Int. J. Res. Undergrad. Math. Ed. (2022) 8:217–221
https://doi.org/10.1007/s40753-022-00183-y

Calculus at the intersection of institutions, disciplines and communities: a special issue Guest Editorial

Irene Biza[1] · Alejandro S. González-Martín[2] · Alon Pinto[3]

Published online: 11 June 2022

Note from the Editors: Following the Calculus in Upper Secondary and Beginning University Mathematics Conference (Kristiansand, August 6–9, 2019), the Editors received a proposal for a Special Issue on the theme of "Calculus at the intersection of institutions, disciplines and communities". The Editors were delighted to accept this proposal. IJRUME co-editor in chief Elena Nardi, also co-chair of the conference with Tommy Dreyfus and John Monaghan, acted as handling editor of the Special Issue. In what follows, Guest Editors Irene Biza, Alejandro S. González-Martín and Alon Pinto introduce the Special Issue.

Calculus courses are widely considered an important gateway, or gate keeper, in many academic and professional paths (Bressoud et al., 2016; Thompson & Harel, 2021). Calculus is also widely recognized as a critical stage in various transition processes, such as the transition from secondary mathematics to tertiary mathematics, the transition within and across university courses, or the transition from university to the workplace (Hochmuth et al., 2021). There is ample evidence that students around the world struggle in their calculus courses, that rates of failure in these courses are comparably high, and that students' experiences in calculus courses are strongly linked to student dropout in STEM programs (Artigue et al., 2007; Faulkner et al., 2019).

Considering the critical role and varied goals of calculus in different contexts, it may seem contradictory that calculus is very often taught de-facto as *one and uniform*

✉ Irene Biza
i.biza@uea.ac.uk

Alejandro S. González-Martín
a.gonzalez-martin@umontreal.ca

Alon Pinto
alon.pinto@weizmann.ac.il

[1] University of East Anglia, Norwich, UK

[2] University of Montreal, Montreal, Canada

[3] Weizmann Institute of Science, Rehovot, Israel

discipline, with similar curricula and teaching approaches across programs, institutions, or even countries. This uniformity can be partly explained by the fact that, historically, calculus is a foundation on which many disciplines are built, and hence traditional calculus courses may be viewed as beneficial for students with varied academic and professional aspirations. However, perception of uniformity is also evident in research on the learning and teaching of calculus. Over the last decades, an extensive body of research has emerged on the teaching and learning of various topics in calculus at various levels, including the tertiary level (Bressoud et al., 2016). By and large, studies in this area tend to treat students and teachers of calculus as uniform cohorts, and not attend explicitly to different goals and needs, or to diverse cultural and institutional characteristics.

In recent years, there is growing awareness of assumptions of uniformity that are made, sometimes implicitly, in research and practice in calculus education. For example, the chapter about calculus in the latest PME Handbook stresses the "need to investigate the relationships between calculus and client disciplines in terms of practices, what should be taught, and what students are learning, to cite just a few" (Hitt & González-Martín, 2016, p. 29). In this vein, and going further, the ZDM special issue on calculus (2014) states: "research [in calculus education] that takes up the institutional and cultural context and how these aspects constrain and enable sustained uptake of advances in calculus learning and teaching is sorely needed […] this represents a new research theme, one yet to be realized to any large extent" (Rasmussen et al., 2014, p. 513). More recently, a ZDM special issue was dedicated to different approaches in calculus education around the world (Thompson & Harel, 2021) and highlighted the diversity in approaches across educational systems and institutions.

Motivated by above calls and observations, this special issue aims to lay the foundations for a long-term exploration of calculus education at the intersection of different institutions, disciplines and communities – an exploration that will provide insight into the rich and complex boundaries, tensions and connections that underlie the teaching and learning of calculus at the post-secondary level. At a time when universities are more accountable than ever, and following two years with dramatic changes imposed on educational systems by the global spread of Covid-19, changes in relation to the teaching of calculus are bound to happen. This special issue is put forth to encourage and inform research that will support changes that look beyond *how* calculus is being taught and *what* is being taught to *by whom* (e.g., mathematics or non-mathematics specialists), and *for what purposes*.

Building on the above and inspired by presentations and discussions at the *Calculus in Upper Secondary and Beginning University Mathematics* Conference (Monaghan et al., 2019), works presented in recent CERME and INDRUM conferences and other journal publications, we identified two potentially interrelated themes for the special issue:

A. *Intra-mathematical*: Calculus at the intersection of mathematical domains (e.g., geometry, algebra, analysis, probability) and mathematical practices (e.g., reasoning and proof).
B. *Interdisciplinary*: Calculus at the intersection of STEM or STEM-related disciplines (e.g., economics, biology, engineering, teacher education, etc.) and

professional communities (e.g., mathematicians, empirical scientists, engineers, economists, teachers of school mathematics).

Following the call for abstracts, we received 53 high quality expressions of interest from researchers based in 24 countries. Of those abstracts, we selected 12 and invited their authors to develop a full manuscript for peer review. Our selection criteria built on the criteria for publication in IJRUME and included also: centrality of calculus to the study, and clear and explicit attention to different institutions, disciplines or communities in calculus education. Following reviews and revisions, the Special Issue comprises seven papers with authors affiliated in eight countries (Canada, Croatia, France, Israel, Norway, Sweden, UK, USA) and one literature overview paper.

In the literature overview paper, Biza et al. review recent literature (mostly since 2015) around the question *How do calculus courses address the varied – sometimes conflicting – goals, values, and needs of different institutions, disciplines, and communities?* and identify the significance of the *scaffolding* and *filtering* roles calculus courses play. These roles are present, to different extents, in four main areas: calculus in the transition across educational levels, calculus and other mathematical areas, calculus for engineers and non-mathematics professionals, calculus for mathematics teachers. This overview highlights gaps in the literature in each of the four areas above, and also identifies a lack of studies in areas such as diversity, equity and inclusion.

The other seven papers in this special issue address topics related to the intra-mathematical and interdisciplinary themes we mentioned above and highlight the value of a research focus on the institutional roles of calculus courses. Viirman et al. analyze the content and structure of calculus courses—providing examples for the case of limits—in three different countries and at three different levels (high school, university, and teacher training). They show how 'a calculus course' can have different meanings across institutions as well as what the impact of this 'course' on student learning can be. In terms of the intra-mathematical theme and the assumption that calculus courses prepare students for further mathematical content, Bašić and Milin Šipuš show the difficult transition from one-variable integration to multivariable integration. They highlight practices involved in the calculation of integrals which require the inclusion of geometric elements usually taken for granted, or not taken into consideration by institutions. In a similar vein, Broley and Hardy analyze the practices of real analysis students when they deal with tasks akin to tasks previously studied in calculus. Their results show that the passage from calculus practices to real analysis practices is not obvious and that some students may find adopting new practices difficult when the old ones seem to still work when solving these tasks.

Regarding the interdisciplinary role of calculus courses, Hitier and González-Martín study how derivatives are used in calculus and mechanics college courses by analysing tasks related to one-dimensional motion in mathematics and mechanics textbooks and interviewing lecturers. The study reports consistencies and inconsistencies between practices employed in those courses. For example, in calculus, tasks do not pay attention to the mechanics aspects of the problems while, in mechanics, the quintessentially mathematical notion of derivative is hardly used. Still in the interdisciplinary theme, but this time in the specific case of teacher education and

professional development, Pinto and Cooper touch on the critical debate around the benefits of tertiary mathematics courses for mathematics teachers. Specifically, they analyze sessions in which mathematicians and experienced secondary mathematics teachers discuss incidents from mathematics lessons and identify opportunities in which teaching can draw on infinitesimal calculus to go beyond the remits of those lessons. The study demonstrates how content and practices of infinitesimal calculus can support pedagogical practices and enrich the repertoire of secondary mathematics teachers. Moreover, the study provides insights into how interactions of mathematicians and teachers act as a catalyst on how tertiary mathematics can be considered in secondary mathematics teaching.

This special issue also addresses the lack of studies in the areas of diversity, equity and inclusion (DEI) – which we stressed in the overview paper – with two papers. Leyva et al. examined the experiences of 34 calculus students from underrepresented groups in relation to teaching practices that are generally perceived as supporting equitable learning, and found that these practices are necessary yet insufficient to carve equitable opportunities for classroom participation and access to content. Tremaine et al. investigate the motivation of various stakeholders (students, faculty, staff, and administrators) in calculus programs to increase DEI in STEM fields. To this end, Tremaine et al. propose a framework for motivation towards diversifying STEM fields and demonstrate how this framework can help highlight strengths and areas in need of growth in relation to DEI – thus fostering and facilitating productive communication about DEI within calculus programs.

We hope that this special issue lays the foundations for holistic explorations of calculus teaching and learning at the intersection of different institutions, disciplines and communities. We would like to encourage researchers to consider this intersection and to question content and approaches in calculus courses, as well as implicit assumptions about their role in teaching. We envisage that the accumulation of research drawing on this perspective will help to challenge the vision of *(the same) calculus for all* and will provide a basis for informed decisions and changes involving calculus courses around the world.

References

Artigue, M., Batanero, C., & Kent, P. (2007). Mathematics thinking and learning at post-secondary level. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1011–1049). Information Age Publishers

Bressoud, D., Ghedamsi, I., Martinez-Luaces, V., & Törner, G. (2016). *Teaching and Learning of Calculus*. SpringerOpen. https://doi.org/10.1007/978-3-319-32975-8

Faulkner, B., Earl, K., & Herman, G. (2019). Mathematical maturity for engineering students. *International Journal of Research in Undergraduate Mathematics Education*, *5*(1), 97–128. https://doi.org/10.1007/s40753-019-00083-8

Hitt, F., & González-Martín, A. S. (2016). Generalization, covariation, functions, and Calculus. In A. Gutiérrez, G. L. Leder & P. Boero (Eds.), *Second Handbook of Research on the Psychology of Mathematics Education. The Journey Continues* (pp. 3–38). Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6_1

Hochmuth, R., Broley, L., & Nardi, E. (2021). Transition to, across and beyond university. In V. Durand-Guerrier, R. Hochmuth, E. Nardi & C. Winsløw (Eds.), *Research and Development in University Mathematics Education* (pp. 193–215). Routledge. https://doi.org/10.4324/9780429346859

Monaghan, J., Nardi, E., & Dreyfus, T. (Eds.). (2019). *Calculus in upper secondary and beginning university mathematics – Conference proceedings.* MatRIC. https://matric-calculus.sciencesconf.org/

Rasmussen, C., Marrongelle, K., & Borba, M. C. (2014). Research on calculus: what do we know and where do we need to go? *ZDM – Mathematics Education*, *46*(4), 507–515. https://doi.org/10.1007/s11858-014-0615-x

Thompson, P. W., & Harel, G. (2021). Ideas foundational to calculus learning and their links to students' difficulties. *ZDM – Mathematics Education*, *53*(3), 507–519. https://doi.org/10.1007/s11858-021-01270-1