Fuzzy Moderation and Moderated-Mediation Analysis

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Abstract In the causal relationship, a mediator variable is a variable that causes mediation in the dependent and the independent variables. If x is a predictor and y is a response variable, then w is a moderator variable that influences the causal relationship of x and y . A moderator variable is a variable that affects the strength of the relationship between a dependent and independent variable. When there are many complicated causal relations, a mediation analysis or a moderation analysis can be performed considering the existence of mediators or moderators. Moreover, when both mediators and moderators exist, a mediation–moderation analysis can be performed. The existence of these variables occurs in many fields, including social science, medical science, and natural science, etc. However, the values of such variables used are often observed as fuzzy numbers rather than as crisp numbers (real numbers). So in many cases, fuzzy analysis is required because observations are observed with ambiguous values, but in the meantime, only models that use crisp numbers rather than fuzzy numbers have been used. This paper proposes fuzzy moderation analysis and fuzzy moderated-mediation analysis as the first attempts of the moderation and moderated-mediation analysis using fuzzy data. The proposed models can also be used for science and engineering, medical data, but it can also be applied to the humanities fields, where a lot of ambiguous data are observed. For example, data from the humanities fields such as marketing, education or psychology, the data are observed based on a human's mind. Nevertheless, they have been analyzed using crisp

data so far. In this paper, we define several fuzzy moderation models and fuzzy mediation–moderation models considering various situations based on fuzzy least squares estimation (FLSE). In addition, the validity of the proposed model is shown in some examples; it compares the results with existing analysis using crisp data.

Keywords Moderator variable · Moderation analysis · Moderated-mediation analysis · Fuzzy moderation analysis · Fuzzy moderated-mediation analysis · Fuzzy number · Fuzzy data · Fuzzy least squares estimation (FLSE)

1 Introduction

In analyzing the statistical models for variables with causal relationship, we generally use regression models where independent variables affect dependent variables. However, while some models are only described as causative relationships between independent and dependent variables, sometimes the regression model may be influenced by another third variable. In other words, sometimes the independent variable or the dependent variable is influenced by the third variable which is called a *mediator*. In other cases, sometimes the third variable affects the model itself. In this case, the third variable is called a *moderator* [1].

A moderator variable is a variable that affects the strength of the relationship between a dependent and independent variable. Most of the moderator variables measure causal relationship using regression coefficient. The moderator variable is found to be significant, can cause an amplifying or weakening effect between the variables X and Y [2]. The effect of X on Y can be said to be

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moderated if its size or direction is dependent on the moderator. It tells us about the conditions that facilitate, enhance, or inhibit the effect, or for whom or in what circumstance the effect is large vs. small, present vs. absent, positive vs. negative vs. zero. A mediator is a variable that lies between the cause and effect in a causal chain. In other words, mediator variables are the mechanisms through which change in one variable causes change in a subsequent variable [3]. A mediator and a moderator were first introduced by Baron and Kenny in 1986 [2], and have been studied by many authors [2, 4–13] in humanities fields. In a mediation analysis, as in any analysis, we are losing some information when we reduce complex responses that no doubt differ from or situation to situation down to a single number or estimate. Combined, these results suggest that the mechanism by which an independent variable (X) may influence a dependent variable (Y) through mediator (M) that need to be managed depends on a moderator (W). This is *moderated-mediation*. The process by which X affects Y through M is conditional on W . This is also called a *conditional process analysis*, which was renamed by Hayes in 2013 [12]. The above three models for the simple cases are shown in Fig. 1.

In many cases, fuzzy analysis is required from above models because observations are observed with ambiguous values, but in the meantime, only models that use crisp numbers rather than fuzzy numbers have been used so far. In the real world, we meet ambiguous or vague data frequently, such as ‘a few’, ‘about 5’, ‘rather greater than 10’, etc. Moreover, the linguistically expressed outcomes such as ‘light’, ‘moderate’, and ‘heavy’ for describing the degree of intensity of a certain event occur frequently in everyday life. Especially, the data from the humanities

fields such as marketing, education or psychology, the data are observed based on a human’s mind. Nevertheless, they have been analyzed using crisp data so far. For example, consider the situation that we measure ‘how much a person feels happy’ with numbers. It is true a crisp number cannot fully express the happiness of a person’s mind. It is clear that it is much more reasonable to express the degree of happiness using a soft number such as a fuzzy number, which was first introduced by Zadeh [14]. In terms of variables in data analysis, above situation can easily arise if there is a chain of relations of some variables when a precedent variable affects a mediator variable, which then affects a response variable. Or sometimes a variable, which is a moderator affects the whole model, even sometimes combined moderated mediation. Using fuzzy data, a fuzzy mediation analysis including inferences using fuzzy data was first introduced mathematically by Yoon in 2020 [15]. This paper proposes fuzzy moderation analysis and fuzzy moderated-mediation analysis as the first attempts of the moderation and moderated-mediation analysis using fuzzy data. The proposed models can also be used for humanities fields, where a lot of ambiguous data are observed as mentioned above, but it can be applied to science and engineering, medical fields. Especially, in the medical field, a lot of data are observed ambiguously. For example, let us consider the situation when a doctor asks patients to measure “how much they feel pain”. It is clear that it cannot be measured using crisp number,

In this paper, a fuzzy moderation analysis and a fuzzy moderated-mediation analysis are proposed using the triangular fuzzy numbers and L_2 -estimation method has been applied for mediation analysis based on author’s previous study [16–20].

Some basic concepts from [14] are introduced. A fuzzy subset of $\mathbb{R}^1$ is a map, so-called the membership function, from $\mathbb{R}^1$ into $[0, 1]$. Thus fuzzy subset $\tilde{A}$ is identified with its membership function, $\mu_{\tilde{A}}(x)$. For any $\alpha \in [0, 1]$, the crisp set $\tilde{A}_\alpha = \{x \in \mathbb{R}^1 : \mu_{\tilde{A}}(x) \geq \alpha\}$ is called the α -cut of $\tilde{A}$. A fuzzy number A is a normal and convex subset of the real line $\mathbb{R}^1$ with bounded support. The set of all fuzzy numbers will be denoted by $F_c(\mathbb{R}^1)$. In fact, there are no general rules to obtain the membership function of a fuzzy observation. As a special case, we often use the following parametric class of fuzzy numbers, the so-called *LR*-fuzzy numbers:

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{l}\right) & \text{if } x \leq m, \\ R\left(\frac{x-m}{r}\right) & \text{if } x > m, \end{cases}$$

where $L, R : \mathbb{R} \rightarrow [0, 1]$ are fixed left-continuous and non-increasing functions with $R(0) = L(0) = 1$ and $R(1) = L(1) = 0$. L and R are called left and right shape functions of X , m the mode of A and $l, r > 0$ are left and

Fig. 1 Simple mediation, moderation and moderated-mediation model

right spread of X , respectively. We abbreviate an LR -fuzzy number by $\tilde{A} = (m, l, r)_{LR}$. The spreads l and r represent the fuzziness of the number and could be symmetric or non-symmetric. If $l = r = 0$, there is no fuzziness of the number, and it is a crisp number. The α -cuts of the fuzzy numbers are given by the intervals.

$$\tilde{A}_\alpha = [m - L^{-1}(\alpha)l, m + L^{-1}(\alpha)r], \quad \alpha \in (0, 1).$$

We denote the set of all LR -fuzzy numbers as $F_{LR}(\mathbb{R}^1)$. In particular, if $L(x) = R(x) = [1 - x]^+$ in $\tilde{A} = (m, l, r)_{LR}$, then A is called a triangular fuzzy number and denoted by $\tilde{A} = (m, l, r)_{LR} = (m - l, m, m + r)$. Thus we can model the vague data mainly by fuzzy numbers. Based on the extension principle in [14], following two operations for fuzzy triangular numbers can be defined:

$$\tilde{X} \oplus \tilde{Y} = (l_x + l_y, x + y, r_x + r_y),$$

$$k\tilde{X} = \begin{cases} (kl_x, kx, kr_x), & k \geq 0, \\ (kr_x, kx, kl_x), & k < 0, \end{cases}$$

where $\tilde{X} = (l_x, x, r_x)$, $\tilde{Y} = (l_y, y, r_y) \in F_T$ for $k \in \mathbb{R}$. This paper is organized as follows: Sect. 2 provides the proposed fuzzy moderation and moderated-mediation models and estimation method, and Sect. 3 provides data analysis using various models proposed in Sect. 2 with four datasets. And we conclude the results in Sect. 4.

2 Fuzzy Moderation, Moderated-Mediation Analysis

In this section, employing some basic concepts from [12], several fuzzy moderation/moderated-mediation models and estimation methods are proposed.

2.1 Fuzzy Simple Moderation Analysis

In the causal relationship, if $\tilde{X}$ is a fuzzy predictor and $\tilde{Y}$ is a fuzzy response variable, then $\tilde{W}$ is a fuzzy moderator variable that influences the causal relationship of $\tilde{X}$ and $\tilde{Y}$, then the coefficient of $\tilde{X}$ can be assumed to be affected by $\tilde{W}$ which means it can be expressed by a function of $\tilde{W}$. Let us consider $\tilde{X}$'s effect can be expressed by fuzzy data as a function of $\tilde{W}$, as in

$$\hat{\tilde{Y}} = b_0 \oplus f(\tilde{W}) \otimes \tilde{X} \oplus b_2 \tilde{W}. \quad (1)$$

If $f(\tilde{W})$ is given by a linear function of $\tilde{W}$, then $f(\tilde{W}) = b_1 \oplus b_3 \tilde{W}$.

$$\hat{\tilde{Y}} = b_0 \oplus (b_1 \oplus b_3 \tilde{W}) \otimes \tilde{X} \oplus b_2 \tilde{W} \quad (2)$$

This can be rewritten as

$$\hat{\tilde{Y}} = b_0 \oplus b_1 \tilde{X} \oplus b_2 \tilde{W} \oplus b_3 \tilde{X} \otimes \tilde{W}. \quad (3)$$

This model allows $\tilde{X}$'s effect on $\tilde{Y}$ to depend linearly on $\tilde{W}$. Here, $\delta_{\tilde{X} \rightarrow \tilde{Y}}$ is the "conditional effect" of $\tilde{X}$ on $\tilde{Y}$ defined by the function $\delta_{\tilde{X} \rightarrow \tilde{Y}} = b_1 \oplus b_3 \tilde{W}$. Also, $\delta_{\tilde{W} \rightarrow \tilde{Y}}$ is the conditional effect of $\tilde{W}$ on $\tilde{Y}$ defined by the function $\delta_{\tilde{W} \rightarrow \tilde{Y}} = b_2 \oplus b_3 \tilde{X}$.

2.2 Moderation of Only the Direct Effect

The simplest moderated-mediation analysis model in Fig. 2 is a simple combination of simple mediation with moderation of the *fuzzy conditional direct effect (FCDE)* of $\tilde{X}$ on $\tilde{Y}$. Assuming linear moderation of the direct effect of $\tilde{X}$ by $\tilde{W}$, above model in Fig. 3 is represented by:

$$\tilde{M} = a_0 \oplus a\tilde{X},$$

$$\hat{\tilde{Y}} = b_0 \oplus c_1 \tilde{X} \oplus c_1 \tilde{W} \oplus c_1 \tilde{X} \otimes \tilde{W} \oplus b\tilde{M}. \quad (4)$$

Or, equivalently,

$$\hat{\tilde{Y}} = b_0 \oplus \delta_{\tilde{X} \rightarrow \tilde{Y}} \tilde{X} \oplus c_2 \tilde{W} \oplus b\tilde{M}, \quad (5)$$

where $\delta_{\tilde{X} \rightarrow \tilde{Y}}$ is the *conditional direct effect* of $\tilde{X}$ on $\tilde{Y}$, defined as $\delta_{\tilde{X} \rightarrow \tilde{Y}} = c_1 \oplus c_1 \tilde{W}$. Here c_i means a coefficient that constitutes the conditional direct effect.

Fig. 2 Moderated-mediation analysis for conditional direct effect

Fig. 3 Effects of $\tilde{X}$ and $\tilde{W}$ on $\tilde{Y}$ in simple moderation model

2.3 Moderation of Only the Indirect Effect

The mechanism linking $\tilde{X}$ to $\tilde{Y}$ can be said to be conditional if the indirect effect of $\tilde{X}$ on $\tilde{Y}$ through $\tilde{M}$ is contingent on a moderator $\tilde{W}$.

Above model in Fig. 4 can be expressed by:

$$\begin{aligned}\hat{\tilde{M}} &= a_0 \oplus a\tilde{X}, \\ \hat{\tilde{Y}} &= b_0 \oplus c\tilde{X} \oplus b_1\tilde{M} \oplus b_2\tilde{W} \oplus b_3\tilde{M} \otimes \tilde{W}.\end{aligned}\quad (6)$$

Or, equivalently,

$$\hat{\tilde{Y}} = b_0 \oplus c\tilde{X} \oplus \delta_{M \rightarrow Y}\tilde{M} \oplus b_2, \quad (7)$$

Fig. 4 Moderated-mediation analysis for conditional indirect effect

where $\delta_{\tilde{M} \rightarrow \tilde{Y}} = b_1 \oplus b_3\tilde{W}$. The indirect effect of $\tilde{X}$ on $\tilde{Y}$ via $\tilde{M}$ is $a\delta_{\tilde{M} \rightarrow \tilde{Y}}$ but as $\delta_{\tilde{M} \rightarrow \tilde{Y}}$ is a conditional effect of $\tilde{M}$, then $a\delta_{\tilde{M} \rightarrow \tilde{Y}} = a(b_1 \oplus b_3\tilde{W}) = ab_1 \oplus ab_3\tilde{W}$ is the *fuzzy conditional indirect effect (FCIDE)* of $\tilde{X}$ on $\tilde{Y}$ via $\tilde{M}$.

2.4 Moderation of the Direct and the Indirect Effects

Let us consider a model with the direct and indirect effects of X moderated, by two moderators. Above model in Fig. 5 can be expressed by:

$$\begin{aligned}\hat{\tilde{M}} &= a_0 \oplus a_1\tilde{X} \oplus a_2\tilde{W}_1 \oplus a_3\tilde{X} \otimes \tilde{W}_1 \\ &= a_0 \oplus \delta_{X \rightarrow M}\tilde{X} \oplus a_2\tilde{W}_1, \\ \hat{\tilde{Y}} &= b_0 \oplus c_1\tilde{X} \oplus c_2\tilde{W}_2 \oplus c_3\tilde{X} \otimes \tilde{W}_2 \oplus b_1\tilde{M} \oplus b_2\tilde{M} \otimes \tilde{W}_2 \\ &= b_0 \oplus c_1\tilde{X} \oplus c_2\tilde{W}_2 \oplus c_3\tilde{X} \otimes \tilde{W}_2 \oplus \delta_{X \rightarrow M}\tilde{M}\end{aligned}\quad (9)$$

where $\delta_{\tilde{X} \rightarrow \tilde{M}}$ is the effect of $\tilde{X}$ on $\tilde{M}$, defined as $\delta_{\tilde{X} \rightarrow \tilde{M}} = a_1 \oplus a_3\tilde{W}_1$, and $\delta_{\tilde{M} \rightarrow \tilde{Y}} = b_1 \oplus b_3\tilde{W}_2$ is the effect of $\tilde{M}$ on $\tilde{Y}$. The indirect effect of $\tilde{X}$ on $\tilde{Y}$ through $\tilde{M}$ is the product of two conditional effects (conditional indirect effect) defined as:

$$\begin{aligned}\partial_{\tilde{X} \rightarrow \tilde{M}} \partial_{\tilde{M} \rightarrow \tilde{Y}} &= (a_1 \oplus a_3\tilde{W}_1) \otimes (b_1 \oplus b_3\tilde{W}_2) \\ &= a_1b_1 \oplus a_1b_1\tilde{W}_2 \oplus a_3b_1\tilde{W}_1 \oplus a_3b_1\tilde{W}_1 \otimes \tilde{W}_2.\end{aligned}\quad (10)$$

And the direct effect of $\tilde{X}$ is moderated only by $\tilde{W}_2$:

Fig. 5 Moderated-mediation analysis for conditional direct and indirect effects

$$\delta_{X \rightarrow Y} = \dot{c}_1 \oplus \dot{c}_3 \tilde{W}_1. \quad (11)$$

2.5 Moderation of the Indirect Effect with Multiple Mediators

Figure 6 is the model with multiple mediators that includes moderation of effects to $\tilde{M}_1$ and $\tilde{M}_2$ by a common moderator $\tilde{W}$.

Above model in Fig. 6 can be expressed by:

$$\begin{aligned} \hat{\tilde{M}}_1 &= a_{10} \oplus a_{11} \tilde{X} \oplus a_{12} \tilde{W} \oplus a_{13} \tilde{X} \otimes \tilde{W} \\ &= a_{01} \oplus \delta_{\tilde{X} \rightarrow \tilde{M}_1} \tilde{X} \oplus a_{12} \tilde{W}, \end{aligned} \quad (12)$$

$$\begin{aligned} \hat{\tilde{M}}_2 &= a_{20} \oplus a_{21} \tilde{X} \oplus a_{22} \tilde{W} \oplus a_{23} \tilde{X} \otimes \tilde{W} \\ &= a_{20} \oplus \delta_{\tilde{X} \rightarrow \tilde{M}_2} \tilde{X} \oplus a_{22} \tilde{W} \end{aligned} \quad (13)$$

where $\delta_{\tilde{X} \rightarrow \tilde{M}_1}$ and $\delta_{\tilde{X} \rightarrow \tilde{M}_2}$ are the effect of $\tilde{X}$ on $\tilde{M}_1$ and $\tilde{M}_2$, respectively, defined as $\delta_{\tilde{X} \rightarrow \tilde{M}_1} = a_{11} \oplus a_{13} \tilde{W}$ and $\delta_{\tilde{X} \rightarrow \tilde{M}_2} = a_{21} \oplus a_{23} \tilde{W}$. And,

$$\hat{Y} = b_0 \oplus \dot{c} \tilde{X} \oplus b_1 \tilde{M}_1 \oplus b_2 \tilde{M}_2. \quad (14)$$

Here, the conditional indirect effect of $\tilde{X}$ on $\tilde{Y}$ through $\tilde{M}_1$ and $\tilde{M}_2$ depends on $\tilde{W}$ are expressed by $\delta_{\tilde{X} \rightarrow \tilde{M}_1} b_1 = a_{11} b_1 \oplus a_{13} b_1 \tilde{W}$, and $\delta_{\tilde{X} \rightarrow \tilde{M}_2} b_2 = a_{21} b_2 \oplus a_{23} b_2 \tilde{W}$.

Figure 7 is another model with multiple mediators that includes moderation of effects to and from $\tilde{M}_1$ by a common moderator $\tilde{W}$.

Fig. 6 Moderated-mediation analysis for conditional indirect and with multiple mediators

Fig. 7 Moderated-mediation analysis for conditional indirect and with multiple mediators

Above model in Fig. 7 can be expressed by:

$$\begin{aligned} \hat{\tilde{M}}_1 &= a_{01} \oplus a_{11} \tilde{X} \oplus a_{21} \tilde{W} \oplus a_{31} \tilde{X} \otimes \tilde{W} \\ &= a_{01} \oplus \delta_{\tilde{X} \rightarrow \tilde{M}_1} \tilde{X} \oplus a_{21} \tilde{W}, \end{aligned} \quad (15)$$

$$\hat{\tilde{M}}_2 = a_{02} \oplus a_{12} \tilde{X}, \quad (16)$$

$$\begin{aligned} \hat{Y} &= b_0 \oplus \dot{c} \tilde{X} \oplus b_{11} \tilde{M}_1 \oplus b_{12} \tilde{M}_2 \oplus b_{21} \tilde{W} \oplus b_{31} \tilde{M}_1 \otimes \tilde{W} \\ &= b_0 \oplus \dot{c} \tilde{X} \oplus \delta_{\tilde{M}_1 \rightarrow \tilde{Y}} \tilde{M}_1 \oplus b_{12} \tilde{M}_2 \oplus b_{21} \tilde{W}, \end{aligned} \quad (17)$$

where $\delta_{\tilde{X} \rightarrow \tilde{M}_1}$ is the effect of $\tilde{X}$ on $\tilde{M}_1$, defined as $\delta_{\tilde{X} \rightarrow \tilde{M}_1} = a_{11} \oplus a_{31} \tilde{W}$, and $\delta_{\tilde{M}_1 \rightarrow \tilde{Y}} = b_{11} \oplus b_{31} \tilde{W}$ is the effect of $\tilde{M}_1$ on $\tilde{Y}$. The product of these conditional effects yields the conditional specific indirect effect of $\tilde{X}$ on $\tilde{Y}$ through $\tilde{M}_1$.

$$\begin{aligned} \delta_{\tilde{X} \rightarrow \tilde{M}_1} \delta_{\tilde{M}_1 \rightarrow \tilde{Y}} &= (a_{11} \oplus a_{31} \tilde{W}) \otimes (b_{11} \oplus b_{31} \tilde{W}) \\ &= a_{11} b_{11} \oplus (a_{11} b_{31} \oplus a_{31} b_{11}) \tilde{W} \oplus a_{31} b_{31} \tilde{W} \otimes \tilde{W} \end{aligned} \quad (18)$$

which is a curvilinear function of $\tilde{W}$. There is a second specific indirect effect of $\tilde{X}$ in this model through $\tilde{M}_2$, but it is unconditional, because none of its constituent paths is

Fig. 8 Some variety of moderated-mediation analysis models

specified as moderated. More complicated moderated-mediation analysis models are provided in Fig. 8.

2.6 Estimation for Fuzzy Moderation and Conditional Process Analysis

For the least squares estimation, a suitable metric is required on the spaces of fuzzy sets. There are several metrics that can be defined on the fuzzy number set. The distance between two fuzzy numbers is commonly based on the distance between their α -cuts. A useful type of

metric can be defined via support functions. The support function of any compact convex set $A \in \mathbb{R}^d$ is defined as a function $s_A : S^{d-1} \rightarrow \mathbb{R}$ given by for all $r \in S^{d-1}$.

$$s_A(r) = \sup_{a \in A} \langle r, a \rangle,$$

where S^{d-1} is the $(d-1)$ -dimensional unit sphere in $\mathbb{R}^d$ and $\langle \cdot, \cdot \rangle$ denotes the scalar product on $\mathbb{R}^d$. Note that for convex and compact $A \in \mathbb{R}^d$ the support function s_A is uniquely determined. A metric on a fuzzy number set is defined by the L_2 -metric on the space of Lebesgue integrable.

$$\delta_2(A, B) = \left[d \int_0^1 \int_{s_0} s_A(\alpha, r) - s_B(\alpha, r) \|^2 \mu(dr) d\alpha \right]^{1/2}.$$

Based on this, an L_2 -metric for fuzzy numbers can be defined by

$$d^2(\tilde{X}, \tilde{Y}) = D_2^2(\text{Supp } \tilde{X}, \text{Supp } \tilde{Y}) + [m_l(\tilde{X}) - m_l(\tilde{Y})]^2 + [m_r(\tilde{X}) - m_r(\tilde{Y})]^2 \quad (18)$$

A fuzzy regression model which was introduced in the author's previous studies [16, 17] is proposed as follows:

$$\tilde{Y}_i = \beta_0 \oplus \beta_1 \tilde{X}_{i1} \oplus \beta_2 \tilde{X}_{i2} \oplus \cdots \oplus \beta_p \tilde{X}_{ip} \oplus \tilde{E}_i \quad (19)$$

They are represented by $\tilde{X}_{ij} = (l_{xij}, x_{ij}, r_{xij})$ and $\tilde{Y}_i = (l_{y_i}, y_i, r_{y_i})$ for $i = 1, \dots, n, j = 1, \dots, p$. It is assumed that $\tilde{E}_i$ are the fuzzy random errors for expressing fuzziness. Note that we can encompass all cases by

$$l_{xij} = \begin{cases} x_{ij} - \xi_{lij}, & \text{if } \beta_j \geq 0, \\ x_{ij} + \xi_{lij}, & \text{if } \beta_j < 0, \end{cases}$$

$$r_{xij} = \begin{cases} x_{ij} + \xi_{rij}, & \text{if } \beta_j \geq 0, \\ x_{ij} - \xi_{rij}, & \text{if } \beta_j < 0, \end{cases}$$

where ξ_{lij} and ξ_{rij} are the left and right spreads of $\tilde{X}_{ij}$, respectively. Now the estimators are obtained if we minimize following objective function:

$$Q(\beta_{k0}, \beta_{k1}, \dots, \beta_{kp}) = \sum_{i=1}^n d^2(\tilde{Y}_i, \sum_{j=0}^p \beta_{kj} \tilde{X}_{ij}), \quad (20)$$

for $k = 1, 2, \dots, h$, where h is the number of the regression model in this fuzzy mediation analysis. And the objective function (20) can be obtained based on the L_2 -metric (18).

To minimize (20), we obtain the normal equation applying

$$\frac{\partial Q}{\partial \beta_{kl}} = 0.$$

And, for each $k = 1, 2, \dots, h$ the normal equation, which has $\hat{\beta}_{kl}$ as solutions, can be obtained as follows:

$$\sum_{j=0}^p \hat{\beta}_{KJ} \sum_{i=1}^n (l_{xil} l_{xij} + x_{il} x_{ij} + r_{xil} r_{xij}) = \sum_{i=1}^n (l_{xil} l_{y_i} + x_{il} y_i + r_{xil} r_{y_i}). \quad (21)$$

To find the solution vector, we define a *triangular fuzzy matrix (t.f.m.)* which is defined by $\tilde{X} = [\tilde{X}_{ij}]_{n \times (p+1)}$, where $\tilde{X}_{ij}$ is a triangular fuzzy number for $i = 1, \dots, n, j = 0, \dots, p$. and we define a triangular fuzzy vector $\tilde{Y} = [\tilde{Y}_i]^t$.

To minimize the above objective function, fuzzy operations fuzzy numbers and estimators which were defined in our previous study [16, 17] have been applied.

$$\tilde{X} \diamond \tilde{Y} = l_{xy} + xy + r_{xy},$$

$$\tilde{X} \otimes \tilde{Y} = (l_{xy}, xy, r_{xy}),$$

$$\tilde{X} \oplus \tilde{Y} = (l_{x \oplus y}, xy, r_{x \oplus y}),$$

$$\text{where } l_{x \oplus y} = \inf\{l_{xy}, l_{xy}, r_{xy}, r_{xy}\},$$

$$r_{x \oplus y} = \sup\{l_{xy}, l_{xy}, r_{xy}, r_{xy}\}.$$

For given two $n \times n$ t.f.m.s, $\tilde{X} = [\tilde{X}_{ij}]$, $\tilde{A} = [\tilde{A}_{ij}]$, and a crisp matrix $A = [a_{ij}]$, the operations are defined as follows:

$$\tilde{X} \diamond \tilde{A} = [\sum_{k=1}^n \tilde{X}_{ik} \diamond \tilde{A}_{kj}], \dots, \tilde{X} \otimes \tilde{A} = [\oplus_{k=1}^n \tilde{X}_{ik} \otimes \tilde{A}_{kj}],$$

$$A \tilde{X} = [\oplus_{k=1}^n a_{ik} \tilde{X}_{kj}], \dots, k \tilde{X} = [\oplus_{k=1}^n a_{ik} \tilde{X}_{ij}].$$

$$\tilde{X} A = [a_{ij} \tilde{X}], \dots, \tilde{X} \diamond \tilde{X} = [\tilde{X} \diamond \tilde{X}_{ij}],$$

$$\tilde{X} \otimes \tilde{X} = [\tilde{X} \otimes X_{ij}].$$

Using the above operations and algebraic properties, the solutions of normal equation fuzzy estimators are derived for each $k = 1, 2, \dots, h$ by

$$\hat{\beta}_k = (\tilde{X} \diamond \tilde{X})^{-1} \tilde{X} \diamond \tilde{Y}, \quad (22)$$

where

$$\tilde{X} \diamond \tilde{X} = \left[\sum_{l=1}^n (l_{xil} l_{xij} + x_{il} x_{ij} + r_{xil} r_{xij}) \right]_{(p+1) \times (p+1)}$$

and $\tilde{X} \diamond \tilde{Y} = \left[\sum_{i=1}^n (l_{xil} l_{y_i} + x_{il} y_i + r_{xil} r_{y_i}) \right]_{(p+1) \times 1}$, for $l = 0, 1, \dots, p$. Note that (22) exists if $\det(\tilde{X} \diamond \tilde{X}) \neq 0$.

3 Data Analysis

3.1 Fuzzy Moderation Analysis for Lawyer Data with Dichotomous Predictor

Lawyer data are collected by Garcia et al. [10] and has been used in [10, 12, 15]. Participants (all female) read a narrative about a female attorney (Catherine) who lost a promotion at her firm to a much less qualified male through unequivocally discriminatory actions of the senior partners. Participants assigned to the 'protest' condition were then told she protested the decision by presenting an argument to the partners about how unfair the decision was. Participants assigned to the 'no protest' condition were told that although she was disappointed, she accepted the decision and continued working at the firm (X : 1 or 0). After reading the narrative, the participants evaluated how appropriate they perceived her response to be, and also evaluated the characteristics of the attorney (Y). Prior to the study, the participants filled out the Modern Sexism Scale (W). According to Garcia et al. [10], many questionnaires were answered in a linguistic way in the dataset. For example, the participants evaluated the likeability of the target by completing six items. These items included: 'I like Catherine', 'I admire Catherine', 'Catherine is the type of person I would like to be friends with', 'Catherine has many positive traits', 'I would like to be a coworker of Catherine', and 'I feel proud of Catherine'. Garcia et al. [10] averaged these items together to create a liking measure, with high scores indicating greater liking. But in this case, because the answered includes linguistic information, a soft measure is much more reasonable to be used to represent the participants' opinion. So, fuzzy data are very useful in this case instead of using crisp data. In this data analysis, the data are fuzzified with spreads 0.3 to include the vague and ambiguous information of the collected information. Above variables are modeled in Fig. 9.

The ordinary moderation analysis (OMA) is used in [10, 12], and it is estimated as follows:

$$\hat{Y} = 7.706 - 4.129X - 0.472W + 0.901XW. \quad (23)$$

Here, the degree of sexism scale that the participants think (M) is somewhat vague to express with a crisp real number. Also, the evaluation degree for the lawyer has the

Fig. 9 Moderation model for lawyer data

same vagueness. Therefore, the data have been fuzzified using 0.5 spread-triangular fuzzy numbers. The result of the fuzzy moderation model is as follows:

$$\hat{Y} = 6.026 \oplus (-3.127)\tilde{X} \oplus (-0.141)\tilde{W} \oplus 0.699\tilde{X} \otimes \tilde{W}. \quad (24)$$

Here, the coefficients of W and X mean conditional effects. The coefficient for W is the conditional effect for $X = 0$, and the coefficient for X is the conditional effect for $W = 0$. From the results, the coefficients for X were -4.129 and -3.127 , respectively, at *OMA* and *FMA*, which can be interpreted as being less favorable to those who were told that Catherine was not protesting among those who thought there was no sex discrimination ($W = 0$). Here, in the case of *FMA*, the difference in favorability was smaller than in the case of *OMA*. Similarly, this interpretation applies to the coefficient of W . For *OMA* and *FMA*, the coefficients of W were statistically significant, which can confirm that people who are more aware of gender discrimination in people who are told that Catherine did not protest against sexism ($X = 0$) are less likely to like Catherine. The regression coefficient for XW is the estimate of the difference between Y values in two cases ($X = 1$ and $X = 0$) where X differs by one unit for every W increment. The coefficient was significant, meaning that the effects of favorability on whether or not Catherine protested depended on the extent to which she believed gender discrimination was widespread in society. More accurately, the belief in sexual discrimination (W) increases by one unit, meaning that the favorability of those who heard that Catherine ($X = 1$) did not protest ($X = 0$) increases by 0.901 (*OMA*) or 0.699 (*FMA*) units. In other words, the coefficient quantifies the differences between the differences. From the results, we can see that the degree of increased favorability of *FMA* case is less sensitive than that of *OMA*.

The conditional effects of X and W on Y , which are defined by $\delta_{X \rightarrow Y} = a_1 + a_3W$ and $\delta_{W \rightarrow Y} = a_2 + a_3X$, are shown in Table 1. *OMA* [10, 12] and the proposed *FMA* (fuzzy moderation analysis) are used compare the results. The last two rows of Table 1 show the examples of

Table 1 Conditional effects of X and M on Y

Method	Conditional effects	
	X on Y	W on Y
<i>OMA</i>	$-4.129 + 0.901W$	$-0.472 + 0.901X$
<i>FMA</i>	$-3.127 \oplus 0.699\tilde{W}$	$-0.141 \oplus 0.699\tilde{X}$
<i>OMA</i> _{mean}	0.5200	- 0.1077
<i>FMA</i> _{mean}	0.4978	- 0.2168

conditional effects when the mean values of X , W and Y are applied. The result shows that it can be concluded that when the *FMA* was used the conditional effect of X on Y is less than that of *OMA*. Also, the conditional effect of W on Y affects more negatively in *FMA* compared with that of *OMA*. If we categorize X into two groups (1: Protest/0: No protest), then the effect of W on Y can be estimated from following model:

$$\hat{Y}_{X_0} = 7.706 - 0.472W \text{ (OMA: No PROTEST, } X = 0)$$

$$\hat{Y}_{X_1} = 3.577 + 0.4429W \text{ (OMA: PROTEST, } X = 1)$$

$$\hat{Y}_{X_0} = 6.026 \oplus (-0.141)\tilde{W} \text{ (FMA: No PROTEST, } X = 0)$$

$$\hat{Y}_{X_1} = 2.889 \oplus 0.558\tilde{W} \text{ (FMA: PROTEST, } X = 1) \quad (25).$$

The above effects of two categories are visualized as follows.

From Fig. 10, in both *OMA* (a) and *FMA* (b) cases, the two lines have positive slopes for the case of “Protest” ($X = 1$). Also, the two other lines have negative slopes for

(a) Effect of W on Y in *OMA*

(b) Effect of W on Y in *FMA*

Fig. 10 The comparison of the effect of W on Y in *OMA* and *FMA*

the case of “No protest” ($X = 0$). Let us interpret the meanings of the two lines in (a) and (b) with positive slopes. In both *OMA* (a) and *FMA* (b) cases, for participants who are told Catherine had protested ($X = 1$), we can see that the lines have positive slopes. It means that the more a participant thought that sexism in society is serious (horizontal axis), the higher score was given to Catherine (vertical axis). Similarly, for participants who are told Catherine had not protested ($X = 0$), we can see that the lines have negative slopes. It means that the more a participant thought that sexism in society is serious (horizontal axis), the lower score was given to Catherine (vertical axis). In addition, the difference between two y intercepts are -4.129 (*OMA*) in (a) and -3.127 (*FMA*) in (b), which are the same as the coefficients of X and $\tilde{X}$. A y intercept means the y value when horizontal value is 0. So, here two y intercepts are two y values (evaluation scores) of the cases “Protest ($X = 1$)” and “No protest ($X = 0$)” for the participants who think that there is no sex discrimination in society ($W = 0$). It means that for the participants who think that there is no sex discrimination in society ($W = 0$), the difference between the evaluation scores of the participants who are told that Catherine had protested ($X = 1$) and Catherine had not protested ($X = 0$) is -4.129 (*OMA*) and -3.127 (*FMA*), respectively.

The two methods, *OMA* and *FMA*, show similar results but are slightly different. The case of *FMA* is likely less sensitive than the *OMA* case. It shows that if we measure these psychological data using crisp numbers, the results can be slightly exaggerated.

3.2 Fuzzy Moderation Analysis for TRAUMA Data with Multiple Predictors

This data were collected by Peltonen et al. [21]. To collect this data, 113 Palestinian boys in Gaza between the ages of 10 and 14 were interviewed in 2006. The independent variable $\tilde{X}$ is the symptoms of PTSD (X) as measured with the Child Posttraumatic Stress Reaction index (CPTS-R) (range 0–54). The dependent variable Y is depressive symptoms measured by the Child Depression Inventory (CDI). (range 1–28). Feelings of loneliness (W) with respect peer relationships were measured with the Children’s Loneliness Questionnaire. (e.g., “I feel alone”, “I don’t have anybody to play with”, “I feel left out of things”; range 1–5). Loneliness was used as a moderator W which can affect the relation between X and Y . Trauma exposure (X_2) means a count of exposure to traumatic events during the AI-Aqsa Intifada (e.g., shelling of home, being shot, losing family members, witness of killing) (range 1–18). Age (X_3) is the child’s age in years. The moderation model for this data is shown in Fig. 11.

Fig. 11 Moderated-mediation analysis model for TRAUMA data

The ordinary moderation model is estimated as follows:

$$\hat{Y} = 0.482 - 0.241X_1 - 0.858W + 0.130X_1W - 0.246X_2 + 1.066X_3. \quad (26)$$

The data Y, X_1, W are fuzzified for fuzzy moderation model. The spreads are calculated with 2.5, 5, 0.5, respectively, for Y, X_1, W . Here, X_2 and X_3 are crisp data. So, they are calculated as special fuzzy data with spreads 0.

$$\begin{aligned} \hat{\tilde{Y}} = & -2.270 \oplus (-0.172)\tilde{X} \oplus 0.247\tilde{W} \oplus 0.100\tilde{X}_1 \otimes \tilde{W} \\ & \oplus 0.003\tilde{X}_2 \oplus 0.894\tilde{X}_3 \end{aligned} \quad (27)$$

The ordinary conditional direct effect (OCDE) of X on Y and fuzzy conditional effect (FCDE) are defined by $\delta_{X_1 \rightarrow Y} = b_1 + b_3W$ and $\delta_{\tilde{X}_1 \rightarrow \tilde{Y}} = b_1 + b_3\tilde{W}$. The ordinary conditional indirect effect (OCIDE) of W on Y and fuzzy conditional effect (FCIDE) are defined by $\delta_{W \rightarrow Y} = b_1 + b_3X$ and $\delta_{\tilde{W} \rightarrow \tilde{Y}} = b_1 + b_3\tilde{X}_1$. The conditional effects are shown in Table 2 and Fig. 12.

It can be shown from the results that the *FCDE* of $\tilde{X}$ on $\tilde{Y}$ is less sensitive than the ordinary *CDE* of X on Y in this data. Ordinary *CIDE* of W on Y is similar to *FCDE* of $\tilde{W}$ on $\tilde{Y}$ in this case.

Table 2 Conditional effects of X_1 and W on Y

Method	Conditional effects	
	X_1 on Y	W on Y
OMA	$-0.241 + 0.130W$	$-0.858 + 0.1302X$
FMA	$-0.172 \oplus 0.100\tilde{W}$	$0.247 \oplus 0.100\tilde{X}$

(a) Conditional Direct Effect of X on Y **(b)** Conditional Indirect Effect of W on Y **Fig. 12** Conditional effects for TRAUMA data

3.3 Fuzzy Moderation Analysis for TEAM PERFORMANCE Data

The effect of dysfunctional behavior on a work team has been proposed by many authors [2, 4, 5, 7, 8]. Also a mediation analysis for fuzzified data has been done in [15]. The variable “Dysfunctional team behavior ($\tilde{X}$)” means how much members of the team do things to weaken the work of others hinder change and innovation. “Negative affective tone of the work climate ($\tilde{M}$)” means how often team members report feeling negative emotion at work such as “angry”, “disgust”, etc. “Team performance($\tilde{Y}$)”

means supervisor’s judgment as to the team’s efficiency, ability to get task done in a timely fashion, etc. In addition, “Negative expressivity ($\tilde{W}$)” means how easy it is to read the nonverbal signal team members emote about how they are feeling. The model is described in Fig. 13.

The ordinary moderated-mediation model is estimated as follows:

$$\hat{M} = 0.026 + 0.620X,$$

$$\hat{Y} = -0.012 + 0.366X - 0.436M - 0.019W - 0.517MW.$$

Here, $\delta_{M \rightarrow Y} = -0.436 - 0.517W$ is a *CIDE* of M on Y . And $a\delta_{M \rightarrow Y} = -0.620(-0.436 - 0.517W) = -0.270 - 0.321W$ is the *CIDE* of $\tilde{X}$ on $\tilde{Y}$ via $\tilde{M}$.

The data have been fuzzified with spreads 0.05 for *FCDE* and *FCIDE* and the results are shown below:

$$\tilde{M} = 0.025 \oplus 0.624\tilde{X},$$

$$\tilde{Y} = -0.013 \oplus 0.386\tilde{X} \oplus (-0.437)\tilde{M} \oplus (-0.016)\tilde{W} \oplus (-0.487)\tilde{M} \otimes \tilde{W}.$$

Here, $\delta_{\tilde{M} \rightarrow \tilde{Y}} = -0.437 \oplus -0.487\tilde{W}$ is a *FCIDE* of $\tilde{M}$ on $\tilde{Y}$. And $a\delta_{\tilde{M} \rightarrow \tilde{Y}} = 0.624(-0.437 \oplus -0.487\tilde{W}) = -0.273 \oplus (-0.304)\tilde{W}$ is the *FCIDE* of $\tilde{X}$ on $\tilde{Y}$ via $\tilde{M}$.

From Table 3 and Fig. 14, it is shown that ordinary *CIDE* and *FCIDE* give similar results.

(a) Conceptual diagram **(b)** Statistical diagram**Fig. 13** Moderated-mediation analysis model for team performance data

3.4 Fuzzy Moderated-Mediation Analysis for MENTOR Data with Multiple Mediators

This data was collected by Chen et al. [22]. A fuzzy multiple mediation analysis has been analyzed in the author's previous study [15]. Here, we consider moderation in this mediation analysis, allowing the indirect effects of formal mentoring on work-family conflict through

Table 3 Conditional indirect effect of M and X on Y

Method	Conditional indirect effects	
	M on Y	X on Y
OMA	$-0.436 - 0.517W$	$-0.270 - 0.321W$
FMA	$-0.437 \oplus -0.487\tilde{W}$	$-0.273 \oplus (-0.304)\tilde{W}$

(a) Conditional Indirect Effect of M on Y

(b) Conditional Indirect Effect of X on Y

Fig. 14 Conditional effects for team performance data

resource access and workload to both differ as a function of a person's work-family orientation. This survey was conducted on 193 employees of machinery and equipment manufacturers company. Data collection occurred in two waves, with a 3-month period between the waves. The model is described in Fig. 15.

$$\hat{M}_1 = 3.639 - 0.614X - 0.317W + 0.261XW,$$

$$\hat{M}_2 = 0.159 + 0.301X + 0.497W - 0.281XW.$$

The effects of X on M_1 and M_2 are $\delta_{X \rightarrow M_1} = -0.614 + 0.261W$ and $\delta_{X \rightarrow M_2} = 0.301 - 0.281W$. And we obtain $\hat{Y} = 3.111 + 0.058X - 0.404M_1 + 0.333M_2$. Here, the conditional indirect effect of $\tilde{X}$ on $\tilde{Y}$ through $\tilde{M}_1$ and $\tilde{M}_2$ depends on $\tilde{W}$ and are expressed by

$$\begin{aligned} \delta_{X \rightarrow M_1 \rightarrow Y} b_1 &= (-0.404)(-0.614 + 0.261W) \\ &= 0.248 - 0.105W. \end{aligned}$$

The data are fuzzified with spreads 0.5, and we obtain following results:

$$\hat{\tilde{M}}_2 = (-0.440) \oplus 1.302\tilde{X} \oplus 0.425\tilde{W} \oplus (-0.207)\tilde{X} \otimes \tilde{W}.$$

The effects of $\tilde{X}$ on $\tilde{M}_1$ and $\tilde{M}_2$ are $\delta_{\tilde{X} \rightarrow \tilde{M}_1} = (-0.240) \oplus 0.154\tilde{W}$ and $\delta_{\tilde{X} \rightarrow \tilde{M}_2} = 1.302 \oplus (-0.207)\tilde{W}$. And we obtain $\hat{\tilde{Y}} = 1.737 \oplus 0.224\tilde{X} \oplus (-0.291)\tilde{M}_1 \oplus 0.451\tilde{M}_2$. Here, the conditional indirect effect of $\tilde{X}$ on $\tilde{Y}$ through $\tilde{M}_1$ and $\tilde{M}_2$ depends on $\tilde{W}$ are expressed by $\delta_{\tilde{X} \rightarrow \tilde{M}_1 \rightarrow \tilde{Y}} b_1 = (-0.291)((-0.240) \oplus 0.154\tilde{W}) = 0.079 \oplus (-0.045)\tilde{W}$,

(a) Conceptual diagram

(b) Statistical diagram

Fig. 15 Moderated-mediation analysis model for MENTOR data

Table 4 Conditional indirect effect of X on Y through M_1 and M_2

Method	Conditional indirect effects	
	X on Y through M_1	X on Y through M_2
OMA	$0.248 - 0.105W$	$0.100 - 0.094W$
FMA	$0.079 \oplus (-0.045)\tilde{W}$	$0.587 \oplus (-0.093)\tilde{W}$

$\delta_{\tilde{X} \rightarrow \tilde{M}_2 \rightarrow \tilde{Y}} b_2 = (0.451)(1.302 \oplus (-0.207)\tilde{W}) = 0.587 \oplus (-0.093)\tilde{W}$. It is shown from Table 4 and Fig. 16 that ordinary *CIDE* of X on Y through M_1 is more sensitive than *FCIDE* of $\tilde{X}$ on $\tilde{Y}$ through $\tilde{M}_1$. And ordinary *CIDE* of X on Y through M_2 is similar to *FCIDE* of $\tilde{X}$ on $\tilde{Y}$ through $\tilde{M}_2$.

(a) Conditional Indirect Effect of X on Y through M_1 **(b)** Conditional Indirect Effect of X on Y through M_2 **Fig. 16** Conditional effects for MENTOR data

4 Conclusions

In this paper, a fuzzy moderation analysis and fuzzy moderated-mediation analysis were proposed using some operations and estimators introduced in the author's previous study [16, 17]. In psychology, there are many cases when the situations cannot be expressed clearly in a real number. Hence it is more reasonable to apply the fuzzy moderation or fuzzy moderated-mediation analysis than classical mediation analysis. Several psychological data have been applied using the proposed fuzzy mediation analysis. It was shown that the fuzzy moderation analysis gives less sensitive results or sometimes similar results to the ordinary methods in terms of the conditional direct and indirect effects through some examples. This shows that if we measure these psychological data using crisp numbers, the results can be slightly exaggerated. Because to represent a human's mind using a crisp number can "lose" lots of information based on the motivation, it is clear that fuzzy numbers can include more information which the original observations have. If we use crisp numbers in these cases, sometimes somewhat different, exaggerated or distorted results can be obtained. And there are possibilities that the results can be misinterpreted or misconstrued. Although only methodologies were presented in this paper, a study on inferences such as confidence interval and hypothesis test will be provided in further research.

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