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An Integrative Decision-Making Mechanism for Consumers' Brand Selection using 2-Tuple Fuzzy Linguistic Perceptions and Decision Heuristics

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Abstract Consumers perform decision-making (DM) processes to select their preferred brands during their entire consumer journeys. These DM processes are based on the multiple perceptions they have about the products available in the market they are aware of. These consumers usually perform different DM strategies and employ diverse heuristics depending on the nature of the purchase, ranging from more pure optimal choices to faster decisions. Therefore, the design of realistic DM approaches for modeling these consumer behaviors requires a good representation of consumer perceptions and a reliable process for integrating their corresponding heuristics. In this work, we use fuzzy linguistic information to represent consumer perceptions and propose four consumer DM heuristics to model the qualitative linguistic information for the consumer buying decision. In particular, we use 2-tuple fuzzy linguistic variables, which is a substantially more natural and realistic representation without falling in a loss of information. The set of selected heuristics differ in the degree of involvement the consumers give to their decisions. Additionally, we propose a heuristic selection mechanism to integrate the four heuristics in a single DM procedure by using a regulation parameter. Our experimental analysis shows that the combination of these heuristics in a portfolio manner improves the performance of our model with a realistic representation of consumer perceptions. The model's outcome matches the expected behavior of the consumers in several real market scenarios.

Keywords Agent-based modeling · Decision-making heuristics · Fuzzy decision-making · Fuzzy linguistic 2-tuples · Marketing · Consumer behavior

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1 Introduction

Understanding how consumers make their buying decisions is the cornerstone for most of the marketing questions in small, medium, and large enterprises. This knowledge about consumers' journey will elicit successful marketing policies for increasing the sales and awareness levels of the brand. But representing and modeling consumers are not a straightforward endeavour because the total sales of a brand comes from the result of multiple consumer interactions and decisions at different stages of their consumer journeys, the so-called marketing funnel [1]. Furthermore, these decisions may be affected by many diverse off-line and on-line marketing campaigns, as well as emergent word-of-mouth processes among the consumers [2].

Agent-based models (ABM) can be used to model this complex behavior of a market [3–5]. In ABM, simple individual behaviors are modeled into agents of a system where they act and interact with other agents and with the environment. Because of the bottom-up nature of ABM, the emergent complex behavior of the system can be inferred from the micro behaviors of its individuals. ABM provides a suitable tool to replicate realistic market conditions because of its simplicity to model these consumer behaviors and their interactions with brands and media [5, 6]. Notice that modeling individual behaviors is often simpler –and in most of the cases, more accurate– than modeling the behavior of the whole system by global top-down rules.

One of the most important micro behaviors of the population of consumers is how they select a brand from the set of all the available options in a market category. Decision-making (DM) [7] is the mental process that produces the selection of an alternative among a set of different choices. In our case, these alternatives represent the available products or brands in the market that the consumer may choose and those consumer DM processes correspond to sales. Consumer decisions are usually guided by their preferences on several attributes of the products, each possibly having a distinct weight.

Moreover, consumer perceptions[1] are usually qualitative rather than quantitative, and hence fuzzy linguistic variables [8–10] and fuzzy linguistic approaches [11–13] are more suitable to represent and manipulate them [14, 15]. Notice that the human way of expressing and using knowledge is generally fuzzy rather than crisp, with a certain degree of uncertainty and/or imprecision. Therefore, a numerical representation and aggregation of such consumer perceptions would imply a consequent loss of information, which can further affect their DM processes [16, 17].

Following these assumptions, [18] presented a marketing ABM where consumer perceptions are modeled by 2-tuple fuzzy linguistic variables [19, 20]. These variables consist of pairs of a linguistic label and a symbolic translation. This fuzzy representation is significantly more realistic than numerical values and do not suffer the loss of information existing in other fuzzy linguistic approaches when aggregating information to take decisions [19]. To the best of our knowledge, [18] provides the first marketing ABM with an accurate representation of consumer perceptions, which matches the human way of expressing information.

The model proposed in [18] includes a basic consumer DM heuristic that aggregates all the consumer's perceptions for every brand attribute, or driver, according to their preferences and selects the brand with the highest aggregated perception. However, it is well-known that consumers do not always use the same DM process with regard to every purchase [21]. In [22], it is argued that consumer DM strategies are ruled by different mechanisms to explore all the possible alternatives (through their attributes) to eventually select one of them. The aggregation of these mechanisms comprises the so-called consumer DM heuristic. There are different heuristics and they can lead to different choices [23]. Although all these works analyze consumers' behaviors from a psychological perspective, they lack a precise algorithmic description of these heuristics from a computation point of view. Therefore, it is necessary to precisely define a diverse set of consumer DM heuristics to capture the different consumer behaviors in a market.

Therefore, the research questions addressed in this work are the following:

1. How consumers' strategies of purchase can be precisely formulated as heuristics to be integrated into a computational model to simulate the complex behavior of a market?
2. How qualitative information representing consumers' perceptions can be handled by these heuristics?
3. How these heuristics can be integrated into a single DM procedure in order to capture the complex behavior of consumers when facing a purchase?

To tackle these questions, this work proposes four fuzzy linguistic purchase heuristics to properly model different real-world consumer DM strategies. They all manipulate fuzzy linguistic 2-tuples, which represent consumer perceptions in a realistic and accurate manner. The proposed heuristics differ in the degree of involvement the consumers give to their purchases and they are inspired by well-established studies on behavioral economics and consumer behaviors [21–23]. In particular, more involved heuristics perform a more exhaustive search in order to maximize the utility of the chosen brand whereas less involved heuristics seek fast satisfying decisions [24]. The current work is the first one proposing a set of diverse consumer decision heuristics based on the 2-tuple fuzzy linguistic computational model.

Additionally, we introduce a mechanism to integrate the four proposed decision heuristics into a single consumer DM procedure based on the degree of involvement of brand selections defined for the specific market. This mechanism combines the use of the four heuristics as a portfolio of versatile heuristics which will be used depending on the marketing context using a constructive process in DM where consumers use a variety of strategies depending on the tasks demand and they have limited information [21]. Therefore, every consumer, even in the same market, may use different heuristics to decide the brand to choose. Our experimental evaluation shows that the combination of these heuristics improves the performance of our model and this performance matches the

[1] In marketing, consumer *perceptions* refer to the assessments given by a consumer to the *attributes* of a product, and these attributes are commonly known as brand *drivers* since they drive consumers' decisions.

expected behavior in several marketing scenarios. These scenarios were built using real data from a pool of diverse industries, from automakers to dairies.

In summary, the contributions of the current work are as follows:

- We model a set of realistic consumer decision heuristics which handle 2-tuple fuzzy linguistic information. As discussed in Sect. 4.1, this is the natural representation of this kind of information in the marketing domain because it does not suffer the loss of information existing in other approaches (e.g., the ordinal fuzzy linguistic representation) when aggregating information.
- We incorporate these heuristics to a realistic ABM of consumer behavior for marketing. In our ABM, agents represent the consumers of a market and their DM processes simulate their purchases in such a market.
- We introduce a heuristic selection mechanism which captures the nature of actual consumers according to the type of product they purchase in each scenario. This mechanism allows consumers to decide which of the four proposed heuristics to use in each situation in a portfolio manner.
- We evaluate the performance of our approach in a comparison with respect to a classical fuzzy linguistic representation based on linguistic labels. Our empirical results show the superiority of our model.
- We analyze the performance of the heuristic selection mechanism under a sensitivity analysis. When this mechanism is at work, each consumer may select their preferred products using different strategies, which is a behavior that can better fit with reality in marketing scenarios.

The rest of this work is organized as follows. Related works are described in Sects. 2, and 3 reviews some preliminary concepts on fuzzy linguistic approaches and particularly, the 2-tuple linguistic modeling. Section 4 describes the ABM of our model and Sect. 5 presents the consumer DM heuristics that handle 2-tuple fuzzy linguistic variables. In Sect. 6, a mechanism to integrate the four heuristics into a single DM procedure is defined. Section 7 describes the experimental results of the proposed approach in real-world market scenarios. We finally discuss the main conclusions of the study and some lines of future work in Sect. 8.

2 Related Works

ABM have been proposed to model the complex behavior of markets [3–5]. Additionally, ABM has been successfully applied in fields such as economics [25], politics [26, 27], trust-based social systems [28, 29], sharing economy [30], and contract farming [31], among others.

Consumers' behaviors and strategies of DM have been extensively studied from a psychological perspective in marketing literature [21–24]. Nevertheless, there is a lack of a precise definition of these heuristics from a computational point of view. To the best of our knowledge, [18] is the first work that proposes an ABM to model consumers' perceptions using fuzzy linguistic information, but it only describes a very simple DM heuristic for those consumers. The present work aims to fill this gap by precisely defining a set of diverse consumers' DM heuristics, which represent the distinct strategies that consumers use when they face a decision (i.e., a purchase). Moreover, these decisions can be seen as a multi-criteria DM (MCDM) problem, and could be faced by state-of-the-art MCDM methods, such as the fuzzy ordinal priority approach [32, 33].

In this work we are considering the original 2-tuple fuzzy linguistic computational model proposed in [34] to design our marketing model. This decision is based on the kind of information available in our real-world application. As justified in Sect. 4.2, this information is collected in consumer surveys where every consumer considers the same linguistic term set. The cardinality of this term set is odd and its linguistic labels are symmetrically placed around a middle term representing a neutral perception about the brand. Every consumer agent in the model considers this linguistic term set to aggregate information in order to get an overall perception about the set of alternative brands. Hence, we are in a scenario with homogeneous information that perfectly matches the requirements of the original 2-tuple fuzzy linguistic computational model.

In the last two decades, a large amount of research have been developed to propose alternative models to the original 2-tuple fuzzy linguistic computational model or to extend it to deal with DM handling unbalanced linguistic information or heterogeneous information contexts (i.e., dealing with multigranular linguistic information and non-homogeneous information) [35, 36]. These models properly allow the implementation of a general decision scheme for computing with words represented in Fig. 1. A good survey of these approaches can be found in [20].

Particularly, [37] introduced a generalized version of the 2-tuple fuzzy linguistic representation model based on the

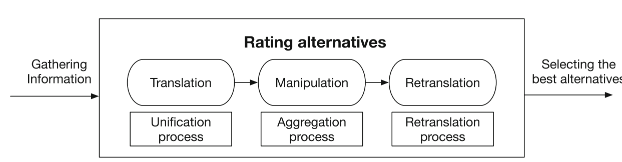

Fig. 1 General decision scheme dealing with heterogeneous contexts in decision-making [20]

concepts of symbolic proportion and canonical characteristic values of linguistic terms. In contrast to the original 2-tuple fuzzy linguistic representation model [19], this model can deal with linguistic variables with linguistic term sets that are not uniformly and symmetrically distributed. Besides, it was extended by incorporating a third parameter to handle incomplete linguistic preferences [38]. But one of the drawbacks of this model is that the semantic of linguistic terms in the linguistic term set used can only be defined by symmetrical trapezoidal fuzzy membership function. This is solved in the numerical scale model [39], which defines a more consistent numerical scale function to make transformations between linguistic 2-tuples and numerical values in different DM situations, which can use any fuzzy membership function shape. This model is further refined in the so-known interval numerical scale model [40], which provides a basis for the linguistic computational model based on 2-tuples and intervals. Overall, numerical scale models generalize the 2-tuple fuzzy linguistic representation model by defining a personalized numerical scale function for the linguistic term set which can be computed by solving an optimization problem.

In summary, all of these models handle different representations of the information used by the decision-maker. Nevertheless, the use of these alternative approaches are out of the scope of the current contribution. In our case, this information comes from marketing surveys whose answers show a linguistic nature but these linguistic answers are constrained within a fixed set of possible responses that are uniformly and symmetrically distributed. Therefore, the original 2-tuple fuzzy linguistic representation model is a natural and convenient representation model for the information handled in a real marketing scenario.

3 Preliminaries

In this section we review some preliminary concepts for modeling fuzzy linguistic information, to be used by the consumer DM heuristics. Linguistic variables are variables whose values are words or sentences in the natural language [8–10]. They are used in *fuzzy linguistic approaches*, where the problem requires to deal with qualitative aspects [41, 42]. The general decision scheme to work with heterogeneous (linguistic) information is depicted in Fig. 1[2]. It includes the translation and retranslation phases defined for this paradigm, which are crucial to manipulate the linguistic information.

[2] In our case, the computational scheme is simpler as every consumer uses the same linguistic scale to establish their preferences since they come from tracking data surveys available at the company (see Sect. 4.1). In such a way, the considered decision scheme is most similar to the one proposed by Yager in [12].

In *linguistic symbolic computational models based on ordinal scales* [43, 44], linguistic variables take values from a predefined totally ordered set of *linguistic labels* $S = \{s_0, \ldots, s_g\}$ of finite size $|S| = g + 1$. In this approach, the semantics of the linguistic labels can be derived from their order [45], i.e., the first label s_0 represents the lowest value, with higher values for the next labels in the ordered set ($\forall s_i, s_j \in S \,.\, s_i \leq s_j \Leftrightarrow i \leq j$.).

Thus, the comparison operators $<, =, >$ can be defined directly from the total order defined in the linguistic term set. Extended aggregation operators compute a convex combination of linguistic labels, working on the label indexes $\{0, \ldots, g\}$ of the linguistic term set S. These operators generate a real value on the granularity interval $\beta \in [0, g]$ [43], that requires to be approximated to a linguistic label of S. The main drawback of this approach is the loss of information produced in this aggregation process [19]. In particular, the aggregation of two distinct sets of linguistic labels may lead to the same value. As a result, it may be hard to asses whether one of these two sets is preferred to the other.

In order to solve the previous problem, the *2-tuple fuzzy linguistic representation model* was proposed [19, 46, 47]. In this approach, linguistic variables are represented by a linguistic label and a symbolic translation.

Definition 1 (Fuzzy Linguistic 2-Tuple [19]) A 2-tuple fuzzy linguistic variable is a pair $\langle s_k, \alpha \rangle$, where $s_k \in S = \{s_0, \ldots, s_T\}$ is a linguistic label from the ordered set of linguistic labels S and $\alpha \in [-0.5, 0.5)$ is a symbolic translation specifying the translation of the fuzzy membership function of the closest linguistic label s_k in case the linguistic label resulting from the symbolic calculus does not exactly correspond to a label in the term set. The 2-tuple set associated with S is defined as $\bar{S} = S \times [-0.5, 0.5)$.

In this work, we consider triangular membership functions for fuzzy linguistic 2-tuple variables [42], although other non piece-wise linear functions could have been also considered instead [41]. In Fig. 2, we represent an example of this fuzzy partition that takes values from the set of linguistic labels $S = \{vlow, low, med, high, vhigh\}$.

As in linguistic symbolic computational models based on ordinal scales, a symbolic computation in the term set S produces a value $\beta \in [0, g]$ (the interval of granularity of S) that must be transformed into a linguistic 2-tuple value. To do so, there is a need to define a numerical-linguistic approximation:

Definition 2 (Numerical-linguistic and linguistic-numerical approximations [19]) Let $S = \{s_0 \ldots s_g\}$ be a set of linguistic terms and $\bar{S}$ the 2-tuple set associated with S

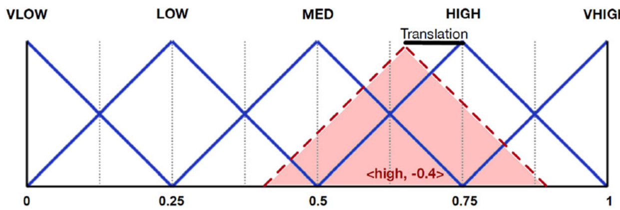

Fig. 2 Triangular membership functions for fuzzy linguistic variables that take values from the set of linguistic labels $\{vlow, low, med, high, vhigh\}$. It also includes the 2-tuple numerical-linguistic approximation $\Delta(2.6) = \langle high, -0.4\rangle$

defined as $\bar{S} = S \times [-0.5, 0.5)$. The function $\Delta_S : [0, g] \rightarrow \bar{S}$ is given by:

$$\Delta_S(\beta) = \langle s_k, \alpha\rangle, \text{with} \begin{cases} \mathrm{i} = \mathrm{round}(\beta) \\ \alpha = \beta - \mathrm{k} \end{cases} \tag{1}$$

with $\beta \in [0, g]$ and where $round(\cdot)$ is the function that assigns the closest integer number $i \in \{0, \ldots, g\}$ to β. In this way, each numerical value in the interval of granularity of S is uniquely identified by a 2-tuple linguistic value $\langle s_k, \alpha\rangle \in \bar{S}$.

Meanwhile, there is a linguistic-numerical approximation assigning a numerical value β to each linguistic 2-tuple defined by the inverse function $\Delta_S^{-1} : \bar{S} \rightarrow [0, g]$ as follows:

$$\Delta_S^{-1}(\langle s_k, \alpha\rangle) = k + \alpha \tag{2}$$

For the sake of simplicity, we remove the subindex S of the approximation functions Δ and Δ^{-1} when it is clear the linguistic term set S they refer to. Besides, we overload Δ for a set of real numbers $R = \{\beta_1, \ldots, \beta_m\}$ and Δ^{-1} for a set of linguistic 2-tuples $X = \{\langle s_1, \alpha_1\rangle \ldots \langle s_m, \alpha_m\rangle\}$ as follows: $\Delta(R) = [\Delta(\beta_i)]_{1 \le i \le n}$ and $\Delta^{-1}(X) = [\Delta^{-1}(\langle s_i, \alpha_i\rangle)]_{1 \le i \le n}$. An example of these approximations can be found in Fig. 2, where it can be seen that $\Delta(2.6) = \langle high, -0.4\rangle$.

The basic operators for the 2-tuple fuzzy linguistic computational model are defined as follows:

Definition 3 (Linguistic 2-Tuple Basic Operators [19]) Let $\langle s_k, \alpha_1\rangle$ and $\langle s_l, \alpha_2\rangle$ be two fuzzy linguistic 2-tuples. The comparison operators $<, =$ and $>$ are defined as follows based on the complete order established in S :

- $\langle s_k, \alpha_1\rangle < \langle s_l, \alpha_2\rangle \Leftrightarrow s_k < s_l \vee (s_k = s_l \wedge \alpha_1 < \alpha_2)$
- $\langle s_k, \alpha_1\rangle = \langle s_l, \alpha_2\rangle \Leftrightarrow s_k = s_l \wedge \alpha_1 = \alpha_2$
- $\langle s_k, \alpha_1\rangle > \langle s_l, \alpha_2\rangle \Leftrightarrow s_k > s_l \vee (s_k = s_l \wedge \alpha_1 > \alpha_2)$

Besides, the negation operator for linguistic 2-tuples is defined by means of the approximation functions Δ and Δ^{-1}:

$$Neg(\langle s_k, \alpha\rangle) = \Delta(g - \Delta^{-1}(\langle s_k, \alpha\rangle) \tag{3}$$

There is a wide variety of aggregation operators to be used in the 2-tuple linguistic model [19, 20], remarking the extension of the OWA operator [48] due to its great versatility. In our scenario, each consumer owns a weight associated to each brand driver, that is specified *a priori* representing her decision preferences. Hence, the weighted average operator for linguistic 2-tuples is the appropriate operator to develop the aggregation process.

Definition 4 (Weighted average operator for linguistic 2-tuples [19]) Given a set of linguistic 2-tuples $X = \{\langle s_1, \alpha_1\rangle, \ldots, \langle s_m, \alpha_m\rangle\}$ to be aggregated and a vector of weights $W = [w_1, \ldots, w_m]$ associated, the 2-tuple weighted average $2TWA : \bar{S}^m \rightarrow \bar{S}$ is defined as:

$$2TWA(X, W) = \Delta\left(\frac{\sum_{i=1}^m \Delta^{-1}(\langle s_i, \alpha_i\rangle) \cdot w_i}{\sum_{i=1}^m w_i}\right)$$

In our scenario the weight vector holds that $w_i \in [0, 1]$ and $\sum_i w_i = 1$ (see Sect. 4.2). Hence, we have a linear combination of the 2-tuples and the denominator value is 1, reducing the expression to $2TWA(X, W) = \Delta(\Delta^{-1}(X \cdot W))$.

4 Decision-Making Representing Consumer Perceptions with the 2-Tuple Fuzzy Linguistic Computational Model

In this section, we first discuss the need of using the qualitative representation of fuzzy linguistic 2-tuples in our framework (see Sect. 4.1), and next we define the ABM used in our model with consumer agents having fuzzy linguistic perceptions (see Sect. 4.2).

4.1 The Need of Fuzzy Linguistic Variables

In our ABM, agents represent consumers of a market. In order to perform realistic simulations of this virtual market, it is necessary to model their perceptions for every brand in the market. This information is commonly obtained from real consumer tracking data and brand health studies from well-established marketing consultants such as Kantar Millward Brown [49]. In these studies, consumers are usually surveyed a set of questions about the available brands in the target market [50].

The most usual scenario is that the answers to those surveys have a linguistic nature, such as linguistic labels, all of defined over the same linguistic term set. In order to process them, these answers are transformed into numerical values by preprocessing the data (see Fig. 3). Alternatively, fuzzy linguistic approaches are the most suitable and natural representation of this kind of qualitative data from

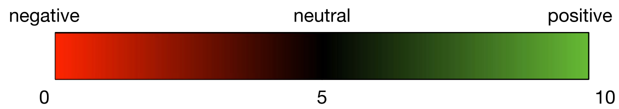

Fig. 3 Representation of consumer perceptions in numerical and linguistic scales

human perceptions [18]. Nevertheless, the representation of consumer perceptions in the computational model is usually the result of the aggregation of many questions to the survey. As a consequence, classical linguistic symbolic computational models based on ordinal scales, which suffer a loss of information when linguistic variables are aggregated, are not an appropriate model to represent the information available in our marketing system. Moreover, although consumer perceptions can be processed and transformed to a numerical scale (with the consequent "loss of information"), our view is that the problem would be better tackled by directly working with the linguistic assessments following a fuzzy linguistic approach instead of transforming them into numerical values. Computing with words definitively provides a more natural representation when dealing with human perceptions, represented as words in natural language, as in our case.

4.2 An ABM with Consumers' Perceptions Based on 2-Tuple Fuzzy Linguistic Variables

In this subsection we briefly describe the structure and components of the marketing ABM used in this work, based on the one proposed in [18]. The emphasis of our model is on the consumer perceptions about the available brands in the market. These perceptions incorporate the heterogeneity and socio-demographic features of the consumers in an implicit way. For example, the income of consumers will determine their perceptions about the price of the brands. Additionally, let us note that these variables can be modified by other marketing mechanisms and events on the consumer journey such as advertising effects, sampling campaigns, or word-of-mouth interactions. In order to have a neat analysis, we fix the perceptions of the consumers and remove the temporal evolution of the consumer brand images.

In the ABM, agents represent consumers who carry out a DM process to select a brand among a set of n available brands $B = \{b_1, \ldots, b_n\}$ (i.e., the set of alternatives). The attributes of each brand are modeled by m drivers $D = \{d_1, \ldots, d_m\}$. As all the brands belong to the same category, they all share the same drivers. In order to represent driver preferences, we define for each agent x a weight vector $W^x = [w_1^x, \ldots, w_m^x]$, such that all weights must be in the interval [0, 1] and their sum must be equal to 1. These weights represent the importance of each driver when the consumer agent x makes a decision.

Each agent has its own perceptions (positive, neutral, or negative) about each driver of each brand. Consumer perceptions are modeled by defining, for each agent x, a matrix of perceptions P^x of dimension $n \times m$, where each element $p_{i,j}^x \in P^x$ represents the perception of agent x on brand $b_i \in B$ about driver $d_j \in D$. As previously stated, these heterogeneous perceptions implicitly capture external variables that previously affected the brand image of the consumers (e.g., consumer habits, loyalty, influence by other consumer, or income). In our model, these perceptions are represented using 2-tuple fuzzy linguistic variables, all of them taking values from a common ordered set of linguistic labels (see Definition 1). This allows us to represent the qualitative view of the consumer on each brand.

Brand awareness is the first dimension and pre-requisite of the brand knowledge system in consumer minds, which helps the brand identification under different conditions [1] and filters the final set of choices when the consumer faces a decision about the brand to buy [21]. Brand awareness can be defined at several levels such as recognition, aided awareness, top of mind, and light knowledge about the brand [51]. It plays a crucial role in disparate markets, from luxury goods [52] to restaurants [53]. Agent-based marketing approaches have also incorporated this natural mechanism into the agent behavioral rules [54, 55]. To model this information, we extend the model of [18] by defining for every agent x a vector $A^x = [a_1^x, \ldots, a_n^x]$ of Boolean variables, where a_i^x represents whether agent x is aware of brand $b_i \in B$ or not. Thus, this vector A representing awareness information is versatile and, depending on the model, could mean a different awareness level (e.g., aided awareness or top of mind).

Definition 5 (Consumer agent) A consumer agent x is defined as the tuple $\langle A^x, W^x, P^x \rangle$, where $A^x = [a_1^x, \ldots, a_n^x]$ is a vector where each element $a_i \in \{0, 1\}$ represents whether agent x is aware of brand $i \in B$ or not; $W^x = [w_1^x, \ldots, w_m^x]$ is a weight vector satisfying that $\forall w_i^x \in W^x \Rightarrow w_i^x \in [0, 1]$ and $\sum_{1 \le i \le m} w_i^x = 1$, with each element w_j representing the importance that agent x gives to driver $j \in D$ in its buying decision; and P^x is an $n \times m$ matrix of fuzzy linguistic 2-tuples, with each element $p_{i,j} \in P^x$ representing the perception that agent x has on brand $i \in B$ about driver $j \in D$.

Notice that perceptions P^x describe the assessments of consumer x on the different attributes of the available products in the market, which will be used by this consumer to perform her DM, where B is the set of alternatives and D is the set of attributes or features, with weights given by W^x.

Table 1 Summary of the ABM components

Component	Description
$B = \{b_1, \ldots, b_n\}$	Set of brands in the analyzed market
$D = \{d_1, \ldots, d_m\}$	Set of drivers of each brand
x	Agent of the ABM
$P^x = [\,-\,p^x_{i,j}\,-\,]$	Agent x's perception about d_j of b_i ($p^x_{i,j} \in \bar{S}, 1 \le i \le n, 1 \le j \le m$)
$A^x = [\,-\,a^x_i\,-\,]$	Agent x's awareness on b_i ($a^x_i \in \{0,1\}, 1 \le i \le n$)
$W^x = [\,-\,w^x_j\,-\,]$	Agent x's weight on driver d_i ($w^x_j \in [0,1], 1 \le j \le m$)

Additionally, notice that the set of drivers D and the set of brands B are market variables globally known by every agent. In particular, they can be inferred by the variables A^x, W^x, and P^x. For instance, W^x defines the weights that agent x gives to each driver $d \in D$ and A^x defines the awareness this agent has about each brand $b \in B$.

In Table 1 we summarize the components of the ABM. For the sake of clarity, in the remaining of this work the elements A^x, W^x, and P^x we will be respectively renamed as A, W, and P (i.e., removing the superindex of the agent x) whenever it is clear the agent they refer to.

5 2-Tuple Fuzzy Linguistic Consumers' Decision-Making Heuristics

In this section we describe four marketing DM heuristics that use fuzzy linguistic information to simulate consumer behaviors when facing brand selection decisions. These fuzzy linguistic purchase heuristics as well as the selection mechanism introduced in Sect. 6 constitute the main proposal of the current work. Namely they are utility maximization (UMAX), majority rule (MAJ), elimination by aspects (EBA), and satisfaction rule (SAT). These heuristics are inspired by well-established studies on behavioral economics and consumer behaviors [21–23].

First, we provide an illustrative example of a market that will allow us to show the trace of each heuristic. In this example market, there is just one consumer agent and four brands having three drivers.

Example 1 Consider a linguistic term set $S = \{vl, l, m, h, vh\}$ (standing for very low, low, medium, high, and very high, respectively). Let us define a consumer agent x and a set of four brands $B = \{b1, b2, b3, b4\}$ with three drivers $D = \{quality, price, comfort\}$ (q, p, and c for short). Let us assume the following matrix of perceptions P, vector of driver weights W, and vector of brand awareness A, for this agent x:

$$W = [\,w_q \quad w_p \quad w_c\,] = [\,0.4 \quad 0.4 \quad 0.2\,]$$

$$A = [\,a_{b1} \quad a_{b2} \quad a_{b3} \quad a_{b4}\,] = [\,\texttt{true} \quad \texttt{true} \quad \texttt{false} \quad \texttt{true}\,]$$

$$P = \begin{bmatrix} p_{b1,q} & p_{b1,p} & p_{b1,c} \\ p_{b2,q} & p_{b2,p} & p_{b2,c} \\ p_{b3,q} & p_{b3,p} & p_{b3,c} \\ p_{b4,q} & p_{b4,p} & p_{b4,c} \end{bmatrix} = \begin{bmatrix} \langle m, 0\rangle & \langle h, -0.45\rangle & \langle h, 0.35\rangle \\ \langle l, 0.45\rangle & \langle vh, 0\rangle & \langle vh, 0\rangle \\ \texttt{null} & \texttt{null} & \texttt{null} \\ \langle h, 0.45\rangle & \langle h, 0.45\rangle & \langle h, 0.45\rangle \end{bmatrix}$$

Notice that the agent is not aware of brand 3 and thus it has no perceptions about that brand.

The variables P, W, and A are the input of every DM heuristic to be applied by the consumer agent. In the unlikely case that an agent is aware of no brand (i.e., the vector A is completely set to `false`), every DM heuristic returns a `null` decision. For the sake of readability, we have omitted this filter in the algorithmic description of the heuristics. We also emphasize that when agents are aware of all the existing brands in the market, we are solving a simple DM problem. However, in a typical marketing scenario, consumer agents are only aware of a subset of the available brands in the market (i.e., A^x is not fully set to `true` for any agent x). In the following subsections we provide the precise formalization of each marketing fuzzy linguistic DM heuristic.

5.1 Utility Maximization

Utility maximization (UMAX) is a probabilistic heuristic that associates a probability to each brand the agent is aware of. Those probabilities are based on the global perceptions the agent has on each brand. In particular, the probability of choosing a brand is the result of aggregating all its driver perceptions, weighted by the driver weights, and normalized among all brands in exponential scale. In the fuzzy linguistic version of this heuristic, the aggregation of consumer perceptions is achieved by the weighted average operator for linguistic 2-tuples (see Definition 4). In Algorithm 1, we provide the pseudocode of this heuristic. Notice that brands the agent is not aware of are assigned a probability equal to 0, i.e., those brands are never chosen. The final decision is returned by the function RndWeightSel (see Algorithm 1) as a random choice based on the probabilities previously computed.

This heuristic requires to aggregate information and thus it uses the approximation functions from Definition 2. As stated in Sect. 3, fuzzy linguistic 2-tuples do not suffer the problem of potential loss of information when they are aggregated.

Algorithm 1 Utility Maximization (UMAX)

```
procedure UMAX(A, W, P)
    float[n] prob   ▷ Prob. of choosing each brand
    float sum ← 0
    for br ← 1 ... n do   ▷ Iterate for every brand
        if A[br] then   ▷ with awareness
            prob[br] ← e^{Δ^{-1}(T2WA(P[br], W))}
                ▷ Prob is T2WA in exponential scale
            sum ← sum + prob[br]
        else
            prob[br] ← 0
        end if
    end for
    for br ← 1 ... n do ▷ Normalize probabilities
        prob[br] ← prob[br] / sum
    end for
    return RndWeightSel(prob)
        ▷ Random selection according to prob
end procedure
```

The following example summarizes the behavior of UMAX:

Example 2 Consider the market described in Example 1. UMAX computes a probability associated to each of the three brands the agent is aware of as:

$$prob(b1) = e^{\Delta^{-1}(2TWA(P[b1],W))} = \\ = e^{2\cdot 0.4+2.55\cdot 0.4+3.35\cdot 0.2} = e^{2.49} = 12.06$$

$$prob(b2) = e^{\Delta^{-1}(2TWA(P[b2],W))} = e^{2.98} = 19.69$$

$$prob(b4) = e^{\Delta^{-1}(2TWA(P[b4],W))} = e^{3.45} = 31.5$$

By defining a probability distribution with these values, we get: $prob(b1) = 0.19$, $prob(b2) = 0.31$, and $prob(b4) = 0.5$. This is, brand $b1$, $b2$ and $b4$ will be chosen with these probability values, respectively. Notice that $b3$ cannot be chosen because the agent has no awareness about this brand.

In the former example, it can be seen how the exponential scale of the probabilities makes some brands much more likely to be chosen. Therefore, although this utility maximization is probabilistic, it gives a much greater weight to those brands with higher utility.

5.2 Majority Rule

The majority rule (MAJ) selects the preferred brand in a pairwise comparison of every brand, comparing their attributes. This process first selects a pair of two brands to compete. In this comparison, each brand is scored according to the drivers that are better with respect to the other brand. Each driver contributes its weight to the score. The winner is randomly selected with probability proportional to their scores and then it is compared to another brand, until no brand is left. The final decision is the brand winning this competition. The pseudocode of this heuristic is presented in Algorithm 2.

Algorithm 2 Majority Rule (MAJ)

```
procedure MAJ(A, W, P)
  ▷ Random shuffling of brands
    int[n] brOrder ← RndShuffle(n)
  ▷ Best brand up to now:
    int bestBrIdx ← argmin_i A[brOrder[i]]
    int bestBr ← brOrder[bestBrIdx]
  ▷ Comparison current vs best brand:
    for curBrIdx ← bestBrIdx + 1 ... n do
        if A[brOrder[curBrIdx]] then
            br_1 ← bestBr
            br_2 ← brOrder[curBrIdx]
            bestBr ← WINNER(br_1, br_2, W, P)
        end if
    end for
    return bestBr   ▷ Final winner
end procedure
function WINNER(b_1, b_2, W, P)
    float[n] prob ← [0, 0, ..., 0]
    for d ← 1 ... m do   ▷ Iterate for each driver
        if P[b_1][d] > P[b_2][d] then   ▷ b_1 is better
            prob[b_1] ← prob[b_1] + W[d]
        else if P[b_1][d] < P[b_2][d] then ▷ b_2 better
            prob[b_2] ← prob[b_2] + W[d]
        else   ▷ Tie in this driver
            prob[b_1] ← prob[b_1] + W[d]/2
            prob[b_2] ← prob[b_2] + W[d]/2
        end if
    end for
    return RndWeightSel(prob)
        ▷ Random selection according to prob
end function
```

Table 2 Execution of MAJ on the consumer of Example 1

Step 1. Comparison between brands $b1$ and $b4$				
Driver (W)	Perception $b1$	Perception $b4$	Prob. $b1$	Prob. $b4$
Quality (0.4)	$\langle m, +0.0\rangle$	$\langle h, +0.45\rangle$	-	+0.4
Price (0.4)	$\langle h, -0.45\rangle$	$\langle h, +0.45\rangle$	-	+0.4
Comfort (0.2)	$\langle h, +0.35\rangle$	$\langle h, +0.45\rangle$	-	+0.2
Probability of choosing the brand:			0.0	1.0
Winner (from roulette wheel selection): brand $b4$				

In the comparisons of two brands, we consider a perception is *better* than another if its fuzzy linguistic 2-tuple is greater than the other. To this end, we use the comparison operator for fuzzy linguistic 2-tuples (see Definition 3 for more details). This way, our approach does not suffer any loss of information existing in other fuzzy linguistic approaches.

Since the winner of each comparison is randomly selected with a probability proportional to its score, this heuristic does not require any criterion to break possible ties in the scores obtained by any pair of brands. Besides, since the probability of choosing a brand is conditioned by its order in the comparisons, we randomly shuffle the order of brands in those comparisons (see Algorithm 2).

Example 3 Consider the market described in Example 1 and assume brands are compared at random order [$b1$, $b4$, $b2$]. Hence, MAJ first compares brands $b1$ and $b4$, and the winner is compared to brand $b2$. We emphasize brands without awareness (e.g., brand $b3$) are not introduced in the comparisons performed with these heuristics. In Table 2 we describe the execution of this heuristic. In particular, assuming that the winner of the first comparison is $b4$, and the winner of the second one is $b2$, the selection of this heuristic would be therefore brand $b2$.

5.3 Elimination by Aspects

Algorithm 3 Elimination By Aspects (EBA)

```
procedure EBA(A, W, P)
    bool[n] active, aux
    ▷ Drivers sorted by descending weight:
    int[m] drS ← SORTBYWEIGHT(W)
    ▷ Random threshold for each driver:
    2-tuple[m] drT ← RNDTHRESH(m)
    repeat
        repeat ← false
        active ← BRANDWITHAWARENESS(A)
        for d ← 1...m do
                            ▷ Eliminate unsatisfying brands
            aux ← ELIMIN(drS[d], P, drT[d])
            active ← AND(active, aux)
                    ▷ If only one brand is remaining, finish:
            if COUNT(active, true)=1 then break
    ▷ If all brands removed, decrease thrs. & repeat:
            else if COUNT(active, true)=0 then
                DECREASETHRESH(drT)
                repeat ← true & break
            end if
        end for
    until repeat = false
    return RANDOMACTIVEBRAND(active)
end procedure
function ELIMIN(driver, P, threshold)
    bool[n] active ← [true, ..., true]
                                ▷ Iterate for every brand
    for br ← 1...n do
        if P[br][driver] < threshold then
            active[br] ← false  ▷ br eliminated
        end if
    end for
    return active
end function
```

Elimination by aspects (EBA) [56] rejects all the brands that do not fulfill some given criteria. In particular, these criteria are random thresholds generated for each driver. First, the drivers are ordered according to their weight. Then, the heuristic iteratively discards, for each driver, all brands whose perception in that driver is below its corresponding threshold. This process is repeated for all the drivers until only one brand is left. If a driver removes all the remaining brands, the threshold is decreased and the process is repeated. If after checking all drivers two or more brands remain unremoved, one of them is randomly chosen. In Algorithm 3, we give the pseudocode of this heuristic.

Again, the comparisons of fuzzy linguistic 2-tuples are performed using the basic comparison operators (see Definition 3 in Sect. 3). In the generation of driver thresholds, we use an uniform distribution of fuzzy linguistic 2-tuples over $\bar{S}$ avoiding extreme values, i.e., avoiding 2-tuples having the linguistic labels with the lowest and the highest values.

Example 4 Consider the market described in Example 1. Assume EBA generates the following random thresholds $T = [\langle m, 0\rangle, \langle h, 0\rangle, \langle l, 0\rangle]$, for the drivers (in order of weights) $\{quality, price, comfort\}$ respectively. Initially, the heuristic starts checking all the brands with awareness with respect to the driver having the highest priority, i.e., *quality*. After this check, there are still more than one brand left, thus this heuristic checks the next driver (i.e., *price*) in the remaining brands. After evaluating the second driver, only one brand is left. Therefore, such a remaining brand $b4$ is the one returned by this heuristic. In Table 3 we describe the execution of this heuristic.

5.4 Satisfaction Rule

The satisfaction rule (SAT) is a heuristic that randomly selects a brand and checks whether it fulfills a given criterion. If it does, such a brand is the selected one; otherwise it checks another. In particular, the criterion is a random threshold generated for each driver, i.e., the selected brand must have perceptions in all its drivers greater or equal than the corresponding thresholds. In Algorithm 4, we provide the pseudocode of this heuristic.

Table 3 Execution of EBA on the consumer of Example 1

Step 1. Evaluating driver *quality*			
Available brands: $\{b1, b2, b4\}$ (brands with awareness)			
Threshold	Perception $b1$	Perception $b2$	Perception $b4$
$\langle m, 0.0\rangle$	$\langle m, 0.0\rangle$	$\langle l, 0.45\rangle$	$\langle h, 0.45\rangle$
Removed brands: $\{b2\}$			
Remaining brands: $\{b1, b4\}$			
2. Evaluating driver *price*			
Available brands: $\{b1, b4\}$			
Threshold	Perception $b1$	Perception $b2$	Perception $b4$
$\langle h, 0.0\rangle$	$\langle h, -0.45\rangle$	-	$\langle h, 0.45\rangle$
Removed brands: $\{b1\}$			
Remaining brands: $\{b4\}$			

Table 4 Execution of SAT on the consumer of Example 1

1. Evaluating brand $b2$		
Driver	Threshold	Perception $b2$
Quality	$\langle m, 0.0\rangle$	$\langle l, +0.45\rangle$
Brand $b2$ does not satisfy a threshold. Remove		
2. Evaluating brand $b1$		
Driver	Threshold	Perception $b1$
Quality	$\langle m, 0.0\rangle$	$\langle m, 0.0\rangle$
Price	$\langle m, 0.1\rangle$	$\langle h, -0.45\rangle$
Comfort	$\langle l, -0.2\rangle$	$\langle l, 0.35\rangle$
Brand $b1$ satisfies all thresholds. Select		

Again, perception comparisons use all the information in the fuzzy linguistic 2-tuples (i.e., their linguistic labels and their symbolic translations). As in MAJ and EBA, there is a random shuffling of brands order and a random generation of driver thresholds, respectively.

Algorithm 4 Satisfaction Rule (SAT)

```
procedure SAT(A, W, P)
    int selected ← −1
    int[n] brOrder ← RNDSHUFFLE(n)
    2-tuple[m] drT ← RNDTHRESH(m)
    repeat
        for br ← 1...n do
            if SATISFYTHRESH(P[br], drT) then
                selected ← brand & break
            end if
        end for
        if selected = −1 then
            DECREASETHRESH(drT)
        end if
    until selected ≠ −1
    return selected
end procedure
function SATISFYTHRESH(P[br], thresh)
    bool satisfies ← true
    for dr ← 1...m do
        if P[br][dr] < thresh[dr] then
            satisfies ← false & break
        end if
    end for
    return satisfies
end function
```

Example 5 Consider the market described in Example 1. Assume SAT generates the following random thresholds $T = [\langle m, 0\rangle, \langle m, 0.1\rangle, \langle l, -0.2\rangle]$, and the random order of brands is [$b2$, $b1$, $b4$]. The heuristic proceeds as shown in Table 4.

The selection is therefore brand $b1$.

6 Selection of Marketing Decision-Making Heuristics

It is clear that the four consumer DM heuristics presented in the previous section may result in different brand choices for each consumer buying event. For instance, in the former examples we have seen that all the three brands with awareness can be chosen by some heuristic (we recall that brands without awareness cannot be chosen). These differences allow us to model the different behaviors of the consumers when facing a DM process according to the importance or the involvement they have on such a decision [21, 24]. In this section we describe how these four heuristics can be combined in order to model the global behavior of a general consumer DM procedure. To this end, we first need to characterize the four proposed heuristics according to their degree of consumer involvement in terms of brand and driver exploration:

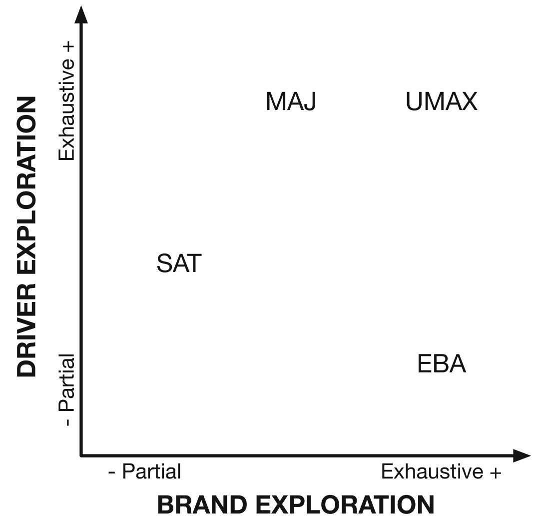

Fig. 4 Distribution of DM heuristics according to their brand and driver exploration degrees (from partial to exhaustive degrees)

- UMAX performs an exhaustive exploration for both brands and drivers, since it compares the utility of every brand and because computing their utility requires to explore every driver.
- MAJ performs an exhaustive exploration for drivers together with a medium degree of brand exploration, since it explores every driver in every comparison between two brands. Although all brands participate in at least one comparison, we would argue that what really drives the DM process is the driver exploration. As an example, see for instance the differences between Example 2 (with UMAX), where brand $b3$ has more than twice the probability of being chosen than brand $b2$, and Example 3 (with MAJ), where brand $b3$ has 0.4 of probability to be chosen in the second comparison with respect to brand $b2$, which has the remaining 0.6 of probability.
- EBA performs an exhaustive exploration of brands with a very low degree of driver exploration. Since the algorithm ends when all but one brands have been discarded, the decision can be achieved with just one driver. Notice also that all brands have to be explored to carry out those discards.
- SAT performs the DM process with a very limited brand exploration and a medium exploration of drivers. Notice that the algorithm ends when a randomly selected brand satisfies all driver thresholds and this can be achieved by the first randomly selected brand. Although every driver participate in the selection

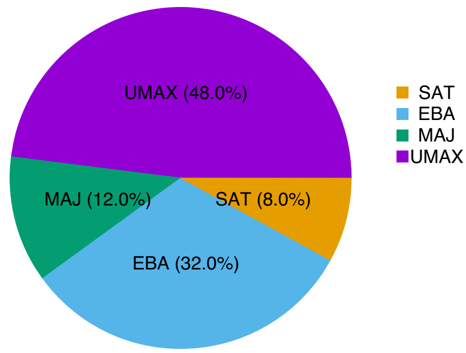

Fig. 5 Example of a probability distribution for choosing DM heuristics, generated with $R = (0.8, 0.6)$

criterion (having a threshold for each driver), we would argue that they partially drive the DM process since those thresholds are randomly generated and may be decreased.

This characterization of the four heuristics is depicted in Fig. 4 according to their degree of brand and driver exploration. Based on it, we define a vector R with two real numbers in [0, 1] where the first component represents the probability of choosing a DM heuristic with exhaustive brand exploration whereas the second one represents the probability of choosing a heuristic with exhaustive driver exploration. This vector allows us to define the probability of choosing each DM heuristic in our selection mechanism as follows:

Definition 6 (Decision heuristic selection with probabilistic roulette) Given a vector $R = (r_1, r_2)$ of two real numbers, with $r_1, r_2 \in [0, 1]$, representing respectively the degree of brand and driver exploration, we define the following probabilities of choosing each DM heuristic:

$$\begin{aligned} &probability(\mathsf{UMAX}) = r_1 \cdot r_2 \\ &probability(\mathsf{MAJ}) = (1 - r_1) \cdot r_2 \\ &probability(\mathsf{EBA}) = r_1 \cdot (1 - r_2) \\ &probability(\mathsf{SAT}) = (1 - r_1) \cdot (1 - r_2) \end{aligned}$$

Notice that the sum of these four probabilities is always 1.

In our work, this vector R is shared by all the consumers in the market and represents the degree of involvement of brand selections in a specific market. Consumers typically do not perform the same heuristic in all their brand selections but adapt them to each purchase. This is called a constructive process in consumer DM [21] where consumers use a variety of strategies depending on the tasks demand and they do not usually have well-defined existing preferences and have limited information [57]. Additionally, the behavior of these consumer agents is not guided by what they are able to compute but by what they happen to see at a given moment [58]. In fact, previous well-known studies from D. Kahneman and A. Tversky already proposed different ways to respond to choices based on the involvement and type of the task, such as having two systems: "system 1" for more intuitive decisions and "system 2" for more rational choices. The proposed framework with the heuristic selection mechanism and the global market vector R takes these these ideas as a base to facilitate choosing the most suitable heuristics depending on the decision context of the market. Notice also that the vector R depends on the market and thus its values are usually provided by the marketer or estimated using historical data of sales. For instance, a consumer would likely apply different strategies to select a car and a bottle of milk, as the former decision implies a significantly higher degree of involvement than the latter.

In Fig. 5, a probabilistic roulette is depicted for a market with $R = (0.8, 0.6)$. Therefore, the probability of choosing each DM heuristic in this market is $p(\mathsf{UMAX}) = 0.48$, $p(\mathsf{MAJ}) = 0.12$, $p(\mathsf{EBA}) = 0.32$, and $p(\mathsf{SAT}) = 0.08$. Notice that this example would correspond to a market where the degree of involvement of the consumer decisions about the brands to choose is relatively high.

7 Empirical Analysis

In this section we present an empirical evaluation of the four consumer DM heuristics in three real markets provided by a Spanish marketing consultancy company. First, we provide a description of the three markets. Second, the behavior of the four heuristics is analyzed separately, including a comparison of our model (based on fuzzy linguistic 2-tuples) w.r.t. a simplified version of it based on ordinal linguistic labels. Finally, we analyze the accuracy of the model when the heuristic selection procedure is in play. All results are compared to real data (i.e., actual sales) in these markets.

7.1 Description of the Markets

In this work, we evaluate the performance of our model in three real markets. Namely, they are *automakers*, *dairy products*, and *luxury automakers*. In what follows, we provide a precise description of each market. For anonymity reasons, we have omitted the name of the brands studied in these markets.

In our model, consumer perceptions, driver weights, and brand awareness are initialized from real data provided by the marketer. This information is usually aggregated into consumer segments, i.e., groups of consumers with a very similar behavior. As done in [18], consumer perceptions are randomly generated following a normal distribution, having a mean equal to the average of the segment the

Table 5 Brand awareness and driver weights for the markets *automakers*, *dairy products*, and *luxury automakers*

	Brand awareness						Driver weights					
	br1	br2	br3	br4	br5	br6	dr1	dr2	dr3	dr4	dr5	dr6
Market automakers	64.0%	88.0%	89.0%	85.0%	61.0%	-	0.290	0.180	0.180	0.150	0.100	0.100
Dairy prod.												
Segment 1	99.3%	98.9%	98.8%	97.7%	–	–	0.607	0.099	0.110	0.115	0.049	0.020
Segment 2	99.5%	99.0%	98.5%	98.1%	–	–	0.586	0.121	0.104	0.096	0.044	0.049
Segment 3	98.8%	97.4%	96.3%	96.0%	–	–	0.584	0.119	0.105	0.097	0.051	0.044
Segment 4	98.9%	97.4%	96.5%	93.7%	–	–	0.557	0.117	0.127	0.104	0.048	0.047
Segment 5	97.2%	95.3%	94.6%	86.6%	–	–	0.525	0.127	0.127	0.103	0.048	0.070
Luxury automak.	50.0%	87.0%	86.0%	83.0%	45.0%	62.0%	0.220	0.120	0.180	0.150	0.150	0.180

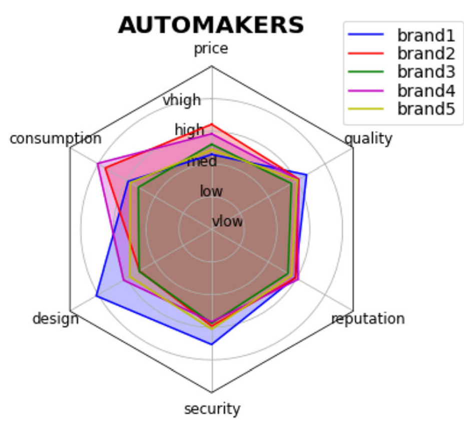

Fig. 6 Radar chart of consumer perceptions of the market *automakers*

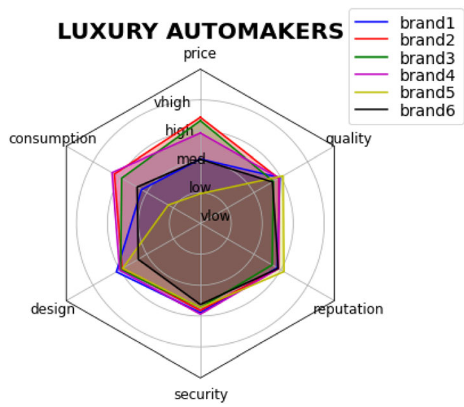

Fig. 7 Radar chart of consumer perceptions of the market *luxury automakers*

consumer belongs to and a low standard deviation. Driver weights are the same for all the consumers belonging to the same segment. The awareness of each brand is usually known for the whole market.

In the *automakers* market, five brands are defined as the brands of interest and consumer perceptions are studied in six drivers: *price, consumption, design, security, reputation,* and *quality*. This market is composed of a single consumer segment. In the *dairy products* market, four brands are studied, five consumer segments are analyzed, and the drivers used are: *price, quality, taste, healthy, natural,* and *reliability*. Finally, in the *luxury automakers* market, six brands are analyzed, a single consumer segment is defined and the drivers are the same as in the *automakers* market.

Brand awareness and driver weights for each market are collected in Table 5. Consumer perceptions for the markets *automakers*, *luxury automakers*, and *dairy product* are represented using radar plots in Figs. 6, 7, and 8, respectively. The values of these radar plots are the averaged perceptions of the consumers about the drivers and brands of the market, where the most negative value is located at the center of the radar plot and most positive value is at the border of the plot for every driver. Since the consumer perceptions are represented as 2-tuples, the values in the plots are defined on the granularity interval $[0, g]$ where the corresponding linguistic terms are placed. These plots thus allow us to easily identify the main characteristics of each market.

7.2 Performance of Fuzzy Linguistic Decision-Making Heuristics

In this subsection, the four consumer fuzzy linguistic DM heuristics are empirically analyzed separately. In particular, we perform four distinct executions of the ABM, each using a distinct heuristic. We carry out this experiment in the three markets considered. Additionally, we compare the performance of each heuristic with respect to a simplified version where the information is expressed using a linguistic symbolic computational model based on ordinal scale, i.e., perceptions are only represented by a

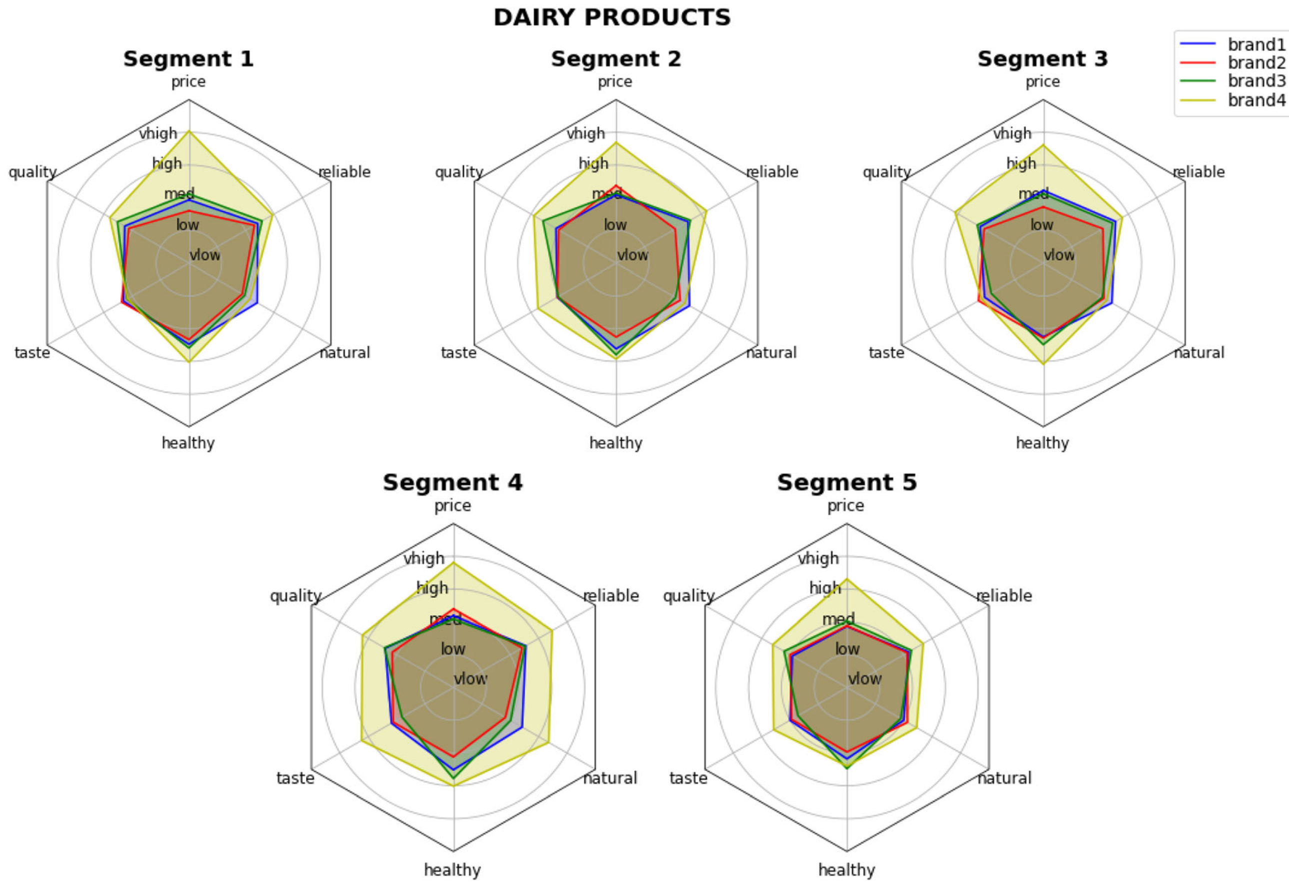

Fig. 8 Radar chart of consumer perceptions of the market *dairy products*

linguistic label from an ordinal linguistic term set and not considering the use of linguistic 2-tuples.

In the market *automakers*, two brands have a market share greater than 30% and a third brand have around a 20%, while the remaining two brands show a lower market share (i.e., below 10%). In the market *dairy products*, there is a clear dominant brand, having a market share above 60%. Finally, in the market *luxury automakers*, the leading brand has almost 40% of the sales, the second one has a market share of almost 30%, and the remaining four brands have less than 30% of the market share altogether.

At each simulation, every consumer performs a unique DM process, i.e., simulating a single brand purchase selection. Those choices of every consumer can be aggregated in order to compute the global performance of each heuristic (comparing them to the actual sales in the market). Since consumer perceptions are randomly generated, two simulations may lead to distinct results. To reduce the possible effects of bias, each execution of our model is composed of 100 Monte Carlo (MC) realizations.

In Fig. 9, the histograms representing the average number of times each brand is selected by each heuristic are depicted as a percentage of the total number of selections in these three markets. Additionally, the real percentage of sales of each brand is also represented. In general, there are small differences between the four consumer DM heuristics in the three markets. No remarkable differences with respect to actual sales are observed either. Nevertheless, some minor differences in some brands can be identified.

The natural question is to know which fuzzy heuristic performs better than the others. To this purpose, we compute several performance indicators. First, the mean absolute error (MAE) of each heuristic with respect to the actual sales is considered. Since MAE weights the same all errors, we also compute the root mean squared error (RMSE), which penalizes variance by giving more weight to the errors with larger absolute values. Finally, we compute the coefficient of determination R^2, which is the proportion of the variance in the dependent variable that is predictable from the independent variables. For two sets of observations X and Y of the same size, these error estimators are computed as:

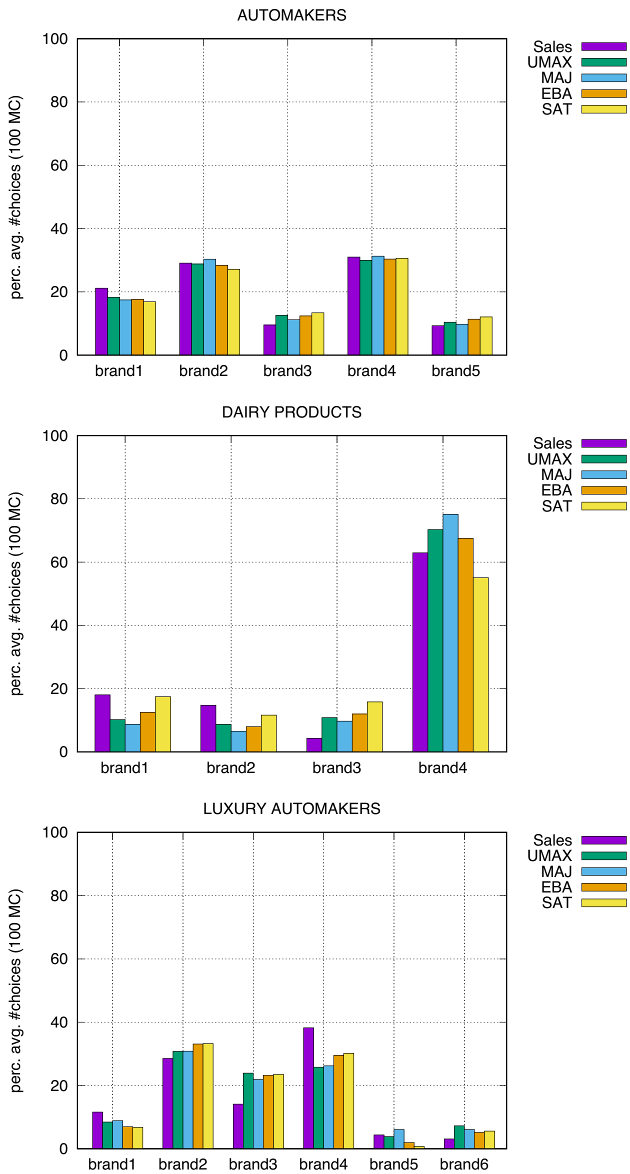

Fig. 9 Results of the fuzzy linguistic DM heuristics in the markets *automakers* (top left), *dairy products* (top right), and *luxury automakers* (bottom). The actual sales of these markets are also depicted for comparison

$$MAE(X,Y) = \frac{\sum_{i=1}^{n} |x_i - y_i|}{n}$$

$$RMSE(X,Y) = \sqrt{\frac{\sum_{i=1}^{n} (x_i - y_i)^2}{n}}$$

$$R^2(X,Y) = 1 - \frac{\sum_{i=1}^{n} (y_i - x_i)^2}{\sum_{i=1}^{n} (y_i - \bar{Y})^2}$$

where x_i (resp. y_i) stands for the i-th element of the set X (resp. Y), n for their size, and $\bar{Y}$ for the mean of Y. In our case, the observations X and Y respectively correspond to the number of choices of each brand for each heuristic in the simulation and to the actual sales in the market.

In our analysis, we use these error estimators (MAE, RMSE, and R^2) to: (i) compare the performance of each heuristic with respect to a simplified version where information is expressed as ordinal linguistic labels, and (ii) compare the performance of the four proposed heuristics. We report the results of this comparison in Table 6.

First, we analyze the differences between the fuzzy linguistic 2-tuple and the ordinal fuzzy linguistic approaches. On the one hand, we observe remarkable differences for the heuristics UMAX and MAJ. This is because they are exhaustive driver exploration heuristics and therefore the loss of information of the ordinal fuzzy linguistic model highly affects the results of the heuristic. In particular, both heuristics exhibit a better performance in the *automakers* and *luxury automakers* markets when consumer perceptions are represented as fuzzy linguistic 2-tuples, but they perform worse in the *dairy products* market. However, notice that this is a market for which exhaustive driver exploration heuristics already perform very poorly, thus these improvements are not significant. On the other hand, there are no remarkable differences in the performance of the heuristics EBA and SAT. This is because of the nature of these heuristics, which only perform a very partial exploration of consumer perceptions and hence their representation has a little impact on the results. This phenomenon can be observed with the three error estimators.

Second, we analyze differences between the four heuristics (in our proposed model that represent consumer perceptions with linguistic 2-tuples). We recall that the MAE, RMSE, and R^2 values are useful to rank the accuracy of each heuristic (the lower, the better for MAE and RMSE; the higher, the better for R^2). According to MAE, in the market *automakers* the two best heuristics are MAJ and UMAX. The same results are obtained with RMSE and R^2. Interestingly, these heuristics perform an exhaustive driver exploration. This is an intuitive result since *automakers* is expected to be an involved market. In the case of *dairy products*, according to MAE the two best heuristics are SAT and EBA, which do not perform an exhaustive driver exploration. Again, these results match the common intuition that a marketer would expect in this market since it is a decision the consumer carries out frequently and hence it may not be very involved. In contrast, according to RMSE and R^2 the best heuristics in this market are EBA and UMAX. This may be due to the large error in the estimated number of sales of *brand*3. This discrepancy is natural since RMSE (and also R^2, to a lesser degree) penalizes variance. Finally, in *luxury automakers*, the two heuristics with the best performance for MAE are MAJ and EBA, whereas for RMSE and R^2 they are EBA and SAT.

Table 6 Comparison of fitting performance of the four heuristics with respect to actual sales, with consumer perceptions represented as 2-tuples (2T) and linguistic labels (LL) in the ordinal fuzzy linguistic model

Error	Market	UMAX		MAJ		EBA		SAT	
		2T	LL	2T	LL	2T	LL	2T	LL
MAE	Automakers	**1.67**	1.99	**1.48**	2.02	1.96	1.96	2.65	2.65
	Dairy products	6.96	6.71	8.80	7.18	**6.15**	**6.15**	**5.76**	**5.76**
	Luxury automakers	5.39	5.61	**4.92**	5.35	**5.25**	**5.25**	5.53	5.53
RMSE	Automakers	**2.01**	2.31	**1.91**	2.54	2.28	2.28	2.99	2.99
	Dairy products	6.99	**6.75**	9.12	7.21	**6.26**	**6.26**	7.15	7.15
	Luxury automakers	6.87	7.16	6.19	6.89	**5.94**	**5.94**	**6.04**	**6.04**
R^2	Automakers	**0.95**	0.94	**0.96**	0.92	0.94	0.94	0.90	0.90
	Dairy products	0.90	**0.91**	0.84	0.90	**0.92**	**0.92**	0.90	0.90
	Luxury automakers	0.71	0.68	0.76	0.71	**0.78**	**0.78**	**0.78**	**0.78**

The two best fitting results for each market, in terms of MAE and RMSE (the lower, the better) and R^2 (the higher, the better), are highlighted in bold

They all are heuristics with a medium degree of involvement. This result is natural since buying a car is usually an involved decision but when it comes to luxury cars this decision may be affected by other less involved factors, such as brand reputation.

Finally, we present a qualitative analysis on the perceptions of the consumers about the brands. In Figs. 6, 8, and 7 we respectively represented the perceptions positioning of the population of consumers in the markets *automakers*, *dairy products*, and *luxury automakers*.

In the *automakers* market, the driver with the highest weight is *price* ($w_{price} = 0.29$), followed by *consumption* and *design*, both with a weight $w_{consumption} = w_{design} = 0.18$. According to these three drivers, the brands with the best perception positioning are *brand*2 and *brand*4 (see Fig. 6). As a consequence, they are the most selected brands in this market and this matches the historical data of actual sales. Notice that *brand*1 has a high perception of its design but its *price* and its *consumption* are worse.

In the *dairy product* market, the *price* has a weight greater than the sum of the weights of the other drivers ($w_{price} = 0.586$). According to Fig. 8, it is clear that *brand*4 has the best perception positioning. Therefore, it is the most selected brand in the ABM and this matches the historical data in this market.

Finally, in the *luxury automakers* market, the three most important drivers are *price* ($w_{price} = 0.22$), *design* ($w_{design} = 0.18$), and *quality* ($w_{quality} = 0.18$). It is interesting to see that *brand*2, *brand*3, and *brand*4 have a similar perception positioning (see Fig. 7). As a consequence, our model estimates a similar number of choices for these three brands. Interestingly, in historical data, *brand*4 is clearly preferred to the other two.

In summary, the analysis of the experimentation developed through the selected performance indicators and graphics allows us to conclude that the proposed fuzzy linguistic DM heuristics allow us to model consumer behavior in the real markets in a natural and accurate way.

7.3 Performance of the Heuristic Selection Procedure

In this subsection we analyze the performance of the model with 2-tuple fuzzy linguistic information when the probabilistic roulette for the heuristic selection mechanism proposed in Sect. 6 is in play. In particular, we carry out several executions of the model differing in the degree of involvement of the market, i.e., with different values of the vector R. Again, we analyze its performance in the markets *automakers*, *dairy products*, and *luxury automakers*, and compare their results to actual sales measuring the MAE, RMSE, and R^2 values of each execution.

In Fig. 10, we represent the results of a sensitivity analysis of the ABM differing in the values of $R = (r_1, r_2)$ for MAE (top), RMSE (center), and R^2 (bottom). In particular, we consider all combinations of these values in the interval [0, 1] with a step size of 0.05. Each cell of these plots is the mean of the 100 MC executions with $R = (r_1, r_2)$, measuring the error between the estimated number of choices of each brand with respect to historical data (i.e., actual sales) with the respective measure.

In the market *automakers*, the ABM performs better for high values of r_2. In particular, the best performance is observed for large r_2 and small r_1, i.e., a probabilistic roulette that mostly chooses MAJ. We recall that MAJ is the case $R = (0, 1)$. In contrast, the performance of the ABM is very poor for small values of both r_1 and r_2, i.e., a probabilistic roulette that mostly chooses SAT that is close to $R = (0, 0)$. These observations can be found for the three error estimators and they are consistent with the results reported in Table 6.

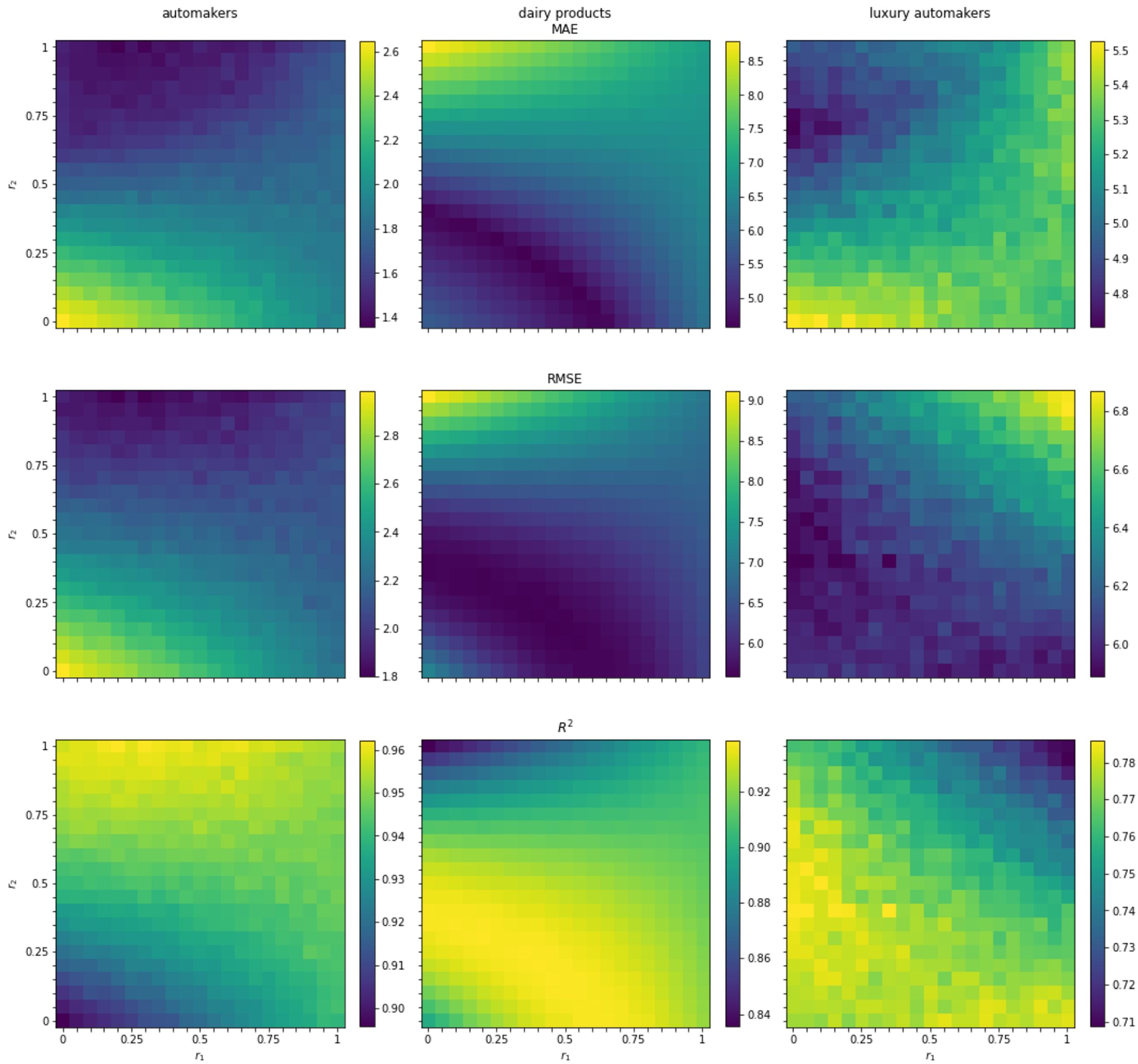

Fig. 10 Sensitivity analysis of the ABM executed with distinct values of $R = (r_1, r_2)$, showing the fitting values of the sales with respect to the fitting measures MAE (top), RMSE (center) and R^2 (bottom), in the markets *automakers* (left), *dairy products* (center), and *luxury automakers* (right)

In the market *dairy products*, the best performance is found in the bottom left corner of the plot, i.e., a probabilistic roulette that mostly chooses a combination of SAT, EBA, and, to a lesser degree, MAJ. This effect is observed for the three error estimators MAE, RMSE, and R^2. This suggests that consumers of this market are driven by non-exhaustive driver exploration heuristics, i.e., they pursue fast (although non-optimal) decisions.

Finally, in the market *luxury automakers*, we find an interesting phenomenon: the error estimator used to measure the performance of the ABM has a great impact on the results. In particular, the less accurate performance according to MAE is a probabilistic roulette that mostly chooses SAT (i.e., the left bottom corner of the plot), whereas according to RMSE and R^2 the less accurate heuristic is UMAX (i.e., the right top corner of the plot). These discrepancies may be due to the large differences in the estimations of sales for *brand*3 and *brand*4, for which UMAX returns the largest error. Nevertheless, the most accurate heuristic (for all the error estimators) seems to be in the left top corner, i.e., a probabilistic roulette that mostly chooses MAJ.

Overall, we observe that, for the three markets, using a combination of heuristics the ABM performs better than when having a single heuristic. Therefore, the probabilistic roulette that integrates the usage of the four proposed consumer DM heuristics is the most promising way to model any market, given the degree of involvement of its consumers. The fact that consumers have a set of heuristics to make their brand selection decisions is not new in the literature [21]. Nevertheless, in this contribution we show that having a set of heuristics and a procedure to select them in a portfolio manner can better fit with reality in marketing scenarios. Our proposed methods can enrich individual-based marketing models where customer interactions and behavioral rules are a cornerstone [5, 59]. Additionally, these heuristic processes can also be used to improve the explanatory modeling process in order to better understand decision-support systems in marketing [60].

8 Conclusions

In this work we have formulated four different fuzzy linguistic DM heuristics for consumer brand selection which handle 2-tuple fuzzy linguistic information that represent consumer perceptions. They differ in the degree of involvement of consumers when the available brands and their features are evaluated, resulting in a diverse set of consumer DM heuristic strategies, ranging from fast decision to optimal choices. This is the natural behavior of a market since consumers unlikely use the same strategy for every purchase event. Instead, they usually have a set of diverse heuristics to face each decision [21–23]. All the four consumer DM heuristics are based on 2-tuple fuzzy linguistic information model, which is a realistic representation of consumer perceptions [18] and does not suffer any loss of information existing in other linguistic symbolic computational models based on ordinal scales [19, 47]. Additionally, we have introduced a mechanism to integrate the proposed consumer heuristics into a single consumer DM procedure. Both the fuzzy DM heuristics and the heuristic selection mechanism can be easily integrated into agent-based simulations for marketing and consumer modeling [5, 6, 18], as done in the current contribution.

Our experimental results with the marketing ABM considering the different DM heuristics and two alternative linguistic symbolic computational models show that the performance of the model matches the expected behavior in several real-world marketing scenarios. This allows us to properly validate the reliability of our proposals for real applications. For instance, in a market of automakers, the most accurate heuristics are the most involved ones whereas in a market of dairy products they are the least involved ones. Moreover, the global performance of the system is improved when the heuristic selection procedure is in play, using the set of diverse heuristics in a portfolio manner. This suggests the importance of incorporating the modeling of the different consumer DM strategies.

In summary, the main obtained results are the following, which are able to answer the introduced research questions and also provide some managerial impacts for marketers and practitioners:

- An algorithmic formulation with four different DM heuristics for consumer purchases was introduced. They are inspired by studies on consumer behaviors. These studies were mainly focus on a psychological perspective and lacked a precise definition of these behaviors. We try to overcome this gap by our proposal (research question #1) and provide a higher support to marketers when understanding consumer behavior.
- The proposed heuristics are designed to handle qualitative information (consumers' perceptions) represented as 2-tuple fuzzy linguistic variables (research question #2). A comparison between two representations of these heuristics was carried out (one based on fuzzy linguistic 2-tuples and another based on linguistic labels). Experimental results show the superiority of our approach with respect to linguistic labeling, being able to extend the scope of research question #2.
- A mechanism to integrate the four heuristics into a single consumer DM procedure was introduced to answer research question #3. Our experimental analysis, enriched with real data about brands and consumers' perceptions, shows that the performance of the model is improved when this mechanism is in play, suggesting the simulated behaviors of consumers are close to the theoretical assumptions of having a portfolio of heuristics [21, 22]. These findings present a clear managerial impact as they can be a cornerstone when building the decision-making processes in decision support systems for marketing [5].

Finally, we propose the following lines of future work based on the limitations of the presented model. The consumer DM heuristics defined in this work are limited to handle fuzzy linguistic 2-tuples. This decision is based on the nature of the information used to initialize the system: real data in the form of uniform and balanced linguistic answers to marketing surveys. Moreover, the proposed ABM is limited to immutable consumers' perceptions that do not vary over time. Therefore, we plan to investigate other fuzzy linguistic representations that may be suitable to other (more general) contexts. Furthermore, we plan to integrate these fuzzy DM heuristics into an ABM with temporal discrete-event simulation. Rather than having a static behavior, consumer opinions and brand preferences will evolve in time, affected by external forces. These forces are the result of marketing campaigns and/or word-of-mouth processes. Thus, more insightful market models can be created by integrating the proposed fuzzy linguistic consumer preferences and DM strategies with temporal marketing events and social network interactions.

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Declarations

Competing interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. This manuscript is the authors' original work and has not been published nor has it been submitted simultaneously elsewhere. All authors have checked the manuscript and have agreed to the submission.

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