An Improved ELECTRE II-Based Outranking Method for MADM with Double Hierarchy Hesitant Fuzzy Linguistic Sets and Its Application to Emergency Logistics Provider Selection

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Abstract The emergency logistics provider selection (ELPS) is an important part of emergency logistics management. In an increasingly complex and highly uncertain evaluation environment, how to effectively select emergency logistics providers has become a hot topic in the field of emergency logistics management. In this paper, by integrating the double hierarchy hesitant fuzzy linguistic term set (DHHFLTS), the normalized projection measure and the improved ELECTRE II method, a practical integrated decision support model is proposed to handle the ELPS problem. First, DHHFLTSs are utilized to represent the complex and uncertain linguistic evaluation information provided by decision makers (DMs) to provide subjective judgments. Meanwhile, a distance measure for DHHFLTSs based on the proposed normalized projection is proposed to reflect both the included angle and the distance between evaluation objects, and a weight-determining method based on the proposed distance measure and the maximizing deviation method is constructed. Then a new decision support framework of double hierarchy hesitant fuzzy linguistic ELECTRE II based on positive and negative ideal solutions is put forward, in which three kinds of preference relations of the concordance and discordance sets are set up to rank the alternatives more reasonably. Finally, a practical example of ELPS is presented to illustrate the effectiveness of the proposed method, and detailed comparative analysis with other representative methods is also conducted to demonstrate the advantages of the proposed method.

Keywords Double hierarchy hesitant fuzzy linguistic term set · Normalized projection measure · ELECTRE II · Multi-attribute decision-making · Emergency logistics provider selection

1 Introduction

In recent years, emergencies such as severe natural disasters, public health emergencies and public security incidents have occurred frequently. Just as recently as December 2019, for example, the COVID-19 which broke out in Wuhan, China forced a national shutdown for a time and the global situation is still not optimistic. Besides, severe flooding brought by the rainy season can affect tens of millions of people in numerous provinces, leaving a large number of them in need of emergency life assistance. The annual typhoon season also brings disasters to southern cities of China. These frequent occurrences of disasters have attracted increasing concerns over emergency logistics.

Different from general commercial logistics, the characteristics of emergency logistics, suddenness and quick response, require timeliness for relief supply and

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distribution, which makes it difficult to meet distribution requirements in a short time based on the power of government or humanitarian organizations alone [1]. Incorporating marketable and socialized power into cooperation with governments allows both the emergency logistics providers and governments to reap benefits. The former can fulfil corporate social responsibility and improve their social image and the latter can be supported more professionally and efficiently in relief work [2, 3]. However, despite its practical significance, there is a relative lack of research on integrated application of methods for emergency logistics provider selection (ELPS), which is something of a surprise.

To select the most suitable provider involves many aspects including product quality, logistics cost, quick response ability, flexible demand, internal and external conditions, etc. Therefore, ELPS can be regarded as a MADM problem. Valuable lessons can be drawn from a large number of studies on existing logistics provider selection, and since the fuzzy set was proposed by Zadeh [4] to describe fuzziness, it has been widely used in combination with MADM methods for different types of logistics provider selection problems [5–9]. While when faced with ELPS, there are two practical problems to consider. On the one hand, considering the hesitation and qualitative expressions of people's cognition, quantitative fuzzy numbers are not proper and intuitive enough to describe the evaluation information, and quantitative expression may cause the distortion of original evaluation information. On the other hand, the non-compensation among attributes should not be ignored. Serious dissatisfaction with one attribute should not be completely compensated by satisfaction with other attributes. For example, for an alternative provider to be evaluated, low cost cannot compensate for slow response, low product level or other unsatisfactory attributes.

Zadeh [10] originally put forward the concept of linguistic variables to describe the assessment information qualitatively. Subsequently, linguistic variables are continuously developed and combined into various forms [11–14], which have been widely used in the field of fuzzy decision making. Among the linguistic variables, the hesitant fuzzy linguistic term set (HFLTS) [12] is a quite useful form that has been extensively expanded from a variety of research directions. Nevertheless, qualitative information expressed using HFLTSs is difficult to effectively meet the needs of DMs for complex and detailed linguistic terms (LTs). For example, HFLTSs cannot be used when DMs expect to express their opinions like "much good", "a little bad", "exactly right medium" or other similar LTs with adverbs of degree to express information in more detail. Thus, in order to elaborate the LTs to represent opinions from DMs more accurately and

reasonably, Gou et al. [15] defined the double hierarchy linguistic term set (DHLTS) in which a second hierarchy linguistic term sets (LTSs) was added to represent the detailed description of the first hierarchy. And considering the hesitation of decision making and combining DHLTS with the form of hesitant fuzzy set, they further introduced the double hierarchy hesitant fuzzy linguistic term set (DHHFLTS).

For alternative selection of MADM problems, various MADM methods have been proposed based on utility value [5–9, 16, 17], aggregation operators [18, 19] and the idea of outranking [20, 21]. Compared with other methods based on utility values and aggregation operators, outranking-based methods have more advantages in ranking alternatives with taking into consideration the non-compensation among attributes. The ELECTRE [22] method is a fairly representative kind of outranking method. Among the family of ELECTRE methods, ELECTRE II method is specifically designed for problems of alternative ranking, whose core idea is constructing strong and weak outranking relations with the concept of concordance and discordance to obtain the ranking result.

One of important concerns in research on extensions of ELECTRE II method focuses on distance measures and comparison methods under different decision-making environments, because as the key to ELECTRE II method, construction of outranking relations depends on the distance and comparison between alternatives. In existing ELECTRE II methods, Hamming distance or Euclidean distance were commonly utilized and extended by researchers to calculate the relative difference between elements [23–25]. However, distance measures do not consider the included angle between elements, which makes it not exact enough to reflect the actual difference between elements. By contrast, the projection measure can reflect the deviations between objects more accurately with the advantage of considering both the distance and the angle between objects, which allows it to be used as an ideal measure [26].

According to the above discussion, constructing an improved ELECTRE II method to deal with MADM problems about ELPS in a DHHFL environment is the motivation of this paper, and the main highlights and contributions of this paper are summed up as follows:

- The DHHFLTS with two hierarchy LTSs is utilized to express evaluation information. Compared with other linguistic variables, DHHFLTS can express complex linguistic evaluation more directly and intuitively for DMs in ELPS problems.
- A normalized projection-based distance measure for DHHFLTSs is defined to identify the difference between double hierarchy hierarchy fuzzy linguistic

elements (DHHFLEs), whose biggest characteristic is that it well integrates the properties of projection and distance measures, and can reflect the difference between DHHFLEs more accurately than the existing distance measures. Meanwhile, the weight-determining method is further established based on the proposed distance measure and the maximizing deviation method.

- An improved ELECTRE II method based on the positive and negative ideal DHHFLEs is put forward to rank the alternatives, in which a comparison method for DHHFLEs based on bidirectional projection is proposed to properly identify the concordance, indifferent and discordance sets.

- A concrete example of ELPS implementation and comparative analysis are provided to show the effectiveness and superiority of the proposed method compared with other existing methods, and a better decision support method for ELPS is presented.

The structure of this paper is organized as follows: In Sect. 2, some related work is reviewed concerning ELPS, DHHFLTSs, and ELECTRE II-based methods, and research gaps are summarized. In Sect. 3, some basic concepts about DHHFLTSs and ELECTRE II method are introduced. In Sect. 4, a distance measure between DHHFLEs based on normalized projection is proposed, and the attribute weight-determining method is defined accordingly. Then a comparison method for DHHFLEs which based on bidirectional projection is presented. Further, an ELECTRE II-based method for MADM with DHHFLTSs is put forward and a decision-making framework is formulated. Section 5 gives an illustrative ELPS example to demonstrate the process of the proposed method, and sensitivity analysis and comparative analysis are conducted. Section 6 discusses the managerial insights and practical implications, and Sect. 7 concludes the paper and points out the deficiencies and future research directions.

2 Literature Review

The attention to emergency logistics research has gradually increased in recent years, especially after the COVID-19 pandemic. Relevant studies involve many aspects including plan selection [16, 27], emergency resource allocation [28], public-private collaborations [29], logistics network design [30, 31], facility location [32, 33], etc., while there are still fewer studies on the ELPS. Kim et al. [2] combined DANP and COPRAS-G to form a multi-attribute decision making (MADM) model for ELPS from a socio-technical systems perspective. AHP and TOPSIS methods were used for partner selection in humanitarian supply chains by Venkatesh et al. [8].

Since DHHFLTS can intuitively express evaluation information, and compared with other linguistic variables, it can describe more detailed information with two hierarchy LTs when evaluating alternatives, much attention has been attached to it recently. The research on DHHFLTS mainly includes three aspects. Firstly, some studies focus on the application of the application of double hierarchy hesitant fuzzy linguistic (DHHFL) preference relations [34, 35]. Secondly, DHHFLTS was extended to more forms to accommodate more decision-making scenarios, such as double hierarchy hesitant bipolar fuzzy linguistic term sets [36], free double hierarchy hesitant fuzzy linguistic term sets [37], and unbalanced double hierarchy linguistic term sets [38]. Thirdly, many MADM methods such as TOPSIS [37], TODIM [39], PROMETHEE [40], ORESTE [41] in DHHFL decision-making environment was constructed.

Among various MADM methods, ELECTRE II method that based on the idea of outranking has been used for various MADM problems under fuzzy environment. Table 2 shows some recent work on ELECTRE II methods for fuzzy MADM. It can be seen that ELECTRE II-based methods have been studied in different fuzzy decision-making scenarios, and the comparison for fuzzy evaluation mostly based on Hamming distance or Euclidean distance.

Table 1 Recent work on ELECTRE II methods for fuzzy MADM

Author	Evaluation forms	Comparison measures	Problem statement
Chen and Xu [42]	Hesitant fuzzy sets	Hamming distance-based	Provider selection
Liao et al. [24]	Hesitant fuzzy linguistic term sets	Hamming distance-based	Servicing agent selection
Lin et al. [43]	Probabilistic linguistic term sets	Euclidean distance-based	Edge node selection
Akram et al. [23]	Hesitant Pythagorean fuzzy sets	Hamming distance-based	Evaluation of facilities
Liu et al. [44]	Linguistic Z-numbers	Hamming distance-based	Provider selection
Zahid et al. [25]	Complex spherical fuzzy sets	Euclidean distance-based	Technology selection
This paper	DHHFLTSs	Bidirectional projection-based	ELPS

Table 2 Symbols and explanations used in the formulas

Symbols	Explanations
ϕ_l	The subscripts of the first hierarchy linguistic terms
φ_l	The subscripts of the second hierarchy linguistic terms
X_i	The ith alternative
$Attr_j$	The jth attribute
$h_{S_O}^{ij}$	The assessment information of the alternative X_i under the attribute $Attr_j$
ϖ_j	The weight of $Attr_j$
$NPrj_{h_{S_O}^a}(h_{S_O}^b)$	The normalized projection of $h_{S_O}^a$ onto $h_{S_O}^b$
$h_{S_O}^{j+}$	The positive ideal DHHFLE
$h_{S_O}^{j-}$	The negative ideal DHHFLE
$\delta_{h_{S_O}}^{ajbj}$	The difference between $h_{S_O}^{aj}$ and $h_{S_O}^{bj}$
$Prj_{\delta_{h_{S_O}}^{j-j+}}(\delta_{h_{S_O}}^{j-aj})$	The bidirectional projections to positive ideal DHHFLE of $h_{S_O}^{aj}$ under the attribute $Attr_j$
$Prj_{\delta_{h_{S_O}}^{aij+}}(\delta_{h_{S_O}}^{j-j+})$	The bidirectional projections to negative ideal DHHFLE of $h_{S_O}^{aj}$ under the attribute $Attr_j$
$\overline{Prj}(\delta_{h_{S_O}}^{aj})$	The relative bidirectional projection of $h_{S_O}^{aj}$ under the attribute $Attr_j$

Based on the preceding reviews, this paper intends to propose an ELECTRE II-based outranking method with evaluation information in the form of DHHFLTSs to handle ELPS problems. The introduction of DHHFLTSs enables DMs to evaluate emergency logistics providers more intuitively. Meanwhile, as an excellent measure tool, bidirectional projection-based comparison measure can identify the difference between two DHHFLTSs more accurately. Furthermore, ELECTRE II method is based on pairwise comparisons, which is appropriate for ELPS with not completely complementary attributes. Therefore, combined with the proposed projection measures and based on the strong and weak outranking relations which derive from comprehensive concordance and discordance indices, ELECTRE II-based method can reasonably and accurately select a reliable emergency logistics provider.

3 Preliminaries

To facilitate readers to better understand this paper, Table 2 lists the symbols and explanations used in the formulas.

3.1 Double Hierarchy Hesitant Fuzzy Linguistic Term Sets

In order to represent complex linguistic evaluation information in more detail, Gou et al. [15] initially defined the concept of DHLTS consisting of two hierarchy LTSs. For a DHLTS whose mathematical form is $S_O = \{\varsigma_{a<\sigma_b>} \mid a = -\alpha, \ldots, -1, 0, 1, \ldots, \alpha; b = -\beta, \ldots, -1, 0, 1, \ldots, \beta\}$, $S = \{\varsigma_a | a = -\alpha, \ldots, -1, 0, 1, \ldots, \alpha\}$ is the first hierarchy LTS to express basic linguistic information like "bad", "good",

"medium" or "perfect". And the second hierarchy LTS $O = \{\sigma_b | b = -\beta, \ldots, -1, 0, 1, \ldots, \beta\}$ indicates the degrees or details of linguistic information included in S.

Attention should be paid to the value of a, which determines whether the second hierarchy LTS is selected in descending order or in ascending order to ensure that the degree of overall evaluation information in a DHHFLTS is increasing. Particularly, only a half of area is valid for the upper limit ς_α and the lower limit $\varsigma_{-\alpha}$, i.e., $O = \{\sigma_b | b = -\beta, \ldots, -1, 0\}$ and $O = \{\sigma_b | b = 0, 1, \ldots, \beta\}$ are utilized to describe ς_α and $\varsigma_{-\alpha}$, respectively. For example, Figure 1 shows the distributions of LTSs for a DHHFLTS with $\alpha = \beta = 3$.

Furthermore, by extending the DHLTS into hesitant fuzzy environment, the DHHFLTS was defined mathematically as follows.

Definition 1 [15] Let S_O be a DHLTS, then a DHHFLTS on a fixed set X is given by

$$H_{S_O} = \{ <x_i, h_{S_O}(x_i)> | x_i \in X \} \quad (1)$$

where $h_{S_O}(x_i)$ is called DHHFLE, denoting the linguistic elements included in S_O, and can be represented by

$$h_{S_O}(x_i) = \{\varsigma_{\phi_l<\sigma_{\varphi_l}>}(x_i)|\varsigma_{\phi_l<\sigma_{\varphi_l}>} \in S_O; l = 1, 2, \ldots, L\} \quad (2)$$

with ϕ_l and φ_l being the subscripts of the first and second hierarchy LTs, respectively, and L being the number of LTs in $h_{S_O}(x_i)$.

Discrete subscripts of LTs would lead to information loss, thus a monotone function f [15] was developed to achieve the transformation between DHHFLEs and real numbers, which is defined as:

$O_1 = \{\sigma_0 = exactly\ right, \sigma_1 = a\ litte, \sigma_2 = rarely, \sigma_3 = far\ from\}$
$O_2 = \{\sigma_{-3} = extremely, \sigma_{-2} = very\ much, \sigma_{-1} = much, \sigma_0 = exactly\ right, \sigma_1 = a\ little, \sigma_2 = rarely, \sigma_3 = far\ from\}$
$O_3 = \{\sigma_{-3} = far\ from, \sigma_{-2} = rarely, \sigma_{-1} = a\ little, \sigma_0 = exactly\ right, \sigma_1 = much, \sigma_2 = very\ much, \sigma_3 = extremely\}$
$O_4 = \{\sigma_{-3} = far\ from, \sigma_{-2} = rarely, \sigma_{-1} = a\ little, \sigma_0 = exactly\ right\}$
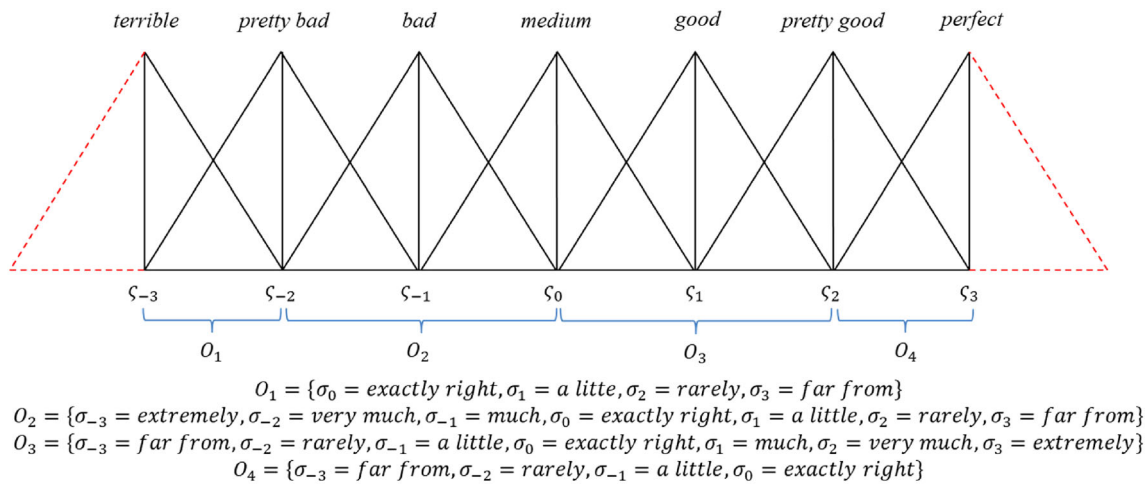
Fig. 1 The distributions of the second hierarchy LTS

$$f : [-\alpha, \alpha] \times [-\beta, \beta] \rightarrow [0, 1],$$
$$f(h_{S_O}^l) = f(\phi_l, \varphi_l) = \frac{\varphi_l + (\alpha + \phi_l)\beta}{2\beta\alpha} = \gamma_l \quad (3)$$

where γ_l is the real number expressing the membership degree.

3.2 Main Concepts Involved in ELECTRE Methods

The core idea of ELECTRE method is to determine the outranking relations which reflect the preference situations among alternatives. There exist four binary preference relations, including I (Indifference), P (Strict Preference), Q (Weak Preference), R (Incomparability). Outranking relations can be obtained through the analysis of the preference relations, which also depend on the concordance and discordance principle.

Definition 2 [45] For the comparison of the two alternatives μ and v, four binary relations are defined as follows.

(1) I corresponds to the indifference relation, and $\mu I v$ expresses the meaning of "μ is indifferent to v".
(2) P corresponds to the strict preference relation, and $\mu P v$ expresses the meaning of "μ is strictly preferred to v".
(3) Q corresponds to the weak preference relation, and $\mu Q v$ expresses the meaning of "μ is weakly preferred to v", which indicates the insufficiency of deducing either I or P.
(4) R corresponds to the incomparability relation, and $\mu R v$ expresses the meaning of "μ is not comparable to v".

The comprehensive outranking relation S is the basis of preferences in ELECTRE methods. In general, S which denotes "at least as good as" is grouped by three binary preference relations P, Q and I, i.e., $S = P \cup Q \cup I$.

Definition 3 [45] For two alternatives μ and v, there are four possible cases by modeling comprehensive preference information:

(1) μ is strictly preferred to v if $\mu S v$ and not $v S \mu$;
(2) v is strictly preferred to μ if $v S \mu$ and not $\mu S v$;
(3) μ is indifferent to v if $\mu S v$ and $v S \mu$;
(4) μ is not comparable to v if not $\mu S v$ and not $v S \mu$.

Definition 4 [45] Concordance and discordance which reflect the condition for or against the outranking relations are the key to all relevant outranking methods.

(1) Concordance: there must exist a sufficient majority of attributes supporting the opinion $\mu S v$ so that it can be validated.
(2) Discordance: if a minority of attributes strongly oppose the opinion $\mu S v$, it cannot be validated.

In other words, the validation of the opinion $\mu S v$ requires both the support of the majority of attributes and the non-strong opposition of the minority of attributes, which distinguishes ELECTRE methods from utility theory-based methods like TOPSIS, MULTIMOORA, VIKOR, etc., and shows the important advantage of ELECTRE methods in dealing with non-compensation among attributes. That is, serious dissatisfaction with the evaluation of one attribute will not be compensated by other highly rated attributes, which conforms to the actual decision-making situation.

4 Positive and Negative Ideal DHHFLEs-Based ELECTRE II Method

4.1 Normalized Projection-Based Distance Measure Between DHHFLEs

In actual decision-making environment, the number of LTs contained in different DHHFLEs may be unequal. To eliminate the influence of this situation and calculate the distance between DHHFLEs correctly, we firstly add LTs to the shorter DHHFLEs after considering the risk attitudes of decision makers to make the number of LTs equivalent. In this paper, optimistic principle is adopted to extend the DHHFLEs where there are less number of LTs by adding the maximum linguistic term in it to make DHHFLEs have same lengths on the same attribute.

Yue [46] provided a normalized projection measure after analyzing the shortcomings and irrationality of some existing projection measures, and provided a new normalized projection measure. In this paper, by extending it into DHHFL environment, the normalized projection measure between two DHHFLEs is defined.

Definition 5 Let $S_O = \{ \varsigma_{a<\sigma_b>} | a = -\alpha, \ldots, -1, 0, 1, \ldots, \alpha; b = -\beta, \ldots, -1, 0, 1, \ldots, \beta \}$ be a DHLTS, $h_{S_O}^1, h_{S_O}^2 \in S_O$ be two DHHFLEs, then the normalized projection of $h_{S_O}^1$ onto $h_{S_O}^2$ is defined as

$$NPrj_{h_{S_O}^2}(h_{S_O}^1) = \frac{\min\left\{ \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2}, \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^2|^2} \right\}}{\max\left\{ \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2}, \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^2|^2} \right\}}. \tag{4}$$

where $h_{S_O}^1 \cdot h_{S_O}^2 = \sum_{l=1}^{\tilde{L}} f(h_{S_O}^{1(l)}) f(h_{S_O}^{2(l)})$, $\tilde{L}$ is the number of DHHFLEs, $|h_{S_O}^1|$ and $|h_{S_O}^2|$ are called the modules of DHHFLEs, which are defined as:

$$|h_{S_O}| = \sqrt{\sum_{l=1}^{\tilde{L}} \left(f(h_{S_O}^{(l)}) \right)^2} = \sqrt{\sum_{l=1}^{\tilde{L}} (f(\phi_l, \varphi_l))^2}$$
$$= \sqrt{\sum_{l=1}^{\tilde{L}} \left(\frac{\varphi_l + (\alpha + \phi_l)\beta}{2\beta\alpha} \right)^2} \tag{5}$$

The normalized projection between two DHHFLEs satisfies the following conditions:

(1) $0 \le NPrj_{h_{S_O}^2}(h_{S_O}^1) \le 1$;

(2) The closer $NPrj_{h_{S_O}^2}(h_{S_O}^1)$ is to 1, the closer $h_{S_O}^1$ is to $h_{S_O}^2$;

(3) $NPrj_{h_{S_O}^1}(h_{S_O}^1) = NPrj_{h_{S_O}^2}(h_{S_O}^2) = 1$;

(4) $NPrj_{h_{S_O}^2}(h_{S_O}^1) = NPrj_{h_{S_O}^1}(h_{S_O}^2)$;

(5) $NPrj_{h_{S_O}^2}(h_{S_O}^1) = 1$ if and only if $h_{S_O}^{1(l)} = h_{S_O}^{2(l)}$ for each l.

Proof

(1) By Eq. (3), we have $0 \le \min\{\frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2}, \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^2|^2}\} \le \max\{\frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2}, \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^2|^2}\}$, hence $0 \le NPrj_{h_{S_O}^2}(h_{S_O}^1) \le 1$.

(2) According to the basic properties of projection, the lager the value of $\frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|}$ is, the closer $h_{S_O}^1$ is to $h_{S_O}^2$. Further, when $h_{S_O}^1$ is equal to $h_{S_O}^2$, $\frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2} = 1$. Thus, the closer $\frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2}$ is to 1, the closer $h_{S_O}^1$ is to $h_{S_O}^2$. Hence, the closer $NPrj_{h_{S_O}^2}(h_{S_O}^1)$ is to 1, the closer $h_{S_O}^1$ is to $h_{S_O}^2$.

(3) and (4) are straightforward, omitted here.

(5) If $h_{S_O}^1 \ne h_{S_O}^2$, then $|h_{S_O}^1| \ne |h_{S_O}^2|$ and $\frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2} \ne \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^2|^2}$. Further, $\min\{\frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2}, \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^2|^2}\} \ne \max\{\frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^1|^2}, \frac{h_{S_O}^1 \cdot h_{S_O}^2}{|h_{S_O}^2|^2}\}$, i.e., $NPrj_{h_{S_O}^2}(h_{S_O}^1) \ne 1$. Hence, $NPrj_{h_{S_O}^2}(h_{S_O}^1) = 1$ if and only if $h_{S_O}^{1(l)} = h_{S_O}^{2(l)}$ for each l.

$\square$

Based on this conditions, the normalized projection-based distance measure for DHHFLEs can be written as

$$dp(h_{S_O}^1, h_{S_O}^2) = 1 - NPrj_{h_{S_O}^2}(h_{S_O}^1). \tag{6}$$

As mentioned above, projection takes both distance and angle into account, which makes it more accurate to reflect degree of closeness between elements. Therefore, the proposed distance measure can get the distance between two DHHFLEs more accurately and properly than the general Hamming distance and Euclidean distance.

Example 1 Suppose that $h_{S_O}^1 = \{\varsigma_{2<\sigma_{-3}>}, \varsigma_{3<\sigma_{-1}>}, \varsigma_3 <\sigma_{-1}>\}$, $h_{S_O}^2 = \{\varsigma_{1<\sigma_2>}, \varsigma_2, \varsigma_{3<\sigma_{-1}>}\}$ and $h_{S_O}^3 = \{\varsigma_{0<\sigma_0>}, \varsigma_1, \varsigma_{2<\sigma_1>}\}$. Table 3 shows the distance $d(h_{S_O}^1, h_{S_O}^3)$ and $d(h_{S_O}^2, h_{S_O}^3)$ obtained by Hamming distance, Euclidean distance and our normalized projection-based distance equations.

It can be seen that the distance $d(h_{S_O}^1, h_{S_O}^3)$ and $d(h_{S_O}^2, h_{S_O}^3)$ calculated by Hamming distance and Euclidean distance are equal, respectively. But actually the distances from $h_{S_O}^1$ and $h_{S_O}^2$ to $h_{S_O}^3$ are different from a projection

Table 3 Comparison among distance measures

	Hamming distance	Euclidean distance	Normalized projection-based distance
$d(h_{S_O}^1, h_{S_O}^3)$	0.167	0.190	0.334
$d(h_{S_O}^2, h_{S_O}^3)$	0.167	0.190	0.323

perspective, which can be obtained by our normalized projection-based distance.

4.2 Attribute Weights Derived by the Maximizing Deviation Method

Considering a MADM problem with DHHFL information where there are m alternatives represented by $X = \{X_1, X_2, \ldots, X_m\}$, n attributes represented by $T = \{Attr_1, Attr_2, \ldots, Attr_n\}$, and $\varpi = \{\varpi_1, \varpi_2, \ldots, \varpi_n\}$ denotes the attribute weights.

The invited experts choose the double hierarchy linguistic terms from the DHLTS S_O to assess the alternatives. By gathering the assessment information and after extending the shorter DHHFLEs to the same length as others, a decision-making matrix with DHHFLEs is established as:

$$H = (h_{S_O}^{ij})_{m \times n} = \begin{matrix} & \begin{matrix} Attr_1 & Attr_2 & \cdots & Attr_n \end{matrix} \\ \begin{matrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{matrix} & \begin{bmatrix} h_{S_O}^{11} & h_{S_O}^{12} & \cdots & h_{S_O}^{1n} \\ h_{S_O}^{21} & h_{S_O}^{22} & \cdots & h_{S_O}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{S_O}^{m1} & h_{S_O}^{m2} & \cdots & h_{S_O}^{mn} \end{bmatrix} \end{matrix} \quad (7)$$

where h_{S_O} denote the DHHFLEs given by DMs according to their own evaluation preferences, and the element $h_{S_O}^{ij}$ refers to the assessment information of the alternative X_i under the attribute $Attr_j$.

The maximizing deviation method [47] advocates that the attribute which has a larger deviation value plays a more important role among all attributes. An optimization model was constructed by Xu et al. [48] based on this method. Since the normalized projection-based distance between DHHFLEs reflects the difference more accurately, it is reasonable and effective to calculate the attribute weights combined the proposed distance measure with the maximizing deviation method. And the attribute weight optimization model under DHHFL environment combined with the normalized projection-based distance can be expressed as

$$(M-1) \begin{cases} max\ dp(\varpi) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{m} \varpi_j dp(h_{S_O}^{ij}, h_{S_O}^{kj}) \\ s.t.\ \sum_{j=1}^{n} \varpi_j^2 = 1,\ \varpi_j \geq 0,\ j = 1, 2, \ldots, n \end{cases} \quad (8)$$

To solve the optimization model, the Lagrange function is given by

$$L(\varpi, \xi) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{m} \varpi_j dp(h_{S_O}^{ij}, h_{S_O}^{kj}) + \frac{\xi}{2} \left(\sum_{j=1}^{n} \varpi_j^2 - 1 \right), \quad (9)$$

where ξ denoting the Lagrange multiplier. After computing the partial derivatives of L and setting them equal to 0, the following equation can be obtained:

$$\varpi_j = \frac{\sum_{i=1}^{m} \sum_{k=1}^{m} dp(h_{S_O}^{ij}, h_{S_O}^{kj})}{\sqrt{\sum_{j=1}^{n} \left(\sum_{i=1}^{m} \sum_{k=1}^{m} dp(h_{S_O}^{ij}, h_{S_O}^{kj}) \right)^2}}. \quad (10)$$

By normalizing ϖ_j, the optimal attribute weights are finally obtained:

$$\varpi_j^* = \frac{\varpi_j}{\sum_{j=1}^{n} \varpi_j} = \frac{\sum_{i=1}^{m} \sum_{k=1}^{m} dp(h_{S_O}^{ij}, h_{S_O}^{kj})}{\sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{m} dp(h_{S_O}^{ij}, h_{S_O}^{kj})}. \quad (11)$$

Based on the related concepts of projection with respect to DHHFLTSs mentioned above, in this section, an ELECTRE II outranking method based on positive and negative ideal DHHFLEs is formulated.

4.3 The Positive Ideal DHHFLE and the Negative Ideal DHHFLE

Considering that the number of LTs for different DHHFLEs may be distinct from each other, to compare two DHHFLEs appropriately and operate correctly, we adopt the optimistic principles to add the upper bound to the shorter DHHFLE until the alternatives on each attribute have the same length $\widetilde{L}$.

Definition 6 For the attribute $Attr_j$, the positive ideal DHHFLE $h_{S_O}^{j+}$ and the negative ideal DHHFLE $h_{S_O}^{j-}$ are defined, respectively:

$$h_{S_O}^{j+} = \left\{ h_{S_O}^{j+(l)} | h_{S_O}^{j+(l)} \right.$$
$$= \left\{ \begin{array}{l} \max\limits_{i=1,2,\ldots m} \{h_{S_O}^{ij(l)}\}, \text{for benefit attribute } Attr_j \\ \min\limits_{i=1,2,\ldots m} \{h_{S_O}^{ij(l)}\}, \text{for cost attribute } Attr_j \end{array} \right., l=1,2,\ldots,\widetilde{L} \right\}$$
$$(12)$$

$$h_{S_O}^{j-} = \left\{ h_{S_O}^{j-(l)} | h_{S_O}^{j-(l)} \right.$$
$$= \left\{ \begin{array}{l} \min\limits_{i=1,2,\ldots m} \{h_{S_O}^{ij(l)}\}, \text{for benefit attribute } Attr_j \\ \max\limits_{i=1,2,\ldots m} \{h_{S_O}^{ij(l)}\}, \text{for cost attribute } Attr_j \end{array} \right., l=1,2,\ldots,\widetilde{L} \right\}$$
$$(13)$$

where $h_{S_O}^{ij(l)}$ denotes the lth linguistic term in the DHHFLE $h_{S_O}^{ij}$.

Example 2 Suppose that an initial decision matrix with three benefit attributes and three alternatives is shown in the upper half part of Table 4. Firstly we add the upper bound to the shorter DHHFLEs to makes all DHHFLEs have the same length as shown in the bottom half of Table 4.

Then the positive ideal DHHFLE of the attribute $Attr_1$ can be computed as:

$$h_{S_O}^{1+} = \left\{ \max\{h_{S_O}^{i1(1)}\}, \max\{h_{S_O}^{i1(2)}\}, \max\{h_{S_O}^{i1(3)}\} \right\}$$
$$= \{ \max\{\varsigma_{0<\sigma_2>}, \varsigma_{0<\sigma_{-2}>}, \varsigma_{1<\sigma_1>}\},$$
$$\max\{\varsigma_{1<\sigma_2>}, \varsigma_1, \varsigma_{1<\sigma_1>}\},$$
$$\max\{\varsigma_{1<\sigma_2>}, \varsigma_{2<\sigma_1>}, \varsigma_{1<\sigma_1>}\} \}$$
$$= \{\varsigma_{1<\sigma_1>}, \varsigma_{1<\sigma_2>}, \varsigma_{2<\sigma_1>}\}$$

Likewise, we can obtain $h_{S_O}^{2+} = \{\varsigma_{0<\sigma_2>}, \varsigma_{1<\sigma_2>}, \varsigma_{2<\sigma_0>}\}$ and $h_{S_O}^{3+} = \{\varsigma_{1<\sigma_1>}, \varsigma_2, \varsigma_{3<\sigma_{-1}>}\}$.

4.4 Construction of Concordance, Discordance and Indifferent Sets

The bidirectional projection measure can simultaneously consider the relations among elements, the positive ideal elements and the negative ideal elements, thus it is more accurate to identify the degree of closeness between the positive and negative ideal DHHFLEs and the alternatives. Accordingly, a comparison method for DHHFLEs based on the bidirectional projection is put forward to compare alternatives on each attribute as follows.

For two DHHFLEs $h_{S_O}^{1j}$ and $h_{S_O}^{2j}$ under the attribute $Attr_j$, let $h_{S_O}^{1j} = \{h_{S_O}^{1j(1)}, h_{S_O}^{1j(2)}, \ldots, h_{S_O}^{2j(\widetilde{L})}\}$ and $h_{S_O}^{2j} = \{h_{S_O}^{2j(1)}, h_{S_O}^{2j(2)}, \ldots, h_{S_O}^{2j(\widetilde{L})}\}$, then

$$\delta_{h_{S_O}}^{1j2j} = \{f(h_{S_O}^{2j(1)}) - f(h_{S_O}^{1j(1)}), f(h_{S_O}^{2j(2)})$$
$$- f(h_{S_O}^{1j(2)}), \ldots, f(h_{S_O}^{2j(\widetilde{L})}) - f(h_{S_O}^{1j(\widetilde{L})})\} \quad (14)$$

And the module of $\delta_{h_{S_O}}^{1j2j}$ is defined as:

$$\left| \delta_{h_{S_O}}^{1j2j} \right| = \sqrt{\sum_{l=1}^{\tilde{L}} \left(f(h_{S_O}^{2j(\tilde{L})}) - f(h_{S_O}^{1j(\tilde{L})}) \right)^2} \quad (15)$$

Subsequently, the bidirectional projection for DHHFLEs on each attribute can be defined as:

Definition 7 Let $h_{S_O}^{aj}$ be a DHHFLE on the attribute $Attr_j$, $h_{S_O}^{j+}$ and $h_{S_O}^{j-}$ be the positive and negative DHHFLEs, respectively. Then the bidirectional projections of $h_{S_O}^{aj}$ with $h_{S_O}^{j+}$ and $h_{S_O}^{j-}$ on the attribute $Attr_j$ are defined as:

$$Prj_{\delta_{h_{S_O}}^{j-j+}}(\delta_{h_{S_O}}^{j-aj}) = \frac{\delta_{h_{S_O}}^{j-j+} \cdot \delta_{h_{S_O}}^{j-aj}}{|\delta_{h_{S_O}}^{j-j+}|}, \quad (16)$$

$$Prj_{\delta_{h_{S_O}}^{ajj+}}(\delta_{h_{S_O}}^{j-j+}) = \frac{\delta_{h_{S_O}}^{ajj+} \cdot \delta_{h_{S_O}}^{j-j+}}{|\delta_{h_{S_O}}^{ajj+}|}, \quad (17)$$

Table 4 An example of a decision matrix		$Attr_1$	$Attr_2$	$Attr_3$
	X_1	$\varsigma_{0<\sigma_2>}, \varsigma_{1<\sigma_2>}$	$\varsigma_{0<\sigma_{-1}>}, \varsigma_1, \varsigma_{2<\sigma_0>}$	$\varsigma_{1<\sigma_1>}, \varsigma_2, \varsigma_{3<\sigma_{-1}>}$
	X_2	$\varsigma_{0<\sigma_{-2}>}, \varsigma_1, \varsigma_{2<\sigma_1>}$	$\varsigma_{-1<\sigma_{-1}>}, \varsigma_0, \varsigma_{1<\sigma_{-1}>}$	$\varsigma_{0<\sigma_1>}, \varsigma_{1<\sigma_2>}$
	X_3	$\varsigma_{1<\sigma_1>}$	$\varsigma_{0<\sigma_2>}, \varsigma_{1<\sigma_2>}$	$\varsigma_{0<\sigma_{-1}>}, \varsigma_{1<\sigma_2>}$
	X_1	$\varsigma_{0<\sigma_2>}, \varsigma_{1<\sigma_2>}, \varsigma_{1<\sigma_2>}$	$\varsigma_{0<\sigma_{-1}>}, \varsigma_1, \varsigma_{2<\sigma_0>}$	$\varsigma_{1<\sigma_1>}, \varsigma_2, \varsigma_{3<\sigma_{-1}>}$
	X_2	$\varsigma_{0<\sigma_{-2}>}, \varsigma_1, \varsigma_{2<\sigma_1>}$	$\varsigma_{-1<\sigma_{-1}>}, \varsigma_0, \varsigma_{1<\sigma_{-1}>}$	$\varsigma_{0<\sigma_1>}, \varsigma_{1<\sigma_2>}, \varsigma_{1<\sigma_2>}$
	X_3	$\varsigma_{1<\sigma_1>}, \varsigma_{1<\sigma_1>}, \varsigma_{1<\sigma_1>}$	$\varsigma_{0<\sigma_2>}, \varsigma_{1<\sigma_2>}, \varsigma_{1<\sigma_2>}$	$\varsigma_{0<\sigma_{-1}>}, \varsigma_{1<\sigma_2>}, \varsigma_{1<\sigma_2>}$

According to the definition of projection, the larger $Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-aj})$ is, the closer $h_{S_O}^{aj}$ is to the positive ideal DHHFLE $h_{S_O}^{j+}$. The smaller $Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-aj})$ is, the more distant $h_{S_O}^{aj}$ is from the $h_{S_O}^{j+}$. Similarly, the larger $Prj_{\delta_{hs_O}^{aj+}}(\delta_{hs_O}^{j-j+})$ is, the closer $h_{S_O}^{aj}$ is to the negative ideal DHHFLE $h_{S_O}^{j-}$. The smaller $Prj_{\delta_{hs_O}^{aj+}}(\delta_{hs_O}^{j-j+})$ is, the more distant $h_{S_O}^{aj}$ is from the negative ideal DHHFLE $h_{S_O}^{j-}$.

Based on the bidirectional projection, a method is presented to properly compare two DHHFLEs. For two DHHFLEs $h_{S_O}^{1j}$ and $h_{S_O}^{2j}$, there are three direct cases for the consequence "$h_{S_O}^{1j}$ is superior to $h_{S_O}^{2j}$":

(1) $Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-1j}) > Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-2j})$ and

$Prj_{\delta_{hs_O}^{1jj+}}(\delta_{hs_O}^{j-j+}) < Prj_{\delta_{hs_O}^{2jj+}}(\delta_{hs_O}^{j-j+})$;

(2) $Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-1j}) > Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-2j})$ and

$Prj_{\delta_{hs_O}^{1jj+}}(\delta_{hs_O}^{j-j+}) = Prj_{\delta_{hs_O}^{2jj+}}(\delta_{hs_O}^{j-j+})$;

(3) $Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-1j}) = Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-2j})$ and

$Prj_{\delta_{hs_O}^{1jj+}}(\delta_{hs_O}^{j-j+}) < Prj_{\delta_{hs_O}^{2jj+}}(\delta_{hs_O}^{j-j+})$.

In addition, under the specific case where $Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-1j})$ and $Prj_{\delta_{hs_O}^{1jj+}}(\delta_{hs_O}^{j-j+})$ are both greater or less than $Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-2j})$ and $Prj_{\delta_{hs_O}^{2jj+}}(\delta_{hs_O}^{j-j+})$, respectively, $h_{S_O}^{1j}$ and $h_{S_O}^{2j}$ cannot be compared directly, we adopt the relative bidirectional projection referring to the traditional TOPSIS method, which is denoted as:

$$\overline{Prj}(\delta_{hs_O}^{ij}) = \frac{Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-ij})}{Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-ij}) + Prj_{\delta_{hs_O}^{ijj+}}(\delta_{hs_O}^{j-j+})}. \quad (18)$$

Then if $\overline{Prj}(h_{S_O}^{1j}) > \overline{Prj}(h_{S_O}^{2j})$, it suggests that $h_{S_O}^{1j}$ is superior to $h_{S_O}^{2j}$.

Besides, if $Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-1j}) = Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-2j})$ and $Prj_{\delta_{hs_O}^{1jj+}}(\delta_{hs_O}^{j-j+}) = Prj_{\delta_{hs_O}^{2jj+}}(\delta_{hs_O}^{j-j+})$, then $h_{S_O}^{1j}$ and $h_{S_O}^{2j}$ are indifferent.

Example 3 Use the decision matrix in Example 2. By Eq. (16) and Eq. (17), calculate the bidirectional projections of $h_{S_O}^{11}$, $h_{S_O}^{21}$ and $h_{S_O}^{31}$ on the attribute $Attr_1$, and it can be obtained that $Prj_{\delta_{hs_O}^{1-1+}}(\delta_{hs_O}^{1-11}) = 0.249$, $Prj_{\delta_{hs_O}^{111+}}(\delta_{hs_O}^{1-1+}) = 0.389$, $Prj_{\delta_{hs_O}^{1-1+}}(\delta_{hs_O}^{1-21}) = 0.068$, $Prj_{\delta_{hs_O}^{211+}}(\delta_{hs_O}^{1-1+}) = 0.373$, $Prj_{\delta_{hs_O}^{1-1+}}(\delta_{hs_O}^{1-31}) = 0.295$ and $Prj_{\delta_{hs_O}^{311+}}(\delta_{hs_O}^{1-1+}) = 0.231$. For $h_{S_O}^{11}$ and $h_{S_O}^{31}$, $Prj_{\delta_{hs_O}^{1-1+}}(\delta_{hs_O}^{1-31}) > Prj_{\delta_{hs_O}^{1-1+}}(\delta_{hs_O}^{1-11})$ and $Prj_{\delta_{hs_O}^{311+}}(\delta_{hs_O}^{1-1+}) < Prj_{\delta_{hs_O}^{111+}}(\delta_{hs_O}^{1-1+})$, so we can obtain that "$h_{S_O}^{31} \succ h_{S_O}^{11}$". For $h_{S_O}^{11}$ and $h_{S_O}^{21}$, the comparison satisfies the case where $Prj_{\delta_{hs_O}^{1-1+}}(\delta_{hs_O}^{1-11}) > Prj_{\delta_{hs_O}^{1-1+}}(\delta_{hs_O}^{1-21})$ and $Prj_{\delta_{hs_O}^{111+}}(\delta_{hs_O}^{1-1+}) > Prj_{\delta_{hs_O}^{211+}}(\delta_{hs_O}^{1-1+})$. Then calculate the relative bidirectional projection, we have $\overline{Prj}(h_{S_O}^{11}) = 0.391$ and $\overline{Prj}(h_{S_O}^{21}) = 0.154$. Since $\overline{Prj}(h_{S_O}^{11}) > \overline{Prj}(h_{S_O}^{21})$, we can obtain that "$h_{S_O}^{11} \succ h_{S_O}^{21}$"

In ELECTRE II method, for each pair of alternatives X_e and X_f, the concordance sets $J_C = \{j | h_{S_O}^{ej} \succ h_{S_O}^{fj}\}$ are the attribute sets containing all those attributes where X_e is superior to X_f. They are categorized into three parts in view of the different superior degree.

(1) The strong DHHFL concordance set $J_{C_{ef}^s}$:

$$J_{C_{ef}^s} = \left\{ j \middle| \begin{array}{l} Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-ej}) > Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-fj}); \\ \text{and } Prj_{\delta_{hs_O}^{ejj+}}(\delta_{hs_O}^{j-j+}) < Prj_{\delta_{hs_O}^{fjj+}}(\delta_{hs_O}^{j-j+}) \end{array} \right\} \quad (19)$$

(2) The medium DHHFL concordance set $J_{C_{ef}^m}$:

$$J_{C_{ef}^m} = \left\{ j \middle| \begin{array}{l} Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-ej}) > Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-fj}) \text{ and } Prj_{\delta_{hs_O}^{ejj+}}(\delta_{hs_O}^{j-j+}) = Prj_{\delta_{hs_O}^{fjj+}}(\delta_{hs_O}^{j-j+}), \\ \text{or } Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-ej}) = Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-fj}) \text{ and } Prj_{\delta_{hs_O}^{ejj+}}(\delta_{hs_O}^{j-j+}) < Prj_{\delta_{hs_O}^{fjj+}}(\delta_{hs_O}^{j-j+}) \end{array} \right\} \quad (20)$$

(3) the weak DHHFL concordance set $J_{C_{ef}^w}$:

$$J_{C_{ef}^w} = \left\{ j \middle| \overline{Prj}(h_{S_O}^{ej}) > \overline{Prj}(h_{S_O}^{fj}) \right\} \quad (21)$$

According to the proposed bidirectional projection-based comparison method for DHHFLEs, it is the degree of closeness to the positive and negative ideal solutions that leads to the different degrees that one alternative is superior to another, and that reflects the difference among the three types of concordance sets. A higher degree of closeness to the positive ideal solution and a lower degree of closeness to the negative ideal solution make one alternative more superior to another. Thus, $J_{C_{ef}^s}$ is more concordant than $J_{C_{ef}^m}$, and $J_{C_{ef}^m}$ is more concordant than $J_{C_{ef}^w}$.

In a similar way, the corresponding DHHFL discordance sets $J_D = \{j | h_{S_O}^{ej} \prec h_{S_O}^{fj}\}$ consisting of all those attributes where X_e is inferior to X_f are divided into three types:

(1)   The strong DHHFL discordance set $J_{D_{ef}^s}$:

$$J_{D_{ef}^s} = \left\{ j \mid Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-ej}) < Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-fj}); \right. \\ \left. \text{and } Prj_{\delta_{hs_O}^{eji+}}(\delta_{hs_O}^{j-j+}) > Prj_{\delta_{hs_O}^{fji+}}(\delta_{hs_O}^{j-j+}) \right\}$$

(22)

(2)   The medium DHHFL discordance set $J_{D_{ef}^m}$:

$$J_{D_{ef}^m} = \left\{ \begin{array}{l} j \mid Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{-ej}) < Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{-fj}) \text{ and } Prj_{\delta_{hs_O}^{eji+}}(\delta_{hs_O}^{j-j+}) = Prj_{\delta_{hs_O}^{fji+}}(\delta_{hs_O}^{j-j+}), \\ \text{or } Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{-ej}) = Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{-fj}) \text{ and } Prj_{\delta_{hs_O}^{eji+}}(\delta_{hs_O}^{j-j+}) > Prj_{\delta_{hs_O}^{fji+}}(\delta_{hs_O}^{j-j+}) \end{array} \right\}$$

(23)

(3)   The weak DHHFL discordance set $J_{D_{ef}^w}$:

$$J_{D_{ef}^w} = \left\{ j \mid \overline{Prj}(h_{S_O}^{ej}) < \overline{Prj}(h_{S_O}^{fj}) \right\}$$

(24)

In addition to the cases mentioned above, the indifferent set is defined as:

$$J_{I_{ef}} = \left\{ j \mid Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-ej}) = Prj_{\delta_{hs_O}^{j-j+}}(\delta_{hs_O}^{j-fj}) \right. \\ \left. \text{and } Prj_{\delta_{hs_O}^{eji+}}(\delta_{hs_O}^{j-j+}) = Prj_{\delta_{hs_O}^{fji+}}(\delta_{hs_O}^{j-j+}) \right\}$$

(25)

4.5 Concordance Index and Discordance Index

According to the concordance sets and discordance sets of each pair of alternatives on each attribute, the DHHFL concordance index I_{ef}^c of X_e and X_f can be computed as Eq. (26).

$$I_{ef}^c = \frac{\omega_C^s \times \sum_{j \in J_{C_{ef}^s}} \varpi_j + \omega_C^m \times \sum_{j \in J_{C_{ef}^m}} \varpi_j + \omega_C^w \times \sum_{j \in J_{C_{ef}^w}} \varpi_j + \omega_I \times \sum_{j \in J_{I_{ef}}} \varpi_j}{\sum_{j=1}^n \varpi_j}$$

$$= \omega_C^s \times \sum_{j \in J_{C_{ef}^s}} \varpi_j + \omega_C^m \times \sum_{j \in J_{C_{ef}^m}} \varpi_j$$

$$+ \omega_C^w \times \sum_{j \in J_{C_{ef}^w}} \varpi_j + \omega_I \times \sum_{j \in J_{I_{ef}}} \varpi_j$$

(26)

where ϖ_j denotes the weight of the attribute $Attr_j$, ω_C^s, ω_C^m, ω_C^w and ω_I denote the weights of the strong, medium, weak and indifferent concordance sets, respectively.

As shown in Eq. (26), the calculation of I_{ef}^c considers the weights of attributes in concordance and indifferent sets, suggesting the relative superior of the alternative X_e to the alternative X_f, and $0 \leq I_{ef}^c \leq 1$. A larger I_{ef}^c denotes a more conspicuous preference of X_e to X_f. Then by using the obtained value of indices I_{ef}^c, the DHHFL concordance matrix C can be constructed as:

$$C = \begin{bmatrix} - & \cdots & I_{1f}^c & \cdots & I_{1(m-1)}^c & I_{1m}^c \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ I_{e1}^c & \cdots & I_{ef}^c & \cdots & I_{e(m-1)}^c & I_{em}^c \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ I_{m1}^c & \cdots & I_{mf}^c & \cdots & I_{m(m-1)}^c & - \end{bmatrix}$$

(27)

The DHHFL discordance index I_{ef}^d denotes the relative inferior of X_e to X_f according to the discordance attributes, and can be computed as Eq. (28)

$$I_{ef}^d = \frac{\max\limits_{j \in J_{D_{ef}^s} \cup J_{D_{ef}^m} \cup J_{D_{ef}^w}} \left\{ \omega_D^s \times \varpi_j dp(h_{S_O}^{ej}, h_{S_O}^{fj}), \omega_D^m \times \varpi_j dp(h_{S_O}^{ej}, h_{S_O}^{fj}), \omega_D^w \times \varpi_j dp(h_{S_O}^{ej}, h_{S_O}^{fj}) \right\}}{\max\limits_j dp(h_{S_O}^{ej}, h_{S_O}^{fj})}$$

(28)

where the ω_D^s, ω_D^m and ω_D^w are the weights of the tree types of DHHFL discordance sets, and $dp(h_{S_O}^{ej}, h_{S_O}^{fj})$ denotes the normalized projection-based distance between two alternatives on the jth attribute.

The discordance index reflects the degree of going against that the alternative is "at least as good as" the others. Using the obtained discordance indices, the DHHFL discordance matrix can be constructed as:

$$D = \begin{bmatrix} - & \cdots & I_{1f}^d & \cdots & I_{1(m-1)}^d & I_{1m}^d \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ I_{e1}^d & \cdots & I_{ef}^d & \cdots & I_{e(m-1)}^d & I_{em}^d \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ I_{m1}^d & \cdots & I_{mf}^d & \cdots & I_{m(m-1)}^d & - \end{bmatrix}$$

(29)

After defining the concordance and discordance matrices, the concordance and discordance Boolean matrices are constructed based on the concordance and discordance level.

Definition 8 For a DHHFL concordance matrix $C = \left[I_{ef}^c \right]_{m \times m}$, $\bar{C}$ is defined as the concordance level which is expressed by $\bar{C} = \sum_{e=1}^m \sum_{f=1}^m I_{ef}^c / m(m-1)$, then the DHHFL concordance Boolean matrix B_C is defined as:

$$B_C = \begin{bmatrix} - & \cdots & c_{1f} & \cdots & c_{1(m-1)} & c_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ c_{e1} & \cdots & c_{ef} & \cdots & c_{e(m-1)} & c_{em} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ c_{m1} & \cdots & c_{mf} & \cdots & c_{m(m-1)} & - \end{bmatrix}$$

(30)

where the element c_{fg} is a zero-one variable, and satisfying: if $I_{ef}^c \geq \bar{C}$, $c_{ef} = 1$, else $c_{ef} = 0$. $c_{ef} = 1$ means that

alternative X_e dominates alternative X_f from a prospective of concordance.

Definition 9 For a DHHFL discordance matrix $D = \left[I_{ef}^d \right]_{m \times m}$, $\bar{D}$ is defined as the the discordance level which is expressed by $\bar{D} = \sum\limits_{e=1}^{m} \sum_{f=1}^{m} I_{ef}^d / m(m-1)$, then the DHHFL discordance Boolean matrix B_D is defined as:

$$
B_D = \begin{bmatrix}
- & \cdots & d_{1f} & \cdots & d_{1(m-1)} & d_{1m} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
d_{e1} & \cdots & d_{ef} & \cdots & d_{e(m-1)} & d_{em} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
d_{m1} & \cdots & d_{mf} & \cdots & d_{m(m-1)} & -
\end{bmatrix} \quad (31)
$$

where the element d_{ef} is a zero-one variable, and satisfying: if $I_{ef}^d < \bar{D}$, $d_{ef} = 1$, else $d_{ef} = 0$. As mentioned before, the discordance index represents the degree to which one alternative is inferior to another. Thus $d_{ef} = 1$ suggests that alternative X_e surpasses alternative X_f from a discordance prospective.

Then the global matrix R is constructed to get the overall outranking relations by means of multiplying the corresponding elements in the two Boolean matrices B_C and B_D, namely its elements $r_{ef} = c_{ef} \times d_{ef}$.

To visualize the outranking information included in the global matrix and obtain the final ranking result of the alternatives, an outranking graph is depicted using the graph theory. Concretely, the outranking graph is defined as $G = (V, P)$. V represents a set of vertices that correspond to alternatives. And P represents a set of arcs reflecting ranking relations between alternatives. On each directional arc, the vertex that the arrow points to denotes the supposed inferior alternative to the other alternative of the start vertex of the arc. Under the situation when two alternatives are incomparable, there is no arc between them.

Specially, considering that the Boolean matrices are constructed by utilizing the mean values as the concordance and discordance level, which may result in the values being not complementary. To address this issue, Lin et al. [43] proposed an improved method. However, it cannot completely cover all the complementary cases which reflect the weak outranking relations between alternatives. Here we further improve the method.

Let R be the global matrix as defined before, whose elements $r_{ef} = c_{ef} \times d_{ef}$. In the outranking graph, if the elements satisfy that $r_{ef} = r_{fe} = 0$ or $r_{ef} = r_{fe} = 1$, there is

no arc between X_e and X_f. In this case, set $r_{ef} = (c_{ef}) \times (d_{ef}) = 1$ and $r_{ef} = (c_{ef}) \times (d_{ef}) = 0$, in which $(c_{ef}) = 1$ and $(d_{ef}) = 1$ when $I_{ef}^c > I_{fe}^c$ and $I_{ef}^c - I_{fe}^c > I_{ef}^d - I_{fe}^d$, which suggests the alternative X_e weakly outperforms alternative X_f.

4.6 The Decision-Making Procedure

Based on what is analyzed above, the decision-making procedure of the proposed ELECTRE II-based outranking method can be formed and expressed as follows.

Step 1. Establish the decision matrix H with DHHFLTSs by Eq. (7).

Step 2. Calculate the normalized projection-based distance between any two DHHFLEs concerning each attribute according to Eq. (6).

Step 3. Compute attribute weights by Eq. (11) and set the weights of DHHFL concordance, discordance, and indifference sets.

Step 4. Identity the positive and negative ideal DHHFLEs by Eq. (12) and Eq. (13).

Step 5. Compute the bidirectional projection between each DHHFLE and the positive and negative ideal DHHFLEs concerning each attribute by Eq. (16) and Eq. (17).

Step 6. Set up the strong, medium and weak DHHFL concordance sets by Eqs. (19)-(21).

Step 7. Set up the strong, medium and weak DHHFL discordance sets, as well as the indifference set by Eqs. (22)-(25).

Step 8. Compute the concordance and the discordance indices by Eq. (26) and Eq. (28) to construct the DHHFL concordance and the DHHFL discordance matrices of each pair of alternatives.

Step 9. Obtain the DHHFL concordance and discordance Boolean matrices according to Definitions 8 and 9.

Step 10. Construct the final global matrix and obtain the outranking relations. And Then depict the outranking graph and give the final ranking result.

To visualize the decision-making process of the proposed method, Fig. 2 is drawn to demonstrate the procedure.
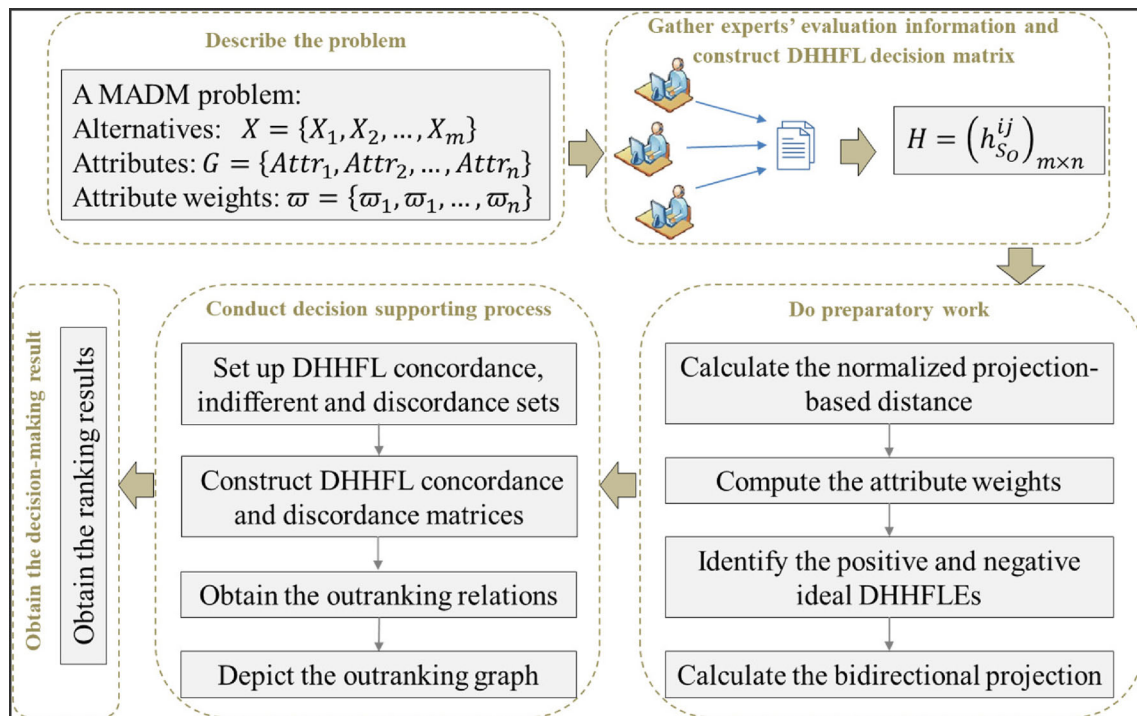
Fig. 2 The decision-making procedure

5 Demonstrative Example

In this section, a numerical example concerning ELPS for natural disaster preparation to demonstrate more details of the proposed outranking method.

5.1 Application Background

China is one of the countries with severe natural disasters whose characteristics include great variety, wide geographical distribution, high frequency of occurrence, heavy disaster losses and high disaster risk. The Ministry of Emergency Management of China has stated that more than 70% of the cities and more than 50% of the population are distributed in areas with high risk of meteorological, seismic, geological and marine disasters. And natural disasters result in an average of 300 billion yuan in direct economic losses each year.

Emergency logistics in China is carried out under the leadership of the government. In Ningbo, Zhejiang Province, a coastal city in south-eastern China which is subject to the seasonal risk of typhoons and floods, emergency logistics management is an important part of government work. In recent years, Ningbo attaches great importance to the orderly emergency logistics management of natural disasters, and focuses on effective prevention from a scientific perspective. And a comprehensive emergency response plan system has been basically established, in which the reserves and delivery of relief materials is a crucial link. Among the various types of relief materials, diet guarantees are essential.

In response to the requirement that "government reserves should be combined with social reserves, physical reserves and capacity reserves", and in order to further comprehensively promote the construction of emergency logistics, the emergency logistics management department of Ningbo is selecting the appropriate emergency logistics providers from five alternative enterprises in the food and beverage industry, namely, NFS (X_1), CC(X_2), GT (X_3), MK (X_4), WHH (X_5). The evaluation is based on six main attributes [2] including one cost attribute ($Attr_1$) and five benefit attributes ($Attr_2$ to $Attr_6$) which is shown in Table 5.

- Cost This attribute mainly includes various costs of emergency logistics providers such as transportation cost, inventory cost, packing cost and quality cost.
- Product level This attribute describes actual situations of products of providers. Factors such as product

Table 5 Evaluation attributes for ELPS

Evaluation attributes	Description
Cost ($Attr_1$)	This attribute refers to the cost incurred in ELPS
Product level ($Attr_2$)	This attribute refers to actual situation of products.
Quick response ability of supply ($Attr_3$)	This attribute reflects the emergency reserve capability of the providers.
Quick response ability of transport ($Attr_4$)	This attribute reflects the emergency transport capability of the providers.
Basic management($Attr_5$)	This attribute includes infrastructure, assets and emergency technical level, etc.
Reputation level($Attr_6$)	This attribute depends on the business performance of the providers.

Table 6 Initial DHHFL decision matrix

	$Attr_1$	$Attr_2$	$Attr_3$
X_1	$\varsigma_{0<\sigma_1>},\varsigma_1,\varsigma_{2<\sigma_{-1}>}$	$\varsigma_{0<\sigma_{-1}>},\varsigma_{1<\sigma_2>}$	$\varsigma_{-1<\sigma_2>},\varsigma_0,\varsigma_{1<\sigma_2>}$
X_2	$\varsigma_{1<\sigma_{-1}>},\varsigma_{2<\sigma_0>}$	$\varsigma_{0<\sigma_{-2}>},\varsigma_1,\varsigma_{2<\sigma_1>}$	$\varsigma_{1<\sigma_0>}$
X_3	$\varsigma_{1<\sigma_{-2}>},\varsigma_2,\varsigma_{3<\sigma_{-2}>}$	$\varsigma_{1<\sigma_{-1}>},\varsigma_{2<\sigma_{-1}>}$	$\varsigma_{-1<\sigma_{-2}>},\varsigma_{0<\sigma_2>}$
X_4	$\varsigma_{0<\sigma_1>},\varsigma_1,\varsigma_{2<\sigma_1>}$	$\varsigma_{0<\sigma_{-1}>},\varsigma_1,\varsigma_{2<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-2}>},\varsigma_{2<\sigma_{-1}>}$
X_5	$\varsigma_{1<\sigma_{-3}>},\varsigma_{2<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-2}>},\varsigma_{2<\sigma_0>}$	$\varsigma_{-2<\sigma_2>},\varsigma_{-1},\varsigma_{0<\sigma_3>}$

	$Attr_4$	$Attr_5$	$Attr_6$
X_1	$\varsigma_{2<\sigma_{-1}>}$	$\varsigma_{-1<\sigma_1>},\varsigma_0,\varsigma_{1<\sigma_2>}$	$\varsigma_{0<\sigma_1>},\varsigma_1,\varsigma_{2<\sigma_2>}$
X_2	$\varsigma_{1<\sigma_{-2}>},\varsigma_2,\varsigma_{3<\sigma_0>}$	$\varsigma_{0<\sigma_{-1}>},\varsigma_1,\varsigma_{2<\sigma_1>}$	$\varsigma_{0<\sigma_{-1}>},\varsigma_1,\varsigma_{2<\sigma_2>}$
X_3	$\varsigma_{2<\sigma_{-3}>},\varsigma_{3<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-2}>},\varsigma_{2<\sigma_{-1}>}$	$\varsigma_{-1<\sigma_2>},\varsigma_{0<\sigma_2>}$
X_4	$\varsigma_{1<\sigma_2>},\varsigma_2,\varsigma_{3<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-1}>},\varsigma_{1<\sigma_1>}$	$\varsigma_{2<\sigma_{-2}>},\varsigma_{2<\sigma_2>}$
X_5	$\varsigma_{0<\sigma_0>},\varsigma_1,\varsigma_{2<\sigma_1>}$	$\varsigma_{1<\sigma_{-2}>},\varsigma_{2<\sigma_1>}$	$\varsigma_{1<\sigma_2>},\varsigma_2,\varsigma_{3<\sigma_{-1}>}$

qualification ratio, product failure ratio, quality assurance ability affect the evaluation of this attribute.

- Quick response ability of supply This attribute mainly assesses rapid production capacity and material storage capacity of the providers.
- Quick response ability of transport This attribute mainly assesses speed of delivery and transport capacity of the providers.
- Basic management This attribute reflects the technical strength of providers which is assessed by infrastructure and assets, completeness of emergency plans, level of informatization and so on.
- Reputation level This attribute reflects long-term cooperation intention and ability of each provider, which can be expressed by factors such as business reputation, business ethics and business stability.

It can be seen that these attributes are not complementary in a practical sense. For example, an alternative cannot be selected if it performs poorly in quick response though it has low cost. And in order to evaluate the alternatives in detail, the experts make evaluations using DHHFLTSs whose first hierarchy LTS is $S = \{\varsigma_{-3} = terrible, \varsigma_{-2} = pretty\ bad, \varsigma_{-1} = bad, \varsigma_0 = medium, \varsigma_1 = good, \varsigma_2 = pretty\ good, \varsigma_3 = perfect\}$, and second hierarchy LTS is

$$O = \begin{cases} \{\sigma_{-3} = far\ from, \sigma_{-2} = rarely, \sigma_{-1} = a\ little, \sigma_0 = exactly\ right \\ \quad \sigma_1 = much, \sigma_2 = very\ much, \sigma_3 = extremely\}, if\ t > 0. \\ \{\sigma_{-3} = extremely, \sigma_{-2} = very\ much, \sigma_{-1} = much, \sigma_0 = exactly\ right, \\ \quad \sigma_1 = a\ little, \sigma_2 = rarely, \sigma_3 = far\ from\}, if\ t < 0. \end{cases}$$

After the evaluation information is gathered and transformed into DHHFLEs, the initial DHHFL decision matrix is obtained and shown in Table 6.

5.2 Method Implementation

Step 1. Adopt the optimistic principle mentioned in sect. 3 and add the maximum LTs into the shorter DHHFLEs to make the numbers of DHHFLEs equivalent on the same attribute. Then the obtained decision matrix is shown in Table 7.

Table 7 DHHFL decision matrix

	$Attr_1$	$Attr_2$	$Attr_3$
X_1	$\varsigma_{0<\sigma_1>}, \varsigma_1, \varsigma_{2<\sigma_{-1}>}$	$\varsigma_{0<\sigma_{-1}>}, \varsigma_{1<\sigma_2>}, \varsigma_{1<\sigma_2>}$	$\varsigma_{-1<\sigma_2>}, \varsigma_0, \varsigma_{1<\sigma_2>}$
X_2	$\varsigma_{1<\sigma_{-1}>}, \varsigma_{2<\sigma_0>}, \varsigma_{2<\sigma_0>}$	$\varsigma_{0<\sigma_{-2}>}, \varsigma_1, \varsigma_{2<\sigma_1>}$	$\varsigma_{1<\sigma_0>}, \varsigma_{1<\sigma_0>}, \varsigma_{1<\sigma_0>}$
X_3	$\varsigma_{1<\sigma_{-2}>}, \varsigma_2, \varsigma_{3<\sigma_{-2}>}$	$\varsigma_{1<\sigma_{-1}>}, \varsigma_{2<\sigma_{-1}>}, \varsigma_{2<\sigma_{-1}>}$	$\varsigma_{-1<\sigma_{-2}>}, \varsigma_{0<\sigma_2>}, \varsigma_{0<\sigma_2>}$
X_4	$\varsigma_{0<\sigma_1>}, \varsigma_1, \varsigma_{2<\sigma_1>}$	$\varsigma_{0<\sigma_{-1}>}, \varsigma_1, \varsigma_{2<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-2}>}, \varsigma_{2<\sigma_{-1}>}, \varsigma_{2<\sigma_{-1}>}$
X_5	$\varsigma_{1<\sigma_{-3}>}, \varsigma_{2<\sigma_{-1}>}, \varsigma_{2<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-2}>}, \varsigma_{2<\sigma_0>}, \varsigma_{2<\sigma_0>}$	$\varsigma_{-2<\sigma_2>}, \varsigma_{-1}, \varsigma_{0<\sigma_3>}$

	$Attr_4$	$Attr_5$	$Attr_6$
X_1	$\varsigma_{2<\sigma_{-1}>}, \varsigma_{2<\sigma_{-1}>}, \varsigma_{2<\sigma_{-1}>}$	$\varsigma_{-1<\sigma_1>}, \varsigma_0, \varsigma_{1<\sigma_2>}$	$\varsigma_{0<\sigma_1>}, \varsigma_1, \varsigma_{2<\sigma_2>}$
X_2	$\varsigma_{1<\sigma_{-2}>}, \varsigma_2, \varsigma_{3<\sigma_0>}$	$\varsigma_{0<\sigma_{-1}>}, \varsigma_1, \varsigma_{2<\sigma_1>}$	$\varsigma_{0<\sigma_{-1}>}, \varsigma_1, \varsigma_{2<\sigma_2>}$
X_3	$\varsigma_{2<\sigma_{-3}>}, \varsigma_{3<\sigma_{-1}>}, \varsigma_{3<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-2}>}, \varsigma_{2<\sigma_{-1}>}, \varsigma_{2<\sigma_{-1}>}$	$\varsigma_{-1<\sigma_2>}, \varsigma_{0<\sigma_2>}, \varsigma_{0<\sigma_2>}$
X_4	$\varsigma_{1<\sigma_2>}, \varsigma_2, \varsigma_{3<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-1}>}, \varsigma_{1<\sigma_1>}, \varsigma_{1<\sigma_1>}$	$\varsigma_{2<\sigma_{-2}>}, \varsigma_{2<\sigma_2>}, \varsigma_{2<\sigma_2>}$
X_5	$\varsigma_{0<\sigma_0>}, \varsigma_1, \varsigma_{2<\sigma_1>}$	$\varsigma_{1<\sigma_{-2}>}, \varsigma_{2<\sigma_1>}, \varsigma_{2<\sigma_1>}$	$\varsigma_{1<\sigma_2>}, \varsigma_2, \varsigma_{3<\sigma_{-1}>}$

Step 2. Calculate the normalized projection-based distance between any two alternatives concerning each attribute as exhibited in Table 8 by Eq. (6).

Step 3. Compute the weight vector of attributes by Eq. (11) as:

$$\varpi = (0.1011, 0.1017, 0.2591, 0.1305, 0.165, 0.2426)^T.$$

Determine the weights of the concordance, discordance and indifference sets through relevant decision analysis:

$$\omega = (\omega_C^s, \omega_C^m, \omega_C^w, \omega_D^s, \omega_D^m, \omega_D^w, \omega_I)$$
$$= (1, 0.9, 0.8, 1, 0.9, 0.8, 0.7) \quad (32)$$

Step 4. Identify the positive and negative ideal DHHFLEs on each attribute shown in Table 9.

Step 5. Use Eq. (16) and Eq. (17) to calculate the bidirectional projection of each alternative concerning each attribute, $(Prj_{\delta_{h_{S_O}}^{j-j+}}(\delta_{h_{S_O}}^{j-ij}), Prj_{\delta_{h_{S_O}}^{ijj+}}(\delta_{h_{S_O}}^{j-j+}))$, as show in Table 10.

Step 6. Construct the strong, medium, weak DHHFL concordance sets and the DHHFL indifference set, denoted as $J_{C^s}, J_{C^m}, J_{C^w}, J_I$:

$$J_{C^s} = \begin{bmatrix} - & 1 & 1,6 & 1,2 & 1,3,4 \\ 3,5 & - & 3,6 & - & 3,4 \\ 2,5 & 2,4 & - & 2 & 3,4 \\ 3,5,6 & 1,4,5,6 & 1,3,6 & - & 3,4 \\ 2,5,6 & 1,2,5,6 & 1,5,6 & 2,5 & - \end{bmatrix},$$

$$J_{C^m} = \begin{bmatrix} - & 6 & - & - & - \\ - & - & - & 2 & - \\ - & 5 & - & - & - \\ - & - & - & - & 1,6 \\ - & - & - & - & - \end{bmatrix},$$

$$J_{C^w} = \begin{bmatrix} - & 2,4 & 3 & - & - \\ - & - & - & - & - \\ 4 & - & - & 5 & 2 \\ 4 & 3 & - & - & - \\ - & - & - & - & - \end{bmatrix}, \quad J_I = \begin{bmatrix} - & - & - & - & - \\ - & - & 1 & - & - \\ - & 1 & - & 4 & - \\ - & - & 4 & - & - \\ - & - & - & - & - \end{bmatrix}$$

Step 7. Construct the strong, medium, weak DHHFL discordance sets, denoted as $J_{D^s}, J_{D^m}, J_{D^w}$:

$$J_{D^s} = \begin{bmatrix} - & 3,5 & 2,5 & 3,5,6 & 2,5,6 \\ 1 & - & 2,4 & 1,4,5,6 & 1,2,5,6 \\ 1,6 & 3,6 & - & 1,3,6 & 1,5,6 \\ 1,2 & - & 2 & - & 2,5 \\ 1,3,4 & 3,4 & 3,4 & 3,4 & - \end{bmatrix},$$

$$J_{D^m} = \begin{bmatrix} - & - & - & - & - \\ 6 & - & 5 & - & - \\ - & - & - & - & - \\ - & 2 & - & - & - \\ - & - & - & 1,6 & - \end{bmatrix}, \quad J_{D^w} = \begin{bmatrix} - & - & 4 & 4 & - \\ 2,4 & - & - & 3 & - \\ 3 & - & 5 & - & - \\ - & - & 2 & - & - \\ - & - & - & - & - \end{bmatrix}$$

Table 8 The normalized projection-based distances

$Attr_1$	$h_{S_O}^{11}$	$h_{S_O}^{21}$	$h_{S_O}^{31}$	$h_{S_O}^{41}$	$h_{S_O}^{51}$	$Attr_2$	$h_{S_O}^{12}$	$h_{S_O}^{22}$	$h_{S_O}^{32}$	$h_{S_O}^{42}$	$h_{S_O}^{52}$
$h_{S_O}^{11}$	–	0.124	0.139	0.120	0.054	$h_{S_O}^{12}$	–	0.111	0.015	0.114	0.171
$h_{S_O}^{21}$	–	–	0.017	0.229	0.172	$h_{S_O}^{22}$	–	–	0.125	0.212	0.067
$h_{S_O}^{31}$	–	–	–	0.243	0.186	$h_{S_O}^{32}$	–	–	–	0.100	0.184
$h_{S_O}^{41}$	–	–	–	–	0.070	$h_{S_O}^{42}$	–	–	–	–	0.265
$h_{S_O}^{51}$	–	–	–	–	–	$h_{S_O}^{52}$	–	–	–	–	–
$Attr_3$	$h_{S_O}^{13}$	$h_{S_O}^{23}$	$h_{S_O}^{33}$	$h_{S_O}^{43}$	$h_{S_O}^{53}$	$Attr_4$	$h_{S_O}^{14}$	$h_{S_O}^{24}$	$h_{S_O}^{34}$	$h_{S_O}^{44}$	$h_{S_O}^{54}$
$h_{S_O}^{13}$	–	0.109	0.413	0.257	0.399	$h_{S_O}^{14}$	–	0.094	0.186	0.172	0.182
$h_{S_O}^{23}$	–	–	0.477	0.338	0.326	$h_{S_O}^{24}$	–	–	0.101	0.086	0.259
$h_{S_O}^{33}$	–	–	–	0.210	0.647	$h_{S_O}^{34}$	–	–	–	0.017	0.334
$h_{S_O}^{43}$	–	–	–	–	0.553	$h_{S_O}^{44}$	–	–	–	–	0.323
$h_{S_O}^{53}$	–	–	–	–	–	$h_{S_O}^{54}$	–	–	–	–	–
$Attr_5$	$h_{S_O}^{15}$	$h_{S_O}^{25}$	$h_{S_O}^{35}$	$h_{S_O}^{45}$	$h_{S_O}^{55}$	$Attr_6$	$h_{S_O}^{16}$	$h_{S_O}^{26}$	$h_{S_O}^{36}$	$h_{S_O}^{46}$	$h_{S_O}^{56}$
$h_{S_O}^{15}$	–	0.297	0.337	0.290	0.467	$h_{S_O}^{16}$	–	0.068	0.426	0.286	0.249
$h_{S_O}^{25}$	–	–	0.057	0.011	0.242	$h_{S_O}^{26}$	–	–	0.384	0.335	0.300
$h_{S_O}^{35}$	–	–	–	0.067	0.196	$h_{S_O}^{36}$	–	–	–	0.590	0.569
$h_{S_O}^{45}$	–	–	–	–	0.250	$h_{S_O}^{46}$	–	–	–	–	0.050
$h_{S_O}^{55}$	–	–	–	–	–	$h_{S_O}^{56}$	–	–	–	–	–

Step 8. Calculate the concordance indices and construct the DHHFL concordance matrix as:

$$C = \begin{bmatrix} - & 0.5052 & 0.5510 & 0.2028 & 0.4907 \\ 0.4241 & - & 0.5725 & 0.0915 & 0.3896 \\ 0.3711 & 0.4515 & - & 0.3251 & 0.4710 \\ 0.7711 & 0.8465 & 0.6942 & - & 0.6989 \\ 0.5093 & 0.6104 & 0.5087 & 0.2667 & - \end{bmatrix}$$

Calculate the discordance indices and construct the DHHFL discordance matrix as:

$$D = \begin{bmatrix} - & 0.1650 & 0.1650 & 0.2399 & 0.1650 \\ 0.1186 & - & 0.1058 & 0.2404 & 0.2426 \\ 0.2426 & 0.2591 & - & 0.2426 & 0.2426 \\ 0.1011 & 0.0576 & 0.1017 & - & 0.1554 \\ 0.2591 & 0.2591 & 0.2591 & 0.2591 & - \end{bmatrix}$$

Table 9 The positive and negative ideal DHHFLEs

	Negative ideal DHHFLEs	Negative ideal DHHFLEs
$Attr_1$	$\varsigma_{0<\sigma_1>}, \varsigma_1, \varsigma_{2<\sigma_{-1}>}$	$\varsigma_{1<\sigma_{-1}>}, \varsigma_{2<\sigma_0>}, \varsigma_{3<\sigma_{-2}>}$
$Attr_2$	$\varsigma_{1<\sigma_{-1}>}, \varsigma_{2<\sigma_0>}, \varsigma_{2<\sigma_1>}$	$\varsigma_{0<\sigma_{-2}>}, \varsigma_1, \varsigma_{2<\sigma_{-1}>}$
$Attr_3$	$\varsigma_{1<\sigma_0>}, \varsigma_{2<\sigma_{-1}>}, \varsigma_{2<\sigma_{-1}>}$	$\varsigma_{-1<\sigma_{-2}>}, \varsigma_{-1}, \varsigma_{0<\sigma_2>}$
$Attr_4$	$\varsigma_{2<\sigma_{-1}>}, \varsigma_{3<\sigma_{-1}>}, \varsigma_{3<\sigma_0>}$	$\varsigma_{0<\sigma_0>}, \varsigma_1, \varsigma_{2<\sigma_{-1}>}$
$Attr_5$	$\varsigma_{1<\sigma_{-1}>}, \varsigma_{2<\sigma_1>}, \varsigma_{2<\sigma_1>}$	$\varsigma_{-1<\sigma_1>}, \varsigma_0, \varsigma_{1<\sigma_1>}$
$Attr_6$	$\varsigma_{1<\sigma_2>}, \varsigma_{2<\sigma_2>}, \varsigma_{3<\sigma_{-1}>}$	$\varsigma_{-1<\sigma_2>}, \varsigma_{0<\sigma_2>}, \varsigma_{0<\sigma_2>}$

Step 9. Calculate and obtain the concordance level $\bar{C} = 0.4876$ and the discordance level $\bar{D} = 0.1941$, then obtain the DHHFL concordance Boolean matrix B_C and the DHHFL discordance Boolean matrix B_D as:

$$B_C = \begin{bmatrix} - & 1 & 1 & 0 & 1 \\ 0 & - & 1 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 1 & 1 & 1 & - & 1 \\ 1 & 1 & 1 & 0 & - \end{bmatrix}, \quad B_D = \begin{bmatrix} - & 1 & 1 & 0 & 1 \\ 1 & - & 1 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 1 & 1 & 1 & - & 1 \\ 0 & 0 & 0 & 1 & - \end{bmatrix}$$

Step 10. Multiply the matrices B_C and B_D component-wise and obtain the global matrix R:

$$R = \begin{bmatrix} - & 1 & 1 & 0 & 1 \\ 0 & - & 1 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 1 & 1 & 1 & - & 1 \\ 0 & 0 & 0 & 0 & - \end{bmatrix}$$

In terms of the global matrix, the strong out ranking relations can be obtained, and the strong outranking graph is drawn as Fig. 3(a). It can be found that there exists elements that are not complementary in the global matrix R, mark them with brackets and then we have:

$$R = \begin{bmatrix} - & 1 & 1 & 0 & 1 \\ 0 & - & 1 & 0 & (0) \\ 0 & 0 & - & 0 & (0) \\ 1 & 1 & 1 & - & 1 \\ 0 & (0) & (0) & 0 & - \end{bmatrix}$$

Then the matrices B_C and B_D are altered as:

$$B_C = \begin{bmatrix} - & 1 & 1 & 0 & 1 \\ 0 & - & 1 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 1 & 1 & 1 & - & 1 \\ 1 & 1 & 1 & 1 & - \end{bmatrix}, B_D = \begin{bmatrix} - & 1 & 1 & 0 & 1 \\ 1 & - & 1 & 0 & (0) \\ 0 & 0 & - & 0 & (0) \\ 1 & 1 & 1 & - & 1 \\ 0 & (1) & (1) & 1 & - \end{bmatrix}$$

Sequentially, the final global matrix R' is obtained:

$$R' = \begin{bmatrix} - & 1 & 1 & 0 & 1 \\ 0 & - & 1 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 1 & 1 & 1 & - & 1 \\ 0 & 1 & 1 & 0 & - \end{bmatrix}$$

Then the overall outranking graph is drawn as Fig. 3(b) according to matrix R'. As depicted in Fig. 3(b), the solid and dotted directional arcs represent strong outranking relations and weak outranking relations between alternatives, respectively.

According to the final outranking graph, the final ranking result of the five alternatives is:

$$X_4 \succ X_1 \succ X_5 \succ X_2 \succ X_3$$

5.3 Sensitivity Analysis

In the decision-making framework constructed in this paper, the maximizing deviation method used in weight determination is based on the proposed normalized projection-based distance. Fig. 4 shows the weight of each remaining attribute obtained after removing one attribute. It can be seen that after removing any attribute, the relative weights of the remaining attributes do not change because the evaluation deviation under each attribute does not change. This shows the validity of the proposed normalized projection-based distance.

Further, the ranking change of each alternative after removing each attribute can also be obtained, as shown in Fig. 5. According to Fig. 5, it can be analyzed which attribute each alternative is more sensitive to, that is, under which attribute each alternative performs better. For X_4, no matter which attribute is removed, it is always the best alternative, indicating that it evaluates well under each attribute. The other alternative have their own advantages and disadvantages under each attribute. For example, X_5 becomes the second best alternative after removing $Attr_1$, but if removing $Attr_5$ or $Attr_6$, it becomes the last one. For X_2, it is obvious that $Attr_2$ and $Attr_3$ are two attributes that have the most significant impact on its ranking.

5.4 Comparative Analysis

5.4.1 Comparison with the positive and negative ideal HFLEs-based ELECTRE II method

Liao et al. [24] proposed an extended ELECTRE II based on the positive and negative ideal hesitant fuzzy linguistic elements (HFLEs), which used a hybrid Hamming distance measure to obtain the distance for the calculation of discordance index between HFLEs, as well as the distance between HFLEs and the positive and negative ideal HFLEs as the basis of comparing two HFLEs.

It is obvious that the original information will be changed if transforming the DHHFLEs to HFLEs in Liao's method by deleting the secondary hierarchy. For further comparison, it is extended to DHHFL environment and is applied to the ELPS example above. Since the attribute weights in the contrast ELECTRE II method are directly given, we let the weight vector be the same as what we calculate in our proposed method to eliminate the impact might happen.

Replace the normalized projection-based distance and the bidirectional projection with hybrid Hamming distance in [24] between DHHFLEs which can be computed as:

Table 10 Bidirectional projection

	$Attr_1$	$Attr_2$	$Attr_3$	$Attr_4$	$Attr_5$	$Attr_6$
X_1	(0.202,0.111)	(0.103,0.282)	(0.309,0.625)	(0.239,0.344)	(0.019,0.475)	(0.289,0.469)
X_2	(0.027,0.223)	(0.041,0.278)	(0.546,0.432)	(0.246,0.373)	(0.220,0.444)	(0.225,0.469)
X_3	(0.027,0.223)	(0.227,0.174)	(0.190,0.595)	(0.356,0.348)	(0.323,0.444)	(0.000,0.577)
X_4	(0.148,0.149)	(0.041,0.296)	(0.574,0.444)	(0.356,0.348)	(0.284,0.393)	(0.545,0.333)
X_5	(0.148,0.167)	(0.237,0.236)	(0.052,0.647)	(0.055,0.438)	(0.452,0.222)	(0.513,0.333)

(a) The strong outranking graph   **(b)** The final outranking graph
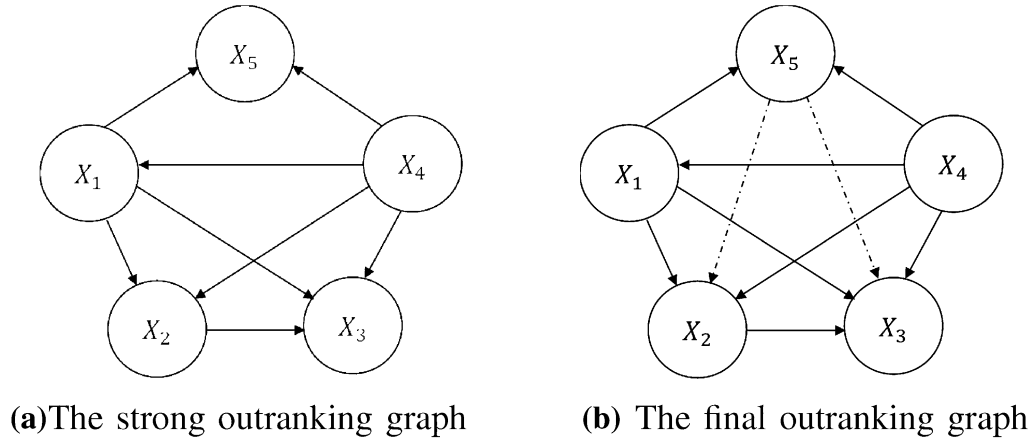
Fig. 3 The strong and final outranking graphs
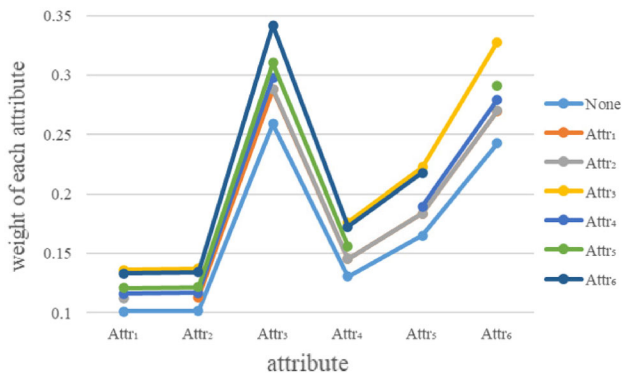
Fig. 4 Weight of each remaining attribute after removing one attribute

$$d_{hdd}(h_{S_O}^1, h_{S_O}^2) = \frac{1}{2}\left(\frac{1}{\tilde{L}}\sum_{l=1}^{\tilde{L}}|f(h_{S_O}^{1(l)}) - f(h_{S_O}^{2(l)})| + \max_{l=1,2,...,\tilde{L}}|f(h_{S_O}^{1(l)}) - f(h_{S_O}^{2(l)})|\right) \quad (33)$$

the distances from the DHHFLEs to the positive and negative ideal DHHFLEs on each attribute are listed in Table 11.

According to the comparison method in [24], the strong, medium, weak concordance sets and the indifference sets can be obtained as:

$$J_{C^s} = \begin{bmatrix} - & 1,2,6 & 1,3,6 & 1,2 & 1,3,4 \\ 3,5 & - & 3,6 & - & 3,4 \\ 2,4,5 & 2,4,5 & - & 2,5 & 3,4 \\ 3,4,5,6 & 1,3,4,5,6 & 1,3,6 & - & 3,4,6 \\ 2,5,6 & 1,2,5,6 & 1,5,6 & 2,5 & - \end{bmatrix},$$

$$J_{C^m} = \begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix},$$

$$J_{C^w} = \begin{bmatrix} - & 4 & - & - & - \\ - & - & - & 2 & - \\ - & - & - & - & - \\ - & - & - & - & 1 \\ - & - & 2 & - & - \end{bmatrix}, J_I = \begin{bmatrix} - & - & - & - & - \\ - & - & 1 & - & - \\ - & 1 & - & 4 & - \\ - & - & 4 & - & - \\ - & - & - & - & - \end{bmatrix}$$

The strong, medium and weak concordance sets can be obtained as:
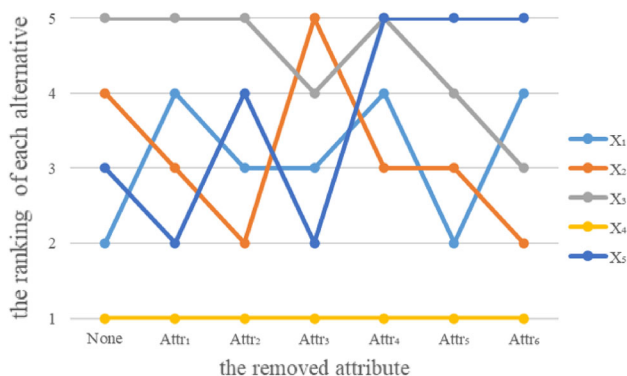
Fig. 5 Ranking of each alternative after removing one attribute

$$J_{D^s} = \begin{bmatrix} - & 3,5 & 2,4,5 & 3,4,5,6 & 2,5,6 \\ 1,2,6 & - & 2,4,5 & 1,3,4,5,6 & 1,2,5,6 \\ 1,3,6 & 3,6 & - & 1,3,6 & 1,5,6 \\ 1,2 & - & 2,5 & - & 2,5 \\ 1,3,4 & 3,4 & 3,4 & 3,4,6 & - \end{bmatrix},$$

$$J_{D^m} = \begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}, J_{D^w} = \begin{bmatrix} - & - & - & - & - \\ 4 & - & - & - & - \\ - & - & - & - & 2 \\ - & 2 & - & - & - \\ - & - & - & 1 & - \end{bmatrix}$$

And then continue following the steps in Sect. 5 to obtain corresponding required values. To keep the paper reasonably concise, the process is no more repeated. Finally the outranking graph based on the contrast ELECTRE II method applying hybrid Hamming distance is depicted as Fig. 6, which shows the ranking result is: $X_4 \succ X_5 \succ X_1 \succ X_2 \succ X_3$.

The optimal alternative is X_4 and overall ranking is pretty much the same, indicating the normalized projection-based distance measure and bidirectional projection applied to the comparison method in our proposed ELECTRE II method with DHHFLTSs are suitable and valid. Differences between these two methods mainly comes from the different distance measures, which shows the superiority of applying normalized projection-based distance and bidirectional projection to DHHFLTS in the ELECTRE II method. Specifically, the differences are analyzed as follows.

(1)   The hybrid Hamming distance measure in [24] is an extension of traditional Hamming distance. The distances from two different DHHFLEs to another identical DHHFLE calculated by hybrid Hamming distance might be the same, such as $d_{hdd}(h_{S_O}^{21}, h_{S_O}^{41}) = d_{hdd}(h_{S_O}^{31}, h_{S_O}^{41}) = 0.130$, $d_{hdd}(h_{S_O}^{14}, h_{S_O}^{54}) = d_{hdd}(h_{S_O}^{34}, h_{S_O}^{54}) = 0.222$, etc. It affects the calculation of

discordance index and ultimately affects the ranking result. While the proposed normalized projection-based distance between DHHFLEs that takes angle into account can accurately reflects the distance between different DHHFLEs, which has more superiority.

(2)   It can be seen that the medium concordance and discordance sets of the contrast ELECTRE II method are empty sets by the comparison method in [24], failing to distinguish some existing medium degree of superior obtained by the proposed comparison method based on the bidirectional projection in this paper. It reflects that the bidirectional projection is more suitable to be used for comparing the distance to the positive and negative ideal DHHFLEs in the proposed ELECTRE II-based outranking method.

5.4.2 Comparison with the DHHFL-MULTIMOORA Method

By extending MULTIMOORA method into DHHFL environment, Gou et al. [15] proposed the DHHFL-MULTIMOORA method. It utilized three measures including the DHHFL ratio system (DHHFLRS), the DHHFL reference point (DHHFLRP) and the DHHFL full multiplicative form (DHHFLFMF) measures to deal with MADM problems. Since the the weights of attributes was also not considered in the DHHFL-MULTIMOORA method, the weight vector is set to be the same as what we calculate in our proposed method to eliminate the impact might happen.

Step 1.   Calculate the DHHFLRS measure. Firstly, calculate the normalization of all DHHFLEs by

$$h_{S_O}^{ij*} = E(h_{S_O}^{ij}) / \sum_{i=1}^{m} E(h_{S_O}^{ij}) \tag{34}$$

where $E(h_{S_O}^{ij}) = \frac{1}{L}\sum_{l=1}^{L} f(h_{S_O}^{ij(l)})$ denoting the expected values of $h_{S_O}^{ij}$. And the results are shown in Table 12. Then the DHHFL ratio Ψ_i^* for each alternative can be computed by

$$\Psi_i^* = \sum_{j=1}^{\theta} h_{S_O}^{ij*} - \sum_{j=\theta+1}^{m} h_{S_O}^{ij*} \tag{35}$$

Then the calculation results are obtained as $\Psi_1^* = 0.1560$, $\Psi_2^* = 0.1680$, $\Psi_3^* = 0.1346$, $\Psi_4^* = 0.1798$, $\Psi_5^* = 0.1594$. Put them in descending order, the ranking order is $X_4 \succ X_2 \succ X_5 \succ X_1 \succ X_3$.

Table 11 Hybrid Hamming distance with the positive and negative ideal DHHFLEs

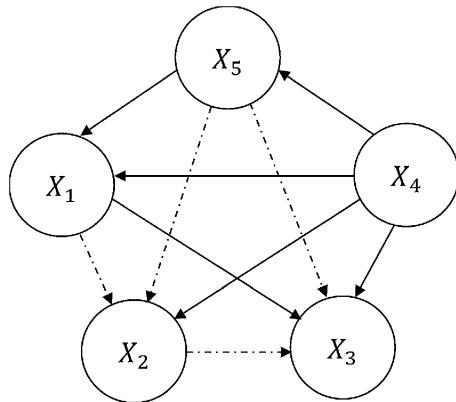| (d_{hdd}^+, d_{hdd}^-)	$Attr_1$	$Attr_2$	$Attr_3$	$Attr_4$	$Attr_5$	$Attr_6$
X_1	(0.037,0.139)	(0.139,0.083)	(0.222,0.204)	(0.176,0.204)	(0.315,0.037)	(0.222,0.250)
X_2	(0.139,0.037)	(0.176,0.074)	(0.093,0.361)	(0.167,0.185)	(0.176,0.148)	(0.269,0.231)
X_3	(0.139,0.037)	(0.083,0.167)	(0.352,0.185)	(0.083,0.241)	(0.102,0.222)	(0.333,0.000)
X_4	(0.083,0.120)	(0.157,0.037)	(0.074,0.380)	(0.083,0.241)	(0.139,0.185)	(0.037,0.324)
X_5	(0.074,0.102)	(0.046,0.148)	(0.380,0.046)	(0.250,0.074)	(0.037,0.315)	(0.074,0.315)

Fig. 6 The outranking graph

Step 2. Calculate the DHHFLRP measure. Figure out the positive ideal DHHFLEs as the the maximal objective reference points $h_{S_O}^{j+}(j = 1, 2, 3, 4, 5, 6)$, and calculate the distance between each DHHFLE $h_{S_O}^{ij}$ and $h_{S_O}^{j+}$ by

$$D(h_{S_O}^{ij}, h_{S_O}^{j+}) = \omega_j \sqrt{\frac{1}{\tilde{L}} \sum_{l=1}^{\tilde{L}} \left(f(h_{S_O}^{ij}) - f(h_{S_O}^{j+}) \right)^2} \quad (36)$$

And the results are shown in Table 13. Then the ranking result can be obtained based on the Min-Max metric, namely $X_4 \succ X_1 \succ X_2 \succ X_3 \succ X_5$.

Step 3. Calculate the DHHFLFMF measure. Using the DHHFLFMF measure, the evaluation value Υ_i of each alternative is calculated by

$$\Gamma_i = \prod_{j=1}^{\theta} (E(h_{S_O}^{ij}))^{\omega_j} / \prod_{j=\theta+1}^{n} (E(h_{S_O}^{ij}))^{\omega_j} \quad (37)$$

Then we obtain $\Gamma_1 = 0.2714$, $\Gamma_2 = 0.2873$, $\Gamma_3 = 0.2396$, $\Gamma_4 = 0.3038$, $\Gamma_5 = 0.2732$. Put them in descending order, the ranking order is $X_4 \succ X_2 \succ X_5 \succ X_1 \succ X_3$.

By combining the calculation results obtained by the three measures and according to the dominancy theory, the ranking result obtained by DHHFL-MULTIMOORA method is $X_4 \succ X_2 \succ X_5 \succ X_1 \succ X_3$. It shows that the optimal alternative is X_4, as same as the result obtained by our proposed method, while the order of the middle three alternatives is different. After further exploration and analysis, the reasons for the different order of the middle three alternatives can be expressed as follows:

(1) The ratio system in DHHFL-MULTIMOORA method is only concerned with the expected values of the DHHFLEs but not with the discrete cases of them, which lacks a certain rationality. If a DHHFLE with a large expected value also has a large deviation degree which is not considered in the DHHFLRS measure in [15], the ranking is not reasonable enough. While the bidirectional projection in the proposed ELECTRE II-based outranking method involves the difference values between DHHFLEs and the positive DHHFLEs, the difference values between DHHFLEs and the negative DHHFLEs, and the difference values between the positive DHHFLEs and the negative DHHFLEs, which reflects not only the closeness between DHHFLEs and the positive and negative DHHFLEs but also the discrete degree of DHHFLEs.

(2) The positive ideal DHHFLEs is taken as the reference point in DHHFL-MULTIMOORA method, while the relations between DHHFLEs and the negative ideal DHHLEs is neglected. While the proposed ELECTRE II-based outranking method in this paper considers both positive and negative ideal DHHFLEs, which makes the method more reasonable and effective.

(3) The distance between the DHHFLEs on each attribute is not considered in DHHFL-MULTIMOORA method, which directly relates to the determination of attribute weights and the limited compensation between alternatives in our proposed method. In practical issues, when the distance between two alternatives achieves a certain level, it may be unacceptable for decision makers to

Table 12 The normalization results of DHHFLEs in DHHFL-MULTIMOORA method

	$Attr_1$	$Attr_2$	$Attr_3$	$Attr_4$	$Attr_5$	$Attr_6$
X_1	0.191	0.186	0.209	0.199	0.169	0.199
X_2	0.207	0.198	0.242	0.203	0.203	0.189
X_3	0.218	0.212	0.152	0.206	0.203	0.146
X_4	0.202	0.192	0.242	0.217	0.203	0.230
X_5	0.183	0.212	0.155	0.175	0.220	0.235

Table 13 The distance between each DHHFLE and the reference points

	$Attr_1$	$Attr_2$	$Attr_3$	$Attr_4$	$Attr_5$	$Attr_6$
X_1	0.003	0.012	0.053	0.021	0.044	0.050
X_2	0.012	0.016	0.024	0.019	0.026	0.061
X_3	0.012	0.007	0.075	0.009	0.016	0.081
X_4	0.007	0.015	0.017	0.009	0.022	0.008
X_5	0.006	0.005	0.090	0.031	0.005	0.016

compensate the loss on an attribute by gaining another. This is exactly part of the superiority of the ELECTRE II method.

6 Managerial Insights and Practical Implications

The comparison results show that the introduction of projection makes the ELECTRE II method more reasonable and accurate in the DHHFL environment, which is a helpful tool for the government to select emergency logistics providers. Besides, as an outranking method, ELECTRE II has more superiority and rationality than utility theory-based methods like MULTIMOORA in dealing with the ELPS problems under DHHFL environment.

Thus, for complex MADM problems, especially like the ELPS problems faced by government departments, decision making needs to be comprehensive and deliberate. Double hierarchy linguistic terms can be applied to express complex linguistic evaluation information more comprehensively and intuitively for DMs. In addition, if an alternative performs severely unsatisfactorily under a certain attribute, it certainly cannot be selected even though it may perform well under most other attributes. That is, the performance of the alternatives under each attribute cannot completely compensate each other. Under such situations, the relation between alternatives can be accurately

calculated by the proposed method. Therefore, it is convincing and practical to choose the improved ELECTRE II-based outranking method for the ELPS problems.

7 Conclusion and Outlook

Emergency logistics management is of great significance for social construction, and how to select appropriate emergency logistics providers is a complex decision-making process for the government. In this paper, an integrated decision-making method based on ELECTRE II is put forward to handle MADM problems with evaluation information in the form of DHHFLTSs. To formulate the method, a new distance measure for DHHFLTSs based on normalized projection is defined and utilized in the maximizing deviation method to calculate the attribute weights. Then a bidirectional projection measure is used to form a new comparison method for DHHFLTSs. Accordingly, the improved ELECTRE II outranking method is presented by integrating the distance measure and the comparison method, and its decision framework is given. In addition, a practical MADM problem concerning ELPS and detailed result analysis are given to illustrate the advantages of the proposed method.

This paper also has limitations. The numerical result based on the proposed normalized projection-based distance measure is larger than traditional Hamming distance or Euclidean distance, although it does not affect the effectiveness of decision-making process, it is what is going to study and improve in the future. Besides, this paper is conducted based on that DMs are entirely rational, research direction in the future is to consider the bounded rationality of DMs. In-depth research is going to be carried out on the combination of the proposed method with behavior theories like prospect theory [49] and regret theory [26]. In addition, DHHFLTSs facilitate group decision making, and with the digital development of society of society, more decision-making problems involve large-scale DMs [50]. Large-scale group decision making (LSGDM) has attracted a lot of attention [51, 52], and the application of DHHFLTSs to LSGDM will be worth exploring in the future.

Declarations

Conflict of interest The authors declare that there is no conflict of interests regarding the publication of this paper.

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