REVIEW

A review of monitoring, calculation, and simulation methods for ground subsidence induced by coal mining

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Abstract

Subsidence data acquisition methods are crucial to mining subsidence research and an essential component of achieving the goal of environmentally friendly coal mining. The origin and history of the existing methods of field monitoring, calculation, and simulation were introduced. It summarized and analyzed the main applications, flaws and solutions, and improvements of these methods. Based on this analysis, the future developing directions of subsidence data acquisition methods were prospected and suggested. The subsidence monitoring methods have evolved from conventional ground monitoring to combined methods involving ground-based, space-based, and air-based measurements. While the conventional methods are mature in technology and reliable in accuracy, emerging remote sensing technologies have obvious advantages in terms of reducing field workload and increasing data coverage. However, these remote sensing methods require further technological development to be more suitable for monitoring mining subsidence. The existing subsidence calculation methods have been applied to various geological and mining conditions, and many improvements have already been made. In the future, more attention should be paid to unifying the studies of calculation methods and mechanical principles. The simulation methods are quite dependent on the similarity of the model to the site conditions and are generally used as an auxiliary data source for subsidence studies. The cross-disciplinary studies between subsidence data acquisition methods and other technologies should be given serious consideration, as they can be expected to lead to breakthroughs in areas such as theories, devices, software, and other aspects.

Keywords Mining subsidence · Subsidence monitoring · Subsidence calculation · Subsidence simulation

1 Introduction

According to the current global energy consumption structure and its development trend, coal, oil, natural gas, and new energy are the four most important components of the world energy system (Zou et al. 2021). In 2019, coal consumption accounted for 27% of global energy consumption. In China, coal resources are the fundamental guarantee of energy security. Coal consumption accounted for 56% of the total primary energy consumption in 2021 (Ministry of Natural Resources PRC 2022), and it is predicted that it will still account for approximately 50% by 2030 (Qian et al. 2018). Unlike other major coal-producing countries, more than 85% of coal production in China comes from underground mining (Hu et al. 2020b). Today, environmentally friendly coal mining, which is an advanced mining concept trying to balance resource exploitation and environmental protection, is increasingly considered, and the research and treatment of coal mining subsidence are its critical subjects (Wang et al. 2020). Mining-induced subsidence and its influence on safety and environment are the basic scientific topics of resource exploitation (Qian and Xu 2019), ground control (Peng 2015a), environmental ecological restoration (Peng and Bi 2020), and land reclamation (Hu et al. 2020b).

This paper is focused on the mining subsidence induced by the excavation of underground coal seams, which is defined as the movement of overlying strata and ground

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surface (Dai 2018). There are several subsidence variables including vertical subsidence (also known as vertical displacement), horizontal displacement, slope (also known as tilt), curvature, horizontal strain (also known as horizontal deformation), twist, and shear strain (Peng 2020). The first five variables are the most often used basic variables.

Data acquisition methods of the aforementioned subsidence variables, including field monitoring (observation) methods, calculation (prediction) methods, and simulation (experiment) methods, are the key objects of mining subsidence research. The purpose of this paper is as follows: (1) to summarize the acquisition methods of mining subsidence data, mainly limited to that on the ground surface; (2) to introduce these methods, including their basic principles, advantages and disadvantages, improvements, and applications; (3) to review and prospect these methods. It is expected to provide a means for future scholars to quickly comprehend this field.

2 Development of mining subsidence research

The study of mining subsidence originated in Belgium at the end of the nineteenth century. At the beginning of the twentieth century, it was developed in Germany, Austria, and Britain, and the first research climax appeared at that time. In 1931, a new science discipline—mining subsidence engineering—was formed in Germany (Edmonds 2018; Kratzsch 1983). Afterward, in the 1950s and 1960s, a second research climax appeared in Poland and Germany. From the end of the twentieth century, there was a third research climax, which mainly happened in China (Jiang et al. 2019), America (Luo and Peng 1992, 2000), and some European countries (Blachowski et al. 2019; Muntean et al. 2015; Ścigała and Szafulera 2019; Vervoort 2020; Vervoort and Declercq 2018). The timeline of significant historical events in the mining subsidence field is shown in Fig. 1, and the theories and methods involved in this figure will be further introduced in the following chapters.

Recent important works around the world include *Stochastic Methods in Mechanics of Granular Bodies* by Litwiniszyn (1974), *Subsidence Engineers' Handbook* by the National Coal Board (1975), *Mining Subsidence Engineering* by Kratzsch (1983), *Subsidence: Occurrence, Prediction and Control* by Reddish and Whittaker (1989), and *Surface Subsidence Engineering: Theory and Practice* by Peng (2020).

Systematic research on mining subsidence in China began with Liu Baochen and Liao Guohua who studied in Poland. In 1965, they published the first monograph in Chinese on mining subsidence—*Basic Laws of Ground Surface Movement in Coal Mines* (Liu and Liao 1965). Later, in 1981, the college textbook *Movement of Rock Strata and Ground Surface in Coal Mines* was written by China Institute of Mining and Technology et al. (1981). Then, several important monographs were published, such as *Laws of Ground Surface Movement and Overburden Failure in Coal Mines and their Applications* (China Coal Research Institute 1981), *Mining Subsidence* (He et al. 1991a), and *Mining Subsidence Engineering* (Zou et al. 2003). In 2014, China University of Mining and Technology re-wrote the college textbook *Deformation Monitoring and Mining Subsidence Engineering* (Deng et al. 2014). In 2017, the State Administration of Work Safety of PRC and other government agencies united the mining subsidence researchers and technicians to re-compile *The Regulations of Coal Mining and Pillar Design for Protecting Buildings, Water Bodies, Railways, Main Mineshafts and Tunnels* (hereinafter called "*The Regulations of Coal Mining*", which has been previously published twice in the years of 1986 and 2000), which summarized the theories and methods of subsidence prediction and control deemed suitable for China's mining conditions (State Administration of Work Safety et al. 2017). Recently, the book *60-year Achievements by Chinese Mine Surveying (1953–2013)* reviewed the historical events and important figures in subsidence development in China (Wang et al. 2016b).

By analyzing the mining subsidence projects supported by the National Natural Science Foundation of China (Chen
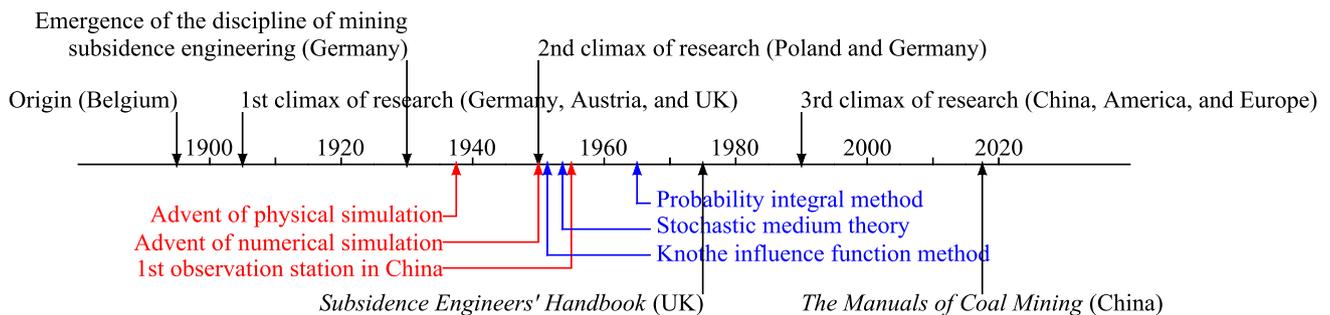
Fig. 1 Timeline of the study of mining subsidence

et al. 2014b; Zou et al. 2017), the impact of mining on the environment and its evolution law, the theory of deep resource exploitation, real-time and high-precision monitoring, and the theory of subsidence prediction are shown to be hotspots in the field of mining subsidence research.

3 Monitoring methods used in mining subsidence

Mining subsidence monitoring provides basic data for studying subsidence laws, identifying or evaluating mining damages, and restoring the geological and ecological environment (Peng and Bi 2020; Wang et al. 2020).

Benefitting from the development of surveying and other technologies, mining subsidence monitoring (observation) methods have developed from conventional ground observation stations to data acquisition methods combining conventional means with space-based and air-based methods. The processing of the monitoring data also shows a trend of integrating multiple methods (Ding et al. 2013; Li 2012; Suresh and Yarrakula 2018; Wang 2017). In field monitoring, conventional geodetic methods (e.g., level, total station, and global navigation satellite system (GNSS)) and non-contact remote sensing methods (e.g., unmanned aerial vehicle (UAV) and interferometric synthetic aperture radar (InSAR)) both have some limitations (Tang et al. 2020; Wang 2017). The former has high accuracy and stability, but the measurement results are scattered (generally along the strike and/or dip directions of a working face) and the field workload is heavy; whereas the latter can obtain areal displacement data (point clouds or grid data) of a whole region, but topographic relief, surface coverage, and large gradient deformation, which are common in subsidence areas, will affect the monitoring accuracy. Summarizing the existing subsidence monitoring research, the main monitoring methods and their comparisons can be found in Fig. 2 and Table 1.

3.1 Ground monitoring methods

(1) Conventional ground observation stations

Ground observation stations are a commonly used and important method to monitor ground surface subsidence caused by underground excavation. A conventional observation station consists of several discrete monitoring points, as shown in Fig. 2a. The point distribution forms of most observation stations are linear, specifically, approximately along the strike and/or dip main sections of a working face (China Coal Research Institute 1981; He et al. 1991a), whereas the points in some stations are along/on the mining-influenced

objects (Marino and Osouli 2020; Orynyak et al. 2019), and only a few stations are mesh-shaped for special research purposes (Yang 1990; Zhang et al. 2021c, 2020). The often-used measurement method is leveling + traverse survey or GNSS (static positioning or real-time kinematic (RTK)). The direct results are three-dimensional (3D) coordinates of the monitoring points at each observation time, and then the subsidence variables, including vertical subsidence, horizontal displacement, and other deformations (slope, curvature, and horizontal strain), can be solved.

The first ground observation station in China was established in Linxi Mine in Kailuan Mining Bureau in 1954 (Cui and Deng 2017). After that, ground observation stations were extensively constructed in most major mining areas. Plenty of mine data and observation data were accumulated and summarized in *The Manuals of Coal Mining and Pillar Design for Protecting Buildings, Water Bodies, Railways, Main Mineshafts and Tunnels* (hereinafter called "*The Manuals of Coal Mining*") (Hu et al. 2017), which mainly included geological and mining conditions, characteristics of overlying strata, angular parameters of subsidence basin, parameters of the probability integral method, and parameters of dynamic subsidence.

The monitoring results obtained by conventional observation stations are reliable in accuracy (up to millimeter/centimeter scales), but there are two unavoidable technical disadvantages: i) the distribution form of monitoring points determines that the measurement results are scattered and cannot cover the whole subsidence area, even most of the results are only linear; ii) conventional measurement methods (leveling + traverse survey or GNSS) cannot acquire subsidence data in real time due to the mode of data transmission and processing, which means these methods cannot be used in pre-warning.

(2) Real-time monitoring by GNSS

Conventional measurement methods have a heavy field workload, and the data are discontinuous and theoretically only exist at exactly the observation time. To realize automatic and real-time monitoring, some researchers have attempted to set continuously operating GNSS devices with electrical power at the monitoring points, transmit the measured data to a data processing center (or server) through a wireless network, and then calculate the subsidence and send it to the user's PC and smartphone (Ding et al. 2013; Lian et al. 2020b; Tao et al. 2018; Yu et al. 2014), as illustrated in Fig. 2b. Some studies on GNSS real-time monitoring also involve other accessory equipment, such as a digital camera (Lian et al. 2020b) or a CCD element (Ding et al. 2013).

This approach still relies on the physical monitoring points of the observation station. Limited by their high price,
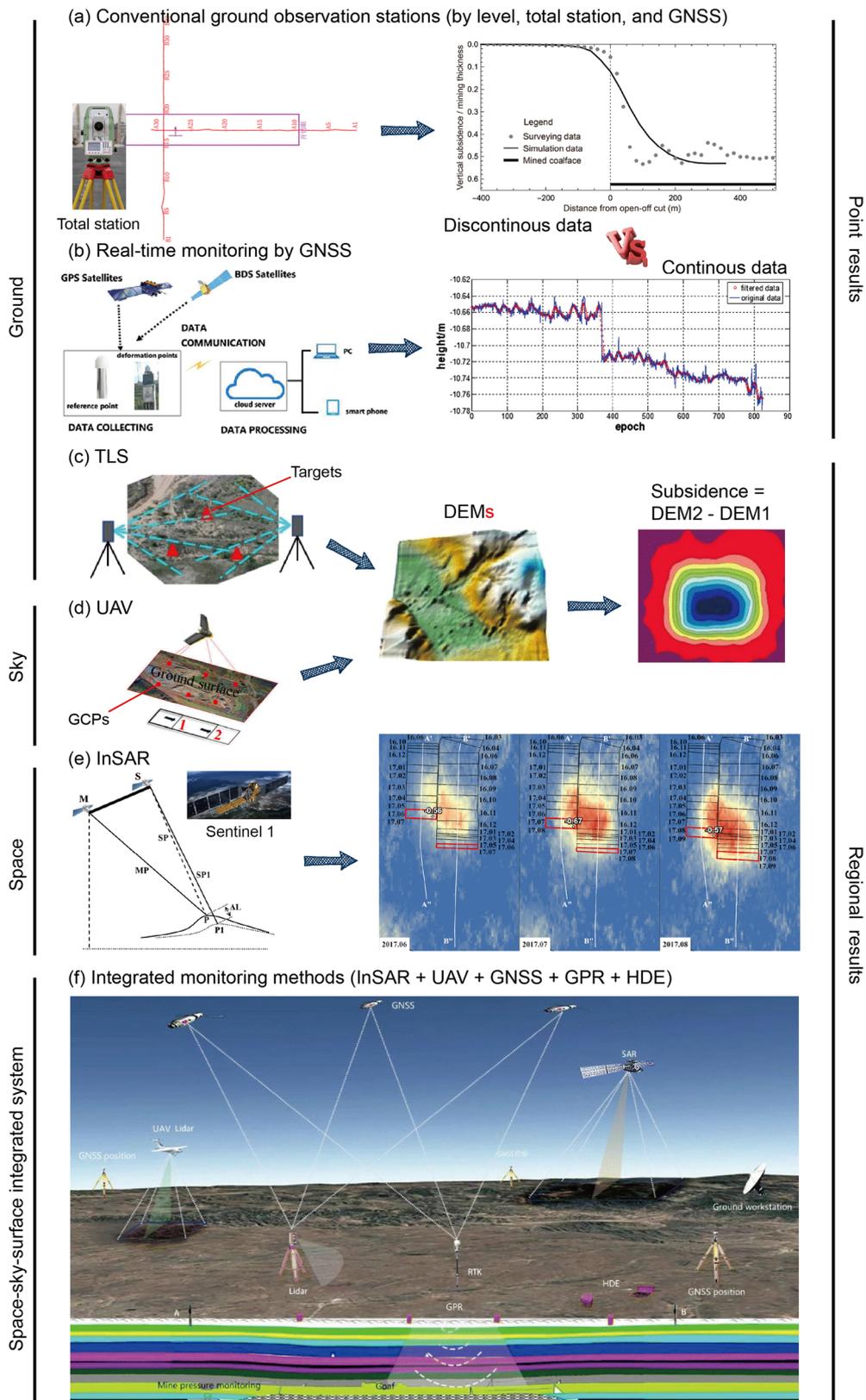
Fig. 2 Mining subsidence monitoring methods (Cai et al. 2020; Gu et al. 2020; Hu et al. 2021; Ilieva et al. 2019; Tao et al. 2018; Zhang et al. 2021c; Zhou et al. 2020)

only a few devices will be arranged at important monitoring points, such as significant ground objects or the key points of a subsidence basin.

(3) Terrestrial laser scanning (TLS)

TLS is a kind of light detection and ranging (LiDAR) technology and can be either stationary or mobile. It can acquire point cloud data of the surface of a whole 3D object with high density and high accuracy. Generally speaking, the precision of a single point obtained by a TLS device is approximately equal to that obtained by a total station (Pirotti et al. 2013; Zhang et al. 2008). TLS has been widely used in deformation and damage studies, such as those of cultural heritage or rock structure (Fanti et al. 2012; Gigli and Casagli 2011; Viero et al. 2012).

In mining subsidence research, TLS has often been employed to monitor subsidence-influenced ground objects, such as buildings (Li and Wang 2021; Li et al. 2019; Lian et al. 2020a), high-voltage towers (Lian and Hu 2017), and roads (Ao et al. 2013). For measuring the subsidence basin, conventional data acquisition techniques cannot retrieve information on the whole area, but TLS can overcome this disadvantage (Song et al. 2015; Szwarkowski and Moskal 2018).

However, because the range of the subsidence basin (several hundred meters to several kilometers in the strike direction of a working face) is generally larger than the surveying range of the TLS device, it has to separate the subsidence range into several subareas (observation stations) and splice them through targets, as shown in Fig. 2c. Errors in target splicing or station transfer measurement will reduce the accuracy of point cloud data. Zhou et al. (2014a) developed a GNSS and TLS combined monitoring technology to enhance the splicing accuracy. Gu et al. (2020) abandoned the target by using existing ground feature points to splice the multi-station point clouds in the same phase, simplify field surveying, and reduce labor intensity.

In TLS data processing, the subsidence basin is normally obtained through the subtraction of two digital elevation models (DEMs), which are reconstructed by TLS point cloud data. Some methods were put forward to improve the DEM modeling results, including a decimal Morton code-based indexing method (Ao et al. 2016) and a curved NURBS (a mathematical model) surface algorithm (Song et al. 2015). Meng et al. (2017) proposed to divide the point cloud data of two periods using the same grid to directly obtain the subsidence surface without DEM modeling.

There are still some other problems that need to be solved in TLS subsidence monitoring, such as difficulty in obtaining surface horizontal displacement and the immaturity of surface vegetation removal technology (Lian et al. 2019; Zhang et al. 2008).

3.2 UAV monitoring methods

UAVs are now one of the most popular platforms for field data collection (Chao et al. 2010; Colomina and Molina 2014; Li 2016; Li and Li 2014; Watts et al. 2012). The year 2015 is even known as ''the first year of civilian UAV'' (Ren et al. 2019). The UAV platforms include fixed-wing and rotor-wing, and the main sensors (loads) include visible band, near-infrared, multi-spectral, hyper-spectral, thermal, laser scanner, and synthetic aperture radar sensors. Recently, the appearance of low-cost UAVs (e.g., DJI Phantom) makes terrain measurement rapid and cheap.

There are many UAV applications in mines for all stages of exploration, exploitation, and restoration (Park and Choi 2020; Ren et al. 2019). Limiting subsidence monitoring to the narrow sense of obtaining the surface displacement and deformation values, visible light cameras (digital cameras) and airborne laser scanning (ALS, i.e., UAV with a load of LiDAR) are popular sensors. Summarizing the recent studies (in the past 5 years) of mining subsidence monitoring by UAV, the typically used platform, sensors, and reported measurement accuracies are listed in Table 2.

As Table 2 illustrates, when using rotor-wing UAVs with digital cameras, if excluding studies using the DJI Phantom series (the reported accuracy is 50–140 mm (Stupar et al. 2020; Suh and Choi 2017; Tian et al. 2018)), the subsidence monitoring accuracy is 20–47 mm, reaching centimeter scale. When using rotor-wing UAVs with ALS, if excluding the study by Miao et al. (2021) with an accuracy of 111–156 mm, the monitoring accuracy is 39–73 mm, also reaching centimeter scale. Compared to TLS, measurement by ALS is less accurate but can obtain the entire subsidence basin faster (Ćwiąkała et al. 2020). Fixed-wing UAVs are less employed in subsidence monitoring because they fly faster than rotor-wing UAVs, and the monitoring accuracy of the former (about 100 mm) is normally lower than that of the latter.

Mining subsidence is obtained by the subtraction of DEMs got by two UAV flight missions, as in Fig. 2d. Therefore, selecting or improving the ground filtering algorithm to get rid of the impact of non-topographic factors (e.g., surface attachments) on DEM is a very important subject in UAV data processing. It is found that no algorithm can perform well in every terrain condition (Serifoglu Yilmaz and Gungor 2016; Zeybek and Şanlıoğlu 2019), especially in areas with large surface variations (Sithole and Vosselman 2004). Tian et al. (2018) used cross-sectional point cloud filtering to remove the impact of vegetation and ponding, where the average mean square error of the sampling points was about

Table 1 Comparison of the typical characteristics of the main mining subsidence monitoring methods

Method	Conventional ground observation station			UAV	InSAR
	Level	Total station	GNSS		
Field workload	High	High	High	Low	–
Office workload	Low	Low	Low	High	High
Data type	Points	Points	Points	Planar data/ point cloud	Planar data/ point cloud
Plane accuracy	–	2 mm	2 mm (static)/ 2 cm (kinematic)	10 cm	–
Height accuracy	2 mm	5 mm	5 mm (static)/ 5 cm (kinematic)	10 cm	10 mm

2 cm lower than that without filtering. Lian et al. (2021a) tried five software to process subsidence data and found that the effects of ground filtering algorithms were significantly dependent on the selection of optimal parameters, although some of those were not easy to adjust. Employing an adaptive triangulated irregular network algorithm achieved the best result and improved the precision of the maximum subsidence value to 98%.

Another issue of concern in UAV monitoring is that ground control points (GCPs) is usually required for geographical correction. It has been proved that using GCPs can be effective in providing highly accurate (to centimeter scale) DEM products (Akturk and Altunel 2019; Padró et al. 2019; Toth et al. 2015). However, the arrangement and measurement of GCPs are time-consuming and laborious work. Therefore, if the UAV is equipped with high-precision positioning modules (e.g., RTK and/or post-processed kinematic (PPK)), it can be considered to monitor the subsidence without setting GCPs (Lian et al. 2021a, b).

Furthermore, as terrain fluctuations reduce topographic monitoring accuracy, some studies attempted to pre-import a low-resolution DEM (relative to the measurement result) of the surveying area or to load other sensors before a flight, and then employed a flight mode of varying absolute altitude (flight height relative to the ground is constant) to ensure consistent image resolution to improve the monitoring accuracy (Lian et al. 2021a, b; Yang et al. 2021).

In general, compared to conventional surveying instruments, UAVs can obtain the data of the subsided area faster but are less accurate. The UAV monitoring accuracy can reach several centimeters to more than ten centimeters. Therefore, for surveying the main subsidence area—most likely with a vertical subsidence of several meters—the accuracy of UAVs can meet the monitoring requirement, but they are insufficient for jobs aimed at obtaining subsidence boundaries where the vertical subsidence equals 10 mm.

3.3 InSAR monitoring methods

InSAR is a kind of active remote sensing technique that interferes (twice or multiple times) SAR data of the same area to obtain the topography or deformation of the ground surface, as in Fig. 2e. It originated in the 1970s and was first used to obtain elevation data (Graham 1974; Rogers and Ingalls 1969). Taking the most commonly used measurement mode of repeat-track SAR interferometry as an example, the expression of the interferometric phase (ϕ_{int}) of a target point is (Ferretti et al. 2001; Hanssen 2001; Hooper et al. 2007; Moreira et al. 2013):

$$\phi_{int} = \phi_{flat} + \phi_{topo} + \phi_{def} + \phi_{atmo} + \phi_{noise} \tag{1}$$

where ϕ_{flat} is the flat phase caused by reference ellipsoid surface, ϕ_{topo} is the terrain phase caused by topography, ϕ_{def} is the deformation phase caused by surface displacement, ϕ_{atmo} is the atmospheric phase caused by the difference in

Table 2 Some latest applications of mining subsidence monitoring by UAV

Literature	Platform	Sensor	Accuracy (mm)
Lian et al. (2021ab), Stupar et al. (2020), Suh and Choi (2017), Tian et al. (2018), Yang et al. (2021)	Rotor-wing	Digital camera	20–140
He et al. (2021a), Miao et al. (2021), Tang et al. (2020), Zheng et al. (2022)	Rotor-wing	ALS	39–156
Wang et al. (2022b), Zhou et al. (2020)	Fixed-wing	Digital camera	100–118

Note: The "Accuracy" column is not strict and is only for reference because i) the hardware performances, flight parameters, and data processing methods in the published works are different; and ii) the accuracy indicators are different—in most works, the error of sampling points or the mean square error is used

atmospheric phase delay between passes, and ϕ_{noise} is the noise phase caused by processing and thermal noise.

The concept of differential InSAR (D-InSAR) was first proposed in 1989 and enabled surface deformation monitoring by removing the contribution of ϕ_{topo} in Eq. (1) to extract ϕ_{def} (Gabriel et al. 1989). Then, the InSAR technique was first applied in 1996 to monitor surface subsidence caused by underground mining by Carnec et al. (1996). Due to its advantages, such as full-time and all-weather operation and wide imaging range, as well as the continuous updating of SAR satellites, this technique has become an important means to monitor subsidence in the past two decades, and its accuracy can reach centimeter or even millimeter scales (Chang et al. 2005; Chen et al. 2021; Zhu et al. 2011). Some of the latest applications of mining subsidence monitoring by InSAR can be found in Table 3.

The following problems still exist in the current use of InSAR techniques to monitor subsidence in mine areas: (1) the surface conditions of a mining area, such as vegetation and topography, are complex, which leads to the phenomenon of decoherence (Tian and Liao 2013); (2) mining subsidence is usually characterized by large gradients of deformation, long duration, and rapid deformation at specific stages. As the maximum displacement that can be monitored by InSAR is very dependent on the SAR wavelength (even less than the theoretical value in practice), some mining subsidence that occurs in the period between two imaging events may be too large to be detected at some monitoring spots (Ng et al. 2009); (3) limited by its measurement principle, the D-InSAR technique can only obtain the displacement in the line of sight (LOS) direction of the radar (Yang et al. 2020; Zhu et al. 2019), that is, only a one-dimensional projection of the true 3D displacement (vertical subsidence plus horizontal displacement vector) is extracted. Furthermore, this technique is very insensitive to the surface displacement in the north–south direction, namely, the flight direction of the satellite, which is perpendicular to the LOS direction.

Many scholars have tried to apply improved InSAR techniques to mining subsidence monitoring. Some multi-temporal InSAR (MT-InSAR) techniques (Berardino et al. 2002; Ferretti et al. 2011; Hooper et al. 2012; Karamvasis and Karathanassi 2020; Morishita et al. 2020), such as small baseline subset InSAR (SBAS-InSAR) (Du et al. 2021; Lanari et al. 2004; Yin et al. 2011; Yuan et al. 2021), persistent scatterer InSAR (PS-InSAR) (Colesanti et al. 2007; Liu et al. 2022), and distributed scatterer InSAR (DS-InSAR) (Li et al. 2021b), have proved effective in decreasing the effects of temporal and spatial decoherence and atmospheric delay (Hu et al. 2021), which makes InSAR more applicable for subsidence monitoring. Some scholars have combined multiple InSAR techniques to monitor mining areas (Chen et al. 2021). For example, InSAR techniques have been combined with the offset-tracking method for monitoring mining-induced large-gradient subsidence, and subsidence along both the LOS and azimuth directions (Chen et al. 2020b; Ou et al. 2018). Some scholars have fused multi-platform SAR imagery or multi-track SAR data to acquire multi-dimensional (2D and 3D) surface displacement (Dong et al. 2021; Samsonov et al. 2013; Zhang et al. 2021b; Zheng et al. 2018).

In addition, some scholars used InSAR monitoring data to invert the parameters of the probabilistic integral

Table 3 Some of the latest applications of mining subsidence monitoring by InSAR

Band	Source	Technique	Study area	Max vertical subsidence (mm)	Accuracy (mm)	Literature
C	RADARSAT-2	TS-InSAR	Bu'ertai Mine (China)	199		Ma et al. (2016)
	Sentinel-1	D-InSAR	Upper Silesian Coalfield (Poland)	> 800	183	Ilieva et al. (2019)
	Sentinel-1	TS-InSAR	Juye Coalfield (China)	218	59	Chen et al. (2021)
	Sentinel-1	D-InSAR	Yulin Coalfield (China)	300	9.28	Dong et al. (2021)
	ENVISAT ASAR	TS-InSAR	Jharia Coalfield (India)	90		Kumar et al. (2020)
L	ALOS-PALSAR	TS-InSAR	Southern Coalfield (Australia)	> 300		Du et al. (2016)
	ALOS-PALSAR	TS-InSAR	Lengshuijiang Mine (China)	> 250		Yin et al. (2011)
	ALOS-2	D-InSAR	Bu'ertai, Appin and Metropolitan Mines (China and Australia)	380		Ng et al. (2017)
X	TerraSAR-X	D-InSAR	Daliuta Mine (China)	656	17	Chen et al. (2014a)
	TerraSAR-X	D-InSAR and offset tracking	Cuijiazhai Mine (China)	2000	74.8	Ou et al. (2018)
	COSMO-SkyMed	D-InSAR	Wannian Mine (China)	160	8.2	Liu and Huang (2021)

Note: The "Accuracy" column is not strict and is only for reference because the accuracy indicators are different among the literature, including the error of sampling points and mean square error

method and temporal function model to improve the subsidence prediction methods (Fan et al. 2014; Jiang et al. 2021; Yang et al. 2017a, b, 2022). Others have used InSAR to monitor residual subsidence, as the InSAR technique is good at monitoring small gradient deformations (Chen et al. 2020a; Modeste et al. 2021). These attempts have broadened the application of InSAR techniques in subsidence studies.

With the increasing channels of SAR data acquisition, the improving spatial and temporal image resolutions, as well as the developing InSAR theories, it can be foreseen that the applicability of the InSAR technique, the accuracy of the obtained subsidence data, and the maximum displacement that can be detected will be further improved.

3.4 Integrated monitoring methods

Space-air-ground integrated earth observation network is the scientific frontier in the field of earth observation (Li 2012). In 2003, the United States, China, the European Union, and other countries (regions) established the Global Earth Observation System of Systems (GEOSS), aiming to build a distributed global earth observation system based on the integration of multiple systems.

In the field of mining subsidence monitoring, many scholars tried to obtain the subsidence values by integrating data from multiple sources. The most common application is to use conventional ground measurement results, which are considered reliable in accuracy, to verify or calibrate remote sensing results. For example, conventional observed data can be used as GCPs for UAV measurement (Lian et al. 2021a), or to correct the results achieved by InSAR technology (Tang 2011).

There are also further comprehensive integration applications. Chen et al. (2013) integrated D-InSAR and TLS technology to monitor subsidence, and the mean absolute error was 23.1 mm (compared to the subsidence obtained by leveling). Wang et al. (2022b) integrated a UAV with a visible light camera, D-InSAR, and SBAS-InSAR technologies, and obtained the complete high-precision deformation field with complete time series. As illustrated in Fig. 2f, National Energy Group Co., Ltd. and China University of Mining Technology (Beijing) established a space-sky-surface integrated system consisting of the technologies of InSAR, UAV (with a visible light camera and an infrared camera), GNSS, ground-penetrating radar (GPR), and high-density electricity (HDE) (Zhang et al. 2021c, 2020). This work realized the high-efficient and high-precision monitoring of mining subsidence under the conditions of complex terrain and large area.

4 Calculation methods used in mining subsidence

The subsidence calculation methods, also known as computation methods or prediction methods, are achieved by summarizing, analyzing, and fitting the observation data. However, in practices, especially those related to security, the calculation cannot take the place of the observation, as there are always some differences between them (Khanal and Hodgkinson 2021).

For analyzing the co-occurrence terms in subsidence calculation studies, a total of 1693 articles related to this subject from 2005 to the present were downloaded from the website Web of Science and managed by the software CiteSpace. The results are shown in Fig. 3, which illustrates that the calculation methods have strong pertinence to certain engineering backgrounds, such as geological conditions, mechanical properties, overlying strata, and surrounding rocks. In other words, these methods are not universally adaptive. The figure also shows that the probability integral method has always been an important method for subsidence computation, but with the development of mathematics and statistics, new methods such as neural networks and data mining are increasingly applied to compute the subsidence. Besides field surveying, numerical and physical simulations are also common methods for obtaining subsidence data, which can be used as auxiliary derivation or verification tools for the calculation methods.

Referring to the traditional classifications of subsidence calculation methods (Bahuguna et al. 1991; China Coal Research Institute 1981; He et al. 1991a), this paper intends to divide the calculation methods into two categories: those with the computational functions fitting the element subsidence and those fitting the full-scale subsidence. The main calculation methods discussed in this paper can be found in Fig. 4.

4.1 Influence function methods

The influence function method computes the subsidence of a mining area in two steps. As illustrated in Fig. 4a, it first uses a function (or functions), which is called the influence function, to fit the subsidence induced by an element excavation and then employs the integration or summation method to calculate the full-scale subsidence induced by the whole excavated zone, a working face or a mine, for instance.

The influence function method originated in Poland and has gone through the stages of method selection, model modification and improvement, and algorithm optimization. From the 1950s to 1960s, research was successively

Fig. 3 Co-occurrence terms analysis of the papers related to mining subsidence calculation via the software CiteSpace (Source of the papers: Web of Science)
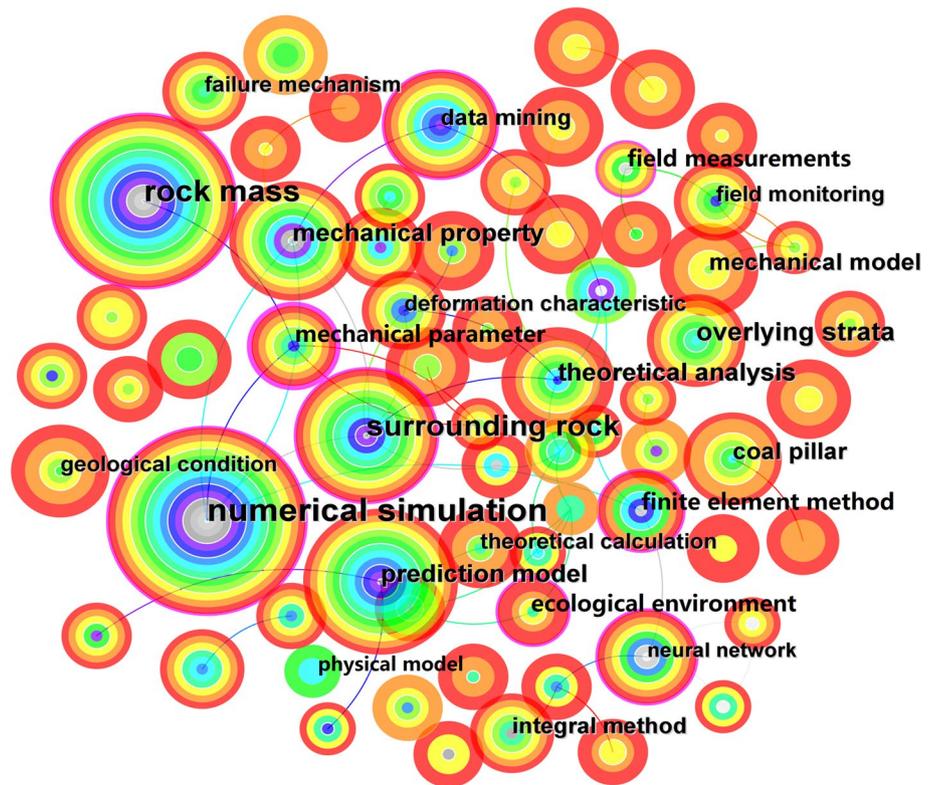
carried out in major coal mining countries such as Poland, the Soviet Union, Germany, and Britain, and significant achievements were made (Liu and Dai 2016; Yang and Dai 2016). For decades, the influence function method has been widely used all around the world. At present, scholars are still trying to improve the applicability of this method in theories and practices (Knothe 2005; Luo and Cheng 2009; Malinowska et al. 2020; Saeidi et al. 2013; Sheorey et al. 2000; Wang et al. 2013; Yang and Luo 2021).

The Knothe influence function method (first published in Knothe's Ph.D. thesis in 1951), the stochastic medium theory by Litwiniszyn (1954), and the probability integral method (based on the stochastic medium theory) by Liu and Liao (1965) are the most often used influence function methods. Although the basic principles of the above three methods are not quite the same, they all can be deduced to the same influence function (by different derivation processes) for simulating the element vertical subsidence (He et al. 1991a; Jiang et al. 2019; Litwiniszyn 1974), which is a probability density function of a Gaussian distribution, as depicted as Eq. (2) in Table 4:where $f(x)$ is the influence function for element vertical subsidence, and R is the main influence radius.

Of course, the expressions of the influence function methods are not unique. Other applicable equations, such as Eqs. (3) and (4) listed in Table 4, can also be used as the

influence functions; their forms are similar to a probability density function of a Gaussian distribution. Moreover, there are some quite different equations that have been used, such as a Weibull distribution based on the clastic theory (He 1988; He et al. 1982).

The influence function method has some shortcomings, and many improvements have been made to extend the applicability of this method.

(1) The influence function method neglects the mechanical relationships within the rock strata but directly employs expressions to describe the geometric shape of the subsided ground surface (or rock stratum). As it simplifies the impact of complex and uncertain geological and mining conditions, the mechanical mechanism of the movement and development in the layers from the extracted coal seam to the ground surface is unclear.

Academician Liu Baochen, one of the presenters of the probability integral method (the most often used influence function method in China), reviewed this method and specified in *Introduction to Rock Mechanics in Mines* (Liu 1982) that it should take the rock rheology model into account to study the stress–strain of rock mass in a unified way. However, the constitutive equation of the stochastic medium theory, which is the foundation of the probability integral
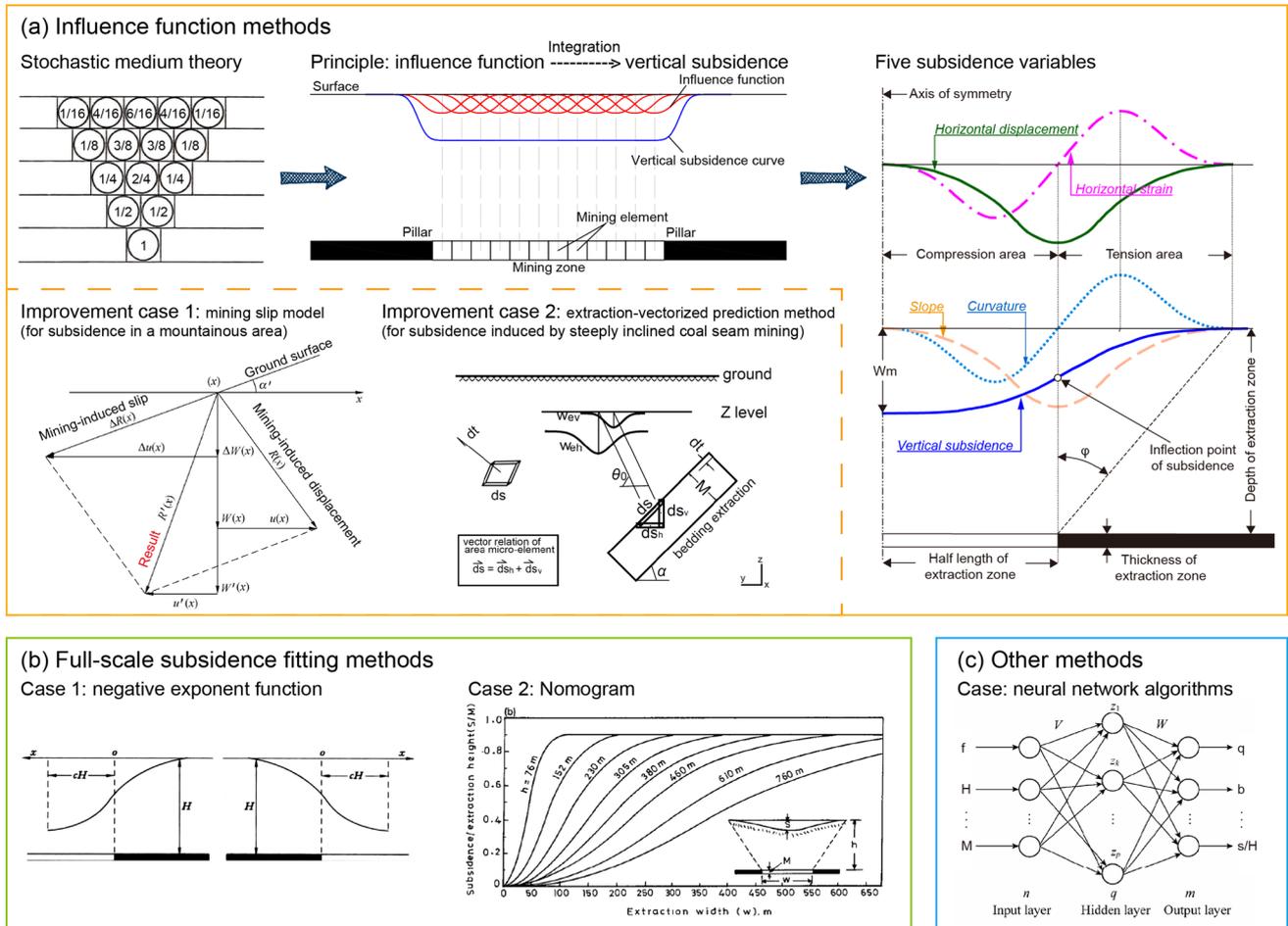
Fig. 4 Mining subsidence calculation methods (Dai et al. 2002b; Guo et al. 2004; He et al. 1991a; He 2003; Liu and Liao 1965; National Coal Board 1975)

method, is still unknown, so the probability integral method, as well as the other influence function methods, has not yet broken through the theoretical limitations to achieve Liu's goal. Yang and Dai (2016) also proposed that mining subsidence calculations should use a comprehensive model of stress–strain–displacement or numerical methods based on

mechanics and displacement theories. Unfortunately, the above-mentioned research on the correlation between surface subsidence and stress–strain in the overlying strata has made slow progress thus far.

(2) The influence function method is a calculation model based on media that are stratiform, horizontal, and isotropic. It also can be adapted to inclined coal seams with a dip angle less than 45° (or 55°). In this case, the element influence functions of vertical and horizontal movements must be derived separately (He et al. 1991a; Luo and Cheng 2009). Theoretically, the method cannot be directly applied to conditions such as mountain surfaces, steeply inclined coal seams, or thick loess layers. Many subsidence studies have been carried out on these non-general conditions to expand the application areas of the influence function method, and some examples are given below:

Table 4 The most commonly used influence functions in subsidence computation

Presenter	Partial users	Influence functions	
Knothe	Knothe (1953, 1957), Zenc (1969), Reddish and Whittaker (1989)	$f(x) = \frac{1}{R} e^{-\pi \left(\frac{x-s}{R} \right)^2}$	(2)
Litwiniszyn	Litwiniszyn (1957), Liu and Liao (1965), Sroka et al. (2006)	$f(x) = \frac{\sqrt{t}}{R} e^{-t\pi \left(\frac{x-s}{R} \right)^2}$	(3)
	Ehrhardt and Sauer (1961)	$f(x) = \frac{\sqrt{4.6/\pi}}{R} e^{-4.6 \left(\frac{x-s}{R} \right)^2}$	(4)

a) Calculating subsidence in a mountainous area

To calculate the subsidence under mountainous topography, He Wanlong's research team suggested to consider the mining-induced ground slip (directed to the downward direction of the surface) within the probability integral method, as in Fig. 4a. In this case, a so-called surface feature coefficient should be determined according to the concavity and convexity of landforms, and then the surface displacement under mountainous terrain is calculated as the vector summation of mining-induced ground displacement and mining-induced ground slip (He 1981, 1983; He and Kong 1986; He et al. 1991b; Ma et al. 2000). These research achievements were arranged and summarized in *Mining Subsidence and its Induced Damage in Mountain Areas* (He 2003) and also included in *The Manuals of Coal Mining* (Hu et al. 2017).

In addition, some studies improved the above-mentioned mining slip model or optimized its parameters (Guo et al. 2016b; Han et al. 2013; Tang et al. 2019; Wang et al. 2014), some attempted to use a new influence function to fit the element subsidence under mountainous conditions and then computed the full-scale subsidence basin (Cai et al. 2014, 2016b), and some worked on specific areas, such as a valley bottom (Guo et al. 2016a).

b) Calculating subsidence induced by steeply inclined coal seam mining

The subsidence laws of steeply inclined coal seams are very different from those of horizontal and gently inclined coal seams. The conventional influence function methods have poor applicability to the subsidence calculation of steeply inclined coal seams. In response, Dai Huayang's research team developed an extraction-vectorized prediction method (Dai et al. 2003, 2002a, b), as in Fig. 4a. This method employs the seam's dip angle as an input variable and the symmetrical subsidence troughs induced by hypothetical horizontal and vertical seam mining as the boundary conditions. In fact, this method can be used to calculate the subsidence of coal seams with arbitrary dip angles from 0° to 90°, which certainly includes steeply inclined coal seams. Later, this team found that the influence of subsidence would accumulate when an extra-thick steeply inclined coal seam is excavated by horizontal slice mining technology. Therefore, a prediction model related to the mining influence propagation angle was established to compute the subsidence under such conditions (Dai et al. 2013a, b; Yi 2009).

Otherwise, the vertical integration method and propagation angle integration method recommended by *The Manuals of Coal Mining* (Hu et al. 2017) are also feasible influence function methods that can be used to solve the subsidence induced by steeply inclined coal seams.

c) Calculating subsidence under other non-general conditions

Some studies improved the influence function methods or corrected the parameters to calculate subsidence under non-general conditions such as a thick unconsolidated layer (Chang et al. 2014; Liu et al. 2009, 2013; Lv and Wang 2019; Tang et al. 2019; Wang et al. 2013), a key stratum (Li et al. 2015; Qian et al. 1996; Yang and Luo 2021), a fault (Zhang and Zhong 1995), or a geologic structure (Li et al. 1995).

Furthermore, there have been some other improvements of the influence function methods aiming at dynamic subsidence computation (Cui et al. 2001; Hu et al. 2016a, 2011; Li et al. 2020; Wang et al. 2022a; Zhang et al. 2021a) and sub-critical subsidence computation (Chang et al. 2014; Dai and Wang 2004).

(3) Compared with the full-scale subsidence fitting method, the influence function method has more complicated formulas and computing process (i.e., integrate the influence function to obtain the final subsidence). This process requires high mathematics and computing abilities. At present, with the fast development of information technologies, this problem has been largely solved. Particularly, mathematical software such as MATLAB (Li et al. 2008; Wang et al. 2021) and Mathematica (Cai et al. 2016a; Cai et al. 2014) are becoming more and more popular in today's scientific work, so that researchers can focus on analyzing and summarizing the subsidence laws into calculation methods, leaving the difficult algorithm implementation process to these mathematical software.

4.2 Full-scale subsidence fitting methods

The full-scale subsidence fitting method uses functions (also curves or tables were used in the past, which can be considered implicit functions) to directly fit the measured subsidence profile data. It has strong pertinence and good fitting effect but limited applicability. In traditional classifications, it can also be divided into the typical curve method and the profile function method.

The most important functions or curves include the following: negative exponent function (Wang 1981), hyperbolic function (Liu and Dai 2019), error function (Tian et al. 2009), Pearson function (Kongji Mine in Huainan Mining Bureau and China Coal Research Institute 1979), typical curve of Fengfeng Coalfield (only for the vertical evidence) (He et al. 1991a), typical curve of Pingdingshan

Table 5 Calculation methods of mining subsidence versus dip angles of coal seam

Calculation methods	Dip angles of coal seam		
	0°–30°	30°–45°	45°–90°
Typical curves	√		
Negative exponential function method	√		
Probability integral method	√	√	
Pearson function method			√
Extraction-vectorized prediction method	√	√	√

Coalfield (Kailuan Coal Research Institute 1978), Nomogram (National Coal Board 1975), and profile-curves used in Britain, Russia, Poland, and Hungary (summarized in *Mining Subsidence Engineering* by Kratzsch (1983)).

The negative exponent function method, as in Fig. 4b, is a commonly used full-scale subsidence fitting method in China. *The Regulations of Coal Mining* (Ministry of Coal Industry PRC 1986) provides the complete calculation process of the subsidence along the main section of a rectangular working face by using this method. Some other studies showed that this function could also be used to predict the surface subsidence in mountain areas (Dai 1990), to compute the movement and deformation of overlying strata (not only those of the ground surface) (Wang et al. 2016a), and to describe different subsidence developing stages by different negative exponential functions (Huang and Wang 2008).

The Manuals of Coal Mining (Hu et al. 2017) recommends employing the Pearson function method to compute subsidence induced by steeply inclined coal seams and summarizes its prediction parameters, including those of Kongji Mine and Jiulong Mine in Huainan Coalfield, Datai Mine in Beijing, and Majiagou Mine in Kailuan. The Pearson function was first put forward by fitting field surveying data obtained in Kongji Mine (Kongji Mine in Huainan Mining Bureau and China Coal Research Institute 1979). Ou and Zhu (1984) further analyzed the physical meanings of the parameters of this function and made some improvements to this method.

Some of the often used subsidence calculation methods, including the influence function method and full-scale subsidence fitting method, are listed in Table 5 with their applicable coal seam dip angles.

4.3 Other methods

Due to the integrated development of various disciplines, many complex mathematical methods have been introduced into subsidence calculation, such as neural networks, grey system mode (Guo and Zhang 1997), fuzzy probability theory (Li et al. 2013), ant colony algorithms (Jia 2015),

quantum particle swarm algorithms (Zhu et al. 2020), and wolves algorithms (Li et al. 2021a).

Among them, neural network algorithms are the most widely used, as in Fig. 4c. The so-called network is usually an approximation of an algorithm or a function, which is composed of a large number of nodes connected with each other. The connection between every two nodes represents a weight. A neural network training algorithm is used to optimize the weight value, so that the whole network has the best prediction effect. The neural network method is normally used in subsidence studies to obtain the numerical subsidence values (Chen et al. 2022; Kumar et al. 2022; Zhang et al. 2011), or to fit (or optimize) the computational parameters of existing methods, such as those of the probability integral method (Chi et al. 2021; Guo et al. 2003, 2004). However, this method requires a lot of data to train the algorithms.

5 Simulation methods used in mining subsidence

Simulation methods are common and effective tools to evaluate complex mechanical behaviors and to study the deformation and failure mechanisms of rock and soil mass (Gao 2019; Hoek and Martin 2014). In simulations, the geological and mining conditions should be built similarly to those in the field; then, the excavation process is carried out, and its impact is studied. Generally, the simulation can be implemented by a computer (software) or by setting up a physical model in the laboratory, which are termed numerical simulation and physical simulation, respectively, as shown in Fig. 5. In mining subsidence studies, this method is often employed as a supplement to field surveying data or as a verification of an improved calculation method, and it rarely appears alone.

5.1 Numerical simulation methods

The geological and mining conditions are complex and fixed in an actual mine, and it is hard to separate specific conditions from them for a given research purpose. As numerical simulation is an experimental method to reproduce the process from excavation to damage by computer, it can adjust those in situ conditions to conduct a series of tests without the restrictions of the real world.

In the process of subsidence simulation, the main steps include the selection of simulation method and software, determination of rock mass parameters and boundary conditions, calibration by the field surveying data, and subsidence data extraction and analysis. Some choices can significantly influence the precision and even correctness of the simulation results, including the constitutive model selection,

(a) Numerical modeling (by softwre FLAC3D)

(c) Physical modeling

(b) Numerical simulation result (slope surface)

(d) Physical simulation result
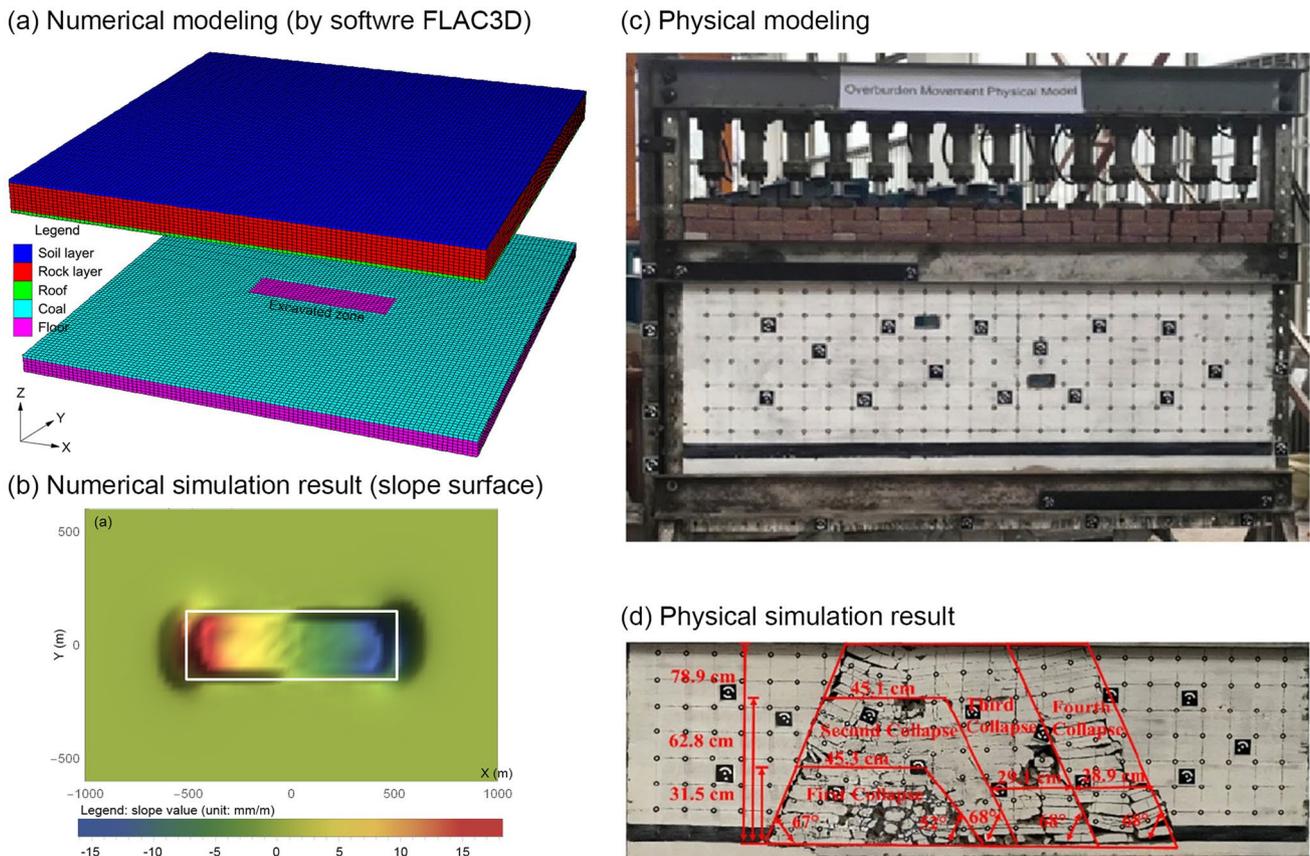
Fig. 5 Mining subsidence simulation methods (Cai et al. 2020; Luo et al. 2020)

continuity of the rock mass (continuous or discrete), lithological parameters, and the relationship between the calculation iteration step and the real-world time (Cui and Deng 2017; Gao 2019; Gong et al. 2002; Wang and Feng 1996). Therefore, the numerical simulations generally require verification with engineering practices.

In mining subsidence studies, the commonly used numerical simulation methods include the finite element method (FEM, including the software of ABAQUS and ANSYS), the finite difference method (FDM, including the software of FLAC and FLAC3D), the discrete element method (DEM, including the software of UDEC, 3DEC, and PFC), and the boundary element method (BEM) (Jing 2003; Ma and Yang 2001). FDM, FEM, and BEM are continuum methods, whereas DEM is a discontinuum method. These methods emerged in the 1950s and 1970s (Deng et al. 2014). With the development of computer technology, there are many mature commercial software options now; in the field of geotechnical engineering, those developed by ITACSA Inc. are the most popular (Xie et al. 2005). As there are many abstruse mechanical principles involved in the numerical simulation methods, mining subsidence researchers and technicians make few contributions to method improvement

and software development, and they mainly participate in applications.

Numerical simulation methods have been applied to the following mining subsidence topics:

(1) Mining subsidence laws under different geological and mining conditions, such as key strata (Xu et al. 2005), faults (Wu 2011; Xie et al. 1999), underground water (Zhao and Konietzky 2020), integral overburden (Sun et al. 2019), inclined coal seams (Sun et al. 2021), steeply inclined coal seams (Gao and Yu 2003), multiple seams (Ghabraie et al. 2015), backfilling mining (Huang et al. 2017; Liu et al. 2019), shallow mining (Cui et al. 2014), and deep mining (Yu et al. 2021);

(2) Inversion for subsidence-related parameters, such as rock mechanics of the overlying strata (Zhang et al. 2017a) and prediction parameters of the influence function methods (Cai et al. 2019);

(3) The influence of subsidence on surface objects, such as underground pipelines (Peng et al. 2012), reservoir dams (Xu et al. 2013), and land resources (Cai et al. 2020).

In the actual mining-damage process, the rock mass is not completely continuous or discrete as defined in the aforementioned simulation software. Some continuum-discontinuum coupling methods have been developed (or applied) to study the stress and strain in the surrounding rocks after mining (Gao and Chen 2016; Wang et al. 2018). At present, these methods are not yet used in subsidence research, but they are arguably worth trying.

5.2 Physical simulation methods

Physical simulation, also known as similar material simulation, originated in the Soviet Union in 1937 and was then widely used in major coal mining countries around the world (Cui and Deng 2017; He et al. 1991a). It is an experimental approach based on the principle of similarity.

The gravity loaded sand–plaster model made with materials of different mixture ratios of sand, plaster, gypsum, lime, and maybe loess is one of the best options for modeling mining subsidence (Cui and Deng 2017; Ghabraie et al. 2015). The most important step is to build the physical model with its overlying strata and geological conditions being as "similar" to the prototype as possible (Luo et al. 2020). Therefore, the built model must meet a series of similarity requirements, including geometry, physical and mechanical properties, boundary conditions, and initial conditions. The theories of similarity, including geometric similarity, kinematic similarity, and dynamic similarity, must be well applied during the model design stage.

During the test, the excavation action is simulated by digging out part of the model, which is assumed as the working face. This action leads to movement inside the model. By observing the model, the movement and damage of the model strata and ground surface are obtained, and then those of the in situ rocks can be deduced. The most commonly used and reliable observation method is the total station (or theodolite) observation method (Liu et al. 2008). With the rapid development of technology, photogrammetry (Luo et al. 2020) and laser scanning technology (Ghabraie et al. 2015) are also often employed now. Particularly, for studying stress and strain, mechanical transducers should be buried in the model.

In practice, the physical simulation method is often used to observe the movement procedure of the overlying strata and ground surface for analyzing or verifying the subsidence rules and mechanisms, as this procedure cannot be visually detected in real rocks. Some researchers used this method to study the influence of mining patterns on subsidence, such as in long-wall mining (Luo et al. 2020; Zhang et al. 2017b), short-wall mining (Liu et al. 2019), the controlling role of isolated pillars (Liu et al. 2011), and comparisons of sub-level mining and slice mining (Cui et al. 1999). Some researchers considered the impact of the resource occurrence

mode on subsidence, such as in shallow coal seams (Cao et al. 2021; Cui et al. 2014; Wang et al. 2004), steeply inclined coal seams (Dai et al. 2000a; Ren et al. 2010), and multiple seams (Ghabraie et al. 2015; Hu et al. 2020a). Some scholars studied the effect of overlying strata and their properties on subsidence, such as in thick loose layers (Dai et al. 2010; Hu et al. 2016b; Liu et al. 2012; Wang et al. 2019; Wu et al. 2002; Zhou et al. 2014b) and key strata (Xu et al. 2005). Some researchers investigated related damage phenomena, such as ground subsidence and landslide induced by the combined influence of mining and topography (Dai et al. 2000b; Zhao et al. 2020) and ground fissures (He et al. 2021b; Yang et al. 2018). A majority of the physical simulation studies were performed with 2D models, and only a few used 3D models, as reported in the literature (Lu et al. 2021; Wang et al. 2004; Zhang et al. 2017b).

Compared with the prototype, the physical model is minified (both in size and lithology) and simplified, and the obtained model movement data may have unexpected deviations from the in situ movement. Compared with the numerical simulation, as the physical simulation is time-consuming and laborious, it cannot perform a series of simulations, which makes the simulated data insufficient to invert the calculation functions or parameters. Therefore, in mining subsidence studies, physical simulations are generally used along with field measurements or numerical simulations. They are rarely used as an independent data source for research. The numerical value of subsidence is not the direct objective of physical simulation; at least, it is not the most important objective.

6 Problems and prospects

6.1 Problems and prospects of monitoring methods

In mining subsidence monitoring, the conventional measurement methods, including leveling, traverse survey, and GNSS, are mature in technology and reliable in accuracy. Although there are some disadvantages like low efficiency and heavy field workload, they are still indispensable in subsidence monitoring in the foreseeable future. Particularly, the continuously operating GNSS system (Yu et al. 2014) will play a more important role in the real-time monitoring of subsidence in the future.

New non-contacting remote sensing technologies such as TLS, UAV (with various sensors), and InSAR will be more frequently applied to subsidence monitoring, but, at least for now, they still need to be verified or calibrated by data obtained by conventional methods. Areas of mining subsidence are mostly located in rural settings. The topographic relief, surface vegetation, and large gradient deformation are the most important factors that affect the measurement

accuracy of remote sensing technologies and therefore the most important directions for improvement. These include, for example, the problems currently encountered in InSAR monitoring such as multi-dimensional displacement and decoherence phenomenon (Zhu et al. 2017). While using remote sensing technology, data acquisition is important, as is data processing (Lian et al. 2021a). It is expected that more studies will be devoted to the improvement of data processing algorithms, the cooperative use of multiple methods and software, and so on. The space-sky-surface integrated observation system will be an effective solution and research hotspot in the field of mining subsidence monitoring and even earth observation, and will promote the transformation from Digital Earth to Intelligent Earth (Li 2012).

Furthermore, with the rapid development of the theories and technologies of the Internet of Things, big data, and cloud computing (Ren et al. 2019; Wang 2017), the combination of surveying and these methods is anticipated.

In general, the development of subsidence monitoring depends on two aspects: the development of surveying instruments and the development of data processing methods and software.

6.2 Problems and prospects of calculation methods

Influence function methods, including the probability integral method used in China, are the most commonly used subsidence calculation methods. These methods ignore the mechanical relationships within the rock strata and cannot reveal the mechanical mechanism of the occurrence and development of mining subsidence (Cui and Deng 2017; Liu and Dai 2016), which are the biggest flaws in their principles. The integration of multiple disciplines, including mining science, mechanics, geology, and surveying and mapping, will be an inevitable choice for the further development and improvement of both influence function methods and other calculation methods, and ultimately to achieve the objective of computing the mining subsidence by using a comprehensive model of stress–strain–displacement or using numerical methods based on mechanics and displacement theories (Yang and Dai 2016). Moreover, the existing subsidence calculation methods take the assumption of isotropic continuous media as the research premise. Therefore, computations under special geological and mining conditions, such as dip angle variations of both the surface and coal seams, thick loess, geological structure, and fully mechanized top coal caving mining, must be further summarized and improved in combination with the monitoring data. Meanwhile, the physical meanings of the prediction parameters need to be well considered in parameter selection or optimization.

With the development of mathematics and computer technologies, methods such as neural networks will be increasingly used in subsidence calculation. However, these methods require a large amount of input data, and the process between input and output is like a black box. How to apply them reliably in subsidence practice needs further study and verification. For now, they can be used as a supplement to the classic calculation methods.

6.3 Problems and prospects of simulation methods

At present, the most significant problem existing in numerical subsidence simulation is that this method is based on complex mechanical principles, which leads to low participation of subsidence researchers in simulation methods research and simulation software development. Solving subsidence problems by simulation heavily depends on the ability to apply the simulation tools and to interpret the simulation results (Gao 2019). The more detailed the model built, the more accurately subsidence can be predicted. However, the geological exploration at the mining site normally cannot support establishing a model consistent with the prototype, so the numerical simulation often requires calibration, which is a step of human intervention (Peng 2015b). In addition, subsidence research often needs to be combined with surface object protection. The overlying strata and surface objects differ greatly in scale and mechanical properties. How to transfer subsidence influence from ground surface to surface object requires further research.

In the actual mining-damage process, the rock mass is not completely continuous or discontinuous. Limitations exist in the commonly used subsidence simulation software, and this paper believes that the continuum-discontinuum coupling method (Gao and Chen 2016; Wang et al. 2018) should be able to more truly reflect the mining-damage process and is worth trying in subsidence research.

It is difficult for physical simulation to restore site conditions (similar to numerical simulation), and it cannot freely adjust the model parameters in modeling, so it can only obtain rough numerical results. Physical simulation is more suitable for studying the process of subsidence development, but its numerical values are not really credible.

6.4 Future needs of mining subsidence data and related studies

The quality, acquisition efficiency, and spatial and temporal coverage of the subsidence data are commonly concerned, which often cannot be balanced under certain time and economic cost constraints.

From the perspective of subsidence research, data quality (accuracy, resolution), temporal continuity (observation frequency, satellite revisit cycle), and coverage are very important, while acquisition efficiency may sometimes be compromised. For future research, we believe that, in

addition to using subsidence data to optimize calculation methods, the mechanism and process of subsidence from occurrence, development to stabilization should be further studied in terms of both mechanics and geological properties of the overlying strata, to obtain a complete description and explanation of stress, strain, and movement within the rock and soil bodies affected by mining.

From the perspective of serving mine production and safety, subsidence data acquisition should be timelier, preferably in real time. This requires the support of equipment for continuous observation, hardware and software for data processing, and transmission networks. In the future, subsidence data should be managed and applied in conjunction with the construction of 3D geological models and smart mine platforms.

From the perspective of environmental protection (including geology, ecology, land resources, humanities, etc.) in mining areas, subsidence data is one of the background materials, and the algorithms or models of re-processing the data into useful information for specific protection objects are the focus of attention.

7 Conclusions

Subsidence data acquisition methods, including monitoring, calculation, and simulation, are summarized in this paper.

Benefiting from the development of surveying and mapping and other professional technologies, the main subsidence monitoring methods now include not only conventional ground measurement methods but also new remote sensing approaches. The accuracy, acquisition efficiency, and spatial and temporal coverage of the monitoring data are indicators of interest in subsidence research and mine production. Improving these indicators is the future development direction of the monitoring methods. Furthermore, with the evolution of technologies and the decrease in cost, the integration of multiple observation techniques is expected to be increasingly applied to subsidence monitoring.

Subsidence calculation methods are divided into two categories: methods with computational functions fitting the element subsidence, and those fitting the full-scale subsidence. The existing calculation methods, such as the Knothe influence function method, probability integral method, and Pearson function, are obtained by fitting the subsidence data. We suggest that the process and mechanism of subsidence occurrence, development, and stabilization should be focused on in the future, to clarify the transformation principle of stress–strain–movement within the rock and soil layers affected by mining, then to improve or even re-derive the subsidence calculation models. In addition, including the Internet of Things, big data, and cloud computing in the calculation of subsidence is a direction worth exploring.

The simulation methods are implemented by computer software or by setting up a physical model in the laboratory, which are termed numerical simulation and physical simulation, respectively. They are generally employed as a supplement to measurement data or for verification of calculation methods.

Declarations

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