A Reliable Solution of Nonlinear Time Dependent Fractional Model of Ebola Virus Disease with Arbitrary Order Derivative in Liouville–Caputo Sense

Vinod Kumar Bhardwaj¹ · Manish Goyal¹

Accepted: 10 November 2021 / Published online: 30 November 2021

© The Author(s), under exclusive licence to Springer Nature India Private Limited 2021

Abstract

In this article, the analysis of an arbitrary order Ebola virus disease model is conducted to find out its reliable solution. The fractional derivative is taken in Liouville–Caputo sense. The solution of this nonlinear model is achieved using fractional variational iteration scheme. The convergence analysis of the obtained solution is also presented which confirms that it is positive, bounded and convergent. The outcomes are discussed with figures explaining variation of susceptible, infected, recovered population and number of disease induced deaths with time. The negligible error in successive iterations of various population shows the competency of the presented scheme. The results endorse that FVIM is extremely effective, powerful and easy in usage.

Keywords Fractional differential equations (FDE) · Ebola disease model · Liouville–Caputo fractional derivative · Fractional variational iteration scheme (FVIM)

List of Symbols

t	Time variable
β	Order of the fractional derivative
$\mathbb{N}$	Set of natural numbers
λ	Lagrangian multiplier
$\tilde{S}_n$	Restricted variation
l	Tolerance limits
$\mathbb{R}$	Set of real numbers
$P_i; 1 \leq i \leq 4$	Operators from Banach space to Banach space
BS	Banach Space

✉ Manish Goyal
manish.goyal@gla.ac.in

Vinod Kumar Bhardwaj
dr.vinodbhardwaj@gmail.com

¹ Department of Mathematics, Institute of Applied Sciences and Humanities, GLA University, Mathura 281406, India

Introduction

Ebola is a fatal and complex pathogen for a human being. It attacks healthy cells of the body of host and replicates itself in it [1, 2]. It is a rare virus that causes bleeding [3–8]. In 1976, it was first detected in two outbreaks, one in South Sudan, and the other in Congo near the river Ebola. In spite of wide search, its reservoir could not be recognized but it may contain bats [1]. The 2014–2016 outbreak of Ebola in West Africa was the biggest since 1976. After starting in Guinea, it moved to Liberia and Sierra Leone. The recent outbreak of Ebola virus disease (EVD) is found in the Republic of Guinea on 14 February 2021. However, on 19 June 2021, the outbreak was declared over. There were 16 confirmed and 7 probable cases, of whom 12 people died [9].

This virus transmits through physical connection with secretions, tissues, body fluids or semen of infected ones. Its early symptom are headache, fever, sore throat, joint ache, muscle ache, weakness etc. In the later stage, the symptom are vomiting, diarrhea, bleeding, stomach pain and organ failure. Its incubation period is between 2–21 days and infection period is about 4–10 days. Its mortality rate ranges from 25 to 90% [10]. Despite high mortality rate, some patients survive and few develop long-lasting manifestations. It is seen that the individual genetic differences have a major role in the deaths by EVD [11]. In 2014, the average incubation time was 11.4 days. The average period from the beginning of symptom to hospitalization is 5 ± 4.7 days. The average period to death after hospitalization was 4.2 ± 6.4 days [12]. Ebola virus remains in semen for a long time in survivors with a risk of transmission [13]. The antivirals like Favipiravir is found to be effective in delaying survival and dropping viral load. The study of Ebola virus is done in [14–16]. The transmission EVD Dynamics is studied by various researchers in [17–19].

Mathematical investigation is done to state several phenomena of biological [20–22], medical [23] and physical [24–28] significance. The fractional derivatives [29–34] are used to describe the real-life problems. The arbitrary order of derivative is an index of memory [35]. For details of work on the fractional EVD models (see, for details, [36–43]). Fractional calculus [44] is a latent tool in electromagnetic theory [45], bio-engineering [46], plasma physics [47], control theory [48], neurophysiology [49], electric technology [50], visco-elasticity [51], disease dynamics [52–54] and many others. Over the years, the researchers have worked on various techniques for the solution of nonlinear FDE such as homotopy perturbation method [55], homotopy analysis method [56], Adomian decomposition method [57], fractional order homotopy perturbation method [58], variational iteration scheme and its modifications [59–66] etc.

However, the consistency in the solution of schemes [67] is vital. Most of the FDE have no exact solution, therefore approximation by FVIM is needed. He [68] established VIM for the solution of nonlinear equations and then applied to FDE. Highly nonlinear equations cannot successfully be solved by general methods. FVIM [69–72] is remarkably competent to address the challenge. It offers solution in an infinite convergent series. It entails no discretization and linearization. In FVIM, initial solution is picked with unknown parameters as searched solutions. It reduces computation significantly and has meticulousness in finding Lagrange's multiplier.

Our goal is to investigate an efficient solution of time fractional Ebola virus model by FVIM. This paper is planned as below. In Sect. 1, there is introduction. In Sect. 2, the definition along with properties of some fractional operators are specified. In Sect. 3, the fractional EVD model is discussed. In Sect. 4, the idea of FVIM is discussed with analysis

of convergence and solution by FVIM is also found. In Sect. 5, we elaborate results and afterwards conclusion is found.

Preliminaries

We give certain properties with definition of Liouville–Caputo operator (see, for details, [73–76]).

Definition 2.1: Liouville–Caputo α -order derivative is defined as:

$${}^{LC}D_{a+}^{\alpha}\varphi(\eta) = \frac{1}{\Gamma(n-\alpha)} \int_a^{\eta} (\eta-\varsigma)^{n-\alpha-1} D_{\varsigma}^n \varphi(\varsigma) d\varsigma, \quad (n-1 < \alpha \leq n; n \in \mathbb{N}),$$

where, $D_{\eta}^n := \frac{d^n}{d\eta^n} (n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\})$.

If α is positive integer, it turns into ordinary derivative,

$${}^{LC}D_{a+}^{\alpha} = D_{\eta}^{\alpha} (\alpha \in \mathbb{N}).$$

Definition 2.2: Liouville–Caputo α -order derivative ($\alpha > 0$) on space $\mathbb{R} = (-\infty, \infty)$ is:

$${}^{LC}D_{-\infty+}^{\alpha}\varphi(\zeta) = \frac{1}{\Gamma(n-\alpha)} \int_{-\infty}^{\zeta} (\zeta-\varsigma)^{n-\alpha-1} D_{\varsigma}^n \varphi(\varsigma) d\varsigma, \quad (n-1 < \alpha \leq n; n \in \mathbb{N}).$$

- $I_t^{\zeta} h(x, t) = \frac{1}{\Gamma_{\zeta}} \int_0^t (t-s)^{\zeta-1} h(x, s) ds; \zeta, t > 0.$
- $D_t^{\nu} V(x, \tau) = I_{\tau}^{m-\nu} \frac{\partial^m V(x, \tau)}{\partial \tau^m}, m-1 < \nu \leq m.$
- $D_t^{\zeta} I_t^{\zeta} h(t) = h(t), m-1 < \zeta \leq m, m \in \mathbb{N}.$
- $I_t^{\zeta} D_t^{\zeta} h(t) = h(t) - \sum_{k=1}^{m-1} h^{(k)}(0^+) \frac{t^k}{k!}, m-1 < \zeta \leq m, m \in \mathbb{N}.$
- $I^{\beta} t^{\alpha} = \frac{\Gamma(\alpha+1)}{\Gamma(\beta+\alpha+1)} t^{\beta+\alpha}.$
- $D_t^{\beta} t^{\alpha} = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1-\beta)} t^{\alpha-\beta}.$

Model Description

Atangana and Goufo [19] developed a model describing the spread of Ebola fever in the West African countries. Following this, we have reconstructed a nonlinear time dependent EVD model to include the features using memory effect. Let N be the total population of West African countries, S be the susceptible, I be the infected, R be the number of recovery and D be the number of disease induced deaths due to EVD in these countries. We consider that the people get susceptible at rate s , infect at rate i , recover at rate r and the disease induced death rate is d . The death rate due to other reasons is assumed as α .

Rates of change in susceptible and infected with time t are,

$$\begin{aligned} D_t^{\beta} S(t) &= -iS(t)I(t) + sR(t) - \alpha N, \\ D_t^{\beta} I(t) &= iS(t)I(t) - (d+r)I(t), \end{aligned}$$

Here, $iS(t)I(t)$ are the people removed from the susceptible.

Rates of change in the recovered and the number of deaths with time are,

$$\begin{aligned} D_t^\beta R(t) &= rI(t) - sR(t), \\ D_t^\beta D(t) &= dI(t) + \alpha N, \end{aligned}$$

Hence the EVD model is written as,

$$\left. \begin{aligned} D_t^\beta S(t) &= -iS(t)I(t) + sR(t) - \alpha N, \\ D_t^\beta I(t) &= iS(t)I(t) - (d+r)I(t), \\ D_t^\beta R(t) &= rI(t) - sR(t), \\ D_t^\beta D(t) &= dI(t) + \alpha N. \end{aligned} \right\} \quad (1)$$

with conditions $S(0) = S_0$, $I(0) = I_0$, $D(0) = D_0$, $R(0) = R_0$ and $0 < \beta \leq 1$. The property of non-locality is main use of working with FDE. It means that the next state of system is dependent on prior states. Therefore, these models adhere to reality. The time dependent fractional derivative submits memory modulation [77] of a system. Clearly, the presented EVD model is impacted by memory in time that confirms aptness of fractional order modeling. Therefore, a systematic and exhaustive study of Eq. (1) is important. Equation (1) is nonlinear and it is visibly tough to find its exact solution. Therefore, it turned into our motivation to solve Eq. (1) by FVIM because it gives reliable results for nonlinear equations without finding special polynomials.

FVIM for the Fractional Ebola Virus Disease Model

Consider a fractional EVD model in Eq. (1). The correctional functionals for Eq. (1) are,

$$\left. \begin{aligned} S_{n+1}(t) &= S_n(t) + \lambda \int_0^t \left(\frac{d^\beta S_n(\tau)}{d\tau^\beta} + i \tilde{S}_n \tilde{I}_n - s \tilde{R}_n + \alpha N \right) (d\tau)^\beta, \\ I_{n+1}(t) &= I_n(t) + \lambda \int_0^t \left(\frac{d^\beta I_n(\tau)}{d\tau^\beta} - i \tilde{S}_n \tilde{I}_n + (d+r) \tilde{I}_n \right) (d\tau)^\beta, \\ R_{n+1}(t) &= R_n(t) + \lambda \int_0^t \left(\frac{d^\beta R_n(\tau)}{d\tau^\beta} - r \tilde{I}_n + s \tilde{R}_n \right) (d\tau)^\beta, \\ D_{n+1}(t) &= D_n(t) + \lambda \int_0^t \left(\frac{d^\beta D_n(\tau)}{d\tau^\beta} - d \tilde{I}_n - \alpha N \right) (d\tau)^\beta. \end{aligned} \right\} \quad (2)$$

Here, λ is Lagrangian multiplier.

By variational theory,

$$\frac{d^\beta \lambda}{d\tau^\beta} \Big|_{\tau=t} = 0, \quad \text{and} \quad 1 + \lambda \Big|_{\tau=t} = 0.$$

We instantly get,

$$\lambda = -1. \quad (3)$$

From Eq. (2),

$$\left. \begin{aligned} S_{n+1}(t) &= S_n(t) - \int_0^t \left(\frac{d^\beta S_n(\tau)}{d\tau^\beta} + i S_n I_n - s R_n + \alpha N \right) (d\tau)^\beta, \\ I_{n+1}(t) &= I_n(t) - \int_0^t \left(\frac{d^\beta I_n(\tau)}{d\tau^\beta} - i S_n I_n + (d + r) I_n \right) (d\tau)^\beta, \\ R_{n+1}(t) &= R_n(t) - \int_0^t \left(\frac{d^\beta R_n(\tau)}{d\tau^\beta} - r I_n + s R_n \right) (d\tau)^\beta, \\ D_{n+1}(t) &= D_n(t) - \int_0^t \left(\frac{d^\beta D_n(\tau)}{d\tau^\beta} - d I_n - \alpha N \right) (d\tau)^\beta. \end{aligned} \right\} \quad (4)$$

Consecutive approximations $S_n(t)$, $I_n(t)$, $R_n(t)$ and $D_n(t)$; $n \geq 0$ are hence calculated. $\tilde{S}_n$, $\tilde{I}_n$, $\tilde{R}_n$ and $\tilde{D}_n$ are the restricted variations so, $\delta \tilde{S}_n = 0$, $\delta \tilde{I}_n = 0$, $\delta \tilde{R}_n = 0$, $\delta \tilde{D}_n = 0$.

We get $S_{n+1}(t)$, $I_{n+1}(t)$, $R_{n+1}(t)$ and $D_{n+1}(t)$, $n \geq 0$.

Solution is,

$$\left. \begin{aligned} S(t) &= \lim_{n \rightarrow \infty} S_n(t), \\ I(t) &= \lim_{n \rightarrow \infty} I_n(t), \\ R(t) &= \lim_{n \rightarrow \infty} R_n(t), \\ D(t) &= \lim_{n \rightarrow \infty} D_n(t). \end{aligned} \right\} \quad (5)$$

and

FVIM Procedure

Step I. find $S_0 = S(0)$, $I_0 = I(0)$, $R_0 = R(0)$, $D_0 = D(0)$, set $n = 0$;

Step II. use S_n , I_n , R_n and D_n to get S_{n+1} , I_{n+1} , R_{n+1} and D_{n+1} from Eq. (4);

Step III. define $S_n := S_{n+1}$, $I_n := I_{n+1}$, $R_n := R_{n+1}$ and $D_n := D_{n+1}$

Step IV. if $\max |S_{n+1} - S_n| < Tol$, $\max |I_{n+1} - I_n| < Tol$, $\max |R_{n+1} - R_n| < Tol$ and $\max |D_{n+1} - D_n| < Tol$ stop, otherwise continue;

Step V. set $S_{n+1} := S_n$, $I_{n+1} := I_n$, $R_{n+1} := R_n$ and $D_{n+1} := D_n$;

Step VI. Set $n = n + 1$, return to step II ;

Convergence Analysis of FVIM

Consider operators P_1 , P_2 , P_3 and P_4 :

$$\left. \begin{aligned} P_1 &= \int_0^t (-1) \left(D_\tau^\beta S_n(\tau) + i S_n(\tau) I_n(\tau) - s R_n(\tau) + \alpha N \right) (d\tau)^\beta, \\ P_2 &= \int_0^t (-1) \left(D_\tau^\beta I_n(\tau) - i S_n(\tau) I_n(\tau) + (d + r) I_n(\tau) \right) (d\tau)^\beta, \\ P_3 &= \int_0^t (-1) \left(D_\tau^\beta R_n(\tau) - r I_n(\tau) + s R_n(\tau) \right) (d\tau)^\beta, \\ P_4 &= \int_0^t (-1) \left(D_\tau^\beta D_n(\tau) - d I_n(\tau) - \alpha N \right) (d\tau)^\beta. \end{aligned} \right\} \quad (6)$$

also, state the components $v_{k(i)}, k = 0, 1, \dots; 1 \leq i \leq 4$,

$$\left. \begin{aligned} S(t) &= \lim_{n \rightarrow \infty} S_n(t) = \sum_{k=0}^{\infty} v_{k(1)}, \\ I(t) &= \lim_{n \rightarrow \infty} I_n(t) = \sum_{k=0}^{\infty} v_{k(2)}, \\ R(t) &= \lim_{n \rightarrow \infty} R_n(t) = \sum_{k=0}^{\infty} v_{k(3)}, \\ D(t) &= \lim_{n \rightarrow \infty} D_n(t) = \sum_{k=0}^{\infty} v_{k(4)}. \end{aligned} \right\} \quad (7)$$

Equation (7) quickly converges [78].

Theorem 1 [79]: Let P_1, P_2, P_3 and P_4 be the operators from Banach space BS to BS . Equation (7) converge [80, 81] if $0 < \xi < 1$ exists in such a way that,

$$\|P_i[v_{0(i)} + v_{1(i)} + v_{2(i)} + \dots + v_{k+1(i)}]\| \leq \xi \|P_i[v_{0(i)} + v_{1(i)} + v_{2(i)} + \dots + v_{k(i)}]\|, \quad 1 \leq i \leq 4,$$

(i.e. $v_{k+1} \leq \xi v_k$), $\forall k \in \mathbb{N} \cup \{0\}$.

Theorem 2 [79]: Let Eq. (7) converge to $S(t), I(t), R(t)$ and $D(t)$. If $\sum_{k=0}^q v_{k(1)}$ is used as approximation to $S(t)$ of Eq. (1), maximum error $E_q(t)$ is,

$$E_q(t) \leq \frac{1}{1 - \xi} \xi^{q+1} \|v_0\|,$$

If $\forall i \in \mathbb{N} \cup \{0\}$, we define,

$$\chi_i = \begin{cases} \frac{\|v_{i+1}\|}{\|v_i\|}, & \|v_i\| \neq 0 \\ 0, & \|v_i\| = 0, \end{cases},$$

then $\sum_{k=0}^{\infty} v_{k(i)}, 1 \leq i \leq 4$ of Eq. (7) converge if $0 \leq \chi_i < 1$.

From Theorem 2,

$$\left\| S(t) - \sum_{k=0}^{\infty} v_{k(1)} \right\| \leq \frac{1}{1 - \chi} \chi^{q+1} \|v_0\|, \text{ where } \chi = \max\{\chi_i, i = 0, 1, 2, \dots, q\}.$$

Implementation of FVIM

By Eq. (4),

$$\begin{aligned} S_{n+1}(t) &= S_n(t) - \int_0^t \left(\frac{d^\beta S_n(\tau)}{d\tau^\beta} + i S_n I_n - s R_n + \alpha N \right) (d\tau)^\beta, \\ I_{n+1}(t) &= I_n(t) - \int_0^t \left(\frac{d^\beta I_n(\tau)}{d\tau^\beta} - i S_n I_n + (d + r) I_n \right) (d\tau)^\beta, \end{aligned}$$

$$R_{n+1}(t) = R_n(t) - \int_0^t \left(\frac{d^\beta R_n(\tau)}{d\tau^\beta} - r I_n + s R_n \right) (d\tau)^\beta,$$

$$D_{n+1}(t) = D_n(t) - \int_0^t \left(\frac{d^\beta D_n(\tau)}{d\tau^\beta} - d I_n - \alpha N \right) (d\tau)^\beta.$$

Placing $n = 0, 1, \dots$ in Eq. (4),

$$S_1(t) = S_0 - (i S_0 I_0 - s R_0 + \alpha N) \frac{t^\beta}{\Gamma(1+\beta)} = S_0 - \rho \frac{t^\beta}{\Gamma(1+\beta)},$$

$$I_1(t) = I_0 + (i S_0 I_0 - d I_0 - r I_0) \frac{t^\beta}{\Gamma(1+\beta)} = I_0 - \xi \frac{t^\beta}{\Gamma(1+\beta)},$$

$$R_1(t) = R_0 + (r I_0 - s R_0) \frac{t^\beta}{\Gamma(1+\beta)} = R_0 + \chi \frac{t^\beta}{\Gamma(1+\beta)},$$

$$D_1(t) = D_0 + (d I_0 + \alpha N) \frac{t^\beta}{\Gamma(1+\beta)} = D_0 + \eta \frac{t^\beta}{\Gamma(1+\beta)},$$

$$S_2(t) = S_0 - \rho \frac{t^\beta}{\Gamma(1+\beta)} + \{i(\rho I_0 - \xi S_0) + s \chi\} \frac{t^{2\beta}}{\Gamma(1+2\beta)} + i \rho \xi \frac{\Gamma(1+2\beta) t^{3\beta}}{(\Gamma(1+\beta))^2 \Gamma(1+3\beta)},$$

$$I_2(t) = I_0 + \xi \frac{t^\beta}{\Gamma(1+\beta)} + \{i(\xi S_0 - \rho I_0) - (d+r)\xi\} \frac{t^{2\beta}}{\Gamma(1+2\beta)} - i \rho \xi \frac{\Gamma(1+2\beta) t^{3\beta}}{(\Gamma(1+\beta))^2 \Gamma(1+3\beta)},$$

$$R_2(t) = R_0 + \chi \frac{t^\beta}{\Gamma(1+\beta)} + (r\xi - s\chi) \frac{t^{2\beta}}{\Gamma(1+2\beta)},$$

$$D_2(t) = D_0 + \eta \frac{t^\beta}{\Gamma(1+\beta)} + d\eta \frac{t^{2\beta}}{\Gamma(1+2\beta)},$$

and so on.

Here, $\rho = (i S_0 I_0 - s R_0 + \alpha N)$, $\xi = (i S_0 I_0 - (d+r) I_0)$, $\chi = (r I_0 - s R_0)$, and, $\eta = (d I_0 + \alpha N)$.

Similarly, we can find the ensuing terms. Consequently, we get the solution as given in Eq. (5).

From Eqs. (6) and (7), the iterations for Eq. (1) are,

$$v_{0(1)} = S_0, \quad v_{0(2)} = I_0, \quad v_{0(3)} = R_0, \quad v_{0(4)} = D_0,$$

$$v_{1(1)} = \frac{-\rho t^\beta}{\Gamma(1+\beta)}, \quad v_{1(2)} = \frac{-\xi t^\beta}{\Gamma(1+\beta)}, \quad v_{1(3)} = \frac{\chi t^\beta}{\Gamma(1+\beta)}, \text{ and } v_{1(4)} = \frac{\eta t^\beta}{\Gamma(1+\beta)},$$

and so on.

If we compute χ_i , we get,

$$\chi_i = \frac{\|v_{i+1(1)}\|}{\|v_{i(1)}\|} < 1,$$

$$\chi'_i = \frac{\|v_{i+1(2)}\|}{\|v_{i(2)}\|} < 1,$$

Fig. 1 Variation of **a** Susceptible $S(t)$, **b** Infected $I(t)$, **c** Recovered $R(t)$, **d** Death $D(t)$ population with time t at diverse values of order β of fractional derivative

Table 1 Value of parameters for simulation [19]

S. no	1	2	3	4	5	6	7	8	9	10
Parameters	N	α	r	i	s	d	S_0	I_0	R_0	D_0
Values	1000	0.01	0.4	0.01	0.02	0.6	900	10	0	0

$$\chi_i'' = \frac{\|v_{i+1(3)}\|}{\|v_{i(3)}\|} < 1, \quad \text{and}$$

$$\chi_i''' = \frac{\|v_{i+1(4)}\|}{\|v_{i(4)}\|} < 1,$$

when for instance, $i > 1$ and $0 < \beta \leq 1$.

It approves that the solution of Eq. (1) by FVIM is bounded, convergent and positive.

Numerical Results and Discussion

Simulations are done for S , I , R and D at distinct fractional values of β and $\beta = 1$ using Maple package The FVIM solution of Eq. (1) is achieved using value of parameters given in Table 1. Figure 1a depicts the behavior of susceptibles with time for distinct β . It shows that the susceptibles increase with increase in value of the arbitrary order β . Figure 1b portrays the behavior of infected ones with time for distinct β . In it, the infected increase with decrease in β . Fig. 1c illustrates the behavior of recovery with time for separate β . In it, this number decreases as we increase the value of β . Fig. 1d shows behavior of number of deaths with time for diverse β . In it, D increases with decreasing values of β . In Table 2 the absolute error between the successive iterations of susceptibles decrease sharply at the orders $\beta = 0.5$ and 1 of the fractional derivative. In Table 3 the absolute error between the consecutive approximations of infected also decrease sharply at orders $\beta = 0.5$ and 1. In Table 4, the absolute error between the succeeding estimates of recovered reduce at orders $\beta = 0.5$ and 1 while in Table 5 the absolute error between the subsequent iterates of disease induced deaths also decrease at the orders $\beta = 0.5$ and 1.

Ebola virus may be used for bio-terrorism so the research on producing effective vaccines is in progress. The detailed study of EVD model given by Eq. (1) is relevant. Prevention from EVD can be done by the early identification of infected and stopping its transmission. A good control of outbreak depends on applying a set of involvements, such as case management, observation and contact tracing, better laboratory facility, safe burials and social mobilization. Ebanga and Imazeb are the two monoclonal antibodies which were approved for the treatment of EVD infection by the US Food and Drug Administration in the late 2020 [10]. A vaccine rVSVdeltaG-ZEBOV-GP is found to be safe and immunogenic till now but more

Table 2 Error between consecutive iterations for $S(t)$ at order β

t	$\beta = 1$		$\beta = 0.50$	
	$ S_1 - S_0 $	$ S_2 - S_1 $	$ S_1 - S_0 $	$ S_2 - S_1 $
0	0	0	0	0
0.005	0.50	0.008870666667	7.978845608	3.522509382
0.010	1.00	0.03546933333	11.28379167	7.022576161
0.015	1.50	0.07977600000	13.81976597	10.50803302
0.020	2.00	0.1417706667	15.95769121	13.98167506
0.025	2.50	0.2214333333	17.84124116	17.44511768

Table 3 Error between consecutive iterations for $I(t)$ at order β

t	$\beta = 1$		$\beta = 0.50$	
	$ I_1 - I_0 $	$ I_2 - I_1 $	$ I_1 - I_0 $	$ I_2 - I_1 $
0	0	0	0	0
0.05	4.00	0.1033208333	20.18506018	4.856180562
0.15	12.00	0.9898875000	34.96154978	16.44994277
0.25	20.00	2.916354167	45.13516668	29.57547985
0.35	28.00	6.042720833	53.40464942	43.86244602
0.45	36.00	10.52898750	60.55518053	59.12587515

Table 4 Error between consecutive iterations for $R(t)$ at order β

t	$\beta = 1$		$\beta = 0.50$	
	$ R_1 - R_0 $	$ R_2 - R_1 $	$ R_1 - R_0 $	$ R_2 - R_1 $
0	0	0	0	0
0.001	0.004	0.00001596	0.1427299293	0.031920
0.002	0.008	0.00006384	0.2018506018	0.063840
0.003	0.012	0.00014364	0.2472154893	0.095760
0.004	0.016	0.00025536	0.2854598586	0.127680
0.005	0.020	0.00039900	0.3191538243	0.15960

Table 5 Error between consecutive iterations for $D(t)$ at order β

t	$\beta = 1$		$\beta = 0.50$	
	$ D_1 - D_0 $	$ D_2 - D_1 $	$ D_1 - D_0 $	$ D_2 - D_1 $
0	0	0	0	0
0.2	3.20	0.1920	8.074024070	1.920
0.4	6.40	0.7680	11.41839434	3.840
0.6	9.60	1.7280	13.98461991	5.760
0.8	12.80	3.0720	16.14804814	7.680
1.0	16.00	4.8000	18.05406667	9.600

studies are required to stop new outbreaks. The present EVD model exposes a new feature of β which was absent in integer order ($\beta = 1$) (Figs. 2, 3, 4 and 5).

Fig. 2 Variation of Susceptibles $S(t)$ with time at different i

Fig. 3 Variation of number of infected $I(t)$ with time at various r

Conclusion

Severe devastation of mankind was observed in Africa by EVD few years ago. A time-fractional EVD model of arbitrary order is examined with efficacious use of FVIM in finding its new reliable solution. FVIM scheme is capable of lessening the size of calculation. The results verify that FVIM scheme is competent even with low iterations. The convergence analysis shows that the FVIM solution is convergent, bounded, positive. Tables 2a – 2d

Fig. 4 Variation of $I(t)$ with time at distinct d

Fig. 5 Variation of $R(t)$ with time at some s

display that the absolute error between consecutive iterations is insignificant for S , I , R and D . As a future work, certain hybrid methods using integral transforms and numerical methods can also be applied to solve the nonlinear fractional EVD models. A comparative study of results can be performed to obtain more accuracy. It is concluded that FVIM is highly efficient

and influential in finding the trustworthy solution to the non-integer order nonlinear models of extreme significance for society.

Author's Contribution VB investigated the results, validated the solution and was major contributor in writing manuscript. MG conceptualized, visualized the solution of fractional Ebola virus disease model and supervised the methodology used. Both the authors read the final manuscript and approved.

Funding No funding was granted to conduct this research work.

Availability of Data and Material Data sharing is not applicable.

Declarations

Conflicts of interest The authors have no conflicts of interest/competing interests to disclose.

References

- Barry, M., Traoré, F.A., Sako, F.B., Kpamy, D.O., Bah, E.I., Poncin, M., Keita, S., Cisse, M., Touré, A.: Ebola outbreak in Conakry, Guinea: epidemiological, clinical, and outcome features. *Méd. Mal. Infect.* **44**(11–12), 491–494 (2014)
- Lewnard, J.A., Mbah, M.L.N., Alfaro-Murillo, J.A., Altice, F.L., Bawo, L., Nyenswah, T.G., Galvani, A.P.: Dynamics and control of Ebola virus transmission in Montserrado, Liberia: a mathematical modelling analysis. *Lancet Infect. Dis.* **14**(12), 1189–1195 (2014)
- Chua, K.B., Crameri, G., Hyatt, A., Yu, M., Tompong, M.R., Rosli, J., McEachern, J., Crameri, S., Kumarasamy, V., Eaton, B.T., Wang, L.F.: A previously unknown reovirus of bat origin is associated with an acute respiratory disease in humans. *Proc. Natl. Acad. Sci.* **104**(27), 11424–11429 (2007)
- Alton, J.: *The Ebola Survival Handbook*. Skyhorse Publishing, New York (2014)
- Légrand, J., Grais, R.F., Boelle, P.Y., Valleron, A.J., Flahault, A.: Understanding the dynamics of Ebola epidemics. *Epidemiol. Infect.* **135**(4), 610–621 (2007)
- Report of an International Commission: Ebola haemorrhagic fever in Zaire, 1976. *Bull. World Health Organ.* **56**(2), 271–293 (1978)
- Okware, S.I., Omaswa, F.G., Zaramba, S., Opio, A., Lutwama, J.J., Kamugisha, J., Rwaguma, E.B., Kagwa, P., Lamunu, M.: An outbreak of Ebola in Uganda. *Trop. Med. Int. Health* **7**(12), 1068–1075 (2002)
- Report of a WHO/International Study Team: Ebola haemorrhagic fever in Sudan, 1976. Report of a WHO/International Study Team. *Bull. World Health Organ.* **56**(2), 247–270 (1978)
- <https://www.who.int/emergencies/situations/ebola-2021-nzerekore-guinea>. Accessed on July 10, 2021
- <https://www.who.int/news-room/fact-sheets/detail/ebola-virus-disease>. Accessed on Sept. 25, 2021
- Gire, S.K., Goba, A., Andersen, K.G., Sealfon, R.S., Park, D.J., Kanneh, L., Jalloh, S., Momoh, M., Fullah, M., Dudas, G., Wohl, S.: Genomic surveillance elucidates Ebola virus origin and transmission during the 2014 outbreak. *Science* **345**(6202), 1369–1372 (2014)
- WHO Ebola Response Team: Ebola virus disease in West Africa—the first 9 months of the epidemic and forward projections. *N. Eng. J. Med.* **371**(16), 1481–1495 (2014)
- Chertow, D.S.: Understanding long-term effects of Ebola virus disease. *Nat. Med.* **25**(5), 714–715 (2019)
- Almuqrin M. A., Goswami P., Sharma S., Khan I., Dubey R.S., Khan A.: Fractional model of Ebola virus in population of bats in frame of Atangana-Baleanu fractional derivative. *Results Phys.* **26**, 104295 (2021)
- Mamo, D.K., Koya, P.R.: Mathematical modeling and simulation study of SEIR disease and data fitting of Ebola epidemic spreading in West Africa. *J. Multidiscip. Eng. Sci. Technol.* **2**(1), 106–114 (2015)
- Rachah, A., Torres, D.F.M.: Dynamics and optimal control of Ebola transmission. *Math. Comput. Sci.* **10**(3), 331–342 (2016)
- Osemwinyen, A.C., Diakhaby, A.: Mathematical modelling of the transmission dynamics of Ebola virus. *Appl. Comput. Math.* **4**(4), 313–320 (2015)
- Rachah, A., Torres, D.F.M.: Analysis, simulation and optimal control of a SEIR model for Ebola virus with demographic effects. *Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.* **67**(1), 179–197 (2018)
- Atangana, A., Goufo, E.F.D.: On the mathematical analysis of Ebola hemorrhagic fever: deathly infection disease in West African countries. *BioMed. Res. Int.* **2014**, 1–7 (2014)

20. Goyal, M., Prakash, A., Gupta, S.: *Mathematical Modeling and Soft Computing in Epidemiology*, pp. 173–198. CRC Press, Boca Raton (2020)
21. Srivastava, H.M., Gunerhan, H.: Analytical and approximate solutions of fractional-order susceptible-infected-recovered epidemic model of childhood disease. *Math. Methods Appl. Sci.* **42**(3), 935–941 (2019)
22. Goyal, M., Baskonus, H.M., Prakash, A.: An efficient technique for a time fractional model of lassa hemorrhagic fever spreading in pregnant women. *Eur. Phys. J. Plus* **134**(10), 482 (2019)
23. Prakash, A., Goyal, M., Gupta, S.: q-homotopy analysis method for fractional Bloch model arising in nuclear magnetic resonance via the Laplace transform. *Ind. J. Phys.* **94**(4), 507–520 (2020)
24. Prakash, A., Goyal, M., Baskonus, H.M., Gupta, S.: A reliable hybrid numerical method for a time dependent vibration model of arbitrary order. *AIMS Math.* **5**(2), 979–1000 (2020)
25. Goyal, M., Prakash, A., Gupta, S.: An efficient perturbation Sumudu transform technique for the time-fractional vibration equation with a memory dependent fractional derivative in Liouville–Caputo sense. *Int. J. Appl. Comput. Math.* **7**(4), 156 (2021)
26. Goyal, M., Baskonus, H.M.: A reliable solution of arbitrary order nonlinear Hunter-Saxton equation with time dependent derivative in Liouville–Caputo sense. *Int. J. Appl. Comput. Math.* **7**(4), 125 (2021)
27. Prakash, A., Goyal, M., Gupta, S.: Numerical simulation of space-fractional Helmholtz equation arising in seismic wave propagation, imaging and inversion. *Pramana* **93**(2), 28 (2019)
28. Goyal, M., Prakash, A., Baleanu, D.: An efficient hybrid computational technique for the time dependent Lane–Emden equation of arbitrary order (in press). *Int. J. Ocean Eng. Sci.* (2021). <https://doi.org/10.1016/j.joes.2021.07.004>
29. Srivastava, H.M.: Fractional-order integral and derivative operators and their applications. *Mathematics* **8**(6), 1016 (2020)
30. Goyal, M., Baskonus, H.M., Prakash, A.: Regarding new positive, bounded and convergent numerical solution of nonlinear time fractional HIV/AIDS transmission model. *Chaos Soliton Fract.* **139**, 110096 (2020)
31. Prakash, A., Veerasha, P., Prakash, D.G., Goyal, M.: A homotopy technique for a fractional order multi-dimensional telegraph equation via the Laplace Transform. *Eur. Phys. J. Plus* **134**(1), 19 (2019)
32. Saad, K.M., Al-Sharif, E.H.F.: Analytical study for time and time-space fractional Burgers' equation. *Adv. Differ. Equ.* **2017**(1), 300 (2017)
33. Prakash, A., Veerasha, P., Prakash, D.G., Goyal, M.: A new efficient technique for solving fractional coupled Navier–Stokes equations using q-homotopy analysis transform method. *Pramana* **93**(1), 6 (2019)
34. Khader, M.M., Saad, K.M.: Numerical treatment for studying the blood ethanol concentration systems with different forms of fractional derivatives. *Int. J. Mod. Phys. C* **31**(3), 2050044 (2020)
35. Du, M., Wang, Z., Hu, H.: Measuring memory with the order of fractional derivative. *Sci. Rep.* **3**(1), 3431 (2013)
36. Mhlhanga, A.: Dynamical analysis and control strategies in modelling Ebola virus disease. *Adv. Differ. Equ.* **2019**(1), 458 (2019)
37. Area, I., Batarfi, H., Losada, J., Nieto, J.J., Shammakh, W., Torres, A.: On a fractional order Ebola epidemic model. *Adv. Differ. Equ.* **2015**(1), 278 (2015)
38. Goufo, E.F.D., Pene, M.K., Mugisha, S.: Stability analysis of epidemic models of Ebola hemorrhagic fever with nonlinear transmission. *J. Nonlinear Sci. Appl.* **9**(6), 4191–4205 (2016)
39. Derakhshan, M.H.: The stability analysis and numerical simulation based on Sinc Legendre collocation method for solving a fractional epidemiological model of the Ebola virus. *Part. Differ. Equ. Appl. Math.* **3**, 100037 (2021)
40. Singh, H.: Analysis for fractional dynamics of Ebola virus model. *Chaos Solitons Fract.* **138**, 109992 (2020)
41. Dokuyusu, M.A., Dutta, H.: A fractional order model for Ebola Virus with the new Caputo fractional derivative without singular kernel. *Chaos Solitons Fract.* **134**, 109717 (2020)
42. Srivastava, H.M., Saad, K.M., Khader, M.M.: An efficient spectral collocation method for the dynamic simulation of the fractional epidemiological model of the Ebola virus. *Chaos Solitons Fract.* **140**, 110174 (2020)
43. Srivastava, H.M., Deniz, S.: A new modified semi-analytical technique for a fractional-order Ebola virus disease model. *RACSAM* **115**(3), 137 (2021)
44. Sun, H.G., Zhang, Y., Baleanu, D., Chen, W., Chen, Y.Q.: A new collection of real world applications of fractional calculus in science and engineering. *Commun. Nonlinear Sci. Numer. Simul.* **64**, 213–231 (2018)
45. Engheta, N.: On the role of fractional calculus in electromagnetic theory. *IEEE Antennas Propag. Mag.* **39**(4), 35–46 (1997)
46. Magin, R.L.: Fractional calculus in bioengineering. *Crit. Rev. Biomed. Eng.* **32**, 1–104 (2004)

47. Zhang, Y.: Travelling wave solutions of (2+1)-dimensional generalised time-fractional Hirota equation. *Pramana* **90**(3), 34 (2018)
48. Bohannan, G.W.: Analog fractional order controller in temperature and motor control applications. *J. Vib. Control* **14**, 1487–1498 (2008)
49. Robinson, A.D.: The use of control systems analysis in neurophysiology of eye movements. *Annu. Rev. Neurosci.* **4**, 463–503 (1981)
50. Ali, M.F., Sharma, M., Jain, R.: An application of fractional calculus in Electrical Engineering. *Adv. Eng. Tec. Appl.* **5**(4), 41–45 (2016)
51. Bagley, R.L., Torvik, P.J.: Fractional calculus in the transient analysis of viscoelasticity damped structures. *AIAA J.* **23**, 918–925 (1985)
52. Pawar, D.D., Patil, W.D., Raut, D.K.: Numerical solution of fractional order mathematical model of drug resistant tuberculosis with two line treatment. *J. Math. Comput. Sci.* **10**(2), 262–276 (2020)
53. Pawar, D.D., Patil, W.D., Raut, D.K.: Fractional-order mathematical model for analysing impact of quarantine on transmission of COVID-19 in india. *Math. Model. Comput.* **8**(2), 253–266 (2021)
54. Pawar, D.D., Patil, W.D., Raut, D.K.: Analysis of malaria dynamics using its fractional order mathematical model. *J. Appl. Math. Inform.* **39**(1–2), 197–214 (2021)
55. He, J.H.: Homotopy perturbation technique. *Comput. Methods Appl. Mech. Eng.* **178**(3–4), 257–262 (1999)
56. Liao, S.: On the homotopy analysis method for the nonlinear problems. *Appl. Math. Comput.* **147**(2), 499–513 (2004)
57. Adomian, G.: A review of the decomposition method in applied mathematics. *J. Math. Anal. Appl.* **135**(2), 501–544 (1988)
58. Pawar, D.D., Patil, W.D., Raut, D.K.: Numerical solution of time fractional order partial differential equations. *J. Math. Comput. Sci.* **10**(4), 1066–1082 (2020)
59. Ahmad, H., Khan, T.A., Ahmad, I., Stanimirović, P.S., Chu, Y.-M.: A new analyzing technique for nonlinear time fractional Cauchy reaction-diffusion model equations. *Results Phys.* **19**, 103462 (2020)
60. Ahmad, H., Akgül, A., Khan, T.A., Stanimirović, P.S., Chu, Y.-M.: New perspective on the conventional solutions of the nonlinear time-fractional partial differential equations. *Complexity* **2020**, 8829017 (2020)
61. Ahmad, H., Khan, T.A.: Variational iteration algorithm-I with an auxiliary parameter for wave-like vibration equations. *J. Low Freq. Noise Vib. Act. Control* **38**(34), 1113–1124 (2019)
62. Ahmad, H., Khan, T.A., Stanimirović, P.S., Chu, Y.-M., Ahmad, I.: Modified variational iteration algorithm-II: convergence and applications to diffusion models. *Complexity* **2020**, 8841718 (2020)
63. Ahmad, H., Seadawy, A.R., Khan, T.A.: Study on numerical solution of dispersive water wave phenomena by using a reliable modification of variational iteration algorithm. *Math. Comput. Simul.* **177**, 13–23 (2020)
64. Ahmad, H., Alam, M.N., Omri, M.: New computational results for a prototype of an excitable system. *Results Phys.* **28**, 104666 (2021)
65. Ahmad, H., Khan, T.A., Durur, H., Ismail, G.M., Yokus, A.: Analytic approximate solutions of diffusion equations arising in oil pollution. *J. Ocean Eng. Sci.* **6**(1), 62–69 (2021)
66. Ahmad, I., Khan, M.N., Inc, M., Ahmad, H., Nisar, K.S.: Numerical simulation of simulate an anomalous solute transport model via local meshless method. *Alex. Eng. J.* **59**(4), 2827–2838 (2020)
67. Prakash, A., Goyal, M., Gupta, S.: Fractional variational iteration method for solving time fractional Newell–Whitehead–Segel equation. *Nonlinear Eng.* **8**(1), 164–171 (2019)
68. He, J.H.: Variational iteration method—a kind of nonlinear analytical technique: Some examples. *Int. J. Nonlinear Mech.* **34**(4), 699–708 (1999)
69. He, J.H.: A short remark on fractional variational iteration method. *Phys. Lett. A* **375**(38), 3362–3364 (2011)
70. Goyal, M., Prakash, A., Gupta, S.: Numerical simulation for time-fractional nonlinear coupled dynamical model of romantic and interpersonal relationships. *Pramana* **92**(5), 82 (2019)
71. Prakash, A., Goyal, M., Gupta, S.: A reliable algorithm for fractional Bloch model arising in magnetic resonance imaging. *Pramana* **92**(2), 18 (2019)
72. Gupta, S., Goyal, M., Prakash, A.: Numerical treatment of Newell–Whitehead–Segel equation. *TWMS. J. App. Eng. Math.* **10**(2), 312–320 (2020)
73. Kilbas, A.A., Srivastava, H.M., Trujillo, J.J.: Theory and Applications of Fractional Differential Equations, pp. 1–540. Elsevier Science, New York (2006)
74. Podlubny, I.: Fractional Differential Equations, pp. 1–366. Academic Press, San Diego (1999)
75. Caputo, M.: Elasticità e Dissipazione, pp. 1–300. Zani-Chelli, Bologna (1969)
76. Diethelm, K.: The Analysis of Fractional Differential Equations, pp. 1–253. Springer, Berlin (2004)
77. Atangana, A., Alqahtani, R.T.: Tumour model with intrusive morphology, progressive phenotypical heterogeneity and memory. *Eur. Phys. J. Plus* **133**(3), 85 (2018)

78. Sakar, M.G., Ergören, H.: Alternative variation iteration method for solving the time–fractional Fornberg–Whitham equation. *Appl. Math. Model.* **39**(14), 3972–3979 (2015)
79. Odibat, Z.M.: A study on the convergence of variational iteration method. *Math. Comput. Model* **51**(9–10), 1181–1192 (2010)
80. Tatari, M., Dehghan, M.: On the convergence of He’s variational iteration method. *J. Comput. Appl. Math.* **207**(1), 121–128 (2007)
81. Goyal, M., Bhardwaj, V.K., Prakash, A.: Investigating new positive, bounded and convergent numerical solution for the nonlinear time-dependent breast cancer dynamic competition model. *Math. Method. Appl. Sci.* **44**(6), 4636–4653 (2021)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.