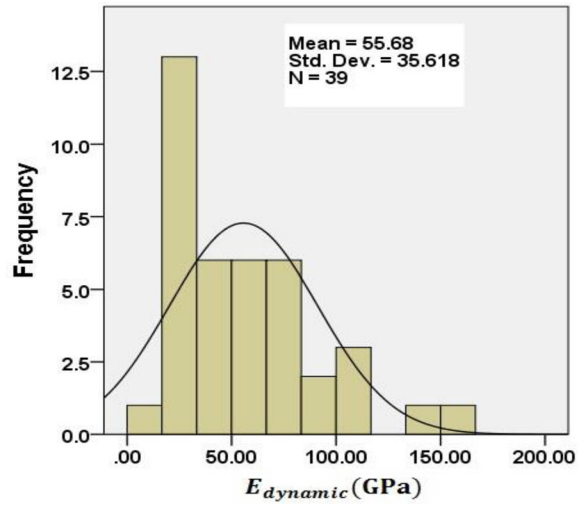
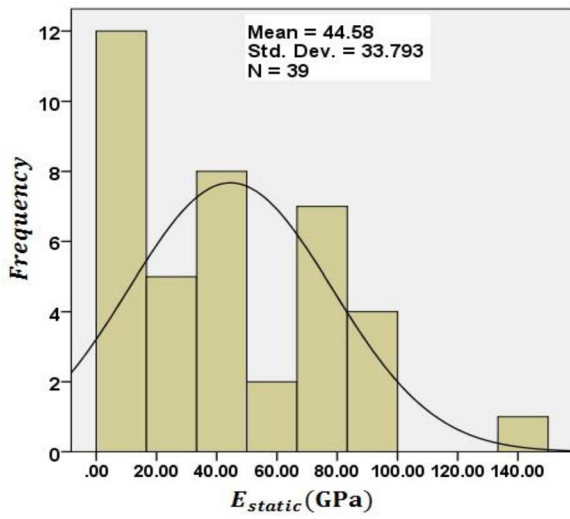
Our achieved results in Fig. 12, with previously published correlations, are illustrated in Fig. 13. It can be concluded that the formulas well fit the data.

According to our analysis, with respect to different published linear correlations between static modulus of elasticity (E_{stat}) and dynamic modulus of elasticity (E_{dyn}), a relationship with high correlation ($R^2 = 0.91$) was observed between a and b parameters as it is shown in Fig. 14. It should be stated that data

published by McCann & Entwisle (1992) for Crystalline rocks were not included in this analysis. The reason is that by applying their equation, the amount of correlation decreases from ($R^2 = 0.91$) to ($R^2 = 0.57$). It might be related to the link between the crystallized structure of rock and wave propagation.

4 Results and discussion

In this research, the basic geomechanical properties of different types of rocks were measured and analyzed. The results show that there is a good correlation between static and dynamic elasticity modulus, rigidity modulus and bulk modulus. Regarding the relationships between static and dynamic modulus of elasticity, the best correlation found to be nonlinear logarithmic and power regression with the value of ($R^2 = 0.91$). Similarly, Brotons et al. (2014, 2016) established the nonlinear correlation between static and dynamic elastic modulus of different types of rocks with the value of $R^2 = 0.99$. Eissa and Kazi (1988) carried out similar research and discovered that the best correlation was to be nonlinear with the value of $R^2 = 0.92$. On the other hand, some other researchers adopted the linear correlation as well-fitted regression line (Belikov et al. 1970) for Granite



◀ Fig. 5 Histogram of investigated parameters

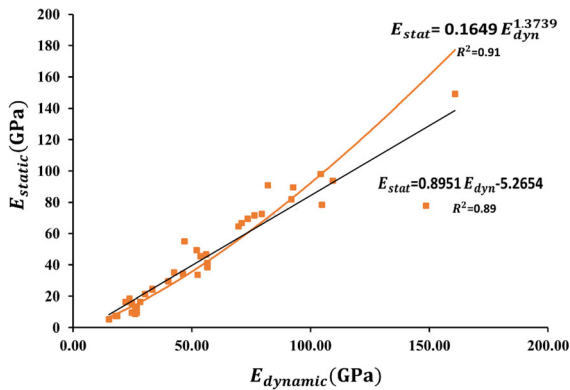


Fig. 6 Relationship between static and dynamic modulus of elasticity for all rocks

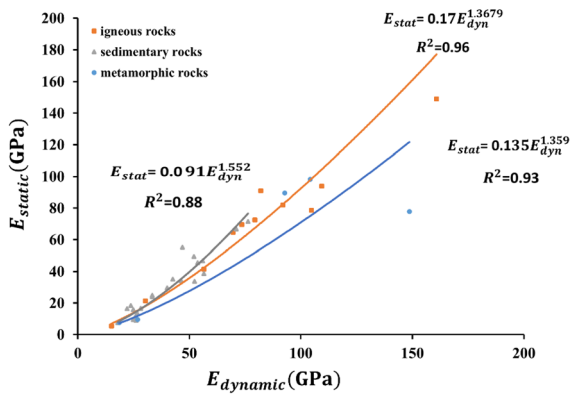


Fig. 7 Relationship between static and dynamic modulus of elasticity based on rock types

with $R^2 = 0.92$, (King 1983) for Igneous and metamorphic rocks with $R^2 = 0.82$, (McCann and Entwisle 1992) for crystalline rocks with $R^2 = 0.82$, (Christaras et al. 1994) for all types of rocks with $R^2 = 0.99$, (Nur and Wang 1999) for all types of rocks with $R^2 = 0.8$). Nevertheless, in the present study, we established the linear correlations for igneous rocks with $R^2 = 0.95$, for sedimentary rocks with $R^2 = 0.90$ and for metamorphic rocks with $R^2 = 0.69$. Considering the relationship between static and dynamic modulus of rigidity, the best correlation observed was nonlinear logarithmic regression with giving the value of ($R^2 = 0.97$); when it comes to bulk modulus, the best correlation was linear with the value of ($R^2 = 0.77$). According to rock types, for igneous rock, the best correlation between static and dynamic modulus of

elasticity (E) was nonlinear logarithmic and power ones with the value of ($R^2 = 0.96$); However, King (1983) found that the best correlation was linear for igneous-metamorphic rocks with the value of ($R^2 = 0.82$). For sedimentary rocks, the best correlation was linear with the value of ($R^2 = 0.88$) and for metamorphic rocks was nonlinear logarithmic and power with the value of ($R^2 = 0.93$). For igneous rocks, the best correlation between static and dynamic modulus of rigidity (G) was nonlinear logarithmic with the value of ($R^2 = 0.96$); for sedimentary rocks was nonlinear logarithmic with the value of ($R^2 = 0.98$); for metamorphic rocks was nonlinear logarithmic with the value of ($R^2 = 0.99$). For igneous rocks, the best correlation between static and dynamic bulk modulus (K) was nonlinear logarithmic with the value of ($R^2 = 0.88$); for sedimentary rocks was linear with the value of ($R^2 = 0.38$); for metamorphic rocks was nonlinear logarithmic with ($R^2 = 0.98$). These values demonstrate an interesting finding that there is a higher correlation between static and dynamic constants in igneous rocks rather than sedimentary and metamorphic rocks except for bulk modulus. Also, based on an analysis of previously obtained linear correlations between static and dynamic modulus of elasticity in Table 1, there is a good logarithmic correlation between constant parameters (a, b). Interestingly enough, our achieved result as shown in Fig. 14 fits well with previously published results with high correlation $R^2 = 0.91$. It means the static modulus of elasticity depends on the dynamic modulus only with a one-parameter formula:

$$E_{stat} = (0.135 \ln(b) + 0.78) - b,$$

where b is rock type - dependent parameter.

Additionally, for a more accurate comparison, root-mean-square (χ) errors between the dynamic and static elastic modulus was calculated as:

$$\chi(m) = \sqrt{\frac{\sum_{j=1}^n [\Pi_{obs(j)} - \Pi_{cal(j)}]^2}{n - 1}} \quad (10)$$

where $\Pi_{obs(j)}$ is the observed value of a parameter in the j th sample, here is the modulus of elasticity (E), $\Pi_{cal(j)}$ is the calculated value of a parameter in the j th sample, $j = 1, 2, \dots, n$, is the number of tested samples. Based on our analyses we received the following correlation,

Fig. 8 Relationship between static and dynamic modulus of rigidity for all rocks

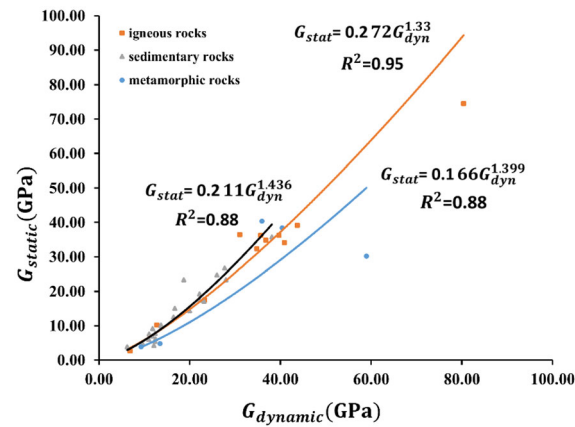
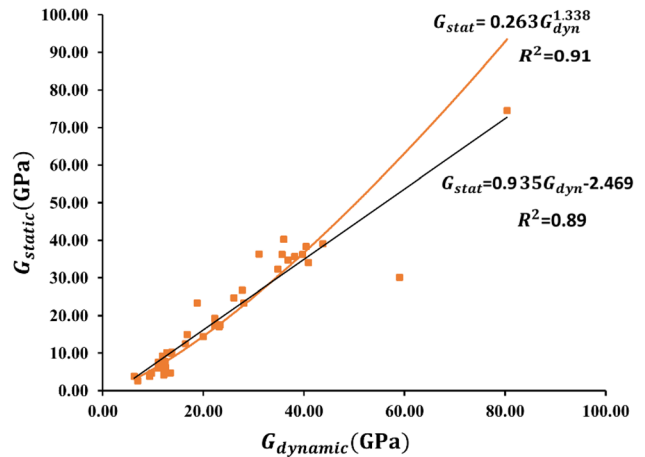


Fig. 9 Relationship between static and dynamic modulus of rigidity based on rock types

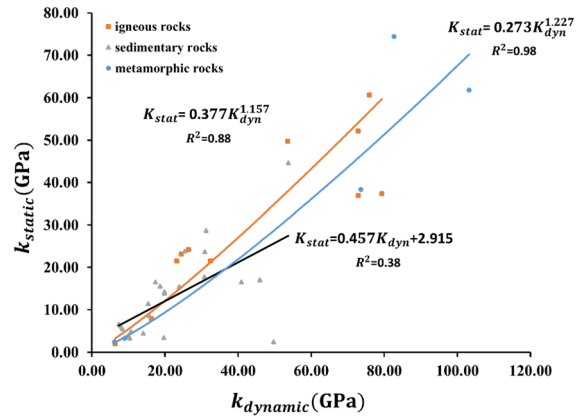


Fig. 11 Relationship between static and dynamic bulk modulus based on rock types

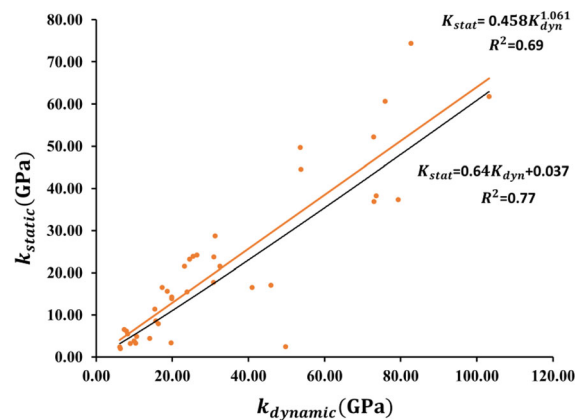


Fig. 10 Relationship between static and dynamic bulk modulus for all rocks

$$\log_{10} E_{stat} = 1.37 \log_{10} (\rho_{bulk} E_{dyn}) - 1.32, R^2 = 0.91, \chi = 10.52$$

$$E_{stat} = 0.164 E_{dyn}^{1.373}, R^2 = 0.91, \chi = 11.58$$

$$E_{stat} = 0.89 E_{dyn} - 5.26, R^2 = 0.89, \chi = 13.17$$

The difference between dynamic and static elastic moduli is highly associated with mineralogical differences, differences in grain/crystal size, differences in porosity. A deviation of dynamic elastic modulus from static elastic modulus can be attributed to the presence of fractures, cracks, cavities and planes of weakness and foliation (Al-Shayea 2004; Guéguen and Palciauskas 1994). In other words, as the number of discontinuities increase, the lower value of Young's modulus and the higher discrepancy between static and dynamic values are expected. The most crucial

Table 7 Linear regression between static and dynamic deformation constants ($E_{stat} = a E_{dyn} - b$)

Rock type	$E_{stat} - E_{dyn}$			$G_{stat} - G_{dyn}$			$K_{stat} - K_{dyn}$		
	<i>a</i>	<i>b</i>	R ²	<i>a</i>	<i>b</i>	R ²	<i>a</i>	<i>b</i>	R ²
Igneous	0.95	5.81	0.95	0.96	1.53	0.96	0.59	- 4.49	0.77
Sedimentary	1.09	13.5	0.90	1.08	5.22	0.91	0.45	- 2.91	0.38
Metamorphic	0.69	- 2.04	0.74	0.69	- 1.55	0.63	0.69	2.04	0.87
All types	0.89	5.26	0.89	0.93	2.46	0.89	0.64	- 0.03	0.77

Table 8 Power regression between static and dynamic deformation constants ($E_{stat} = \alpha E_{dyn}^\beta$)

Rock type	$E_{stat} - E_{dyn}$			$G_{stat} - G_{dyn}$			$K_{stat} - K_{dyn}$		
	R ²			R ²			R ²		
Igneous	0.17	1.36	0.96	0.272	1.332	0.95	0.377	1.157	0.88
Sedimentary	0.091	1.55	0.88	0.211	1.436	0.88	1.131	0.739	0.31
Metamorphic	0.135	1.359	0.93	0.166	1.399	0.88	0.237	1.227	0.98
All types	0.164	1.373	0.91	0.263	1.338	0.91	0.458	1.061	0.69

Table 9 Logarithmic regression between static and dynamic deformation constants ($\log_{10} E_{stat} = c \log_{10}(\rho_{bulk} E_{dyn}) - d$)

Rock type	$E_{stat} - E_{dyn}$			$G_{stat} - G_{dyn}$			$K_{stat} - K_{dyn}$		
	<i>c</i>	<i>d</i>	R ²	<i>c</i>	<i>d</i>	R ²	<i>c</i>	<i>d</i>	R ²
Igneous	1.36	1.31	0.96	2.15	- 0.28	0.96	1.15	0.87	0.88
Sedimentary	1.55	1.65	0.88	2.27	- 0.13	0.98	0.73	0.24	0.3
Metamorphic	1.35	1.40	0.93	2.36	- 0.16	0.99	1.22	1.11	0.98
All types	1.37	1.32	0.91	2.33	- 0.14	0.97	1.06	0.76	0.69

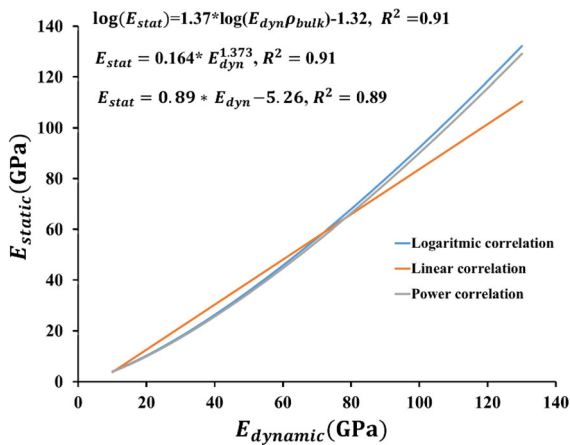


Fig. 12 Linear and non-linear achieved correlations for all rock types

petrographic parameter that influences the deformation behavior of rock is porosity. The trend observed in porous rocks was that the static elastic modulus was inversely proportional to porosity (Brotons et al. 2016;

Garcia-del-Cura et al. 2012). However, Crystalline rocks exhibit lower values of elastic moduli due to the fact that their porous system is constituted by a dense microcrack network. That is, crystalline rocks act as non-continuous solid, while porous rocks with inter-particle porosity behave as a more continuous solid due to the presence of cement and matrix between grains. The effect of mineralogy on elastic moduli of rocks have been studied by several authors and found to be much less than other factors such as porosity and crystal size (Heap and Faulkner 2008; Palchik and Hatzor 2002). Regarding the effect of grain size on the elastic modulus of nanocrystalline, with the decrease of grain size, the elastic modulus decreases (Kim and Bush 1999; Chaim 2004; Zhang and Tahmasebi 2019; Tugrul and Zarif 1999).

It is worth mentioning that more detailed material models beyond ideal elasticity give an exact relationship between the elastic and static moduli. Notably, the observed relations can be explained in a universal thermodynamic framework where internal variables

Fig. 13 Previously published correlations

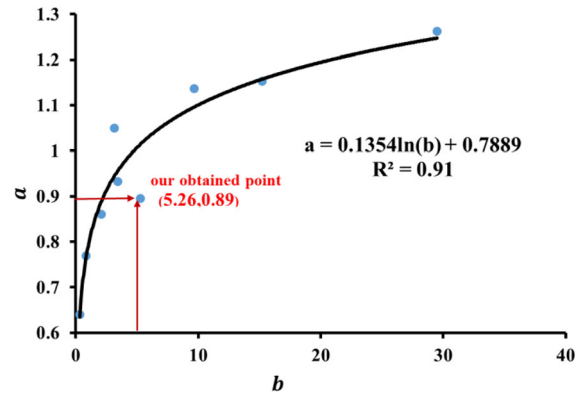
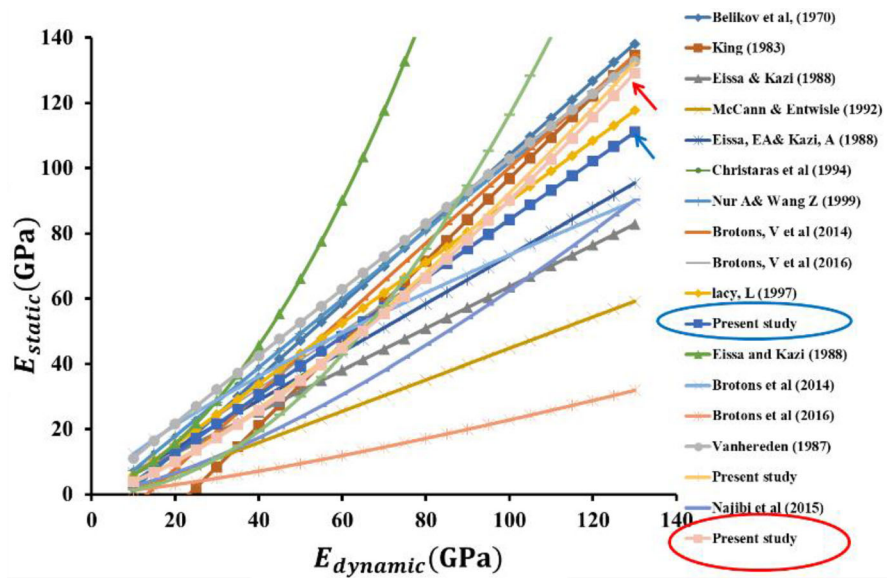


Fig. 14 Relationship between *a* and *b* constants

are characterizing the structural changes (Asszonyi et al. 2015; Berezovski and Ván 2017). The difference of dynamic and elastic moduli is natural in this framework as it is clear from the corresponding dispersion relation and related laboratory experiments (Barnaföldi et al. 2017; Ván et al. 2019) These constitutive models are based only on universal principles of thermodynamics, are independent of particular mechanisms and are successful in characterizing rheological phenomena in rocks including and beyond simple creep and relaxation. This is in accordance with the difficulty for finding a very detailed quantitative mesoscopic mechanism for the dynamics of dissipative phenomena in rocks as well.

5 Conclusion

Generally, the performance difference between the linear and power-law formulas was small, the linear relationship fitted well the data. Therefore, considering also the possibility of one parameter formulation, given in (11), the linear relationship between dynamic and static elastic moduli is suitable for rock engineering. It was expected, that the modulus of rigidity and bulk modulus, as original theoretical Lamé parameters, would correlate better. However, the correlation between the static and dynamic Young modulus ($R^2 = 0.89$) is as good as for the other ones ($R^2 = 0.89$ and 0.77), in spite of the fact that the Young's modulus is a composite parameter. The reason could be the uncertainty in the Poisson ration measurements and also the difference in rock types for the bulk modulus measurements is remarkable. It is worth mentioning that thermodynamic principles more detailed material models beyond ideal elasticity give detailed relationships between the elastic and static moduli in general. Particularly, the observed relations can be explained in a universal thermodynamic framework where internal variables are characterizing the structural changes in the rock.

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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