Interval Oriented Entropy Based Multi-item Solid Transportation Problem with Budget and Breakability

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Abstract To maximization the total profit and minimization the total cost we investigate two transportation models with interval transportation parameters and we present a solution of our newly developed models using appropriate methods. The solid transportation problem has been formulated with and without interval entropy function defined by Shannon's measure of entropy. Also an additional constraint on the total budget at each destination is imposed. When we transport the goods from plants to destination then few items should be damage due to bad condition of route and vehicles, for this reason we introduce the breakability in our respective models. We employ two different approaches (Hu and Wang's Approach and Mahato and Bhunia's Approach) based on interval ranking and made a comparison on it. To convert the multi-objective programming problem into single-objective problem we apply weighted sum method. The models are illustrated with numerical examples and solved using the LINGO.13 optimization software. The results of the models with and without entropy are also obtained and compared.

Keywords Solid transportation problem (STP) · Breakability · Budget constraint · Interval order relation · Entropy function · Weighted sum method

Introduction

Interval number is the generalization of a real number. Just as real numbers have an associated arithmetic and mathematical analysis, interval numbers have a distinct interval arithmetic and

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interval analysis [3, 7–10]. Kulpa [13, 14] described the space of intervals diagrammatically and applying this concept he introduced a new paradigm for studying the interval relations. Some basic interval ranking definitions were prescribed using the general concept of intervals. It has been observed that the definitions given in [9] are not applicable for all pairs of intervals. Ishibuchi and Tanaka [6] proposed some improved definitions of interval order relations in comparison to that defined in [9]. Chanas and Kuchta [11] tried to generalize the work in [6] with the notion of t_0, t_1 – cut of the intervals and a modified version of order relations for intervals has been proposed by Hu and Wang [3]. Mahato and Bhunia [11] proposed a modified version of Ishibuchi and Tanaka's [6]. The transportation problem (TP) was originally developed by Hitchcock [5]. The classical TP deals with transportation of goods from some sources to certain destinations. An extension of TP, called solid transportation problem (STP) was stated by Schell [4]. Entropy function was first introduced by ojha et al. [16] in 2009. Recently Baidya et al. [1, 2] solves two problems on safety factor in STP. Karmakar and Bhunia [18] in 2012 proposed a comparative study of different order relations of intervals. A fixed charge TP with interval parameters solved by Safi and Razmjoo [19] in 2013 and kundu et al. [20] discussed a TP with type-2 fuzzy variables. So many researchers [17, 21–29] developed and discussed about interval ranking and interval number and very few discussed the interval number to solve any transportation problem.

Interval uncertainty is the most important invention in the uncertainty world. Also when we transport the breakable items (glass-goods, toys, ceramic-goods) from source to destination using different mode of transport, sometime it is not possible to transport the commodities smoothly from plant to customer due to bad condition of road and vehicle. After careful study of the literature we have few lacks in STP. These are:

- (i) So many researchers [3, 6, 9, 11] discuss so many ranking operation on the interval inequalities. But nobody can utilize this concept into the TP to make the decision about the transportation cost and total profit gain by the customer.
- (ii) Very few researchers in the literature solve few STPs taking breakability but nobody can solve a multi- item STP with interval ranking as well as breakability.
- (iii) Nobody can solve entropy based STP in interval environment with breakability and budget at each destination.

The problem treated here deals with ordering of interval numbers in solving decision making problems. After careful study of literature we see that nobody can solve a STP with interval unit transportation costs, demands, conveyances capacities, purchasing cost, selling price, budget and breakability etc. In this manuscript we investigate a solution of interval valued STP with total budget and breakability in TP using 'Hu and Wang' and 'Mahato and Bhunia' approaches in interval order relation. We deduce the interval STP using appropriate interval order relation. The entropy function is added to the interval objective and the resulting problem is eventually reduced to single objective problem using weighted sum technique and gradient based optimization-generalized reduced gradient (GRG) method. The models are illustrated with specific numerical data. An assessment of different results of the respective models is presented. Also the solutions of the respective models and the effect of the entropy function on the system using two approaches have been illustrated numerically.

Preliminaries

Interval Numbers

An interval number A is defined as, $A = [a_L, a_R] = \{x : a_L \leq x \leq a_R, x \in \mathbb{R}\}$.

Here $a_L, a_R \in \mathbb{R}$ are the lower and upper bounds of the interval A , respectively. An interval number can also be expressed by its mean and width. In this form, an interval number $A = [a_L, a_R]$ is denoted by $\langle a_M, a_W \rangle$, where $a_M = \frac{a_R + a_L}{2}$ and $a_W = \frac{a_R - a_L}{2}$ are known as the center and radius of the interval, respectively.

Finite Interval Mathematics

Let, $A = [a_L, a_R]$ and $B = [b_L, b_R]$ are two intervals. Here, we shall give basic formulas of interval mathematics as follows:

$$\text{Addition } A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R]$$

$$\text{Subtraction } A - B = [a_L, a_R] - [b_L, b_R] = [a_L - b_R, a_R - b_L]$$

Order Relations of Intervals

Let $A = [a_L, a_R]$ and $B = [b_L, b_R]$ be a pair of arbitrary interval. These can be classified as follows:

Type-I Non-overlapping intervals; *Type-II* Partially overlapping intervals; *Type-III* Completely overlapping intervals;

General Interval Ranking Definitions

Some basic interval ranking definitions were prescribed using the general concept of intervals. These were defined by means of upper, lower bound, mean and width of the intervals.

Hu and Wang's Approach

To introduce this, let us take a simple decision making situation regarding interval ranking. Their interval ranking relation " $\leq$ " is defined as follows:

Definition 1 For any two intervals $A = [a_L, a_R] = \langle a_M, a_W \rangle$ and $B = [b_L, b_R] = \langle b_M, b_W \rangle$,

$$A \leq B \text{ if and only if } \begin{cases} a_M < b_M & \text{for } a_M \neq b_M \\ a_W \geq b_W & \text{for } a_M = b_M \end{cases}.$$

Furthermore $A < B$ if and only if $A \leq B$ and $A \neq B$.

Mahato and Bhunia's Approach

Only upper bound-lower bound form and mean-width form are used to define this order relation. Let $A = [a_L, a_R] = \langle a_M, a_W \rangle$ and $B = [b_L, b_R] = \langle b_M, b_W \rangle$ be two intervals costs/times for the minimization problems.

Optimistic Decision-Making

Definition 2 For the minimization problems, the order relation " $\leq_{omin}$ " between the intervals A and B is (i) $A \leq_{omin} B$ if and only if $a_L \leq b_L$, (ii) $A <_{omin} B$ if and only if $A \leq_{omin} B$ and $A \neq B$.

Definition 3 For the maximization problems, the order relation " $\geq_{omax}$ " between the intervals A and B is (i) $A \geq_{omax} B$ if and only if $a_R \geq b_R$, (ii) $A >_{omax} B$ if and only if $A \geq_{omax} B$ and $A \neq B$.

Pessimistic Decision-Making

In this case, the decision maker determines the minimum cost/time for minimization problems according to the principle “Less uncertainty is better than more uncertainty”.

Definition 4 For minimization problems, the order relation $<_{pmin}$ between the interval $A = [a_L, a_R] = \langle a_M, a_W \rangle$ and $B = [b_L, b_R] = \langle b_M, b_W \rangle$ for pessimistic decision making are (i) $A <_{pmin} B$ if and only if $a_M < b_M$, for Type-I and Type-II intervals, (ii) $A <_{pmin} B$ if and only if $a_M \leq b_M$ and $a_W < b_W$, for Type-III intervals. However, for Type-III intervals with $a_M < b_M$ and $a_W > b_W$, pessimistic decisions cannot be determined. In this case, the optimistic decision is to be considered.

Solution Techniques

Weighted Sum Method

The weighted sum method salaries a set of objectives into a single objective by multiplying each objective with users supplied weights. The weights of an objective are usually chosen in proportion to the objective's relative importance in the problem. However setting up an appropriate weight vector depends on the scaling of each objective function. It is likely that different objectives take different orders of magnitude. When such objectives are weighted to form a composite objective function, it would be better to scale them appropriately so that each objective possesses more or less the same order of magnitude. This process is called normalization of objectives. After the objectives are normalized, a composite objective function F can be formed by summing the weighted normalized objectives and the MOSTP is then converted to a single-objective optimization problem as follows:

$$\text{Optimize } F = \sum_{l=1}^L \omega_l f_l, \quad \omega_l \in [0, 1].$$

Here, ω_l is the weight of the l -th objective function. Since the optimum of the above problem does not change if all the weights are multiplied by a constant, it is the usual practice to choose weights such that their sum is one, i.e., $\sum_{l=1}^L \omega_l = 1$.

Entropy Function

For a random variable with outcomes, the Shannon entropy, a measure of uncertainty and denoted by, is defined as

$$H(x) = - \sum_{i=1}^n p(x_i) \log(x_i)$$

Let T be the transported amount i.e., $T = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q$. We consider a function $F(x)$ which represents the number of possible assignment for the state $X = (x_{ijk}^q)$. The (Shannon) entropy function of a variable X is defined as follows:

$F(X)$ = the number of ways selecting x_{111}^1 from T , multiplied by the number of ways selecting x_{121}^1 from $T - x_{111}^1, \dots$, multiplied by the number of ways selecting x_{MKN}^Q from

$$T - x_{111}^1 - x_{121}^1 - \dots - x_{MN-1K}^Q = \binom{T}{x_{111}^1} \cdot \binom{T - x_{111}^1}{x_{121}^1} \cdot \binom{T - x_{111}^1 - x_{121}^1}{x_{131}^1} \dots \dots \dots \binom{T - x_{111}^1 - x_{121}^1 - \dots - x_{MN-1K}^Q}{x_{MNK}^Q} = \frac{T!}{\prod_{q=1}^Q \prod_{i=1}^M \prod_{j=1}^N \prod_{k=1}^K x_{ijk}^q!}$$

$$\begin{aligned} \text{i.e., } \ln(F(X)) &= \ln(T!) - \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \ln(x_{ijk}^q!) \\ &= \ln(e^{-T} T^T) - \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \ln(e^{-x_{ijk}^q} x_{ijk}^q x_{ijk}^q x_{ijk}^q) \\ &\quad (\text{By using Stirlings approximation formula}) \\ &= T \cdot \ln(T) - \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q \ln(x_{ijk}^q) \\ \frac{\ln(F(X))}{T} &= \ln(T) - \frac{1}{T} \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q \ln(x_{ijk}^q) \end{aligned}$$

i.e., $E_n(X) = \frac{\ln(F(X))}{T}$, which is known as entropy function.

This entropy function (Shannon) can be expressed as

$E_n(X) = -\sum_x f(x)$, where $f(x) = \begin{cases} p(x) \ln(p(x)) & \text{if } p(x) \neq 0 \\ 0 & \text{if } p(x) = 0 \end{cases}$, $p(x)$ being the probability that x is in the state X .

In STP, normalizing the trip number x_{ijk}^q by dividing the total number of trips T , a probability distribution, $p(x) = \frac{x_{ijk}^q}{T}$ is formulated.

Therefore,

$$\begin{aligned} E_n(X) &= -\sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \frac{x_{ijk}^q}{T} \ln\left(\frac{x_{ijk}^q}{T}\right) \\ &= \ln(T) - \frac{1}{T} \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q \ln(x_{ijk}^q). \end{aligned}$$

Assumption and Notation

- (i) $C_{ijk}^q = [C_{ijkL}^q, C_{ijkR}^q]$ is the interval valued unit transportation cost from i -th origin to j -th destination by k -th conveyance of q -th item.
- (ii) $Z = [Z_L, Z_R]$ is the interval valued objective function.
- (iii) x_{ijk}^q is the unknown quantity which is to be transported from i -th origin to j -th destination by k -th conveyance of q -th item.
- (iv) $a_i^q = [a_{iL}^q, a_{iR}^q]$ is the interval availability at i -th origin of q -th item.

- (v) $b_j^q = [b_{jL}^q, b_{jR}^q]$ is the interval requirement at the j -th destination of q -th item.
- (vi) $e_k = [e_{kL}, e_{kR}]$ is the interval conveyances capacity of the k -th conveyance.
- (vii) $B = [B_L, B_R]$ is the interval total budget.
- (viii) $p_i^q = [p_{iL}^q, p_{iR}^q]$ is the interval purchasing cost at i -th origin of q -th item.
- (ix) $s_j^q = [s_{jL}^q, s_{jR}^q]$ is the interval selling price at j -th destination of q -th item.
- (x) α_{ijk}^q is the rate of breaking of the units due to transportation from i -th plant to j -th DC of q -th item by k -th conveyances.

Model Formulation

Model-1 Formulation of Cost Minimization Multi-item interval STP with interval unit transportation costs, availabilities, demands, conveyance capacities, purchasing cost, budget and crisp breakability

$$\text{Minimize } Z = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K (p_i^q + C_{ijk}^q) x_{ijk}^q,$$

Subject to the constraints,

$$\sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q = a_i^q, \quad i = 1, 2, \dots, M; \quad q = 1, 2, \dots, Q, \quad (1)$$

$$\sum_{i=1}^M \sum_{k=1}^K (1 - \alpha_{ijk}^q) x_{ijk}^q \leq b_j^q, \quad j = 1, 2, \dots, N; \quad q = 1, 2, \dots, Q, \quad (2)$$

$$\sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N x_{ijk}^q = e_k, \quad k = 1, 2, \dots, K, \quad (3)$$

$$\sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K p_i^q x_{ijk}^q \leq B, \quad (4)$$

$$x_{ijk}^q \geq 0, \quad i = 1, 2, \dots, M; \quad j = 1, 2, \dots, N; \quad k = 1, 2, \dots, K, \quad q = 1, 2, \dots, Q.$$

The problem is feasible if and only if $A \cap B \cap E \neq \emptyset$, where

$$\begin{aligned} A &= \sum_{q=1}^Q \sum_{i=1}^M a_i^q = \left[\sum_{q=1}^Q \sum_{i=1}^M a_{iL}^q, \sum_{q=1}^Q \sum_{i=1}^M a_{iR}^q \right], \\ B &= \sum_{q=1}^Q \sum_{j=1}^N b_j^q = \left[\sum_{q=1}^Q \sum_{j=1}^N b_{jL}^q, \sum_{q=1}^Q \sum_{j=1}^N b_{jR}^q \right], \\ E &= \sum_{k=1}^K e_k = \left[\sum_{k=1}^K e_{kL}, \sum_{k=1}^K e_{kR} \right]. \end{aligned}$$

Crisp transformation of the objective function of Model-1

Since it is desired to obtain an optimal interval value for the above model, we may minimize two characteristic of objective function, the left objective interval, Z_L , and its right, Z_R

$$Z_L = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \left(p_{iL}^q + C_{ijkL}^q \right) x_{ijk}^q,$$

$$Z_R = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \left(p_{iR}^q + C_{ijkR}^q \right) x_{ijk}^q.$$

Using weighted sum method we have to minimize the following objective function without and with entropy function:

$$\text{Minimize } Z = \frac{w_1 \times Z_L + w_2 \times Z_R}{w_1 + w_2} \text{ and}$$

$$\text{Minimize } Z = 7 \frac{w_1 \times Z_L + w_2 \times Z_R - w_3 \times E_n(X)}{w_1 + w_2 + w_3} \text{ respectively.}$$

where $E_n(X) = \ln(T) - \frac{1}{T} \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q \ln(x_{ijk}^q)$ and where $T = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q$.

In the section stated below we introduce the crisp conversion of the constraints of the respective model using different order relation of the intervals such as Hu and Wang's Approach and Mahato and Bhunia's Approach.

Crisp transformation of the constraints Model-1 using Hu and Wang's approach on interval order relation

Using Hu and Wang's approach we have the following crisp conversion of the constraints (1), (2), (3) and (4) of the above model:

$$a_{iL}^q \leq \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q \leq a_{iR}^q, \quad (5)$$

$$b_{jL}^q \leq \sum_{i=1}^M \sum_{k=1}^K (1 - \alpha_{ijk}^q) x_{ijk}^q \leq b_{jR}^q, \quad (6)$$

$$e_{kL} \leq \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N x_{ijk}^q \leq e_{kR}, \quad (7)$$

$$\text{and } \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \frac{p_{iL}^q + p_{iR}^q}{2} x_{ijk}^q \leq \frac{B_L + B_R}{2}, \quad (8)$$

respectively.

Crisp transformation of the constraints of the above model using Mahato and Bhunia's Approach on interval order relation

Using Mahato and Bhunia's Approach we have the following crisp conversion of the constraints (1), (2), (3), and (4) of Model-1: (5), (6), (7)

$$\text{and } \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K p_{iL}^q x_{ijk}^q \leq B_L \quad (9)$$

respectively.

Model-2 Formulation of Profit Maximization interval STP with interval unit transportation costs, availabilities, demands, conveyance capacities, purchasing cost, selling price, budget and crisp breakability

$$\text{Maximize } Z = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \left(s_j^q - p_i^q - C_{ijk}^q \right) x_{ijk}^q. \quad (10)$$

Subject to the constraints (1), (2), (3) and (4)

$$x_{ijk}^q \geq 0, \quad i = 1, 2, \dots, M; \quad j = 1, 2, \dots, N; \quad k = 1, 2, \dots, K, \quad q = 1, 2, \dots, Q.$$

The feasibility condition is same as Model-1.

Crisp Transformation of the objective function of Model-2

$$Z_L = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \left(s_{jL}^q - p_{iR}^q - C_{ijkR}^q \right) x_{ijk}^q,$$

$$Z_R = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \left(s_{jR}^q - p_{iL}^q - C_{ijkL}^q \right) x_{ijk}^q.$$

Using weighted sum method we have to minimize the following objective function without and with entropy function:

$$\text{Maximize } Z = \frac{w_1 \times Z_L + w_2 \times Z_R}{w_1 + w_2} \text{ and}$$

$$\text{Maximize } Z = \frac{w_1 \times Z_L + w_2 \times Z_R + w_3 \times E_n(X)}{w_1 + w_2 + w_3} \text{ respectively.}$$

where $E_n(X) = \ln(T) - \frac{1}{T} \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q \ln(x_{ijk}^q)$ and where $T = \sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^q$.

Also the crisp transformations of the constraints of the Model-2 are exactly same as Model-1 and using Mahato and Bhunia's approach we have the crisp reduction of the budget constraint is

$$\sum_{q=1}^Q \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K p_{iR}^q x_{ijk}^q \leq B_R, \quad (11)$$

respectively.

Numerical Illustration

Two products produced in two factories and sent to two destinations by two different modes of conveyances. The unit transportation cost, supply, demand, conveyance capacity, budget, purchasing cost and selling price are all intervals in nature and are given below:

Supplies $a_1^1 = [30, 40]$, $a_2^1 = [80, 90]$, $a_1^2 = [60, 90]$, $a_2^2 = [70, 98]$; *Demands* $b_1^1 = [68, 92]$, $b_2^1 = [55, 95]$, $b_1^2 = [49, 96]$, $b_2^2 = [71, 81]$; *Conveyances capacities*

$e_1 = [120, 150]$, $e_2 = [130, 200]$; Total interval budget $B = [799, 1390]$; Purchasing cost $p_1^1 = [2, 5]$, $p_2^1 = [4, 5]$; $p_1^2 = [4, 8]$, $p_2^2 = [2, 4]$. Selling price $s_1^1 = [26, 39]$, $s_2^1 = [23, 37]$; $s_1^2 = [36, 38]$, $s_2^2 = [39, 49]$; Weights $w_1 = 0.5$, $w_2 = 0.5$ (for the models without entropy); $w_1 = 0.3$, $w_2 = 0.4$, $w_2 = 0.3$ (for the models with entropy).

We consider a practical example of a transport company where the possible values of the parameters such as the unit transportation costs, supplies, demands, conveyance capacities and budget precisely determined. These linguistic data can be transferred into interval numbers.

With the above input data we have the following form of our given Model-1:

$$Z_L = \sum_{q=1}^2 \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \left(p_{iL}^q + C_{ijkL}^q \right) x_{ijk}^q \quad \text{and}$$

$$Z_R = \sum_{q=1}^2 \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \left(p_{iR}^q + C_{ijkR}^q \right) x_{ijk}^q$$

Subject to the constraints

$$\begin{aligned} a_{1L}^1 &\leq x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1 \leq a_{1R}^1, & a_{2L}^1 &\leq x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1 \leq a_{2R}^1, \\ a_{1L}^2 &\leq x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2 \leq a_{1R}^2, & a_{2L}^2 &\leq x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2 \leq a_{2R}^2, \\ b_{1L}^1 &\leq (1 - \alpha_{111}^1)x_{111}^1 + (1 - \alpha_{112}^1)x_{112}^1 + (1 - \alpha_{211}^1)x_{211}^1 + (1 - \alpha_{212}^1)x_{212}^1 \leq b_{1R}^1, \\ b_{2L}^1 &\leq (1 - \alpha_{121}^1)x_{121}^1 + (1 - \alpha_{122}^1)x_{122}^1 + (1 - \alpha_{221}^1)x_{221}^1 + (1 - \alpha_{222}^1)x_{222}^1 \leq b_{2R}^1, \\ b_{1L}^2 &\leq (1 - \alpha_{111}^2)x_{111}^2 + (1 - \alpha_{112}^2)x_{112}^2 + (1 - \alpha_{211}^2)x_{211}^2 + (1 - \alpha_{212}^2)x_{212}^2 \leq b_{1R}^2, \\ b_{2L}^2 &\leq (1 - \alpha_{121}^2)x_{121}^2 + (1 - \alpha_{122}^2)x_{122}^2 + (1 - \alpha_{221}^2)x_{221}^2 + (1 - \alpha_{222}^2)x_{222}^2 \leq b_{2R}^2, \\ e_{1L} &\leq x_{111}^1 + x_{121}^1 + x_{211}^1 + x_{221}^1 + x_{111}^2 + x_{121}^2 + x_{211}^2 + x_{221}^2 \leq e_{1R}, \\ e_{2L} &\leq x_{112}^1 + x_{122}^1 + x_{212}^1 + x_{222}^1 + x_{112}^2 + x_{122}^2 + x_{212}^2 + x_{222}^2 \leq e_{2R}. \end{aligned}$$

Using *Hu and Wang's approaches* we have the following crisp form of the budget constraint:

$$\begin{aligned} &\frac{(p_{1L}^1 + p_{1R}^1)}{2} (x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1) + \frac{(p_{2L}^1 + p_{2R}^1)}{2} (x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1) \\ &+ \frac{(p_{1L}^2 + p_{1R}^2)}{2} (x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2) + \frac{(p_{2L}^2 + p_{2R}^2)}{2} (x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2) \\ &\leq \frac{B_L + B_R}{2}. \end{aligned}$$

Using *Mahato and Bhunia's approaches* we have the following crisp form of the budget constraint:

$$\begin{aligned} &p_{1L}^1 (x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1) + p_{2L}^1 (x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1) \\ &+ p_{1L}^2 (x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2) + p_{2L}^2 (x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2) \leq B_L \end{aligned}$$

The problem is feasible since, $A \cap B \cap E = [270, 368] \cap [303, 364] \cap [250, 350] \neq \emptyset$.

Again with the above input we have the following form of Model-2:

$$Z_L = \sum_{q=1}^2 \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \left(s_{jL}^q - p_{iR}^q - C_{ijkR}^q \right) x_{ijk}^q$$

$$Z_R = \sum_{q=1}^2 \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \left(s_{jR}^q - p_{iL}^q - C_{ijkL}^q \right) x_{ijk}^q$$

Also, the crisp transformation of the budget constraint of the Model-2 using Hu and Wang's approach is same as Model-1 but the using Mahato and Bhunia's approach we have the following form of the budget constraint:

$$p_{1R}^1 (x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1) + p_{2R}^1 (x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1) \\ + p_{1R}^2 (x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2) + p_{2R}^2 (x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2) \leq B_R$$

Results

The above constrained optimization problems are executed using LINGO 13.0 optimization software and the results of models are presented below:

Optimal result of the model under Hu and Wang's approach

The optimal compromise solution of the respective models using weighted sum method is as follows (Tables 1, 2, 3, 5):

Table 1 Interval unit transportation cost

i	q=1				q=2			
	k=1		k=2		k=1		k=2	
	j							
	1	2	1	2	1	2	1	2
1	[2,6]	[3,6]	[3,5]	[5,8]	[2,5]	[4,9]	[2,5]	[4,6]
2	[4,7]	[1,3]	[5,7]	[3,5]	[2,5]	[2,5]	[5,8]	[5,7]

Table 2 Rate of breakability (%)

i	q=1				q=2			
	k=1		k=2		k=1		k=2	
	j							
	1	2	1	2	1	2	1	2
1	1 %	4 %	2 %	1 %	2 %	9 %	1 %	2 %
2	3 %	2 %	3 %	1 %	1 %	1 %	1 %	4 %

Table 3 Optimal result of Model-1 (without entropy)

i	q=1				q=2				Interval minimum cost
	k=1		k=2		k=1		k=2		
	j								
	1	2	1	2	1	2	1	2	
<i>Hu and Wang's approach</i>									
1	0	0	40	0	0	0	58.26	1.73	[1,391.9, 2,617.6]
2	0	55.81	29.69	0.31	0	70	0	0	
<i>Mahato and Bhunia's approach</i>									
1	0	0	40	0	0	0	53.26	6.73	[1,401.9, 2,622.6]
2	0	55.81	29.69	0.30	0	70	0	0	

Table 4 Optimal result of Model-1 (with entropy)

i	q=1				q=2				Interval minimum cost	Entropy
	k=1		k=2		k=1		k=2			
	j									
	1	2	1	2	1	2	1	2		
<i>Hu and Wang's approach</i>										
1	17.73	0	22.26	18.22	23.56	0	25.93	34.06	[1,472.1, 2,699.83]	2.15
2	0	37.71	29.51	0	0	46.44	0	0		
<i>Mahato and Bhunia's approach</i>										
1	0	0	40	0	0	0	26.21	33.79	[1,456.34, 2,649.68]	1.91
2	0	56.12	0	0	23.29	46.60	0	0		

Table 5 Optimal result of Model-2 (without entropy)

i	q=1				q=2				Interval maximum profit
	k=1		k=2		k=1		k=2		
	j								
	1	2	1	2	1	2	1	2	
<i>Hu and Wang's approach</i>									
1	0	0	69.39	60.19	0	0	0	0	[5,611.5, 12494.37]
2	0	0	0	0	49.49	80.41	0	0.41	
<i>Mahato and Bhunia's approach</i>									
1	0	0	69.39	95.96	0	0	0	0	[7,008.27, 11278.4]
2	0	0	0	0	69.98	80.02	0	0.82	

Discussion

In the first time we solve profit maximization and cost minimization interval valued STP using weighted sum method with different interval approaches, budget constraint and breakability.

Table 6 Optimal result of Model-2 (with entropy)

i	q=1				q=2				Interval Maximum Profit	Entropy
	k=1		k=2		k=1		k=2			
	j									
	1	2	1	2	1	2	1	2		
<i>Hu and Wang's approach</i>										
1	0	0	94.90	63.89	0	0	0	0	[6,764.6, 11844.25]	1.53
2	0	0	0	0	96.97	53.03	0	29.58		
<i>Mahato and Bhunia's approach</i>										
1	0	0	69.39	0	0	0	0	0	[6,575.61, 11006.52]	1.54
2	0	0	0	64.94	96.97	53.03	0	29.58		

In this manuscript we formulate two models one is to minimize the cost and another is to maximize the whole profit with different interval approaches as Hu and Wang's and Mahato and Bhunia's approaches and we observed that the total cost of Model-1 using Hu and Wang's is minimum than the cost obtained by Mahato and Bhunia's approach. Again solving the Model-2 to maximizing the profit using Mahato and Bhunia's gives the minimum profit than the Hu and Wang's approach. So we may conclude that to minimize the total cost and to maximize the total profit the Hu and Wang's approach is better than the Mahato and Bhunia's approach. So if a decision maker want to optimize the cost and profit of an multi-item interval valued STP under breakability then it has no important to solve the respective models using Mahato and Bhunia's approaches since in both the cases Hu and Wang's approach gives the optimal result of the TP. From Tables 4 and 6, it is observed that the amounts transported from origins to destinations by different modes of conveyances are well balanced when entropy is considered as an additional objective, though this process costs more than the process without entropy however when we solve the Model-2 without entropy we have the maximum profit than solving the model with entropy. Actually, in reality, balanced distributed quantities in different cells from the origins are expected.

Conclusion

Very few have formulated the TP as a maximization problem but nobody can solve a multi-item interval valued cost minimization and profit maximization STP with entropy function, breakability and budget constraint using different interval order relations. Till now, none has formulated the interval optimization problem and solve following weighted sum method. Moreover, fully interval constraints, different forms of dissimilar interval order relations are presented and used for optimization. The present problem will be useful for the different area of research who don't have sufficient or have the interval data for the transportation parameters. It is to note that the imposition of budget constraints gives different results for the profits and costs, which is different from the results as expected. The problem can be extended to include price/quantity discount, transportation of breakable items such as items made of glass, ceramics etc. In the first time we formulate and solve a STP taking interval unit transportation cost, availabilities, demand, conveyances capacities, purchasing cost and budget with breakability and entropy function. The effects of entropy function to solve the STP

are clearly shown in our manuscript. When items are transported through different routes then the transported amounts are not always well-balanced, so to balance the transported amount with interval transportation parameters we use the entropy function and we see that if we consider the entropy in our models then the transported amounts are well-balanced.

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