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ARTICLE

Learning to Teach Mathematics Through Problem Solving

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Abstract
While there has been much research focused on beginning teachers; and mathematical problem solving in the classroom, little is known about beginning primary teachers' learning to teach mathematics through problem solving. This longitudinal study examined what supported beginning teachers to start and sustain teaching mathematics through problem solving in their first 2 years of teaching. Findings show 'sustaining' required a combination of three factors: (i) participation in professional development centred on problem solving (ii) attending subject-specific complementary professional development initiatives alongside colleagues from their school; and (iii) an in-school colleague who also teaches mathematics through problem solving. If only one factor is present, in this study attending the professional development focussed on problem solving, the result was little movement towards a problem solving based pedagogy. Recommendations for supporting beginning teachers to embed problem solving are included.

Keywords Beginning teachers · Mathematical problem solving · Professional development · Problem solving lesson structure

Introduction

For many years curriculum documents worldwide have positioned mathematics as a problem solving endeavour (e.g., see Australian Curriculum, Assessment and Reporting Authority, 2018; Ministry of Education, 2007). There is evidence however that even with this prolonged emphasis, problem solving has not become a significant presence in many classrooms (Felmer et al., 2019). Research has reported on a multitude of potential barriers, even for experienced teachers (Clarke et al., 2007; Holton, 2009). At the same time it is widely recognised that beginning teachers encounter many challenges as they start their careers, and that these challenges

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are particularly compelling when seeking to implement ambitious methods of teaching, such as problem solving (Wood et al., 2012).

Problem solving has been central to mathematics knowledge construction from the beginning of human history (Felmer et al., 2019). Teaching and learning mathematics through problem solving supports learners' development of deep and conceptual understandings (Inoue et al., 2019), and is regarded as an effective way of catering for diversity (Hunter et al., 2018). While the importance and challenge of mathematical problem solving in school classrooms is not questioned, the promotion and enabling of problem solving is a contentious endeavour (English & Gainsburg, 2016). One debate centres on whether to teach mathematics *through* problem solving or to teach problem solving in and of itself. Recent scholarship (and this research) leans towards teaching mathematics *through* problem solving as a means for students to learn mathematics and come to appreciate what it means to do mathematics (Schoenfeld, 2013).

Problem solving has been defined in a multitude of ways over the years. Of central importance to problem solving as it is explored in this research study is Schoenfeld's (1985) proposition that, "if one has ready access to a solution schema for a mathematical task, that task is an exercise and not a problem" (p. 74). A more recent definition of what constitutes a mathematical problem from Mamona-Downs and Mamona (2013) also emphasises the centrality of the learner not knowing how to proceed, highlighting that problems cannot be solved by procedural effort alone. These are important distinctions because traditional school texts and programmes often position problems and problem solving as an 'add-on' providing a practice opportunity for a previously taught, specific procedure. Given the range of learners in any education setting an important point to also consider is that what constitutes a problem for some students may not be a problem for others (Schoenfeld, 2013).

A research focus exploring what supports beginning teachers' learning about teaching mathematics through problem solving is particularly relevant at this time given calls for an increased curricular focus on mathematical practices such as problem solving (Grootenboer et al., 2021) and recent recommendations from an expert advisory panel on the English-medium Mathematics and Statistics curriculum in Aotearoa (Royal Society Te Apārangi, 2021). The ninth recommendation from this report advocates for the provision of sustained professional learning in mathematics and statistics for all teachers of Years 0–8. With regard to beginning primary teachers, the recommendation goes further suggesting that 'mathematics and statistics professional learning' (p. 36) be considered as compulsory in the first 2 years of teaching. This research explores what the nature of that professional learning might involve, with a focus on problem solving.

Scoping the Context for Learning and Sustaining Problem Solving

The literature reviewed for this study draws from two key fields: the nature of support and professional development effective for beginning teachers; and specialised supports helping teachers to employ problem solving pedagogies.

Beginning Teachers, Support and Professional Development

A teacher's early years in the profession are regarded as critical in terms of constructing a professional practice (Feiman-Nemser, 2003) and avoiding high attrition (Karlberg & Bezzina, 2020). Research has established that beginning teachers need professional development opportunities geared specifically to their needs (Fantilli & McDougall, 2009) and their contexts (Gaikhorst, et al., 2017). Providing appropriate support is not an uncontentious matter with calls for institutions to come together and collaborate to provide adequate and ongoing support (Karlberg & Bezzina, 2020). The proposal is that support is needed from both within and beyond the beginning teacher's school; and begins with effective pre-service teacher preparation (Keese et al., 2022).

Within schools where beginning teachers regard the support they receive positively, collaboration, encouragement and 'involved colleagues' are considered as vital; with the guidance of a 'buddy' identified as some of the most valuable in-school support activities (Gaikhorst et al., 2014). Cameron et al.'s (2007) research in Aotearoa reports beginning teachers joining collaborative work cultures had greater opportunities to talk about teaching with their colleagues, share planning and resources, examine students' work, and benefit from the collective expertise of team members.

Opportunities to participate in networks beyond the beginning teacher's school have also been identified as being important for teacher induction (Akiri & Dori, 2021; Cameron et al., 2007). Fantilli & McDougall (2009) in their Canadian study found beginning teachers reported a need for many support and professional development opportunities including subject-specific (e.g., mathematics) workshops prior to and throughout the year. Akiri and Dori (2021) also refer to the need for specialised support from subject-specific mentors. This echoes the findings of Wood et al. (2012) who advocate that given the complexity of learning to teach mathematics, induction support specific to mathematics, and rich opportunities to learn are not only desirable but essential.

Akiri and Dori (2021) describe three levels of mentoring support for beginning teachers including individual mentoring, group mentoring and mentoring networks with all three facilitating substantive professional growth. Of relevance to this paper are individual and group mentoring. Individual mentoring involves pairing an experienced teacher with a beginning teacher, so that a beginning teacher's learning is supported. Group mentoring involves a group of teachers working with one or more mentors, with participants receiving guidance from their mentor(s) (Akiri & Dori, 2021). Findings from Akiri and Dori suggest that of the varying forms of mentoring, individual mentoring contributes the most for beginning teachers' professional learning.

Teachers Learning to Teach Mathematics Through Problem Solving

Learning to teach mathematics through problem solving begins in pre-service teacher education. It has been shown that providing pre-service teachers with opportunities to engage in problem solving as learners can be productive (Bailey, 2015). Opportunities to practise content-specific instructional strategies such as problem solving during student teaching has also been positively associated with first-year teachers' enactment of problem solving (Youngs et al., 2022).

The move from pre-service teacher education to the classroom can be fraught for beginning teachers (Feiman-Nemser, 2003), and all the more so for beginning teachers attempting to teach mathematics through problem solving (Wood et al., 2012). In a recent study (Darragh & Radovic, 2019) it has been shown that an individual willingness to change to a problem-based pedagogy may not be enough to sustain a change in practice in the long term, particularly if there is a contradiction with the context and 'norms' (e.g., school curriculum) within which a teacher is working. Cady et al. (2006) explored the beliefs and practices of two teachers from pre-service teacher education through to becoming experienced teachers. One teacher who initially adopted reform practices reverted to traditional beliefs about the learning and teaching of mathematics. In contrast, the other teacher implemented new practices only after understanding these and gaining teaching experience. Participation in mathematically focused professional development and involvement in resource development were thought to favourably influence the second teacher.

Lesson structures have been found to support teachers learning to teach mathematics through problem solving. Sullivan et al. (2016) explored the use of a structure comprising four phases: launching, exploring, summarising and consolidating. Teachers in Australia and Aotearoa have reported the structure as productive and feasible (Ingram et al., 2019; Sullivan et al., 2016). Teaching using challenging tasks (such as in problem solving) and a structure have been shown to accommodate student diversity, a pressing concern for many teachers. Student diversity has often been managed by grouping students according to perceived levels of capability (called ability grouping). Research identifies this practice as problematic, excluding and marginalising disadvantaged groups of students (e.g., see Anthony & Hunter, 2017). The lesson structure explored by Sullivan et al. (2016) caters for diversity by deliberately differentiating tasks, providing enabling and extending prompts. Extending prompts are offered to students who finish an original task quickly and ideally elicit abstraction and generalisation. Enabling prompts involve reducing the number of steps, simplifying the numbers, and/or varying forms of representation for students who cannot initially proceed, with the explicit intention that students then return to the original task.

Recognising the established challenges teachers encounter when learning about teaching mathematics through problem solving, and the paucity of recent research focussing on beginning teachers learning about teaching mathematics in this way, this paper draws on data from a 2 year longitudinal study. The study was guided by the research question:

What supports beginning teachers' implementation of a problem solving pedagogy for the teaching and learning of mathematics?

Research Methodology and Methods

Data were gathered from three beginning primary teachers who had completed a 1 year graduate diploma programme in primary teacher education the previous year. The beginning teachers had undertaken a course in mathematics education (taught by the author for half of the course) as part of the graduate diploma. An invitation to be involved in the research was sent to the graduate diploma cohort at the end of the programme. Three beginning teachers indicated their interest and remained involved for the 2 year research period. The teachers had all secured their first teaching positions, and were teaching at different year levels at three different schools. Julia (pseudonyms have been used for all names) was teaching year 0–2 (5–6 years) at a small rural school; Charlotte, year 5–6 (9–10 years) at a large urban city school; and Reine, year 7–8 (11–12 years), at another small rural school. All three beginning teachers taught at their respective schools, teaching the same year levels in both years of the study. Ethical approval was sought and given by the author's university ethics committee. Informed consent was gained from the teachers, school principals and involved parents and children.

Participatory action research was selected as the approach in the study because of its emphasis on the participation and collaboration of all those involved (Townsend, 2013). Congruent with the principles of action research, activities and procedures were negotiated throughout both years in a responsive and emergent way. The author acted as a co-participant with the teachers, aiming to improve practice, to challenge and reorient thinking, and transform contexts for children's learning (Locke et al., 2013). The author's role included facilitating the research-based problem solving workshops (see below), contributing her experience as a mathematics educator and researcher. The beginning teachers were involved in making sense of their own practice related to their particular sites and context.

The first step in the research process was a focus group discussion before the beginning teachers commenced their first year of teaching. This discussion included reflecting on their learning about problem solving during the mathematics education course; and envisaging what would be helpful to support implementation. It was agreed that a series of workshops would be useful. Two were subsequently held in the first year of the study, each for three hours, at the end of terms one and two. Four workshops were held during the second year, one during each term. At the end of the first year the author suggested a small number of experienced teachers who teach mathematics through problem solving join the workshops for the second year. The presence of these teachers was envisaged to support the beginning teachers' learning. The beginning teachers agreed, and an invitation was extended to four teachers from other schools whom the author knew (e.g., through professional subject associations). The focus of the research remained the same, namely exploring what supported beginning teachers to implement a problem solving pedagogy.

Each workshop began with sharing and oral reflections about recent problem solving experiences, including successes and challenges. Key workshop tasks included developing a shared understanding of what constitutes problem solving, participating in solving mathematical problems (modelled using a lesson structure (Sullivan et al., 2016), and learning techniques such as asking questions. A time for reflective writing was provided at the end of each workshop to record what had been learned and an opportunity to set goals.

During the first focus group discussion it was also decided the author would visit and observe the beginning teachers teaching a problem solving lesson (or two) in term three or four of each year. A semi-structured interview between the author and each beginning teacher took place following each observed lesson. The beginning teachers also had an opportunity to ask questions as they reflected on the lesson, and feedback was given as requested. A second focus group discussion was held at the end of the first year (an approximate midpoint in the research), and a final focus group discussion was held at the end of the second year.

All focus group discussions, problem solving workshops, observations and interviews were audio-recorded and transcribed. Field notes of workshops (recorded by the author), reflections from the beginning teachers (written at the end of each workshop), and lesson observation notes (recorded by the author) were also gathered. The final data collected included occasional emails between each beginning teacher and the author.

Data Analysis

The analysis reported in this paper drew on all data sets, primarily using inductive thematic analysis (Braun & Clarke, 2006). The research question guided the key question for analysis, namely: What supports beginning teachers' implementation of a problem solving pedagogy for the teaching and learning of mathematics? Alongside this question, consideration was also given to the challenges beginning teachers encountered as they implemented a problem solving pedagogy. Data familiarisation was developed through reading and re-reading the whole body of data. This process informed data analysis and the content for each subsequent workshop and focus group discussions. Colour-coding and naming of themes included comparing and contrasting data from each beginning teacher and throughout the 2-year period. As a theme was constructed (Braun & Clarke, 2006) subsequent data was checked to ascertain whether the theme remained valid and/or whether it changed during the 2 years. Three key themes emerged revealing what supported the beginning teachers' developing problem solving pedagogy, and these constitute the focus for this paper.

Mindful of the time pressures beginning teachers experience in their early years, the author undertook responsibility for data analysis. The author's understanding of the unfolding 'story' of each beginning teacher's experiences and the emerging themes were shared with the beginning teachers, usually at the beginning of a workshop, focus group discussion or observation. Through this process the author's understandings were checked and clarified. This iterative process of member checking (Lincoln & Guba, 1985) began at a mid-point during the first year, once a significant body of data had been gathered. At a later point in the analysis and writing, the beginning teachers also had an opportunity to read, check and/or amend quotes chosen to exemplify their thinking and experiences.

Findings and Discussion

In this section the three beginning teachers' experiences at the start of the 2 year research timeframe is briefly described, followed by the first theme centred on the use of a lesson structure including prompts for differentiation. The second and third themes are presented together, starting with a brief outline of each beginning teacher's 'story' providing the context within which the themes emerged. Sharing the 'story' of each beginning teacher and including their 'voice' through quotes acknowledges them and their experiences as central to this research.

A Start

The beginning teachers' pre-service teacher education set the scene for learning about teaching mathematics through problem solving. A detailed list brainstormed during the first focus group discussion suggested a developing understanding from their shared pre-service mathematics education course. In their first few weeks of teaching, all three beginning teachers implemented a few problems. It transpired however this inclusion of problem solving occurred only while children were being assessed and grouped. Following this, all three followed a traditional format of skill-based (with a focus on number) mathematics, taught using ability groups. The beginning teachers' trajectories then varied with Julia and Reine both eventually adopting a pedagogy primarily based on problem solving, while Charlotte employed a traditional skill-based mathematics using a combination of whole class and small group teaching.

A Lesson Structure that Caters for Diversity Supports Early Efforts

Data show that developing familiarity with a lesson structure including prompts for differentiation supported the beginning teachers' early efforts with a problem solving pedagogy. This addressed a key issue that emerged during the first workshop. During the workshop while a 'list' of ideas for teaching a problem solving lesson was co-constructed, considerable concern was expressed about catering for a range of learners when introducing and working with a problem. For example, Charlotte queried, "*Well, what happens when you are trying to do something more complicated, and we're* (referring to children) *sitting here going, 'I've no idea what you're talking about"*? Reine suggested keeping some children with the teacher, thinking he would say, "*If you're unsure of any part stay behind*". He was unsure however about how he would then maintain the integrity of the problem.

It was in light of this discussion that a lesson structure with differentiated prompts (Sullivan et al., 2016) was introduced, experienced and reflected on during the second workshop. While the co-constructed list developed during the first workshop had included many components of Sullivan's lesson structure, (e.g., a consideration of 'extensions') there had been no mention of 'enabling prompts'. Now, with the inclusion of both enabling and extending prompts, the beginning teachers' discussion revealed them starting to more fully envisage the possibilities of using a problem solving approach, and being able to cater for all children. Reine commented that, *"… you can give the entire class a problem, you've just got to have a plan, [and] your enabling and extension prompts"*. Charlotte was also now considering and valuing the possibility of having a whole class work on the same problem. She said, *"I think … it's important and it's useful for your whole class to be working on the same thing. And … have enablers and extenders to make sure that everyone feels successful"*. Julia also referred to the planning prompts. She thought it would be key to *"plan it well so that we've got enabling and extending prompts"*.

Successful Problem Solving Lessons

Following the second workshop all three beginning teachers were observed teaching a lesson using the structure. These lessons delighted the beginning teachers, with them noting prolonged engagement of children, the children's learning and being able to cater for all learners. Reine commented on how excited and engaged the children were, saying they were, "*just so enthusiastic about it*". In Charlotte's words, "*it really worked*", and Julia enthusiastically pondered this could be "*the only way you teach maths*!".

During the focus group discussion at the end of the first year, all three reflected on the value of the lesson structure. Reine called it a 'framework' commenting,

> I like the framework. So from start to finish, how you go through that whole lesson. So how you set it up and then you go through the phases… I like the prompts that we went through…. knowing where you could go, if they're like, 'What do I do?' And then if they get it too easy then 'Where can you go?' So you've got all these little avenues.

Charlotte also valued the lesson structure for the breadth of learning that could occur, explaining,

> … it really helped, and really worked. So I found that useful for me and my class 'cause they really understood. And I think also making sure that you know all the ins and outs of a problem. So where could they go? What do you need to know? What do they need to know?

While the beginning teachers' pre-service teacher education and the subsequent research process, including the use of the lesson structure, supported the beginning teachers' early efforts teaching mathematics through problem solving, two key factors further enabled two of the beginning teachers (Julia and Reine) to *sustain* a problem solving pedagogy. These were:

(i) Being involved in complementary mathematics professional development alongside members of their respective school staff (a form of group mentoring); and

(ii) Having a colleague in the same school teaching mathematics through problem solving (a form of individual mentoring).

Charlotte did not have these opportunities and she indicated this limited her implementation. Data for these findings for each teacher are presented below.

Complementary Professional Development and Problem Solving Colleague in Same School

Julia

Julia began to significantly implement problem solving from the second term in the first year. This coincided with her attending a 2-day workshop (with staff from her school) that focused on the use of problem solving to support children who are not achieving at expected levels (see ALiM: Accelerated Learning in Maths—Ministry of Education, 2022). She explained, "*… I did the PD with (colleague's name), which was really helpful. And we did lots of talking about rich learning tasks and problem solving tasks…. And what it means*". Following this, Julia reported using rich tasks and problem solving in her mathematics teaching in a regular (at least weekly) and ongoing way.

During the observation in term three of the first year Julia again referred to the impact of having a colleague also teaching mathematics through problem solving. When asked what she believed had supported her to become a teacher who teaches mathematics in this way she firstly identified her involvement in the research project, and then spoke about her colleague. She said, "*I'm really lucky one of our other teachers is doing the ALiM project… So we're kind of bouncing off each other a little bit with resources and activities, and things like that. So that's been really good*".

At the beginning of the second year, Julia reiterated this point again. On this occasion she said having a colleague teaching mathematics through problem solving, "*made a huge difference for me last year*", explaining the value included having someone to talk with on a daily basis. Mid-way through the second year Julia repeated her opinion about the value of frequent contact with a practising problem solving colleague. Whereas her initial comments spoke of the impact in terms of being "*a little bit*", later references recount these as '*huge*' and '*enabling*'. She described:

> a huge effect… it enabled me. Cause I mean these workshops are really helpful. But when it's only once a term, having [colleague] there just enabled me to kind of bounce ideas off. And if I did a lesson that didn't work very well, we could talk about why that was, and actually talk about what the learning was instead…. . It was being able to reflect together, but also share ideas. It was amazing.

Julia's comments raise two points. It is likely that participating in the ALiM professional development (which could be conceived as a form of group mentoring) consolidated the learning she first encountered during pre-service teacher education and later extended through her involvement in the research. Having a colleague (in essence, an individual mentor) within the same school teaching mathematics through problem solving appears to be another factor that supported Julia to implement problem solving in a more sustained way. Julia's comments allude to a number of reasons for this, including: (i) the more frequent discussion opportunities with a colleague who understands what it means for children to learn mathematics through problem solving; (ii) being able to share and plan suitable activities and resources; and (iii) as a means for reflection, particularly when challenges were encountered.

Reine

Reine's mathematics programme throughout the first year was based on ability groups and could be described as traditional. He occasionally used some mathematical problems as 'extension activities' for 'higher level' children, or as 'fillers'. In the second year, Reine moved to working with mixed ability groups (where students work together in small groups with varying levels of perceived capability) and initially implemented problem solving approximately once a fortnight. In thinking back to these lessons he commented, "*We weren't really unpacking one problem properly, it was just lots of busy stuff*". A significant shift occurred in Reine's practice to teaching mathematics primarily by problem solving towards the last half of the second year. He explained, "*I really ramped up towards terms three and four, where it's more picking one problem across the whole maths class but being really, really conscious of that problem. Low entry, high ceiling, and doing more of it too*".

Reine attributed this change to a number of factors. In response to a question about what he considered led to the change he explained,

> … having this, talking about this stuff, trialling it and then with our PD at school with the research into ability grouping... We've got a lot of PD saying why it can be harmful to group on ability, and that's been that last little kick I needed, I think. And with other teachers trialling this as well. Our senior teacher has flipped her whole maths program and just does problem solving.

Like Julia, Reine firstly referred to his involvement in the research project including having opportunities to try problems in his class and discuss his experiences within the research group. He then told of a colleague teaching at his school leading school-wide professional development focussed on the pitfalls of ability grouping in mathematics (e.g., see Clarke, 2021) and instead using problem solving tasks. He also referred to having another teacher also teaching mathematics through problem solving. It is interesting to consider that having positive experiences in pre-service teacher education, the positive and encouraging support of colleagues (Reine's principal and co-teacher in both years), regular participation in ongoing professional development (the problem solving workshops), and having a highly successful one-off problem solving teaching experience (the first year observation) were not enough for Reine to meaningfully sustain problem solving in his first year of teaching.

As for Julia, pivotal factors leading to a sustaining of problem solving teaching practice in the second year included complementary mathematics professional development (a form of group mentoring) and at least one other teacher (acting as an individual mentor) in the same school teaching mathematics through problem solving. It could be argued that pre-service teacher education and the problem solving workshops 'paved the way' for Julia and Reine to make a change. However, for both, the complementary professional development and presence of a colleague also teaching through problem solving were pivotal. It is also interesting to note that three of the four experienced teachers in the larger research group taught at the same level as Reine (see Table 1 below) yet he did not relate this to the significant change in his practice observed towards the end of the second year.

Charlotte

Charlotte's mathematics programme during the first year was also traditional, teaching skill-based mathematics using ability groups. At the beginning of the second year Charlotte moved to teaching her class as a whole group, using flexible grouping as needed (children are grouped together in response to learning needs with regard to a specific idea at a point in time, rather than perceived notions of ability). She reported that she occasionally taught a lesson using problem solving in the first year, and approximately once or twice a term in the second year. Charlotte did not have opportunities for professional development in mathematics nor did she have a colleague in the same school teaching mathematics through problem solving. Pondering this, Charlotte said,

> It would have been helpful if I had someone else in my school doing the same thing. I just thought about when you were saying the other lady was doing it [referring to Julia's colleague]. You know, someone that you can just kind of back-and-forth like. I find with Science, I usually plan with this other lady, and we share ideas and plan together. We come up with some really cool stuff whereas I don't really have the same thing for this.

Based on her experiences with teaching science it is clear Charlotte recognised the value of working alongside a colleague. In this, her view aligns with what Julia and Reine experienced.

Table 1 provides a summary of the variables for each beginning teacher, and whether a sustained implementation of teaching mathematics through problem solving occurred.

The table shows two variables common to Julia and Reine, the beginning teachers who began and sustained problem solving. They both participated in complementary professional development with colleagues from their school, and the presence of a colleague, also at their school, teaching mathematics through problem solving. Given that Julia was able to implement problem solving in the absence of a 'research workshop colleague' teaching at the same year level, and Reine's lack of comment about the potential impact of this, suggests that this was not a key factor enabling a sustained implementation of problem solving.

Attributing the changes in Julia and Reine's teaching practice primarily to their involvement in complementary professional development attended by members of their school staff, and the presence of at least one other teacher teaching mathematics through problem solving in their school, is further supported by a consideration

Table 1 Variables experienced by three beginning teachers; and sustained implementation of teaching mathematics through problem solving

	Beginning teacher participation in action research problem solving workshops, including a focus on a lesson structure	Participation in complementary, school based professional development with members of school staff (form of group mentoring)	Colleague in same school teaching mathematics through problem solving (form of individual mentoring)	Experienced teacher at same level within action research problem solving workshops in second year	Sustained implementation of teaching mathematics through problem- solving
Julia	✓	✓	✓	–	✓
Reine	✓	✓	✓	✓	✓
Charlotte	✓	–	–	–	–

of the timing of the changes. The data shows that while Julia could be considered an 'early adopter', Reine changed his practice reasonably late in the 2 year period. Julia's early adoption of teaching mathematics through problem solving coincided with her involvement, early in the 2 years, in the professional development and opportunity to work alongside a problem solving practising colleague. Reine encountered these similar conditions towards the end of the 2 years and it is notable that this was the point at which he changed his practice. That problem solving did not become embedded or frequent within Charlotte's mathematics programme tends to support the argument.

Conclusion

Understanding what supports primary teachers to teach mathematics through problem solving at the beginning of their careers is important because all students, including those taught by beginning teachers, need opportunities to develop high-level thinking, reasoning, and problem solving skills. It is also important in light of recent calls for mathematics curricula to include more emphasis on mathematical practices (such as problem solving) (e.g., see Grootenboer et al., 2021); and the Royal Society Te Apārangi report (2021). Findings from this research suggest that learning about problem solving during pre-service teacher education is enough for beginning teachers to trial teaching mathematics in this way. Early efforts were supported by gaining experience with a lesson structure that specifically attends to diversity. The lesson structure prompted the beginning teachers to anticipate different children's responses, and consider how they would respond to these. An increased confidence and sense of security to trial teaching mathematics through problem solving was enabled, based on their more in-depth preparation. Beginning teachers finding the lesson structure useful extends the findings of Sullivan et al. (2016) in Australia and Ingram et al. (2019) in Aotearoa to include less experienced teachers.

In order for teaching mathematics through problem solving to be sustained however, a combination of three factors, subsequent to pre-service teacher education, was needed: (i) active participation in problem solving workshops (in this context provided by the research-based problem solving workshops); (ii) attending complementary professional development initiatives alongside colleagues from their school (a form of group mentoring); and (iii) the presence of an in-school colleague who also teaches mathematics through problem solving (a form of individual mentoring). It seems possible these three factors acted synergistically resulting in Julia and Reine being able to sustain implementation. If only one factor is present, in this study attending the problem solving workshops, and despite a genuine interest in using a problem based pedagogy, the result was limited movement towards this way of teaching.

Akiri and Dori (2021) have reported that individual mentoring contributes the most to beginning teachers' professional growth. In a manner consistent with these findings, an in-school colleague (who in essence was acting as an individual mentor) played a critical role in supporting Reine and Julia. However, while Akiri and

Dori, amongst others (e.g., Cameron et al., 2007; Karlberg & Bezzina, 2020), have identified the value of supportive, approachable colleagues, for both Julia and Reine it was important that their colleague was supportive and approachable, *and* actively engaged in teaching mathematics through problem solving. Having supportive and approachable colleagues, as Reine experienced in his first year, on their own were not enough to support a sustained problem solving pedagogy.

Implications for Productive Professional Learning and Development

This study sought to explore the conditions that supported problem solving for beginning teachers, each in their unique context and from their perspective. The research did not examine how the teaching of mathematics through problem solving affected children's learning. However, multiple sets of data were collected and analysed over a 2-year period. While it is neither possible nor appropriate to make claims as to generalisability some suggestions for productive beginning teacher professional learning and development are offered.

Given the first years of teaching constitute a particular and critical phase of teacher learning (Karlberg & Bezzina, 2020) and the findings from this research, it is imperative that well-funded, subject-focussed support occurs throughout a beginning teacher's first 2 years of teaching. This is consistent with the ninth recommendation in the Royal Society Te Apārangi report (2021) suggesting compulsory professional learning during the induction period (2 years in Aotearoa New Zealand). Participation in subject-specific professional development has been recognised to favourably influence new teachers' efforts to adopt reform practices such as problem solving (Cady et al., 2006).

Findings from this study suggest professional development opportunities that complement each other support beginning teacher learning. In the first instance complementarity needs to be with what beginning teachers have learned during their pre-service teacher education. In this study, the research-based problem solving workshops served this role. Complementarity between varying forms of professional development also appears to be important. Furthermore, as indicated by Julia and Reine's experiences, subsequent professional development need not be on exactly the same topic. Rather, it can be complementary in the sense that there is an underlying congruence in philosophy and/or focus on a particular issue. For example, it emerged in the problem solving workshops, that being able to cater for diversity was a central concern for the beginning teachers. Attending to this issue within the problem solving workshops via the introduction of a lesson structure that enabled differentiation, was congruent with the nature of the professional development in the two schools: ALiM in Julia's school, and mixed ability grouping and teaching mathematics through problem solving in Reine's school. All three of these settings were focussed on positively responding to diversity in learning needs.

The presence of a colleague within the same school teaching mathematics through problem solving also appears to be pivotal. This is consistent with Darragh and Radovic (2019) who have shown the significant impact a teacher's school context has on their potential to sustain problem based pedagogies in mathematics. Given that problem solving is not prevalent in many primary classrooms, it would seem clear that colleagues who have yet to learn about teaching mathematics through problem solving, particularly those that have a role supporting beginning teachers, will also require access to professional development opportunities. It seems possible that beginning and experienced teachers learning together is a potential pathway forward. Finding such pathways will be critical if mathematical problem solving is to be consistently implemented in primary classrooms.

Finally, these implications together with calls for institutions to collaborate to provide adequate and ongoing support for new teachers (Karlberg & Bezzina, 2020) suggest there is a need for pre-service teacher educators, professional development providers and the Teaching Council of Aotearoa New Zealand to work together to support beginning teachers' starting and sustaining teaching mathematics through problem solving pedagogies.

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