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ORIGINAL PAPER

Retailer's Optimal Ordering Policy for Deteriorating Inventory having Positive Lead Time under Pre-Payment Interim and Post-Payment Strategy

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Abstract
Management of inventory and its control for the retailers involves the procurement and storage of items for the smooth functioning of day-to-day business affairs. The procurement of goods depends on lead time and the payment mechanism. Thus, these components have a vital role in the optimal strategy for inventory control and management. As a result, these two components have a very high priority in recent developments on inventory models. However, a few researchers investigated such inventory problems with the integration of these components. Also, most of the researchers overlooked the impreciseness of different parameters while developing the inventory models. Thus, it is highly required to consider lead-time, payment strategy, and the impreciseness of parameters for inventory problems to provide an insight into real practice. Therefore, keeping these aspects in view, we develop two inventory models for Weibull deteriorating items having selling-price-dependent demand with positive lead-time in prepayment, interim payment, and post-payment scenarios. In the first model (crisp), the inventory constraints and costs are assumed to be deterministic. However, in the second model (fuzzy), only cost parameters are considered as imprecise, and the rest are all deterministic in nature. We first describe the solution strategies for obtaining the optimality of both crisp and fuzzy models. Subsequently, we perform numerical experiments with various sets of inventory constraints to investigate the efficiency of the proposed models. Finally, through sensitivity analysis of crucial parameters, we provide a ready reference to managerial insights making essential decisions in handling various circumstances which arise during the inventory cycle due to alteration of different constraints or parameters.

Keywords Lead time · Payment strategy · Imprecised parameters · Fuzzy inventory model

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Introduction

In the present day, the most valuable and vulnerable factor is time, and it plays a crucial part in the success of all business firms, industries, organizations, and all types of business affairs. Moreover, it is trivial that inventory management is the backbone for the smooth functioning of all most all business affairs. Thus, in this regard, the effect of time cannot be negligible. Mostly, inventory management is affected by the time in the procurement of goods and settlement of the purchase cost. The amount of time spent on the procurement of goods by placing the order and receiving them is called Lead Time. Similarly, the time of payment is based on the agreement, which is known as the payment mechanism.

During the last few decades, a considerable improvement has been achieved in transportation, communication, and production of inventories due to the advancement of science and technology. Although, the procurement of different products takes a minimum time for the retailer's inventory. Further, the lead-time of various inventory problems is different and is usually uncertain. Generally, the delivery of items takes on or before the delivery date, or sometimes it gets delayed. For instance, the actual delivery of the items ordered to e-commerce platforms generally differs from the provided delivery date. The delivery of items on or before the due time has a good impact on both supplier and retailer's business. However, in case of a delay in delivery, the supplier may lose the present or future orders, and the brand value may decrease. On the other hand, a few or all the customers may or may not wait during the lead time until the delivery of items from the supplier's end. So, the retailer loses demand partially or fully during the lead time depending upon the customer's behavior. Also, the deterioration will be considered as zero because of the non-presence of physical stock in inventory during the lead time. That means, the lead time makes the inventory problems more complex and may increase the cost of inventory maintenance. Thus, the lead-time acts as a vital constraint for inventory management. Moreover, the lead-time may be deterministic, probabilistic, stochastic, or uncertain.

The payment mechanism has a crucial part in all business affairs as it is highly required for sellers and buyers to maintain their positive cash flow. So, they follow several payment strategies to settle their business transactions. Some well-known payment strategies are as follows: (i) Advance Payment, that is, the payment to be made in advance before the order delivery (ii) Cash Payment, that is, the payment to be made instantly upon the receiving the delivery (iii) Credit Payment, that is, the payment to be made instantly after maturity of credit period (iv) Advance-Cash, that is, a partial amount to be paid in advance before the delivery and the remaining amount to be paid at the time of order delivery (v) Advance-Credit, that is, a partial amount to be paid before the delivery as advance and the remaining amount to be paid instantly on completion of credit period (vi) Cash-Credit, that is, a partial payment to be made at the time of delivery and the remaining payment will make instantly upon the maturity of the credit period and (vii) Advance-Cash-Credit, that is, the total cost of the purchase to be paid in three installments and the ratio of payment in each installment decided by seller and buyer's agreement (The first installment is before the delivery of the order, the second installment is at the time of receiving the order, and the last installment will be after the maturity of the credit period).

Generally, the advance payment scenario arises when the items have very high demand, or the volume of order quantity is high, or the retailer is a new one. The advance payment brings various advantages to the seller as follows: (i) protection against nonpayment (ii) preventing order cancellation (iii) earning more profit by re-investing the pre-payment (iv) may decrease in estimation errors in demand, etc. On the other hand, the retailer's inventory cost may increase, and he (or she) may get the assurance for the on-time delivery and price discount of the order quantity (if any). Cash on Delivery (COD or Cash Payment) is the most commonly used payment mechanism. In which, the supplier gets prevention from the nonpayment and can maintain positive cash flow. But, the seller may face the risk of order cancellation from the buyer. From the retailer's perspective, (i) he may need to invest higher working capital, and (ii) he can cancel the order if the supplier fails the on-time delivery. Credit payment or trade credit financing is helpful for both suppliers and retailers under some default risk. The supplier uses credit payment as a marketing strategy to improve sales, reduce on-hand inventory, and attract new customers. However, the credit payment reduces the working capital of the retailer and acts as short-term financing. Which also helps to earn the interest on accumulated revenue during the credit period. But, the seller may not get the payment, and the positive cash flow may slow down. Similarly, the retailer has to pay penalty interest on the credit amount if he fails to pay it at the time of maturity of the credit period. Moreover, different hybrid payment mechanisms, such as Advance-Cash, Advance-Credit, Cash-Credit, and Advance-Cash-Credit, etc. are adopted by both the sellers and buyers as a mixture of Advance-Payment, Cash-Payment, and Credit-Payment. Also, the benefits and risks of these hybrid payment mechanisms are a mixture of their original payment methods.

Several researchers formulate the classical inventory models under deterministic constraints with constant cost parameters. The optimal solutions of such models apply to the inventory problems which have no external influences in minimizing the cost or maximizing the profit. However, most of the constraints and cost parameters of inventory undergo an alteration during the real-time inventory cycle involving many socio-economical factors. For instance, the demand rate may alter by the selling price, substitute products, and quality of services. The rate of deterioration may fall or rise due to the available storage facility and the fluctuation in environmental conditions. Also, various associated costs change with the amendment of government policies or taxes. As a result, classical inventory models are inadequate to deal with most real-world inventory problems. Hence, the researchers of the present era employ the fuzzy set theory to obtain the more appropriate optimal strategy for inventory problems with imprecise costs or constraints.

The above discussion motivated us to develop an inventory model with positive lead-time under different payment strategies for const and imprecise cost parameters. Hence, we propose two inventory models for Weibull deteriorating items having selling-price-dependent demand in pre-payment, interim-payment, and post-payment scenarios. One of them is a crisp model, and another one is a fuzzy model. In the crisp model, the lead-time is positive, and the cost parameters are deterministic. Similarly, in the fuzzy model, the positive lead-time and impreciseness of cost parameters are considered. In the fuzzy model, the impreciseness of various parameters is measured via triangular-fuzzy numbers, and the corresponding total cost, initial ordering quantity, and sales revenue functions are defuzzified by the Graded Mean Integration Representation method. More precisely, we propose the solution algorithms for both models to investigate the optimal strategy. Also, we conduct a numerical examination of different sets of constraints to assess the efficiency of both models. Finally, we perform the sensitivity analysis of the main parameters to draw managerial insights. Also, more such similar inventory problems have been undertaken by various researchers. However, our proposed model is unique in the sense that, we have developed it under both crisp and fuzzy environments with certain realistic inventory constraints, such as,

(i) Positive lead time is considered.
(ii) The cases of pre-payment, cash on delivery, interim-payment and post-payment are taken into account.
(iii) The cost parameters are taken as deterministic in the crisp environment.
(iv) The cost parameters are taken as imprecise in the fuzzy environment.

Also, fuzzy set theory has been used to deal with the imprecise costs.

Literature Review

The replenishment of items is a quite common phenomenon in every retailer's day-to-day business affairs. But, the replenishment may occur instantaneously or may take some time (that is, the lead time is zero or positive) depending on the retailer's inventory. Thus, we categorize the inventory models broadly into two classes. That is, zero lead time inventories as one class and positive lead time inventories as another class. Recently, the authors like Khan *et al.* [10], Kumar *et al.* [12–14], Li *et al.* [16], Nayak *et al.* [21], Samadi *et al.* [24], Taleizadeh [27], Zang *et al.* [29], and Zou and Tian [30] obtained optimal strategies for their inventory problems having zero lead time. Furthermore, we refer the interested readers to the works of Barik *et al.* [1], Mishra *et al.* [19, 20], Routray *et al.* [23], Singh *et al.* [26] and Yu [28] for the inventory models having zero lead-time. However, Braglia *et al.* [2], Kouki *et al.* [11], Rahdar *et al.* [22], Maiti *et al.* [17], and Li [15] found the optimal strategies for retailer's inventory problem with various positive lead-time.

Furthermore, the time of payment over the purchase cost of products to the supplier has a considerable impact on the optimal inventory strategy, and it may require the pre-agreement between the retailers and suppliers. In this case, (i) Cash on Delivery, (ii) Advance Payment, and (iii) Credit Payments are some of the basic payment strategies. Also, a combination of two or more of these may be adopted by both retailers and suppliers. In this context, many inventory models have recently been developed by Samadi *et al.* [24], Rahdar *et al.* [22], Braglia *et al.* [2] and Nayak *et al.* [21] based on the cash on delivery payment mechanism. Subsequently, Maiti *et al.* [17] proposed a stochastic inventory model for time-dependent demand items under the assumption of probabilistic lead-time and advance payment mechanism. Taleizadeh [27] formulated a mathematical procedure for obtaining the economic ordering quantity for evaporating items under the consideration of partial backlogging and multiple equi-sized prepayments. Khan *et al.* [10] presented a inventory model having the items whose demand depends on selling-price and frequency of advertisements under an advance payment mechanism and established optimal strategy for the retailers. Khakzad and Gholamian [7] discussed an inventory model with advance payment, where the average rate of deterioration is linked to the number of inspection times. Further, the investment of preservation technology and green technology were considered by Mashud *et al.* [18] in their inventory problem to reduce both carbon emission and product deterioration under the prepayment scheme. Whereas, Kumar *et al.* [12–14], Jaggi *et al.* [5], Kaliraman *et al.* [6] and Shaikh *et al.* [25] derived the optimal policies for different inventory problems under the assumption of trade credit financing (partial and complete). However, Zang *et al.* [29] studied two inventory models for optimal strategy under advance payment mechanism and advance-credit payment mechanism. Afterwards, Li *et al.* [16] examined the supplier-retailer-customer chain inventory problem having an advance-cash-credit payment mechanism and proposed the optimal strategies. Zou and Tian [30] investigated the optimal result for supply chain inventory with an advance-credit payment facility. Khan *et al.* [9] formulated an EOQ

Table 1 Overview of inventory models of literature

Author(s)	Year	Lead time	Payment Mechanism	Impreciseness
Maiti *et al.* [17]	2009	Stochastic	Advance-Cash	No
Samadi *et al.* [24]	2013	Zero	Cash	All parameters
Taleizadeh [27]	2014	Zero	Advance-Cash	No
Zhang *et al.* [29]	2014	Zero	(i)Advance (ii)Advance-Credit	No
Rahdar *et al.* [22]	2018	Uncertainty	Cash	No
Kumar *et al.* [13]	2019	Zero	Credit	Cost Parameters
Braglia *et al.* [2]	2019	Deterministic/Stochastic	Cash	No
Khan *et al.* [10]	2020	Zero	Advance-Cash	No
Zou and Tian [30]	2020	Zero	Advance-Credit	No
Kumar *et al.* [12]	2020	Zero	Credit	Demand Parameters Deterioration parameters
Kumar *et al.* [14]	2020	Zero	Credit	Cost Parameters
Li *et al.* [16]	2021	Zero	Advance-Cash-Credit	No
Nayak *et al.* [21]	2021	Zero	Cash-Payment	Cost parameters
This paper		Deterministic	(i) Pre-Payment (ii) Interim-Payment (iii) Post-Payment	Cost parameters

model to investigate whether a rented warehouse is needed or not for an inventory having an advance-credit payment mechanism. Duary *et al.* [3] studied a two-warehouse inventory problem, where the supplier offers a price discount for partial advance payment as well as a delay in payment for the rest amount. Again, Khan *et al.* [8] developed a mathematical model for inventory having hybrid advance-cash payment for non-instantaneous deteriorating items in two scenarios (one with back-ordering and other having no shortages).

In general, the market experts propose the deterministic inventory constraints and constant cost parameters based upon available historic data for developing the inventory models. Thus, the optimal strategies obtained in these inventory models are appropriate when the constraints and cost parameters unalter during real-time inventory management. But, in the actual scenario, the cost parameters and constraints may be influenced by many factors and become uncertain. That is, the available information is imprecised or known with uncertainty. Hence, various researchers applied fuzzy set theory for inventory models with imprecised information or values known with uncertainty to obtain optimal strategy. In the past decade, Samadi *et al.* [24], Inrajithsingha *et al.* [4], Kumar *et al.* [12–14], Nayak *et al.* [21] employed fuzzy set theory in their inventory models to obtain optimal strategies that are fit to real inventory problems with imprecised information.

The inventory model for the positive lead-time of uncertain type was considered by Rahdar *et al.* [22], and Braglia *et al.* [2] studied the deterministic (or stochastic) type inventory model under the cash-payment strategy. Subsequently, the stochastic lead-time was considered by Maiti *et al.* [17] under the advance-cash strategy. However, these authors overlooked the impreciseness of parameters in their inventory models. On the other hand, the impreciseness of parameters was taken into account by Kumar *et al.* [12–14] with credit-payments, and Samadi *et al.* [24] and Nayak *et al.* [21] with cash-payment in their inventory models with zero lead-time. Whereas, Taleizadeh [27], Zang *et al.* [29], Khan *et al.* [10], Zou and Tian [30], Li *et al.* [16] developed inventory models under various payment strategies with zero lead-time and neglected the impreciseness of parameters. It has been observed that none of the above researchers investigated the optimal results for the inventory model having positive lead-time with or without imprecised parameters under pre-payment, interim-payment, and post-payment strategies together. For quick insight into the literature, we draw the attention of the readers to Table 1.

Hence, motivated essentially by the above-mentioned investigations, we develop an inventory model for Weibull deteriorating items having selling-price-dependent demand in both crisp and fuzzy environments under pre-payment, interim-payment, and post-payment strategies.

Broadly, our investigation presents:

(i) An inventory model for the Weibull deteriorating items having selling-price-dependent demand with the integration of positive lead-time with pre-payment, interim-payment, and post-payment strategy in a crisp environment to investigate the optimal strategy. The results of this model are more appropriate for real inventory problems with no external influences for minimization of cost.

(ii) The model proposed in the crisp environment further reconstructed in the fuzzy environment to investigate the optimal strategy with known parameter imprecision. The results of this model are quite appropriate for the real inventory problems as most of the parametric values of inventory in the real world are known with uncertainty.

(iii) The solution procedure presented for both the models, and the fuzzy set theory is applied to cope with the uncertainty of parameters in the fuzzy model.

(iv) Extensive numerical experiments are conducted to verify the effectiveness of the suggested model to draw management insights.

Notations

The terminology for the formulation of the inventory model presented here in Table 2.

Assumptions

The present model is developed under the following assumptions.

(i) The inventory system manages homogeneous items.
(ii) The inventory cycle starts at time $t = 0$ and terminates at time $t = T$.
(iii) The $\mathcal{W}$ items are to order at the beginning of the cycle.
(iv) At the time $t = \mathcal{L}$, the retailers receive the items.
(v) During the lead time retailer does not lose any customers.
(vi) On mutual understanding between retailer and distributor, the retailer pays the total purchase amount to the supplier at time $t = \mathcal{M}$.
(vii) The items in the inventory follows the demand rate $D(s)$ depends on the selling price s. Here, $D(s) = ms^{-n}$ and $m, n > 0$.
(viii) Items in the inventory follow two-parameter Weibull-distribution deterioration. That is, the number of items that deteriorate at any time t is $\alpha\beta t^{\beta-1}$.
(ix) The cost parameters in the crisp model are deterministic and in the fuzzy model are imprecise.
(x) The imprecised parameters in the fuzzy environment are measured by the triangular-fuzzy numbers, and the Graded Mean Integration technique is applied to defuzzify the fuzzy expressions and fuzzy functions.

Crisp Model

Mathematical Formulation

The retailer orders an amount of $\mathcal{W}$ items to the supplier at the beginning of each inventory cycle. But, due to lead time $\mathcal{L}$, these items are delivered to the retailer at time $t = \mathcal{L}$. Thus, the backlogged demand during the lead time $[0, \mathcal{L}]$ is cleared from these $\mathcal{W}$ items first. Then, the remaining $\mathcal{Q}$ items are send to the warehouse. After this, the inventory level $\mathcal{Q}$ starts diminishing due to demand and deterioration from time $t = \mathcal{L}$ and reaches zero at time $t = T$.

Let $y(t)$ be the inventory at time t, and $y(t + \Delta t)$ be the inventory at time $t + \Delta t$.

We have,

$$y(t + \Delta t) = y(t) - \alpha\beta t^{\beta-1} y(t)\Delta t - ms^{-n}\Delta t$$
$$\Rightarrow \frac{y(t + \Delta(t) - y(t)}{\Delta t} + \alpha\beta t^{\beta-1} y(t) = -ms^{-n}.$$

Table 2 Notations

Notation	Description
$y(t)$	Inventory level at any time t
$\mathcal{W}$	Initial ordering quantity
$\mathcal{Q}$	Maximum inventory level during the cycle
$\mathcal{L}$	Lead time
T	Total cycle time
$\mathcal{M}$	Timing at which the payment is to be made
$\mathcal{L}_{\mathcal{D}}$	Total demand during the lead time
s	Selling price per unit item
p	Purchase cost per unit item
d	Deterioration cost per unit item
h	Holding cost per unit item per unit time
θ	Rate of interest payable
ϑ	Rate of interest earn
$\mathcal{O}_{\mathcal{C}} = A$	Ordering cost per cycle
$\mathcal{P}_{\mathcal{C}}$	Total purchasing cost
$\mathcal{H}_{\mathcal{C}}$	Total holding cost
$\mathcal{D}_{\mathcal{C}}$	Total deteriorating cost
$\mathcal{S}_{\mathcal{R}}$	Total sales revenue during the cycle
$IP1$	Total interest payable by the retailer during the advance payment strategy $(0 \leq \mathcal{M} \leq \mathcal{L} < T)$
$IP2$	Total interest payable by the retailer during the interim payment strategy $(\mathcal{L} \leq \mathcal{M} \leq T)$
$IP3$	Total interest payable by the retailer during the credit payment strategy $(\mathcal{L} < T \leq \mathcal{M})$
$EI1$	Total interest earned by the retailer during the advance payment strategy $(0 \leq \mathcal{M} \leq \mathcal{L} < T)$
$EI2$	Total interest earned by the retailer during the interim payment strategy $(\mathcal{L} \leq \mathcal{M} \leq T)$
$EI3$	Total interest earned by the retailer during the credit payment strategy $(\mathcal{L} < T \leq \mathcal{M})$
$\mathcal{TC}1$	Total inventory cost during the advance payment strategy $(0 \leq \mathcal{M} \leq \mathcal{L} < T)$
$\mathcal{TC}2$	Total inventory cost during the interim payment strategy $(\mathcal{L} \leq \mathcal{M} \leq T)$
$\mathcal{TC}3$	Total inventory cost during the credit payment strategy $(\mathcal{L} < T \leq \mathcal{M})$
$\tilde{A}$	Fuzzy ordering cost per cycle
$\tilde{h}$	Fuzzy holding cost per unit item per unit time
$\tilde{p}$	Fuzzy purchase cost per unit item
$\tilde{s}$	Fuzzy Selling price per unit item
$\tilde{d}$	Fuzzy deterioration cost per unit item
$\widetilde{\mathcal{TC}}1$	Fuzzy total inventory cost during the advance payment strategy $(0 \leq \mathcal{M} \leq \mathcal{L} < T)$
$\widetilde{\mathcal{TC}}2$	Fuzzy total inventory cost during the interim payment strategy $(\mathcal{L} \leq \mathcal{M} \leq T)$
$\widetilde{\mathcal{TC}}3$	Fuzzy total inventory cost during the credit payment strategy $(\mathcal{L} < T \leq \mathcal{M})$
$\tilde{\mathcal{S}}_{\mathcal{R}}$	Fuzzy total sales revenue during the entire cycle
$G\mathcal{TC}1$	Defuzzified total inventory cost during the advance payment strategy $(0 \leq \mathcal{M} \leq \mathcal{L} < T)$
$G\mathcal{TC}2$	Defuzzified total inventory cost during the interim payment strategy $(\mathcal{L} \leq \mathcal{M} \leq T)$
$G\mathcal{TC}3$	Defuzzified total inventory cost during the credit payment strategy $(\mathcal{L} < T \leq \mathcal{M})$
$G\mathcal{S}_{\mathcal{R}}$	Defuzzified total sales revenue during the entire cycle

Taking limit $\Delta t \to 0$, we obtain the following differential equation:

$$\frac{dy(t)}{dt} + \alpha\beta t^{\beta-1} y(t) = -ms^{-n} \tag{1}$$

with boundary condition $y(T) = 0$, which illustrates the inventory level at any instant of time t.
It is easy to see the solution of the eq. (1) as

$$y(t) = ms^{-n}\big[(T-t) + \frac{\alpha}{\beta+1}(T^{\beta+1} - t^{\beta+1})\big](1-\alpha t^{\beta}). \tag{2}$$

Associated Expressions and Costs

Following expressions or costs can be obtained by using eq. (2), and those are useful for calculating the total inventory cost.

Maximum Inventory

The maximum inventory level is

$$\mathcal{Q} = y(\mathcal{L}) = ms^{-n}\big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha\mathcal{L}^{\beta}). \tag{3}$$

Lead Time Demand

The total demand during the lead time $[0, \mathcal{L}]$ is

$$\mathcal{L}_{\mathcal{D}} = \mathcal{L} * D(s) = \mathcal{L} * ms^{-n}. \tag{4}$$

Initial Ordering Quantity

The inventory replenishment quantity is

$$\begin{aligned}\mathcal{W} &= \mathcal{L}_{\mathcal{D}} + \mathcal{Q}\\ &= ms^{-n}\Big[\mathcal{L} + \big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha t^{\beta})\Big].\end{aligned} \tag{5}$$

Ordering Cost

The ordering cost per cycle is

$$\mathcal{O}_{\mathcal{C}} = A. \tag{6}$$

Purchase Cost

The total payment to be made by the retailer to the supplier towards purchase quantity is

$$\begin{aligned}\mathcal{P}_{\mathcal{C}} &= p * \mathcal{W}\\ &= p * \Big\{ms^{-n}\Big[\mathcal{L} + \big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha t^{\beta})\Big]\Big\}.\end{aligned} \tag{7}$$

Holding Cost

The total holding cost of the inventory during the cycle is

$$\begin{aligned}\mathcal{H}_{\mathcal{C}} &= h * \int_{\mathcal{L}}^{T} y(t)\,dt\\ &= h * \int_{\mathcal{L}}^{T} \Big[ms^{-n}\big[(T-t) + \frac{\alpha}{\beta+1}(T^{\beta+1} - t^{\beta+1})\big](1-\alpha t^{\beta})\Big]\,dt\\ &= hms^{-n}\Bigg[\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} - \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{L}^{\beta+2}}{\beta+2}\Big)\\ &\quad - \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2}\Big)\\ &\quad - \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{T^{\beta+1}\mathcal{L}^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2}\Big)\Bigg]. \end{aligned} \tag{8}$$

Deterioration Cost

The total deteriorating cost of the inventory during the cycle is

$$\begin{aligned}\mathcal{D}_{\mathcal{C}} &= d\Big[\mathcal{Q} - \Big(T - \mathcal{L}\Big)D(s)\Big]\\ &= d\Big[ms^{-n}\big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha\mathcal{L}^{\beta}) - \Big(T-\mathcal{L}\Big)ms^{-n}\Big]\\ &= dms^{-n}\Big[\frac{\alpha}{\beta+1}\big(T^{\beta+1} - \mathcal{L}^{\beta+1}\big)\big(1-\alpha\mathcal{L}^{\beta}\big) - \alpha\mathcal{L}^{\beta}\big(T-\mathcal{L}\big)\Big]. \end{aligned} \tag{9}$$

Payment Strategy

In practice, the retailers borrow a short-term loan from the bank or any financial sector to invest in the inventory, and they need to pay a fixed interest on the loan. Similarly, in some scenarios, the supplier provides trade credit financing to the retailer. Thus, it acts as an asset to retailers by reducing the investment and earning interest on accumulated revenue. Also, the retailers and suppliers follow so many payment strategies in their business transactions. Each payment strategy has benefits and drawbacks for the retailers as well as suppliers. Hence, we consider that the supplier offers various payment strategies to different retailers as per their priority. In detail, pre-payment to the new retailers, cash on delivery for bad retailers, and credit-payment (interim or post-payment) to the old and well-behaved retailers. The effect of these payments on the total cost of the retailer's inventory is as below.

Pre-Payment Strategy ($0 \le \mathcal{M} \le \mathcal{L} < T$)

The retailer makes the total payment of the ordered quantity to the supplier (at $t = \mathcal{M}$) before the stock delivery (at $t = \mathcal{L}$). Thus, the retailer borne an interest on investment from $t = \mathcal{M}$ up to $t = \mathcal{L}$ and then stock left in the inventory till $t = T$. Also, it does not earn any interest. As a result, the total cost of inventory may increase. The calculations of interest are as below.

The total interest payable by the retailer in the pre-payment strategy is

$$IP1 = \Big[\big(\mathcal{L} - \mathcal{M}\big)\big(\mathcal{Q} + \mathcal{L}D(s)\big) + \int_{\mathcal{L}}^{T} y(t)\,dt\Big]p\theta$$

$$
\begin{aligned}
&= ms^{-n}\Bigg[(\mathcal{L}-\mathcal{M})\Big(\mathcal{L}+\Big((T-\mathcal{L})+\frac{\alpha}{\beta+1}(T^{\beta+1}-\mathcal{L}^{\beta+1})\Big)(1-\alpha\mathcal{L}^{\beta})\Big)\\
&+\Big(\frac{T^2}{2}-\mathcal{L}T+\frac{\mathcal{L}^2}{2}+\frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1}-\mathcal{L}T^{\beta+1}-\frac{T^{\beta+2}}{\beta+2}-\frac{\mathcal{L}^{\beta+2}}{\beta+2}\Big)\\
&-\frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2}-T^{\beta+2}\Big)\\
&-\frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1}-\frac{T^{2\beta+2}}{2\beta+2}-\frac{(T\mathcal{L})^{\beta+1}}{\beta+1}+\frac{\mathcal{L}^{2\beta+2}}{2\beta+2}\Big)\Big)\Bigg]p\theta. \qquad (10)
\end{aligned}
$$

The total interest earned by the retailer in the pre-payment strategy is

$$EI1 = 0. \qquad (11)$$

Note: When $\mathcal{M} = \mathcal{L}$, the pre-payment strategy becomes cash on delivery.

Interim-Payment Strategy ($\mathcal{L} \leq \mathcal{M} \leq T$)

The interim payment acts as a partial trade credit to the retailer. So, the retailer earn the interest on accumulated sales from $t = \mathcal{L}$ up to $t = \mathcal{M}$ and borne the interest on stock left in the inventory from $t = \mathcal{M}$ to $t = T$. Thus, the total inventory cost may increase or decrease depending upon the difference of interest earned and payable. The calculations of interest are given below.

The total interest payable by the retailer in the interim-payment strategy is

$$
\begin{aligned}
IP2 &= \Big(\int_{\mathcal{M}}^{T} y(t)\,dt\Big)p\,\theta\\
&= ms^{-n}\Bigg[\frac{T^2}{2}-\mathcal{M}T+\frac{\mathcal{M}^2}{2}+\frac{\alpha}{\beta+1}\Big(T\mathcal{M}^{\beta+1}-\mathcal{M}T^{\beta+1}-\frac{T^{\beta+2}}{\beta+2}+\frac{\mathcal{M}^{\beta+2}}{\beta+2}\Big)\\
&-\frac{\alpha}{\beta+2}\Big(\mathcal{M}^{\beta+2}-T^{\beta+2}\Big)\\
&-\frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1}-\frac{T^{2\beta+2}}{2\beta+2}-\frac{(T\mathcal{M})^{\beta+1}}{\beta+1}+\frac{\mathcal{M}^{2\beta+2}}{2\beta+2}\Big)\Bigg]p\theta. \qquad (12)
\end{aligned}
$$

The total interest earned by the retailer in the interim-payment strategy is

$$
\begin{aligned}
EI2 &= \Big((\mathcal{M}-\mathcal{L})\mathcal{L}D(s)+\int_{\mathcal{L}}^{\mathcal{M}} tD(s)\,dt\Big)s\vartheta\\
&= ms^{-n}\Big((\mathcal{M}-\mathcal{L})\mathcal{L}+\Big(\frac{\mathcal{M}^2}{2}-\frac{\mathcal{L}^2}{2}\Big)\Big)s\vartheta. \qquad (13)
\end{aligned}
$$

Note: When $\mathcal{L} = \mathcal{M}$, the interim-payment strategy becomes cash on delivery. As a result, the retailer does't earn any interest.

Post-Payment Strategy ($\mathcal{L} < T \leq \mathcal{M}$)

The post-payment acts as a full trade credit to the retailer. So, the retailer earns the interest on accumulated sales from $t = \mathcal{L}$ to $t = T$ and the total sales revenue from $t = T$ to $t = \mathcal{M}$. In this case, the retailers do not bear any interest. As a result, the total cost of inventory may decrease. The calculations of interest are as below.

The total interest payable by the retailer in the post-payment strategy is

$$IP3 = 0. \tag{14}$$

The total interest earned by the retailer in the post-payment strategy is

$$\begin{aligned} EI3 &= \left[(\mathcal{M} - \mathcal{L})\mathcal{L}ms^{-n} + \int_{\mathcal{L}}^{T} tms^{-n}\,dt + (\mathcal{M} - T)(T - \mathcal{L})ms^{-n} \right] s\vartheta \\ &= ms^{-n}\left(\frac{-3\mathcal{L}^2}{2} - \frac{T^2}{2} + (\mathcal{M} + \mathcal{L})T \right) s\vartheta. \end{aligned} \tag{15}$$

Total Costs

Taking all the costs of inventory together, the total cost of the inventory cycle under various payment strategies are as follows.

Pre-Payment Strategy

The total inventory cost of retailers with a pre-payment strategy is

$$\begin{aligned} \mathcal{TC}1 &= \frac{1}{T}\Big[\mathcal{O_C} + \mathcal{P_C} + \mathcal{H_C} + \mathcal{D_C} - IP1 + EI1 \Big] \\ &= \frac{1}{T}\Bigg[A + pms^{-n}\Big[\mathcal{L} + \big[(T - \mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1}) \big](1 - \alpha t^{\beta}) \Big] \\ &\quad + hms^{-n}\Bigg[\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} - \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{L}^{\beta+2}}{\beta+2} \Big) \\ &\quad - \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2} \Big) - \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} \\ &\quad - \frac{T^{2\beta+2}}{2\beta+2} - \frac{T^{\beta+1}\mathcal{L}^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2} \Big) \Bigg] \\ &\quad + dms^{-n}\Big[\frac{\alpha}{\beta+1}\big(T^{\beta+1} - \mathcal{L}^{\beta+1} \big)\big(1 - \alpha\mathcal{L}^{\beta} \big) - \alpha\mathcal{L}^{\beta}\big(T - \mathcal{L} \big) \Big] \\ &\quad - p\theta ms^{-n}\Bigg[(\mathcal{L} - \mathcal{M})\Big(\mathcal{L} + \Big((T - \mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1}) \Big)\big(1 - \alpha\mathcal{L}^{\beta}\big) \Big) \\ &\quad + \Bigg(\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} \\ &\quad - \frac{T^{\beta+2}}{\beta+2} - \frac{\mathcal{L}^{\beta+2}}{\beta+2} \Big) - \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2} \Big) \\ &\quad - \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{(T\mathcal{L})^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2} \Big) \Bigg) \Bigg] \Bigg]. \end{aligned} \tag{16}$$

Interim-Payment Strategy

The total inventory cost of retailers with an interim-payment strategy is

$$\mathcal{TC}2 = \frac{1}{T}\Big[\mathcal{O_C} + \mathcal{P_C} + \mathcal{H_C} + \mathcal{D_C} - IP2 + EI2 \Big]$$

$$
\begin{aligned}
&= \frac{1}{T}\Bigg[A + pms^{-n}\Big[\mathcal{L} + \big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha t^{\beta})\Big] \\
&+ hms^{-n}\Bigg[\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} - \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{L}^{\beta+2}}{\beta+2}\Big) \\
&- \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2}\Big) \\
&- \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{T^{\beta+1}\mathcal{L}^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2}\Big)\Bigg] \\
&+ dms^{-n}\Big[\frac{\alpha}{\beta+1}\big(T^{\beta+1} - \mathcal{L}^{\beta+1}\big)\big(1 - \alpha\mathcal{L}^{\beta}\big) \\
&- \alpha\mathcal{L}^{\beta}\big(T - \mathcal{L}\big)\Big] - p\theta ms^{-n}\Bigg[\frac{T^2}{2} - \mathcal{M}T + \frac{\mathcal{M}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{M}^{\beta+1} - \mathcal{M}T^{\beta+1} \\
&- \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{M}^{\beta+2}}{\beta+2}\Big) - \frac{\alpha}{\beta+2}\Big(\mathcal{M}^{\beta+2} - T^{\beta+2}\Big) \\
&- \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{(T\mathcal{M})^{\beta+1}}{\beta+1} + \frac{\mathcal{M}^{2\beta+2}}{2\beta+2}\Big)\Bigg] \\
&+ s\vartheta ms^{-n}\Bigg[\big(\mathcal{M} - \mathcal{L}\big)\mathcal{L} + \Big(\frac{\mathcal{M}^2}{2} - \frac{\mathcal{L}^2}{2}\Big)\Bigg]\Bigg]. \qquad (17)
\end{aligned}
$$

Post-Payment Strategy

The total inventory cost of retailers with a post-payment strategy is

$$
\begin{aligned}
\mathcal{TC}3 &= \frac{1}{T}\Big[\mathcal{O_C} + \mathcal{P_C} + \mathcal{H_C} + \mathcal{D_C} - IP3 + EI3\Big] \\
&= \frac{1}{T}\Bigg[A + pms^{-n}\Big[\mathcal{L} + \big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha t^{\beta})\Big] \\
&+ hms^{-n}\Bigg[\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} - \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{L}^{\beta+2}}{\beta+2}\Big) \\
&- \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2}\Big) \\
&- \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{T^{\beta+1}\mathcal{L}^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2}\Big)\Bigg] \\
&+ dms^{-n}\Big[\frac{\alpha}{\beta+1}\big(T^{\beta+1} - \mathcal{L}^{\beta+1}\big)\big(1 - \alpha\mathcal{L}^{\beta}\big) \\
&- \alpha\mathcal{L}^{\beta}\big(T - \mathcal{L}\big)\Big] + s\vartheta ms^{-n}\Bigg[\frac{-3\mathcal{L}^2}{2} - \frac{T^2}{2} + (\mathcal{M} + \mathcal{L})T\Bigg]\Bigg]. \qquad (18)
\end{aligned}
$$

Sales Revenue

The total sales revenue of the inventory cycle is

$$
\mathcal{S_R} = T * D(s) * s = T * m * s^{-n} * s = Tms^{1-n}. \qquad (19)
$$

Solution Algorithm

The procedure for obtaining the optimal strategy for the crisp model is as follows.

Pre-Payment or Cash-Payment

In the pre-payment or cash-payment strategy ($\mathcal{M} \leq \mathcal{L}$), the optimal strategy can be obtained as follows:

Step 1 Using the eq. (16), find T^* such that $\frac{d\mathcal{TC}1}{dT}(T^*) = 0$ and $\frac{d^2\mathcal{TC}1}{dT^2}(T^*) > 0$.
Step 2 Set $T = T^*$, then by using the eqs. (16), (5), and (19) obtain the total cost $\mathcal{TC}1$, initial ordering quantity $\mathcal{W}$ and the sales revenue $\mathcal{S_R}$.
Step 3 Finaly, the simplification leads to the optimal strategy, that is, total cycle time T, total inventory cost $\mathcal{TC}1$, initial ordering quantity $\mathcal{W}$ and total sales revenue $\mathcal{S_R}$.

Interim-payment or Post-Payment

In general, the suppliers offer trade credit (that is, $\mathcal{L} < \mathcal{M}$) to their old and good-credit retailers. However, the trade credit may act as an interim payment or post-payment strategy to the retailer depending upon his (or her) inventory constraints. Thus, the optimal results and payment strategy for the retailer's inventory can be obtained as follows:

Step 1 Using the eq. (17), find $T2^*$ such that $\frac{d\mathcal{TC}2}{dT}(T2^*) = 0$ and $\frac{d^2\mathcal{TC}2}{dT^2}(T2^*) > 0$.
Step 2 Using the eq. (18), find $T3^*$ such that $\frac{d\mathcal{TC}3}{dT}(T3^*) = 0$ and $\frac{d^2\mathcal{TC}3}{dT^2}(T3^*) > 0$.
Step 3 If $\mathcal{L} < \mathcal{M} \leq T2^*$, then

(i) by setting $T = T2^*$, and using the eqs. (17), (5) and (19) obtain the total cost $\mathcal{TC}2$, initial ordering quantity $\mathcal{W}$ and the sales revenue $\mathcal{S_R}$
(ii) obtain the optimal results, that is, total cycle time T, inventory cost $\mathcal{TC}2$, initial ordering quantity $\mathcal{W}$ and total sales revenue $\mathcal{S_R}$
(iii) the retailer can opt for the interim-payment strategy.

Step 4 If $\mathcal{L} < T3^* \leq \mathcal{M}$, then

(i) setting $T = T3^*$, and by using the eqs. (18), (5) and (19) obtain the total cost $\mathcal{TC}3$, initial ordering quantity $\mathcal{W}$ and sales revenue $\mathcal{S_R}$
(ii) obtain the optimal results, that is, total cycle time T, inventory cost $\mathcal{TC}3$, initial ordering quantity $\mathcal{W}$ and total sales revenue $\mathcal{S_R}$
(iii) the retailer can opt for the post-payment strategy.

The entire crisp model can be visualized from the flow chart given below.

Fuzzy Model

While developing the mathematical model for the inventory system to minimize the total cost, the associated parameters are considered deterministic. But, in a real scenario, the parameters such as ordering cost, holding cost, purchase cost, selling price, deterioration cost, demand, lead time, etc. involved in the model are fuzzy. We know that the ordering cost consists of loading, unloading, and transportation cost. Thus, the inflation in fuel rates

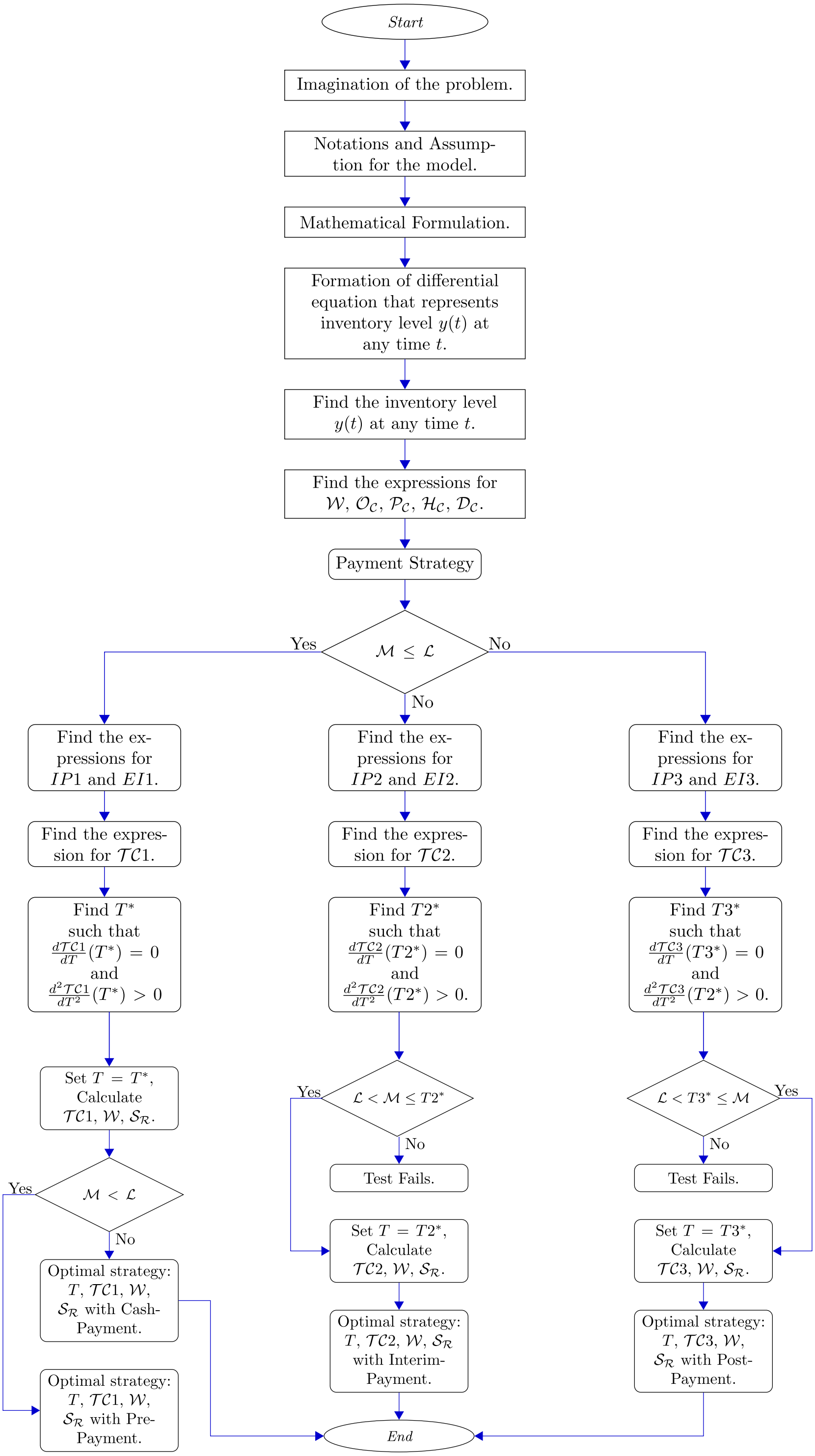


Fig. 1 Flow Chart of Crisp Model

indirectly causes impreciseness in ordering costs. In some scenarios, the suppliers offer a price discount depending on the order quantity. As a result, the purchase cost may be imprecise. The deterioration cost may also be imprecise due to technical advancement and unexpected climate changes. Similarly, the holding cost, selling price, and other constraints may be imprecise due to several factors. Taking all these shortcomings into account, here we consider the order cost A, holding cost h, purchasing cost p, selling price s and the deteriorating cost d as imprecise parameters, and are represented by the fuzzy numbers $\tilde{A}$, $\tilde{h}$, $\tilde{p}$, $\tilde{s}$ and $\tilde{d}$ respectively.

Total Costs

On approaching in the similar lines of the crisp model, we get the following total costs in each payment strategy and the sales revenue for the inventory model in a fuzzy environment.

Pre-Payment Strategy

The total cost function for the inventory in pre-payment strategy with impreciseed parameters is

$$\begin{aligned}
\widetilde{TC}1 &= \frac{1}{T}\Big[\mathcal{O_C} + \mathcal{P_C} + \mathcal{H_C} + \mathcal{D_C} - IP1 + EI1\Big] \\
&= \frac{1}{T}\Big[\tilde{A} + \tilde{p}m\tilde{s}^{-n}\Big[\mathcal{L} + \big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha t^{\beta})\Big] \\
&+ \tilde{h}m\tilde{s}^{-n}\Big[\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} - \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{L}^{\beta+2}}{\beta+2}\Big) \\
&- \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2}\Big) - \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} \\
&- \frac{T^{2\beta+2}}{2\beta+2} - \frac{T^{\beta+1}\mathcal{L}^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2}\Big)\Big] + \tilde{d}m\tilde{s}^{-n}\Big[\frac{\alpha}{\beta+1}\big(T^{\beta+1} - \mathcal{L}^{\beta+1}\big)\big(1-\alpha\mathcal{L}^{\beta}\big) \\
&- \alpha\mathcal{L}^{\beta}\big(T-\mathcal{L}\big)\Big] \\
&- \tilde{p}\theta m\tilde{s}^{-n}\Big[\big(\mathcal{L}-\mathcal{M}\big)\Big(\mathcal{L} + \Big((T-\mathcal{L}) + \frac{\alpha}{\beta+1}\big(T^{\beta+1} - \mathcal{L}^{\beta+1}\big)\Big)\big(1-\alpha\mathcal{L}^{\beta}\big)\Big) \\
&+ \Big(\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} - \frac{T^{\beta+2}}{\beta+2} - \frac{\mathcal{L}^{\beta+2}}{\beta+2}\Big) \\
&- \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2}\Big) \\
&- \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{(T\mathcal{L})^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2}\Big)\Big)\Big]\Big]. \qquad (20)
\end{aligned}$$

Interim-Payment Strategy

The total cost function for the inventory in interim-payment strategy with impreciseed parameters is

$$\widetilde{TC}2 = \frac{1}{T}\Big[\mathcal{O_C} + \mathcal{P_C} + \mathcal{H_C} + \mathcal{D_C} - IP2 + EI2\Big]$$

$$
\begin{aligned}
&= \frac{1}{T}\Big[\tilde{A} + \tilde{p}m\tilde{s}^{-n}\Big[\mathcal{L} + \big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha t^{\beta})\Big] \\
&+ \tilde{h}m\tilde{s}^{-n}\Big[\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} - \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{L}^{\beta+2}}{\beta+2}\Big) \\
&- \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2}\Big) - \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{T^{\beta+1}\mathcal{L}^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2}\Big)\Big] \\
&+ \tilde{d}m\tilde{s}^{-n}\Big[\frac{\alpha}{\beta+1}\big(T^{\beta+1} - \mathcal{L}^{\beta+1}\big)\big(1-\alpha\mathcal{L}^{\beta}\big) \\
&- \alpha\mathcal{L}^{\beta}\big(T-\mathcal{L}\big)\Big] - \tilde{p}\theta m\tilde{s}^{-n}\Big[\frac{T^2}{2} - \mathcal{M}T + \frac{\mathcal{M}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{M}^{\beta+1} - \mathcal{M}T^{\beta+1} \\
&- \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{M}^{\beta+2}}{\beta+2}\Big) - \frac{\alpha}{\beta+2}\Big(\mathcal{M}^{\beta+2} - T^{\beta+2}\Big) \\
&- \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{(T\mathcal{M})^{\beta+1}}{\beta+1} + \frac{\mathcal{M}^{2\beta+2}}{2\beta+2}\Big)\Big] \\
&+ \tilde{s}\vartheta m\tilde{s}^{-n}\Big[\big(\mathcal{M}-\mathcal{L}\big)\mathcal{L} + \Big(\frac{\mathcal{M}^2}{2} - \frac{\mathcal{L}^2}{2}\Big)\Big]\Big]. \qquad (21)
\end{aligned}
$$

Post-Payment Strategy

The total cost function for the inventory in a post-payment strategy with impreciséd parameters is

$$
\begin{aligned}
\widetilde{\mathcal{TC}}3 &= \frac{1}{T}\Big[\mathcal{O}_{\mathcal{C}} + \mathcal{P}_{\mathcal{C}} + \mathcal{H}_{\mathcal{C}} + \mathcal{D}_{\mathcal{C}} - IP3 + EI3\Big] \\
&= \frac{1}{T}\Big[\tilde{A} + \tilde{p}m\tilde{s}^{-n}\Big[\mathcal{L} + \big[(T-\mathcal{L}) + \frac{\alpha}{\beta+1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1-\alpha t^{\beta})\Big] \\
&+ \tilde{h}m\tilde{s}^{-n}\Big[\frac{T^2}{2} - \mathcal{L}T + \frac{\mathcal{L}^2}{2} + \frac{\alpha}{\beta+1}\Big(T\mathcal{L}^{\beta+1} - \mathcal{L}T^{\beta+1} - \frac{T^{\beta+2}}{\beta+2} + \frac{\mathcal{L}^{\beta+2}}{\beta+2}\Big) \\
&- \frac{\alpha}{\beta+2}\Big(\mathcal{L}^{\beta+2} - T^{\beta+2}\Big) - \frac{\alpha^2}{\beta+1}\Big(\frac{T^{2\beta+1}}{\beta+1} - \frac{T^{2\beta+2}}{2\beta+2} - \frac{T^{\beta+1}\mathcal{L}^{\beta+1}}{\beta+1} + \frac{\mathcal{L}^{2\beta+2}}{2\beta+2}\Big)\Big] \\
&+ \tilde{d}m\tilde{s}^{-n}\Big[\frac{\alpha}{\beta+1}\big(T^{\beta+1} - \mathcal{L}^{\beta+1}\big)\big(1-\alpha\mathcal{L}^{\beta}\big) \\
&- \alpha\mathcal{L}^{\beta}\big(T-\mathcal{L}\big)\Big] + \tilde{s}\vartheta m\tilde{s}^{-n}\Big[\frac{-3\mathcal{L}^2}{2} - \frac{T^2}{2} + (\mathcal{M}+\mathcal{L})T\Big]\Big]. \qquad (22)
\end{aligned}
$$

Sales Revenue

The total sales revenue of the inventory cycle in the fuzzy environment is

$$\widetilde{\mathcal{S}}_{\mathcal{R}} = Tm\tilde{s}^{1-n}. \qquad (23)$$

Defuzzyfication

In this work, we assumed that the impreciseness of cost parameters follows the triangular-fuzzy numbers. Thus, we represent them as $\tilde{A} = (A_1, A_2, A_3)$, $\tilde{h} = (h_1, h_2, h_3)$, $\tilde{d} =$

(d_1, d_2, d_3), $\tilde{p} = (p_1, p_2, p_3)$, and $\tilde{s} = (s_1, s_2, s_3)$. Thereafter, the Graded Mean Integration Representation technique is applied to defuzzify the total costs and the sales revenue function. Thus, the resultant defuzzified total costs, sales revenue, and initial ordering quantity are mentioned below.

Total Costs

The defuzzified total costs in different payment strategies are:

Pre-Payment Strategy
The total defuzzified cost of inventory in pre-payment strategy is

$$G\mathcal{TC}1 = \frac{1}{4}\big[TC_{11} + 2TC_{12} + TC_{13}\big] \quad (0 \leq \mathcal{M} \leq \mathcal{L} < T). \tag{24}$$

Interim-Payment Strategy
The total defuzzified cost of inventory in interim-payment strategy is

$$G\mathcal{TC}2 = \frac{1}{4}\big[TC_{21} + 2TC_{22} + TC_{23}\big] \quad (\mathcal{L} \leq \mathcal{M} \leq T). \tag{25}$$

Post-Payment Strategy
The total defuzzified cost of inventory in the post-payment strategy is

$$G\mathcal{TC}3 = \frac{1}{4}\big[TC_{31} + 2TC_{32} + TC_{33}\big] \quad (\mathcal{L} < T \leq \mathcal{M}). \tag{26}$$

Here TC_{ij} in each payment strategy are obtained by replacing $\tilde{A}$, $\tilde{h}$, $\tilde{d}$, $\tilde{p}$ and $\tilde{s}$ in $\widetilde{\mathcal{TC}i}$ by A_j, h_j, d_j, p_j, and s_j for $i = 1, 2, 3$ and $j = 1, 2, 3$ respectively.

Sales Revenue

The defuzzified sales revenue of inventory is

$$G\mathcal{S}_{\mathcal{R}} = \frac{1}{4}[Tms_1{}^{1-n} + 2Tms_2{}^{1-n} + Tms_3{}^{1-n}]. \tag{27}$$

Initial Ordering Quantity

The inventory replenishment quantity (see eq. 5) in the fuzzy environment is

$$G\mathcal{W} = ms^{-n}\Big[\mathcal{L} + \big[(T - \mathcal{L}) + \frac{\alpha}{\beta + 1}(T^{\beta+1} - \mathcal{L}^{\beta+1})\big](1 - \alpha t^{\beta})\Big]. \tag{28}$$

Solution Algorithm

The procedure for obtaining the optimal strategy for the fuzzy model is as follows.

Pre-Payment or Cash-Payment

In the pre-payment or cash-payment strategy ($\mathcal{M} \leq \mathcal{L}$), the optimal strategy can be obtained as follows:

Step 1 Using the eq. (24), find T^* such that $\frac{dG\mathcal{TC}1}{dT}(T^*) = 0$ and $\frac{d^2G\mathcal{TC}1}{dT^2}(T^*) > 0$.

Step 2 Set $T = T^*$, then by using the eqs. (24), (28) and (27), obtain the total cost $G\mathcal{TC}1$, initial ordering quantity $G\mathcal{W}$, and the sales revenue $G\mathcal{S}_{\mathcal{R}}$.

Step 3 The simplification leads to the optimal results, that is, total cycle time T, inventory cost $G\mathcal{TC}1$, initial ordering quantity $G\mathcal{W}$, and total sales revenue $G\mathcal{S}_{\mathcal{R}}$.

Interim-payment or Post-Payment

In general, the suppliers offer trade credit (that is, $\mathcal{L} < \mathcal{M}$) to their old and good-credit retailers. However, the trade credit may act as an interim payment or post-payment strategy to the retailer depending upon his (or her) inventory constraints. Thus, the optimal results and payment strategy for the retailer's inventory can be obtained as follows:

Step 1 Using the eq. (25), find $T2^*$ such that $\frac{dG\mathcal{TC}2}{dT}(T2^*) = 0$ and $\frac{d^2G\mathcal{TC}2}{dT^2}(T2^*) > 0$.

Step 2 Using the eq. (26), find $T3^*$ such that $\frac{dG\mathcal{TC}3}{dT}(T3^*) = 0$ and $\frac{d^2G\mathcal{TC}3}{dT^2}(T3^*) > 0$.

Step 3 If $\mathcal{L} < \mathcal{M} \leq T2^*$, then

(i) setting $T = T2^*$, and by using the eqs. (25), (28) and (27), obtain the total cost $G\mathcal{TC}2$, initial ordering quantity $G\mathcal{W}$, and thre sales revenue $G\mathcal{S}_{\mathcal{R}}$

(ii) The simplification leads to the optimal results, that is, total cycle time T, inventory cost $G\mathcal{TC}2$, initial ordering quantity $G\mathcal{W}$ and total sales revenue $G\mathcal{S}_{\mathcal{R}}$

(iii) the retailer can opt for the interim-payment strategy.

Step 4 If $\mathcal{L} < T3^* \leq \mathcal{M}$, then

(i) setting $T = T3^*$, and by using the eqs. (26), (28) and (27), obtain the total cost $G\mathcal{TC}3$, initial ordering quantity $G\mathcal{W}$, and the sales revenue $G\mathcal{S}_{\mathcal{R}}$

(ii) The simplification leads to the optimal results, that is, total cycle time T, inventory cost $G\mathcal{TC}2$, initial ordering quantity $G\mathcal{W}$ and total sales revenue $G\mathcal{S}_{\mathcal{R}}$

(iii) the retailer can opt for the post-payment strategy.

The entire fuzzy model can be visualized from the flow chart given below.

Numerical Examples

Here, we illustrate the validity of the proposed model with various sets of numerical constraints.

Example 1 **(Pre-Payment Strategy)**

(A) Crisp Model

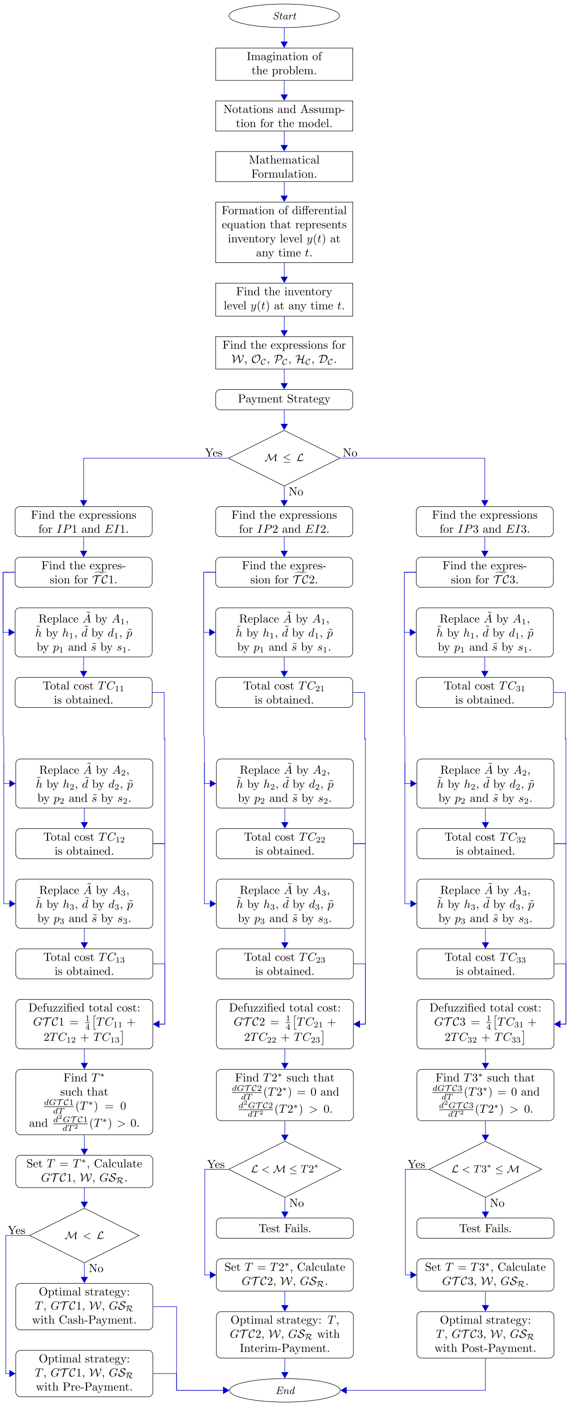
We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 19$, timing of the payment $\mathcal{M} = 15$, demand parameters are $m = 7$ and $n = 0.42$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.02$, rate of payable interest $\theta = 0.52$, rate of earning interest $\vartheta = 0.2$, the ordering cost per cycle $A = 60$, holding cost per unit $h = 4$, deterioration cost per unit $d = 2$, purchase cost per unit $p = 5$ and selling price per unit $s = 20$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm of the sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 38.2691$ items with a total cost of $\mathcal{TC}1 = 33.7709$ and the total sales revenue of $\mathcal{S}_{\mathcal{R}} = 39.7828$ for the cycle time $T = 19.239$.

(B) Fuzzy Model

Fig. 2 Flow Chart of Fuzzy Model



We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 19$, timing of the payment $\mathcal{M} = 15$, demand parameters are $m = 7$ and $n = 0.42$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.02$, rate of payable interest $\theta = 0.52$, rate of earning interest $\vartheta = 0.2$; next the different inventory costs with impreciseness as tringular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (55, 60, 65)$, holding cost per unit $\tilde{h} = (2, 4, 6)$, deterioration cost per unit $\tilde{d} = (1, 2, 3)$, purchase cost per unit $\tilde{p} = (3, 5, 7)$ and selling price per unit $s = (15, 20, 25)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm of the sect. 6.4, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 38.6494$ items with a total cost of $G\mathcal{TC}1 = 33.4016$ and the total sales revenue of $G\mathcal{S}_{\mathcal{R}} = 39.6286$ for the cycle time $T = 19.2427$.

Example 2 **(Pre-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 25$, timing of the payment $\mathcal{M} = 18$, demand parameters are $m = 4$ and $n = 0.002$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.04$, rate of payable interest $\theta = 0.50$, rate of earning interest $\vartheta = 0.25$, the ordering cost per cycle $A = 500$, holding cost per unit $h = 2$, deterioration cost per unit $d = 5$, purchase cost per unit $p = 8$ and selling price per unit $s = 45$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm of the sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 102.521$ items with a total cost of $\mathcal{TC}1 = 162.583$ and the total sales revenue of $\mathcal{S}_{\mathcal{R}} = 178.635$ for the cycle time $T = 25.8261$.

(B) Fuzzy

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 25$, timing of the payment $\mathcal{M} = 18$, demand parameters are $m = 4$ and $n = 0.002$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.04$, rate of payable interest $\theta = 0.50$, rate of earning interest $\vartheta = 0.25$; next the different inventory costs with impreciseness as triangular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (400, 500, 600)$, holding cost per unit $\tilde{h} = (1, 2, 3)$, deterioration cost per unit $\tilde{d} = (2, 5, 8)$, purchase cost per unit $\tilde{p} = (5, 8, 11)$ and selling price per unit $s = (40, 45, 50)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm of the sect. 6.4, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 102.521$ items with a total cost of $G\mathcal{TC}1 = 162.578$ and the total sales revenue of $G\mathcal{S}_{\mathcal{R}} = 178.634$ for the cycle time $T = 25.8261$.

Example 3 **(Pre-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 30$, timing of the payment $\mathcal{M} = 22$, demand parameters are $m = 8$ and $n = 0.001$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.03$, rate of payable interest $\theta = 0.32$, rate of earning interest $\vartheta = 0.15$, the ordering cost per cycle $A = 300$, holding cost per unit $h = 2$, deterioration cost per unit $d = 3$, purchase cost per unit $p = 10$ and selling price per unit $s = 42$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm of the sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 241.02$ items with a total cost of $\mathcal{TC}1 = 293.698$ and the total sales revenue of $\mathcal{S}_{\mathcal{R}} = 334.746$ for the cycle time $T = 30.2403$.

(B) Fuzzy

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 30$, timing of the payment $\mathcal{M} = 22$, demand parameters are $m = 8$ and $n = 0.001$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.03$, rate of payable interest $\theta = 0.32$, rate of earning interest $\vartheta = 0.15$; next the different inventory costs with impreciseness as triangular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (200, 300, 400)$, holding cost per unit $\tilde{h} = (1.5, 2, 2.5)$, deterioration cost per unit $\tilde{d} = (1, 3, 5)$, purchase cost per unit $\tilde{p} = (5, 10, 15)$ and selling price per unit $s = (35, 42, 49)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm of the sect. 6.4, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 241.022$ items with a total cost of $G\mathcal{TC}1 = 293.688$ and the total sales revenue of $G\mathcal{S}_{\mathcal{R}} = 334.744$ for the cycle time $T = 30.2403$.

Example 4 **(Cash-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 15$, timing of the payment $\mathcal{M} = 15$, demand parameters are $m = 7$ and $n = 0.42$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.02$, rate of payable interest $\theta = 0.52$, rate of earning interest $\vartheta = 0.2$, the ordering cost per cycle $A = 60$, holding cost per unit $h = 4$, deterioration cost per unit $d = 2$, purchase cost per unit $p = 5$ and selling price per unit $s = 20$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 30.4371$ items with a total cost of $\mathcal{TC}1 = 13.9059$ and the total sales revenue of $\mathcal{S}_{\mathcal{R}} = 39.7828$ for the cycle time $T = 15.3017$.

(B) Fuzzy

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 15$, timing of the payment $\mathcal{M} = 15$, demand parameters are $m = 7$ and $n = 0.42$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.02$, rate of payable interest $\theta = 0.52$, rate of earning interest $\vartheta = 0.2$; next the different inventory costs with impreciseness as tringular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (55, 60, 65)$, holding cost per unit $\tilde{h} = (2, 4, 6)$, deterioration cost per unit $\tilde{d} = (1, 2, 3)$, purchase cost per unit $\tilde{p} = (3, 5, 7)$ and selling price per unit $s = (15, 20, 25)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 6.4, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 30.743$ items with a total cost of $G\mathcal{TC}1 = 13.7855$ and the total sales revenue of $G\mathcal{S}_{\mathcal{R}} = 39.6286$ for the cycle time $T = 15.3063$.

Example 5 **(Interim-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 15$, timing of the payment $\mathcal{M} = 17$, demand parameters are $m = 5$ and $n = 0.6$, deterioration parameters $\alpha = 0.005$ and $\beta = 0.09$, rate of payable interest $\theta = 0.5$, rate of earning interest $\vartheta = 0.2$, the ordering cost per cycle $A = 350$, holding cost per unit $h = 4$, deterioration cost per unit $d = 0.3$, purchase cost per unit $p = 5$ and selling price per unit $s = 20$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 14.4295$ items with a total cost of $\mathcal{TC}2 = 13.0091$ and the total sales revenue of $\mathcal{S}_{\mathcal{R}} = 16.5723$ for the cycle time $T = 17.414$.

(B) Fuzzy

Let's consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 15$, timing of the payment $\mathcal{M} = 17$, demand parameters are $m = 5$ and $n = 0.6$, deterioration parameters $\alpha = 0.005$ and $\beta = 0.09$, rate of payable interest $\theta = 0.5$, rate of earning interest $\vartheta = 0.2$; next the different inventory costs with impreciseness as tringular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (250, 350, 450)$, holding cost per unit $\tilde{h} = (1.5, 4, 6.5)$, deterioration cost per unit $\tilde{d} = (0.1, 0.3, 0.5)$, purchase cost per unit $\tilde{p} = (2, 5, 8)$ and selling price per unit $s = (10, 20, 30)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 6.4, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 15.5996$ items with a total cost of $G\mathcal{TC}2 = 13.036$ and the total sales revenue of $G\mathcal{S}_{\mathcal{R}} = 16.2986$ for the cycle time $T = 17.5136$.

Example 6 **(Interim-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 8$, timing of the payment $\mathcal{M} = 10$, demand parameters are $m = 9$ and $n = 0.5$, deterioration parameters $\alpha = 0.001$ and $\beta = 0.004$, rate of payable interest $\theta = 0.5$, rate of earning interest $\vartheta = 0.2$, the ordering cost per cycle $A = 800$, holding cost per unit $h = 2$, deterioration cost per unit $d = 0.8$, purchase cost per unit $p = 7$ and selling price per unit $s = 35$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 21.0569$ items with a total cost of $\mathcal{TC}2 = 48.8771$ and the total sales revenue of $\mathcal{S_R} = 53.2447$ for the cycle time $T = 13.8416$.

(B) Fuzzy

Let's consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 8$, timing of the payment $\mathcal{M} = 10$, demand parameters are $m = 9$ and $n = 0.5$, deterioration parameters $\alpha = 0.001$ and $\beta = 0.004$, rate of payable interest $\theta = 0.5$, rate of earning interest $\vartheta = 0.2$; next the different inventory costs with impreciseness as triangular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (550, 800, 1050)$, holding cost per unit $\tilde{h} = (1.5, 2, 2.5)$, deterioration cost per unit $\tilde{d} = (0.5, 0.8, 0.11)$, purchase cost per unit $\tilde{p} = (4, 7, 10)$ and selling price per unit $s = (25, 35, 45)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 6.4, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 21.4846$ items with a total cost of $G\mathcal{TC}2 = 48.7879$ and the total sales revenue of $G\mathcal{S_R} = 52.9658$ for the cycle time $T = 13.8964$.

Example 7 **(Interim-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 18$, timing of the payment $\mathcal{M} = 19$, demand parameters are $m = 19$ and $n = 1.5$, deterioration parameters $\alpha = 0.001$ and $\beta = 0.04$, rate of payable interest $\theta = 0.6$, rate of earning interest $\vartheta = 0.3$, the ordering cost per cycle $A = 80$, holding cost per unit $h = 2$, deterioration cost per unit $d = 1$, purchase cost per unit $p = 9$ and selling price per unit $s = 12$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 8.94017$ items with a total cost of $\mathcal{TC}2 = 4.98368$ and the total sales revenue of $\mathcal{S_R} = 5.51399$ for the cycle time $T = 19.0451$.

(B) Fuzzy

Let's consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 18$, timing of the payment $\mathcal{M} = 19$, demand parameters are $m = 19$ and $n = 1.5$, deterioration parameters $\alpha = 0.001$ and $\beta = 0.04$, rate of payable interest $\theta = 0.6$, rate of earning interest $\vartheta = 0.3$; next the different inventory costs with impreciseness as triangular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (55, 80, 105)$, holding cost per unit $\tilde{h} = (1.5, 2, 2.5)$, deterioration cost per unit $\tilde{d} = (0.5, 1.0, 1.5)$, purchase cost per unit $\tilde{p} = (4, 9, 14)$ and selling price per unit $s = (10, 12, 14)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 6.4, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 8.94017$ items with a total cost of $G\mathcal{TC}2 = 4.98368$ and the total sales revenue of $G\mathcal{S_R} = .51399$ for the cycle time $T = 19.0451$.

Example 8 **(Post-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 11$, timing of the payment $\mathcal{M} = 20$, demand parameters are $m = 8$ and $n = 0.7$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.9$, rate of payable interest $\theta = 0.3$, rate of earning interest $\vartheta = 0.18$, the ordering cost per cycle $A = 390$, holding cost per unit $h = 20$, deterioration cost per unit $d = 4$, purchase cost per unit $p = 7$ and selling price per unit $s = 12$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 18.299$ items with a total cost of $\mathcal{TC}3 = 12.1762$ and the total sales revenue of $\mathcal{S}_{\mathcal{R}} = 16.8595$ for the cycle time $T = 13.0245$.

(B) Fuzzy

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 11$, timing of the payment $\mathcal{M} = 20$, demand parameters are $m = 8$ and $n = 0.7$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.9$, rate of payable interest $\theta = 0.3$, rate of earning interest $\vartheta = 0.18$; next the different inventory costs with impreciseness as tringular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (300, 390, 480)$, holding cost per unit $\tilde{h} = (10, 20, 30)$, deterioration cost per unit $\tilde{d} = (2, 4, 6)$, purchase cost per unit $\tilde{p} = (5, 7, 9)$ and selling price per unit $s = (6, 12, 18)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 20.0474$ items with a total cost of $G\mathcal{TC}3 = 12.8946$ and the total sales revenue of $G\mathcal{S}_{\mathcal{R}} = 16.6133$ for the cycle time $T = 13.0387$.

Example 9 **(Post-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 21$, timing of the payment $\mathcal{M} = 40$, demand parameters are $m = 18$ and $n = 1.7$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.9$, rate of payable interest $\theta = 0.3$, rate of earning interest $\vartheta = 0.18$, the ordering cost per cycle $A = 220$, holding cost per unit $h = 20$, deterioration cost per unit $d = 4$, purchase cost per unit $p = 7$ and selling price per unit $s = 22$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 2.62902$ items with a total cost of $\mathcal{TC}3 = 1.45896$ and the total sales revenue of $\mathcal{S}_{\mathcal{R}} = 2.06812$ for the cycle time $T = 27.965$.

(B) Fuzzy

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 21$, timing of the payment $\mathcal{M} = 40$, demand parameters are $m = 18$ and $n = 1.7$, deterioration parameters $\alpha = 0.0001$ and $\beta = 0.9$, rate of payable interest $\theta = 0.3$, rate of earning interest $\vartheta = 0.18$; next the different inventory costs with impreciseness as triangular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (180, 220, 260)$, holding cost per unit $\tilde{h} = (10, 20, 30)$, deterioration cost per unit $\tilde{d} = (2, 4, 6)$, purchase cost per unit $\tilde{p} = (5, 7, 9)$ and selling price per unit $s = (15, 22, 29)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 3.00573$ items with a total cost of $G\mathcal{TC}3 = 1.15778$ and the total sales revenue of $G\mathcal{S}_{\mathcal{R}} = 2.13619$ for the cycle time $T = 28.1515$.

Example 10 **(Post-Payment Strategy)**

(A) Crisp

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 9$, timing of the payment $\mathcal{M} = 15$, demand parameters are $m = 8$ and $n = 0.5$, deterioration parameters $\alpha = 0.002$ and $\beta = 0.8$, rate of payable interest $\theta = 0.35$, rate of earning interest $\vartheta = 0.18$, the ordering cost per cycle $A = 600$, holding cost per unit $h = 5$, deterioration cost per unit $d = 4$, purchase cost per unit $p = 7$ and selling price per unit $s = 16$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $\mathcal{W} = 29.416$ items with a total cost of $\mathcal{TC}3 = 17.6249$ and the total sales revenue of $\mathcal{S}_{\mathcal{R}} = 32$ for the cycle time $T = 14.6928$.

(B) Fuzzy

We consider a retailer's inventory having the following constraints: lead time $\mathcal{L} = 9$, timing of the payment $\mathcal{M} = 15$, demand parameters are $m = 8$ and $n = 0.5$, deterioration parameters $\alpha = 0.002$ and $\beta = 0.8$, rate of payable interest $\theta = 0.35$, rate of earning interest $\vartheta = 0.18$; next the different inventory costs with impreciseness as triangular fuzzy numbers, the ordering cost per cycle $\tilde{A} = (300, 600, 900)$, holding cost per unit $\tilde{h} = (2, 5, 8)$, deterioration cost per unit $\tilde{d} = (2, 4, 6)$, purchase cost per unit $\tilde{p} = (5, 7, 9)$ and selling price per unit $s = (10, 16, 22)$.

Solution

By using Mathematica 11.1.1 software and following the solution algorithm as mentioned in sect. 5.6, we obtain that the optimal strategy for inventory is as follows: the retailer needs to order a quantity of $G\mathcal{W} = 30.4743$ items with a total cost of $G\mathcal{TC}3 = 17.7065$ and the total sales revenue of $G\mathcal{S}_{\mathcal{R}} = 31.7054$ for the cycle time $T = 14.7859$.

Sensitivity Analysis

Generally, a retailer needs to handle the situation that may arise due to the effects of change in parametric values during the inventory cycle. Thus, we conduct the sensitivity analysis using Example 1 to draw the ramifications of a change in parametric values in the inventory cycle for the required managerial insights.

The increase in value of m effects the optimal results as follows (see Table 3 and Fig. 3):

(i) The total cycle time T of inventory decreases.

Table 3 The effect of sensitivity behavior of different parameters on optimal results

Sensitivity of m					*Sensitivity of n*				
m	T	$\mathcal{GTC}1$	$\mathcal{GW}$	$\mathcal{GS_R}$	n	T	$\mathcal{GTC}1$	$\mathcal{GW}$	$\mathcal{GS_R}$
3	19.5617	16.0821	16.8386	16.9837	0.42	19.2427	33.4016	38.6494	39.6286
4	19.4228	20.4167	22.2920	22.6449	0.43	19.2501	32.5012	37.5345	38.4581
5	19.3390	24.7469	27.7448	28.3061	0.44	19.2578	31.6276	36.4523	37.3223
6	19.2829	29.0749	33.1972	33.9674	0.45	19.2656	30.78	35.4019	36.2201
7	19.2427	33.4016	38.6494	39.6286	0.46	19.2738	29.9577	34.3823	35.1506
Sensitivity of β					*Sensitivity of $\mathcal{L}$*				
β	T	$\mathcal{GTC}1$	$\mathcal{GW}$	$\mathcal{GS_R}$	$\mathcal{L}$	T	$\mathcal{GTC}1$	$\mathcal{GW}$	$\mathcal{GS_R}$
0.02	19.2427	33.4016	38.6494	39.6286	16	16.2875	18.6519	32.7138	39.6286
0.22	19.2427	33.4016	38.6494	39.6286	17	17.2709	23.5463	34.6889	39.6286
0.42	19.2427	33.4016	38.6494	39.6286	18	18.256	28.4641	36.6676	39.6286
0.62	19.2427	33.4016	38.6493	39.6286	19	19.2427	33.4016	38.6494	39.6286
0.82	19.2427	33.4017	38.6493	39.6286	20	20.2307	38.356	40.6338	39.6286

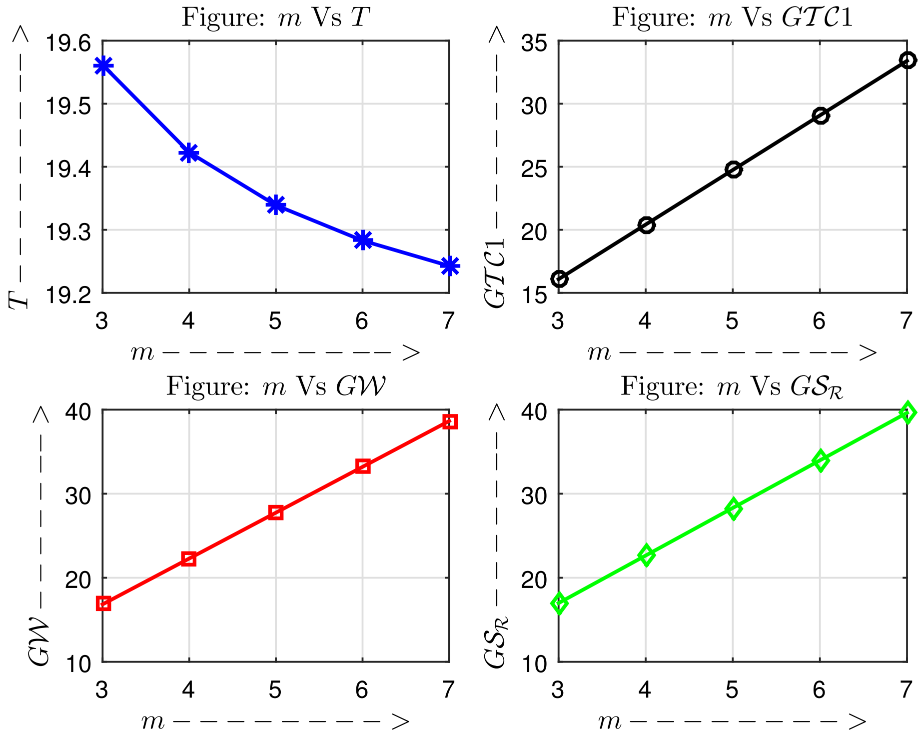


Fig. 3 Sensitivity effect of *m* on optimal results

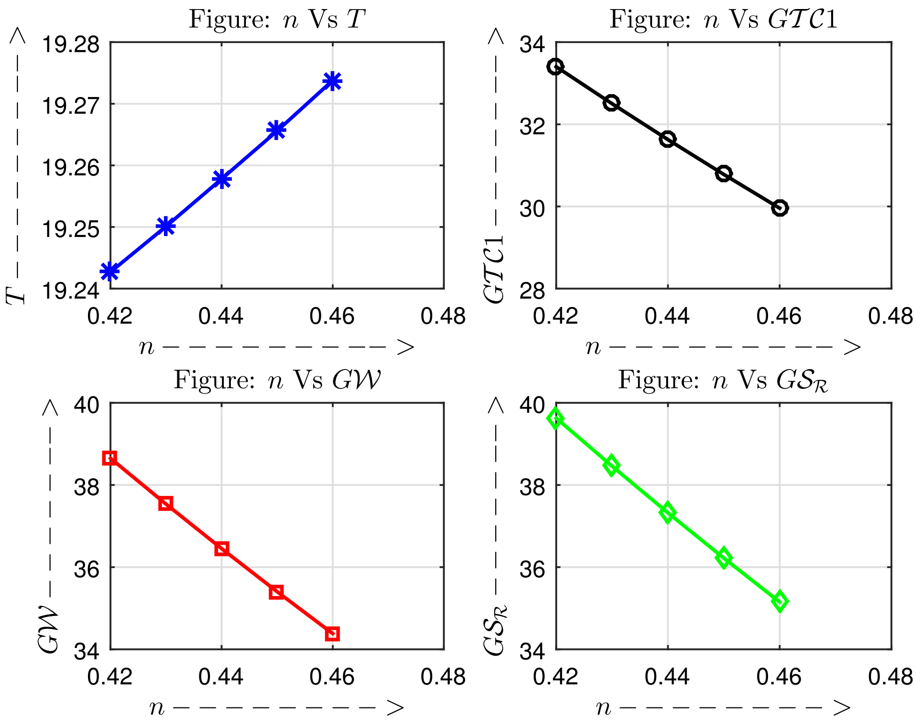


Fig. 4 Sensitivity effect of n on optimal results

(ii) The inventory cost $G\mathcal{TC}1$ increases.
(iii) The initial ordering quantity $G\mathcal{W}$ increases.
(iv) The total sales revenue $G\mathcal{S}_{\mathcal{R}}$ of inventory increases.

The increase in value of n effects the optimal results as follows (see Table 3 and Fig. 4):

(i) There is an increase in cycle time T of inventory.
(ii) The total cost $G\mathcal{TC}1$ of inventory decreases.
(iii) The initial ordering quantity $G\mathcal{W}$ for inventory decreases.
(iv) The total sales revenue $G\mathcal{S}_{\mathcal{R}}$ of inventory decreases.

The increase in value of β effects the optimal results as follows (see Table 3 and Fig. 5):

(i) There is no change in the total cycle time T of inventory.
(ii) The total cost $G\mathcal{TC}1$ of inventory becomes constant and then increases.
(iii) The initial ordering quantity $G\mathcal{W}$ for inventory becomes constant and then rises slightly.
(iv) There is no change in total sales revenue $G\mathcal{S}_{\mathcal{R}}$ of inventory.

The increase in value of $\mathcal{L}$ effects the optimal results as follows (see Table 3 and Fig. 6) :

(i) There is an increase in cycle time T of inventory.
(ii) The inventory cost $G\mathcal{TC}1$ increases.
(iii) The initial ordering quantity $G\mathcal{W}$ increases.
(iv) There is no change in total sales revenue $G\mathcal{S}_{\mathcal{R}}$ of inventory.

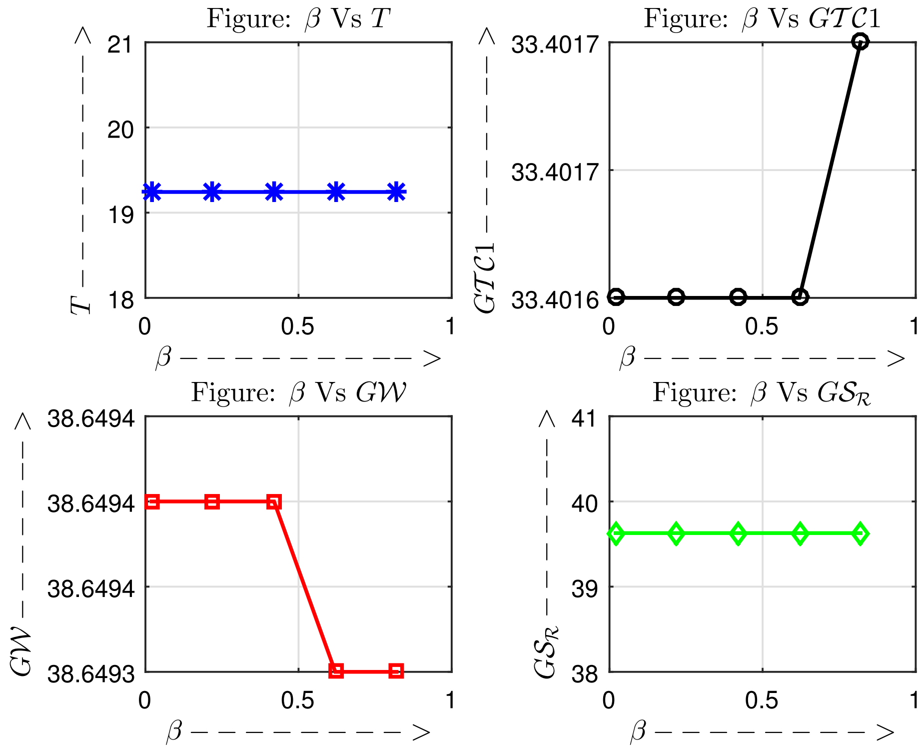


Fig. 5 Sensitivity effect of β on optimal results

Managerial insights

The following managerial insights have been found from the sensitivity examination of various parameters (refer Table 3).

(i) The rise in the value of parameter m leads to an increase in demand. As a result, the total cost, the order quantity, the sales revenue increase, and the business cycle time decreases (refer Table 3 and Fig. 3). Thus, the retailer may earn more profit in less time. Also, the result directs the retailers to take the necessary steps to place the order in advance for the next business cycle and earn more profit.

(ii) The rise in the value of parameter n leads to a decrease in demand. As a result, the total cost, the order quantity, the sales revenue decrease, and the business cycle time increases (refer Table 3 and Fig. 4). In this scenario, the retailer may get a reduction in profit. Also, the retailers need to take the necessary preventive measures to decline the deterioration rate in the inventory.

(iii) The small increment in the value of parameter β leads to a change in the deterioration pattern. As a result, the total cost, the order quantity, the sales revenue, and the business cycle time have a negligible effect (refer Table 3 and Fig. 5). But, a significant change in the value of parameter β results in an increase in inventory cost. In this scenario, the retailer may get a reduction in profit by a considerable increment in β. Thus, the retailers need to take the necessary steps to prevent the increment in the deterioration rate of inventory.

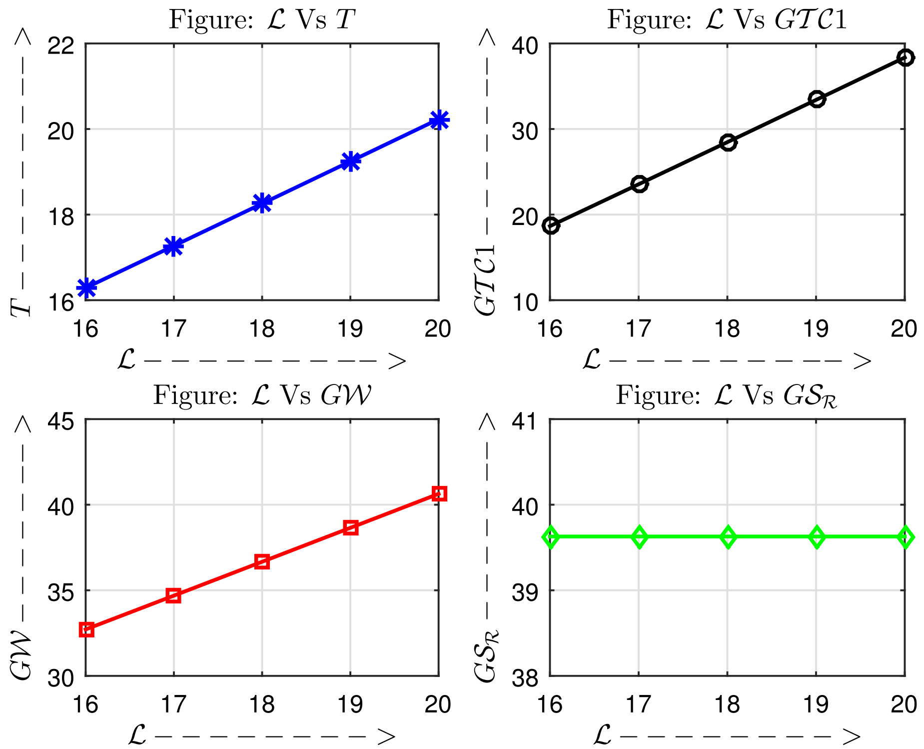


Fig. 6 Sensitivity effect of $\mathcal{L}$ on optimal results

(iv) As lead-time increases, the total cost, the order quantity, and the business cycle time increase relatively. However, the sales revenue remains unchanged (refer Table 3 and Fig. 6). As a result, the retailer may lose profit significantly. Thus, retailers need to implement a suitable strategy to minimize the lead-time for inventory.

Conclusion

In day-to-day business affairs, many inventory models have different payment strategies as well as lead times. During the last three decades, several researchers have obtained optimal strategies for inventory models involving various payment strategies with different lead-time. However, due to the COVID-19 pandemic as well as the direct or indirect war among some countries, and for many other reasons, the global imports and exports have been getting delayed. As a result, in most of the inventory problems a rise in lead-time has occurred. Also, the payment mechanism for inventory problems is getting significantly changed during this pandemic period. Mostly, the suppliers prefer to have advance payment or cash payment during this period. However, the suppliers of a few products need to have trade credit facilities. On the other hand, the costs of inventory may possess certain impreciseness due to several socio-economic factors. As a result, the ideal performance of the optimal strategy may be affected substantially. Thus, it shows that the lead time, payment strategy, and impreciseness of costs play a vital role in the determination of optimal policies for inventory problems. None of the researchers considered the positive lead-time, payment strategies, and impreciseness of cost parameters simultaneously for the inventory model for Weibull deteriorating items with selling-price-dependent demand. Under these circumstances, our present model is the first such realistic inventory model for the retailers availing the pre-payment, cash-payment, interim-payment, and post-payment strategies under the positive lead-time with selling-price-dependent demand having Weibull deterioration under the assumption of deterministic costs and imprecise costs in the crisp and fuzzy environment respectively. Moreover, we have shown numerically that the optimal scheme for the inventory model can be found by following the proposed analytical solution procedures in both the crisp and fuzzy environments. Finally, several decision-making findings are presented via managerial insights in the sensitivity analysis section to handle the situations that may arise during the inventory cycle.

Additionally, we also present below the managerial suggestions, managerial implications, and future scope of the proposed model.

Managerial Suggestions

The crisp and fuzzy models in the proposed investigation had different strategies for the same set of inventory constraints under the assumption of deterministic and imprecise costs. This difference of optimal strategy is not identical for all the inventory constraints in all the scenarios (That is, the retailers in the fuzzy model have a lower or higher cost than the retailers in the crisp model). Thus, it indicates that the impreciseness of cost parameters has a significant impact on optimal outcomes. Also, the results of the crisp model are applicable for inventory problems with the parameters known with complete certainty only, and the fuzzy model is appropriate when the inventory problems have the parameters known with uncertainty. That means, the optimal strategies of the crisp model are inadequate for the real-life inventory problems as they may have impreciseness in costs and other parameters. Hence, we suggest the retailers to apply the optimal strategies of fuzzy models by taking the measures of impreciseness in the parameters and constraints to minimize the cost or maximize the profit.

Managerial Implications

In the present investigation, besides the model in the crisp environment, we developed the model in the fuzzy environment by measuring the impreciseness of cost parameters with the triangular-fuzzy numbers. Moreover, the imprecise cost functions are required to obtain the optimal strategy that has been defuzzified by the Graded mean integration method. However, the triangular-fuzzy numbers may not be adequate to quantify the impreciseness of the parameters in all the scenarios. Thus, the researchers may use different fuzzy numbers to quantify the impreciseness in the associated parameters. Also, they may avail the use of various techniques for defuzzification. Hence, we suggest the curious researchers to make use of the work done in this paper for their inventory problems by choosing appropriate fuzzy numbers and defuzzification techniques in obtaining the optimal strategy.

Future Scope

The present model can be extended by incorporating the downstream trade credit and partial backlogging during the lead-time. One may also extend this model by considering the preservation as well as green technology to reduce the deterioration and so also the carbon emission. Furthermore, the researchers may incorporate the imprecise costs and other parameters to obtain the optimal strategies for their inventory problems in an imprecise environment by following the idea presented in this paper.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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