

# Modeling of most efficient channel form: a quantitative approach

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**Abstract** A fundamental index of channel shape is the form ratio ( $w/d$ ) which is determined by energy-load relation and bank and bed materials of the channel concerned. Average river channels tend to develop their channel-cross sectional form in a way to produce an approximate equilibrium between the channel and the water and sediment it transport. But how far it is deviated from the ideal cross-sectional form is determined only by knowing the *ideal form* which was calculated ‘2’ by Hickin (2004) for *rectangular channel*. This ideal cross-sectional form of ‘maximum efficiency’ is virtually a theoretical one and by attaining of which a river transports its water and load with least friction with its bed. But *ideal form ratios* for *triangular and semicircular channels* have not been calculated earlier. Present paper has found out *ideal form ratios* 2.544 and 2.31 for semicircular and triangular channels respectively.

**Keywords** Form-ratio · Ideal channel form · Hydraulic radius · Wetted perimeter · Semicircular channel · Triangular channel

## Introduction

Width:depth ratio ( $w/d$ ), the most important indicator of channel form is not only a mere numeric fraction. The controlling influence of discharge upon channel form, flow resistance and flow velocity is explored in the concept of

hydraulic geometry (Huggett 2007). Width:depth ( $w/d$ ) ratio of a river increases downstream. Moreover,  $w/d$  ratio is a good reflector of *driving variables* (discharge, quantity and size of sediment load) and boundary conditions (valley confinement, channel substrate, valley slope and riparian vegetation) that controls the form of a channel reach (Charlton 2008). A river with given slope tries to shape its channel to minimize the flow resistance. It is estimated that 95 % of a river’s energy is used in overcoming flow resistance, leaving just 5 % to carry out geomorphological work (Charlton 2008). Flow resistance is determined by channel shape and an ideal channel reach that attain a shape of minimum flow resistance is called *most efficient channel*. Cross-sectional form of a river’s channel is primarily adjusted by bed and bank erosion (Charlton 2008) and lateral channel migration (Simon and Castro 2003). Width/depth ratio represents dominant measures of channel response (Simon 1992; Simon and Darby 1997) and  $w/d$  alone does not define cross-sectional shape (Hey 1978). Form ratio  $w/d$  or  $w/d_{\max}$  (Schumm 1960), section asymmetry  $a_l/a_r$  (Milne 1979), channel asymmetry (Knighton 1981) regarding cross-sectional form give absolute measure. For study of channel’s cross-sectional form, knowing of *ideal form-ratio* is indispensable (Das 2014) and Hickin (2004) calculated it ‘2’ for rectangular channel. *But ideal form-ratios for semicircular and rectangular channels are not yet known*. That is why ideal values of form-ratio of semicircular and rectangular channels are calculated in this paper. At the last section of findings and discussions, ideal form ratio ( $\dot{f}_r$ ) was tested to 29 cross-sections to have a comparative picture of ideal form ratio and observed form ratio.

The conventional belief of the v-shaped cross-sectional form of the rivers is far from the reality (Sen 1993).

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Circular (Leopold and Wolman 1969) and parabolic (Lane 1955) forms are also theoretical. Rather trapezoidal form represents the reality (Sen 1993). But all these forms, whether theoretical or practical, are not obvious for the all channels or entire reach of a same channel. Straight course of a river is impossible (Leopold 1966) which makes another impossibility of uniformity of cross-sectional form of the channel. Width increases faster than depth in downstream and cross-sectional form becomes increasingly rectangular (Sen 1993). But sometimes opposite is also the reality (Knighton 1998; Das 2013).

The conditions for efficiency of the cross-sectional characteristics of the channels are closely related to their capacity of maximum flow. Maximum flow (water and sediment load) is only possible when the cross-sectional form attains the semi-circular or parabolic shape (Knighton 1998) or equilateral-triangular or rectangular (Hickin 2004) shape. These shapes generate the minimum turbulence and shear stress hence channel becomes the 'most efficient'. Thus ideal channel form is considered as the 'best conveyance characteristics' (Crickmay 1974). Relationship between channel form and processes operating in the channels were studied as hydraulic geometry by Leopold and Maddock (1953), Wolman (1955), Leopold and Miller (1956) and others. They computed cross-sectional variables of mean-depth ( $d$ ) and width ( $w$ ) in terms of discharge ( $Q$ ).

$$w = aQ^b \quad \text{and} \quad d = cQ^f$$

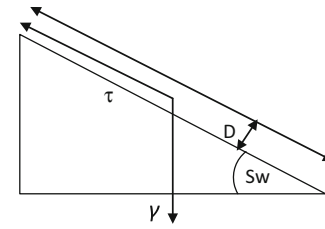
Different average exponent values for  $b$  and  $f$  of different rivers have been calculated by Leopold and Maddock (1953), Wolman (1955), Leopold and Miller (1956), Lewis (1969), Wilcock (1971) and Harvey (1975). So again,  $w/d$  ratio bears much more importance than a mere form ratio of a channel's cross-section. Present paper was aimed in modeling ideal form ratio of semicircular and triangular channel to compare with real channels. To test the ideal form ratio, 11 cross-sections of a river and 18 cross-sections of an ox-bow lake were taken and compared.

## Methodology

Manning (1891) flow resistance equation  $v = (1/\tau) R_h^{2/3} S^{1/2}$  (Simon and Castro 2003) suggests that flow resistance determines the velocity of a river. Flow resistance in turn is determined by channel form. With given volume; velocity is proportional to hydraulic radius ( $R_h^{2/3}$ ) and slope ( $S^{1/2}$ ) but inversely proportional to shear stress ( $\tau$ ). Shear stress (Fig. 1) is calculated as:

$$\tau = \gamma DS$$

(Knighton 1998; Richards 1982; Chanson 2004) where,  $\tau$  = Shear Stress ( $N/m^2$ ),  $\gamma$  = Weight Density of Water



**Fig. 1** Shear stress ' $\tau$ ' of a stream of water depth ' $D$ ', slope ' $S_w$ ' and weight density of water ' $\gamma$ '

( $N/m^3$ ,  $lb/ft^3$ ),  $D$  = Average water depth (m, ft) and  $S$  = Water Surface slope (m/m, ft/ft).

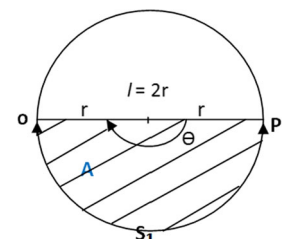
However, to find out channel form of maximum efficiency, computation of minimum shear stress or flow resistance is essential. Because where the shear stress is the least, channel is maximum efficient. With given slope, a minimum flow resistance is only possible when hydraulic radius is the maximum. Hydraulic radius is defined as cross-sectional area ( $A$ ) divided by wetted perimeter ( $P$ ). To find out lowest wetted perimeter of a cross-section with given cross-sectional area, three alternatives model shapes were considered against each of the semicircular and triangular channel. Maximum hydraulic radius ( $A/P$ ) or minimum flow resistance from channels with given cross-sectional area was calculated using trigonometry, method of bisection and Pythagorean Theorem.

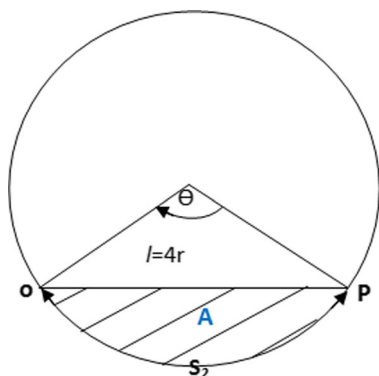
## Findings and discussions

### The most efficient semicircular cross-section

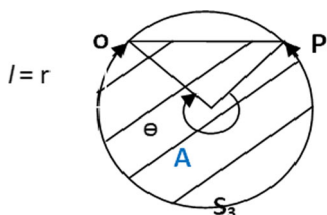
Channel with given cross-sectional area ( $A$ ) and slope ( $S$ ) having highest hydraulic radius is the *ideal channel* form with maximum efficiency. Let cross-sectional shape be semicircular (Fig. 2), minor segment (Fig. 3) and major segment (Fig. 4) of a circle. In all cases, cross-sectional area ' $A$ ' is constant but lengths of wetted perimeters vary and represented by  $S_1$ ,  $S_2$  and  $S_3$  respectively. First type of semicircular channel has a width of  $2r$ , second type of channel has a width of  $4r$  and third type of channel has a width of  $r$ . Different channel widths  $OP$  is represented by the letter ' $l$ '. Now let ratio of wetted perimeters of three types of channel be determined.

**Fig. 2** Semicircular cross-sectional form with constant area ( $A$ ) but minimum wetted perimeter ( $S_1 = \pi r$ ) and medium width ( $2r$ )





**Fig. 3** Minor segment cross-sectional form with constant area (A) but maximum wetted perimeter (S2) and maximum width (4r)



**Fig. 4** Major segment cross-sectional form with constant area (A) but medium wetted perimeter (S3) and minimum width (r)

In any case,  
Constant area

$$A = \frac{\pi r^2}{2} \theta - \frac{1}{2} r^2 \sin \theta$$

$$= \frac{1}{2} r^2 (\theta - \sin \theta)$$

Now  $\frac{l}{\sin \theta} = \frac{r}{\sin(\frac{\pi}{2} - \frac{\theta}{2})}$

Or,  $r = \frac{l}{2 \sin \frac{\theta}{2}}$  (1)

Therefore,  $A = \frac{l^2}{8 \sin^2 \frac{\theta}{2}} (\theta - \sin \theta)$

Or,  $(\theta - \sin \theta) = \left(\frac{8A}{l^2}\right) \sin^2 \frac{\theta}{2} = 0$  (2)

From this equation the value of  $\theta$  for a given value of  $l$  can be calculated. Then the value of  $s$  can be extracted from the relation  $S = r \theta$  as

$S = \frac{l\theta}{2 \sin \frac{\theta}{2}}$  (3)

In first case  $l = 2r$ ,  $A = \pi r^2/2$  and  $S_1 = \pi r$   
 In second case  $l = 4r$ ,  $A = \pi r^2/2$   
 From Eq. (2),

$$\theta - \sin \theta - \frac{8 \frac{\pi r^2}{2}}{16r^2} \sin^2 \frac{\theta}{2} = 0$$

or,  $\theta - \sin \theta - \frac{\pi}{4} \sin^2 \frac{\theta}{2} = 0$

Solving by the ‘method of bisection’, one can get  $\theta = 1.12798$  which is correct upto 5 decimal places.

Now from Eq. (3),

$$S_2 = \frac{4r \times 1.12798}{2 \sin 0.56399} \simeq 4.2201986929r$$

$$S_2/S_1 = \frac{4.2201986929}{\pi} \simeq 1.34333$$

In third case  $l = r$ ,  $A = \pi r^2/2$

From Eq. (2),  $\theta - \sin \theta - 4\pi \sin^2 \theta/2 = 0$

or,  $\theta = 4.79125$  which is correct upto 5 decimal places.

Now from Eq. (3),  $S_3 \simeq 3.52981538r$

Therefore  $S_3/S_1 = 3.52981538/\pi \simeq 1.2357$

Hence,  $S_1:S_2:S_3 = 1:1.34333:1.12357$

Therefore, semicircular form is the most efficient ideal cross-sectional form, having ‘best conveyance characteristics’ with minimum wetted perimeter and maximum hydraulic radius.

**Ideal width (w) and width index (Iw)**

Ideal width ( $\acute{w}$ ) provides tool to compare width of a natural channel with given area to that of the ideal width which the channel tries to attain to be *most efficient*.  $\acute{w}$  is defined as  $(2r)$  and width index ( $I_w$ ) is defined as  $I_w = w/\acute{w}$ .

Where  $w$  = observed width of the channel

$r = \sqrt{2A/\pi}$  ( $A$  = cross-sectional area of semi-circular channel and expressed as  $A = \pi r^2/2$ )

Derivation of  $\acute{w}$ :

$A = \pi r^2/2$

Or,  $\acute{w} = 2r = 2 \times 0.798\sqrt{A}$  [as  $\acute{w} = 2r$ ]

Or,  $\acute{w} = 1.595\sqrt{A}$  (4)

Width index ( $I_w$ ) is a numerical tool to compare the shape of the river cross-sectional form, whether its width matches the width of *most efficient* channel or how much deviated from it. If  $I_w = 1$ , the width matches perfectly with the width of a *most efficient* channel. If  $I_w > 1$ , it indicates wider unconfined channel with negligible slope, non-cohesive substrate and lack of riparian vegetation (Charlton 2008). When  $I_w < 1$ , then the channel is narrower with confined channel, steeper slope, cohesive substrate and or presence of riparian vegetation. In both cases, where  $I_w > 1$  or  $I_w < 1$ , channels are less efficient than ideal channel.

**Ideal depth ( $\hat{D}$ ) and depth index ( $I_d$ )**

Ideal depth ( $\hat{D}$ ) provides tool to compare depth of a natural channel with given area ( $A$ ) to that of the ideal depth which the channel tries to attain to be *most efficient*.

$\hat{D}$  is defined as  $A/\hat{w}$  and depth index ( $I_d$ ) is defined as  $d/\hat{D}$ .

Where  $d$  = observed mean depth of the channel

*Derivation of  $\hat{D}$ :*

$$\hat{D} = A/2r$$

$$\text{Or, } \hat{D} = \pi r^2/4r$$

$$\text{Or, } \hat{D} = 0.627 \sqrt{A} \tag{5}$$

Depth index ( $I_d$ ) is a numerical tool to compare the shape of the river cross-sectional form, whether its depth matches the depth of *most efficient* channel or how much deviated from it. If  $I_d = 1$ , the depth matches perfectly with the depth of a *most efficient* channel. If  $I_d > 1$ , it indicates deeper confined channel (Charlton 2008) with steep valley-side slope, cohesive or bed-rock substrate and presence of riparian vegetation. When  $I_d < 1$ , then the channel is shallower with non-confined channel, gentle slope, non-cohesive substrate and or absence of riparian vegetation. In both cases, where  $I_d > 1$  or  $I_d < 1$ , channels are less efficient than ideal channel.

**Ideal form number ( $\hat{f}_r$ ) and channel-form index ( $C_f I$ )**

Ideal Form Ratio is defined as

$$\hat{f}_r = \hat{w}/\hat{D}$$

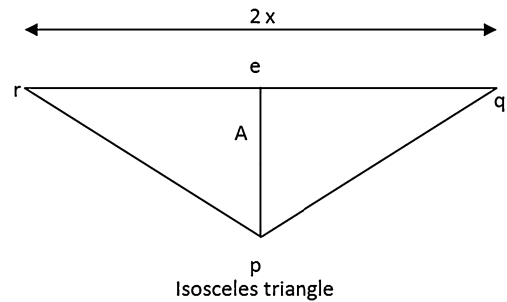
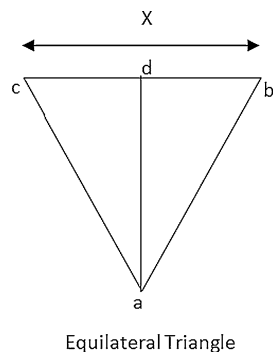
$$\hat{f}_r = 1.595\sqrt{A}/0.627\sqrt{A}$$

$$\hat{f}_r = 2.544 \tag{6}$$

Now Channel-form index ( $C_f I$ ) is defined as  $(w/d)/\hat{f}_r$ .

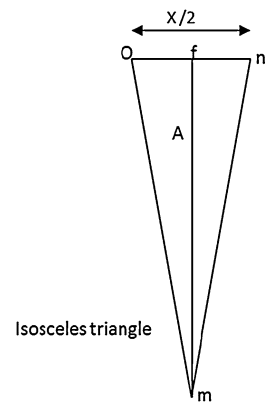
If  $C_f I = 1$ , channel is ideal and semicircular with maximum efficiency. Higher value indicates aggraded and or shallower channels whereas lower value indicates

**Fig. 5** Equilateral triangular cross-sectional form with constant area ( $A$ ) but minimum wetted perimeter



**Fig. 6** Wide isosceles triangular cross-sectional form with constant area ( $A$ ) but medium wetted perimeter

**Fig. 7** Narrow isosceles triangular cross-sectional form with constant area ( $A$ ) but maximum wetted perimeter



degraded and or deeper channel respectively. In both cases, where  $C_f I > 1$  or  $C_f I < 1$ , channels are less efficient than ideal channel.

**The most efficient triangular cross-section**

Let cross-sectional shape be equilateral triangle (Fig. 5), isosceles triangular (Fig. 6) and narrow isosceles triangular (Fig. 7). In all cases, cross-sectional area ‘ $A$ ’ is constant and length of wetted perimeter is represented by  $2ac$ ,  $2pr$  and  $2mo$  respectively. First type of triangular channel has a width of  $x$ , second type of triangular channel has a width of  $2x$  and third type of triangular channel has a width  $x/2$ . Now let ratio of wetted perimeters of three types of channel be determined.

Suppose, area of the equilateral  $\Delta abc$  (Fig. 5) is  $A$ .

$$\text{Therefore, } A = \frac{\sqrt{3}}{4}(ab)^2$$

$$\text{Or, } A = \frac{\sqrt{3}}{4}x^2 \quad [\text{as } ab = x]$$

In  $\Delta pqr$  (Fig. 6), area =  $A$  and  $pq = pr$

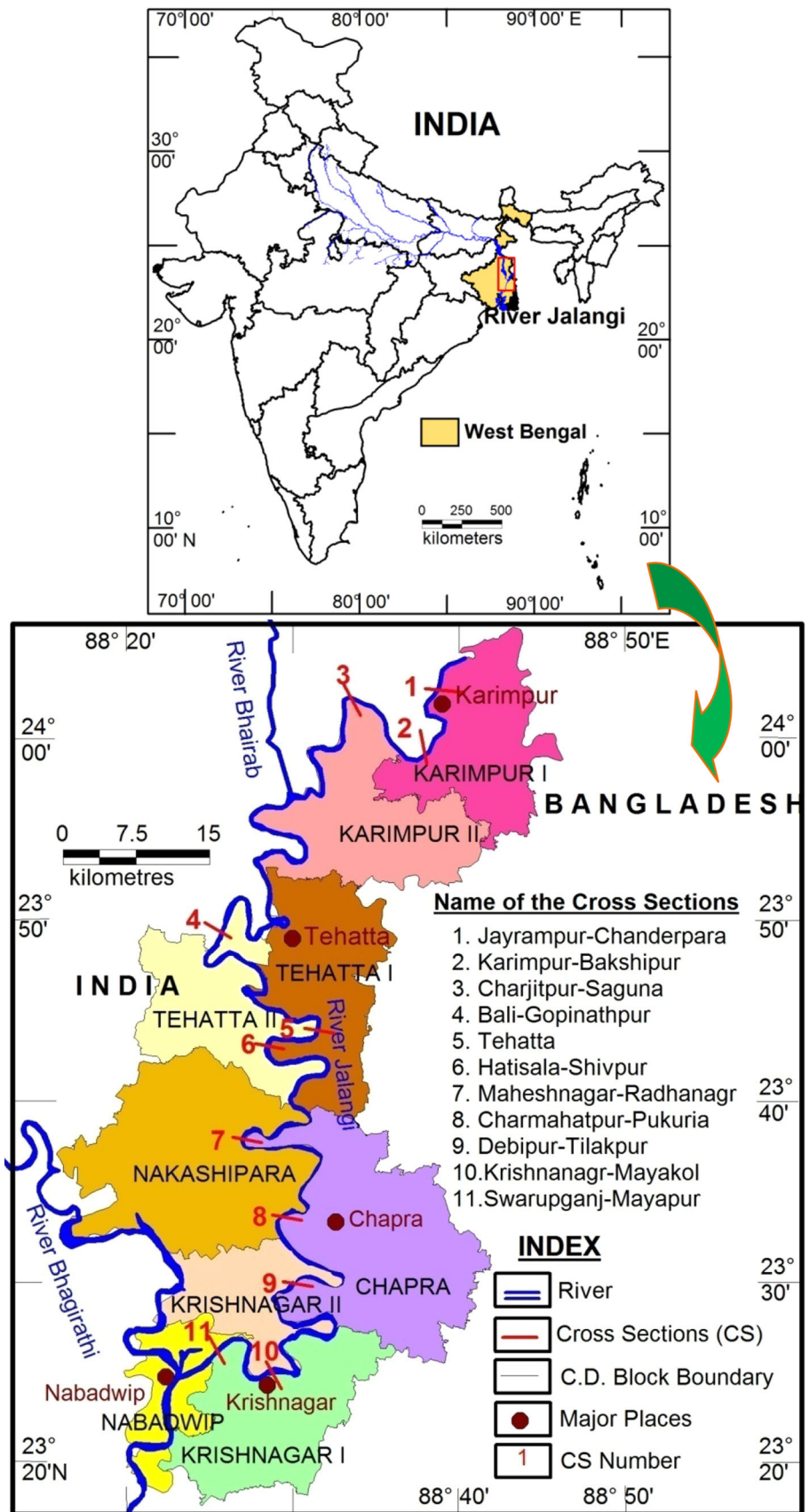
$$\text{Area of } \Delta pqr = 1/2(qr \times ep)$$

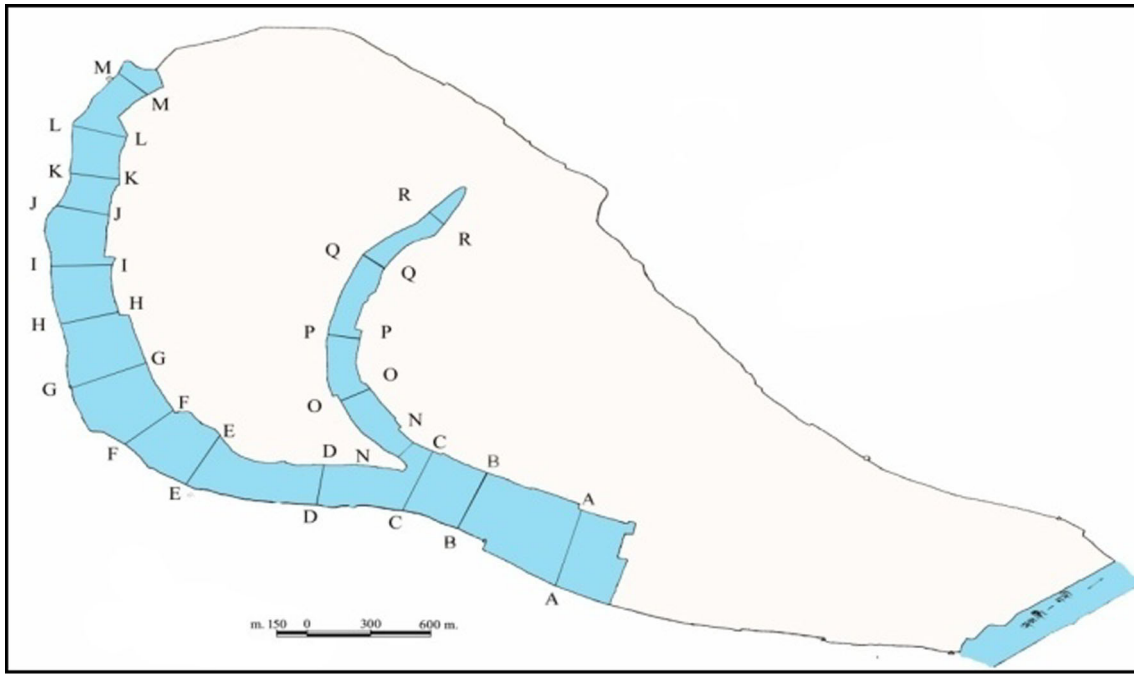
$$= 1/2(2x \times ep) [as, 2x = qr]$$

$$= x \times ep$$

$$\text{Therefore, } x \times ep = \frac{\sqrt{3}}{4}x^2$$

**Fig. 8** Sites of 11 cross sections on river Jalangi





**Fig. 9** Sites of cross-sections on the Hansadanga Beel

$$\text{Or, } ep = \frac{\sqrt{3}}{4}x$$

$$\text{In } \Delta \text{ per, } (pr)^2 = (ep)^2 + (er)^2 \\ = (ep)^2 + x^2 \quad [\text{as } er = x]$$

$$= \left(\frac{\sqrt{3}}{4}x\right)^2 + x^2 \quad \left[\text{as } ep = \frac{\sqrt{3}}{4}x\right]$$

$$= \frac{3}{16}x^2 + x^2$$

$$= (3x^2 + 16x^2) \div 16$$

$$pr = \frac{\sqrt{19}}{4}x = \frac{4.3}{4}x$$

$$4pr = 4.3x$$

$$\frac{x}{4} = \frac{pr}{4.3}$$

(7)

In  $\Delta$  mno (Fig. 7),  $mn = mo$

Area of  $\Delta$  mno =  $1/2(on \times fm)$

$$= \frac{1}{2} \times \frac{x}{2} \times fm [\text{as, } on = x/2]$$

$$= \frac{x}{4} \times fm$$

$$\text{Now, } = \frac{x}{4} \times fm = \frac{\sqrt{3}}{4}x^2 [\text{as Areas of all three triangles are equal}]$$

$$fm = \sqrt{3}x$$

$$\text{In } \Delta \text{ fmo, } (om)^2 = (fm)^2 + (fo)^2$$

$$= (fm)^2 + (x/4)^2 \quad [\text{as } fo = x/4]$$

$$= (\sqrt{3}x)^2 + (x/4)^2 \quad [\text{as } fm = \sqrt{3}x]$$

$$= (48x^2 + x^2) \div 16$$

$$om = 7x/4$$

$$\text{Or, } \frac{x}{4} = \frac{om}{7} \tag{8}$$

Therefore,

$$\frac{x}{4} = \frac{pr}{4.3} = \frac{om}{7} = k \tag{9}$$

$$x = 4k, \quad pr = 4.3k \quad \text{and } om = 7k$$

Therefore, ratio of wetted perimeter of channel types, i.e.

$$1\text{st} : 2\text{nd} : 3\text{rd} = 4 : 4.3 : 7 \tag{10}$$

So, hydraulic radius with given cross-sectional area of wide V-shaped and narrow V-shaped triangular channels are smaller than that of an equilateral triangular channels. Therefore, ideal cross-sectional form of a river having maximum efficiency is either semicircular or equilateral triangular and rectangular with w/d ratio 2:1 (Hickin 2004) as well.

**Table 1** Cross sectional variables of Jalangi River

Name of CS	A	w	d	w/d	$\dot{W} = 1.595\sqrt{A}$	$\check{D} = 0.672\sqrt{A}$	$C_f I = (w/d)/\dot{f}_r$
1	172.21	116.64	1.47642	79.0017	20.93099	8.818576	31.05414
2	453.66	181.01	2.50627	72.2229	33.97238	14.31313	28.38949
3	30.11	47.12	0.63901	73.7394	8.752176	3.687437	28.98563
4	568.9	135	4.21407	32.0355	38.04334	16.02829	12.59257
5	524.78	98.72	5.31584	18.5709	36.53838	15.39423	7.299882
6	860.22	134.2	6.40999	20.9361	46.78056	19.70943	8.229595
7	672.63	117.22	5.73819	20.4281	41.3665	17.42839	8.029898
8	1016.32	184	5.52348	33.3123	50.84824	21.42321	13.09447
9	687.99	144.4	4.76447	30.3077	41.83615	17.62627	11.91338
10	827.68	144.04	5.74618	25.0671	45.88724	19.33306	9.853412
11	786.24	124.2	6.33044	19.6195	44.72375	18.84286	7.712072
Average	600.067273	129.686	4.42403	38.6583	37.2436	15.6914	15.1959
SD	296.0400595	37.6657	1.997753	23.9252	12.38747	5.219049	9.404559
CV	0.493344785	0.290437	0.451568	0.618889	0.332607	0.332607	0.618889

Source: Field survey, 2007–2010

**Table 2** Cross sectional variables of Hansadanga Beel (ox-bow lake)

CS name	A	w	d	w/d	$\dot{W} = 1.595\sqrt{A}$	$\check{D} = 0.672\sqrt{A}$	$C_f I = (w/d)/\dot{f}_r$
AA	151.5802	179.5	0.844458	212.5624	19.6373068	8.273523617	83.55439
BB	420.9537	219.92	1.914122	114.8934	32.7248642	13.78752899	45.1625
CC	512.0287	215.71	2.37369	90.87539	36.09174162	15.20605039	35.72146
DD	412.3411	152.47	2.704408	56.37833	32.38836118	13.64575468	22.16129
EE	482.9629	146	3.307965	44.1359	35.05238496	14.76815216	17.34902
FF	498.9046	181	2.756379	65.66586	35.62619507	15.00990789	25.81205
GG	470.1287	203	2.315905	87.65472	34.58351003	14.57060736	34.45547
HH	392.407	181	2.167994	83.48732	31.59577849	13.31182642	32.81734
II	769.4845	255	3.017586	84.50463	44.24463589	18.6410002	33.21723
JJ	294.1858	186	1.581644	117.5992	27.35719106	11.52603912	46.22608
KK	95.584	171.7	0.55669	308.4302	15.59384768	6.569947112	121.2383
LL	18.435	55	0.335182	164.09	6.848291822	2.885299125	64.50077
MM	61.93925	152.55	0.406026	375.7149	12.55288813	5.288740329	147.6867
NN	136.0183	112	1.214449	92.22289	18.60198453	7.837325144	36.25114
OO	196.4173	136.5	1.438954	94.86057	22.35375571	9.418008676	37.28796
PP	380.7795	128	2.97484	43.02752	31.12414868	13.11312095	16.91333
QQ	317.8545	116	2.740125	42.33384	28.43641669	11.98073481	16.64066
RR	204.9383	109.7	1.86817	58.72057	22.83348766	9.620127716	23.08198
Average	323.1635	161.1694	1.917699	118.731	27.09148834	11.41409415	46.67098
SD	195.0544	48.07964	0.949329	92.59937	9.663063475	4.071209188	36.39913
CV	0.603578	0.298317	0.495035	0.779909	0.356682636	0.356682636	0.779909

Source: Field survey, January 2015

**Ideal width ( $\check{w}$ ) and width index ( $I_w$ )**

Ideal width ( $\check{w}$ ) of triangular channel provides tool to compare natural channel width ( $w$ ) with ideal width which the channel tries to attain to be *most efficient*.

Derivation of  $\check{w}$ :

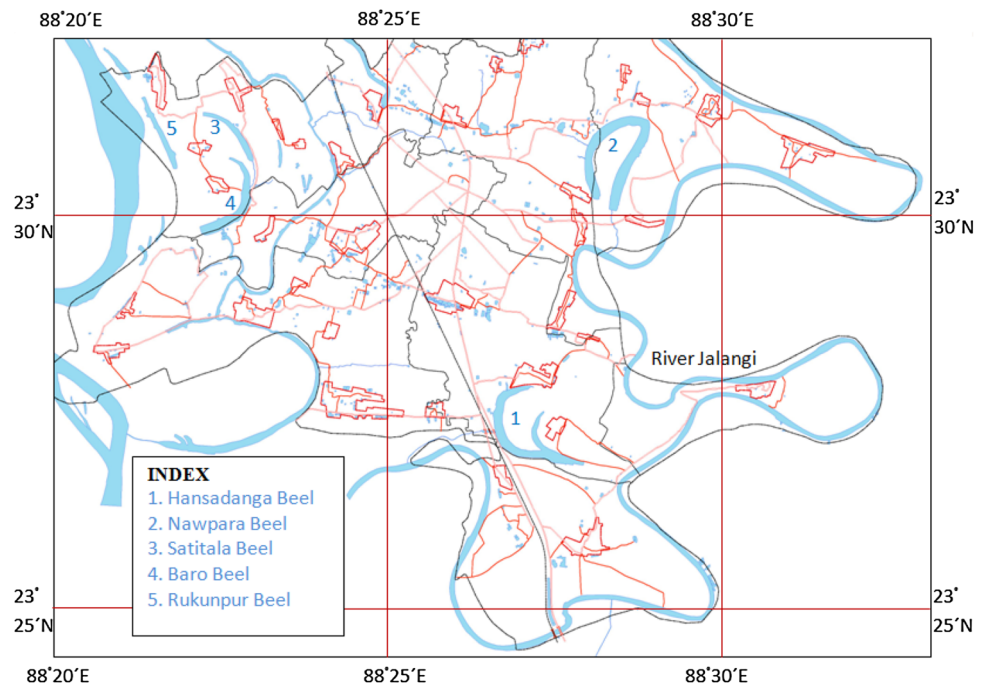
$\check{w} = x$ , [ $x$ , is the length of a side of an equilateral triangle]

Now, if the cross-sectional area of the concerned channel is ‘A’, then

$$A = \frac{\sqrt{3}}{4}x^2$$

$$\text{Or, } x^2 = 2.3094A$$

**Fig. 10** Location of Hansadanga Beel on the right bank of the river Jalangi



$$\begin{aligned}
 \text{Or, } x &= 1.52\sqrt{A} \\
 \text{Or, } \ddot{w} &= 1.52\sqrt{A} \\
 \text{Width index } (I_w) &= w/\ddot{w}.
 \end{aligned}
 \tag{11}$$

Significance of width index in triangular channel form is as width index in semicircular channel form (see under the section “Ideal width ( $\ddot{w}$ ) and width index ( $I_w$ )”)

*Ideal depth ( $\underline{d}$ ) and depth index ( $I_d$ )*

Ideal depth ( $\underline{d}$ ) provides tool to compare mean depth ( $d$ ) of a natural channel of given area to that of the ideal mean depth ( $\underline{d}$ ) which the channel tries to attain to be *most efficient*.

$\underline{d}$  is defined as  $A/\ddot{w}$  and depth index ( $I_d$ ) is defined as  $I_d = d/\underline{d}$

*Derivation of  $\underline{d}$ :*

$$\begin{aligned}
 \underline{d} &= A/1.52\sqrt{A} \\
 \text{Or, } \underline{d} &= \sqrt{A}/1.52
 \end{aligned}
 \tag{12}$$

Significance of depth index in triangular channel form is as depth index in semicircular channel form.

*Ideal form number ( $\Delta_3$ ) and channel-form index ( $C_fI$ )*

$$\begin{aligned}
 \Delta_3 &= \ddot{w}/\underline{d} \\
 \text{Or, } \Delta_3 &= \frac{1.52\sqrt{A}}{\sqrt{A}/1.52} \\
 \text{Or, } \Delta_3 &= 2.31
 \end{aligned}
 \tag{13}$$

Now triangular Channel-form index ( $C_fI_\Delta$ ) is defined as  $(w/d)/\Delta_3$ .

If  $C_fI = 1$ , channel is ideal and equilateral triangular with maximum efficiency. Lesser the value, narrower the shape and implies the channel of upper course or youth stage. If the value is greater than 1, it implies the wider v-shaped channel of lower reach or old stage.

**Testing of  $C_fI$  on river channel and ox-bow lake channel**

$C_fI$  was applied to 29 cross sections of channels. For this empirical test, 11 sections were selected on river Jalangi (Fig. 8), a deltaic distributary of the river Ganges and 18 cross-sections were selected on an ox-bow lake ‘Hansadanga Beel’ (Fig. 9) which is originated during late eighteenth or early nineteenth century (Das 2012) as a neck cut-off of a meander (Fig. 10) of the river Jalangi.

Result shows that average form ratio ( $w/d$ ) of the river Jalangi is 38.6583 (Table 1) and  $C_fI$  is 15.2 which imply that the width/depth ratio is 15.2 times greater than the ideal form ratio (2.55). On the other hand, average form ratio of the ox-bow lake is 118.731 (Table 2) and  $C_fI$  is 46.7. It implies that ox-bow lake channel is many times wider and shallower than the river channel. The finding is very interesting because although there is a great difference between two sets of observations, both set of cross sections are basically on the river Jalangi as the ox-bow lake is also a rejected channel of the same river.  $C_fI$  of the ox-bow lake is many folds higher than the river because it



has lost its depth heavily due to severe silting and no scouring. But the river scours its bed annually during floods.

## Conclusions

The ratio of width to depth is the function of channel shape. But mere width:depth ratio ( $w/d$ ) does not define cross-sectional shape (Hey 1978) even though it is a widely used index. So to have comparison, instead of simple width/depth ratio, Ideal Form Numbers  $f_r = 2.544$  or  $\Delta_3 = 2.31$  gives better explanation. Moreover, with a given area of channel cross-section, how far its *width* and *depth* are deviated from the ideal width or depth can be determined by ideal numbers  $\check{w} = 1.595\sqrt{A}$  or  $\check{w} = 1.52\sqrt{A}$  and  $\check{D} = 0.627\sqrt{A}$  or  $\check{d} = \sqrt{A}/1.52$  respectively.

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