Mathematical Prerequisites for STEM Programs: What do University Instructors Expect from New STEM Undergraduates?

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Abstract

In many countries STEM degree programs face high dropout rates, which are often attributed to insufficient mathematical preparation of incoming undergraduates. However, it is largely unclear which mathematical prerequisites universities expect from their new STEM undergraduates. There are hardly official documents and it is even an open question whether there is a consensus among university instructors. The main goal of this study is therefore to describe the expected mathematical prerequisites for new STEM undergraduates. We conducted a Delphi study with German university instructors for first semester mathematics courses in STEM programs. The participants identified 179 prerequisites addressing: (1) mathematical content, (2) mathematical processes, (3) views about the nature of mathematics, and (4) personal characteristics. For 140 of these prerequisites there was a consensus regarding their necessity among the university instructors. The expected mathematical prerequisites are mainly on a basic level and do not require formalistic or abstract mathematical knowledge or abilities.

Keywords New undergraduates · STEM education · School mathematics · University mathematics · Delphi study

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Introduction

The transition from secondary to tertiary education is of crucial importance for student retention and success in higher education (Gueudet 2008; Rach and Heinze 2017). Students' transitions come with difficulties, in particular for students taking mathematics courses in their first semester at university (Gueudet 2008; Thomas et al. 2015; Engelbrecht 2010). As a result, many students – especially in STEM degree programs – fail the transition and drop out (e.g., Australia: Rylands and Coady 2009; Germany: Heublein et al. 2014; UK: National Audit Office 2007 and Cox 2001; US: McCormick and Lucas 2011). From the perspective of person-environment-fit theory (Lubinski and Benbow 2000), these findings indicate a misfit between the abilities and interests of freshmen (person side) and the requirements of the university studies (environment side). In order to gain more insight into the misfit and, thus, in order to successively decrease the negative consequences of misfit (such as student drop-out), it is important to know more about both the person and the environment side. Prior research has already provided insights into the person side, that is, into the abilities and interests of students. In contrast, the environment side has been investigated far less systematically. There is especially a lack of systematic descriptions in the university instructors' expectations of mathematical requirements for STEM studies— expectations which can be assumed to strongly influence the environment side and which are essential for mathematics courses. This study aims to address this lack of research and reports about a Delphi-Study with university instructors investigating the question: What do university instructors for mathematics courses in STEM programs expect from first semester STEM students?

Theoretical Background

The Transition Problem: Differences between School and University Mathematics

A large body of research indicates fundamental differences between the mathematics education at school and at university (e.g., Engelbrecht 2010; Gueudet and Pepin 2018; De Guzmán et al. 1998; Luk 2005; Rach and Heinze 2017; Selden 2005), which are regarded as problematic when transitioning from high school to university (Gueudet 2008; Engelbrecht 2010). For example, *mathematical activities* such as problem-solving, reasoning and proof, or mathematical modelling are employed in both school mathematics (e.g., CCSSI 2010; KMK 2012) and university STEM programs (e.g., Gueudet 2008). However, the activities have different foci. In school, typically, the solving of real-life problems is emphasized (see the PISA studies, e.g., OECD 2016), whereas in university mathematics the focus is on inner-mathematical problems (e.g., Engelbrecht 2010; Leviatan 2008; De Guzmán et al. 1998). Similarly, the focus shifts from describing concepts to precisely defining concepts or from convincing via argumentation to mathematical proving (Tall 1991; Thomas et al. 2015).

School and university mathematics also differ with respect to *introducing mathematical concepts and terms*. In school, mathematical concepts are typically introduced starting from students' everyday experiences rather than using abstract formal definitions (Tall 1992). The idea is to support students in creating a mental image so that they can attach meaning to the new concept. There are varying traditions in different

countries on the emphasis of real-world references in mathematics lessons. Particularly in several European countries like Germany or the Netherlands real-world examples play an essential role in mathematics lessons to create learning opportunities for conceptual knowledge, whereas in the U.S., for example, there is often a stronger focus on procedures and real-world problems play a role but a less prominent one than in European countries. At university, typically all mathematical concepts are introduced by a formal definition that describes a logical relation of abstract properties (Tall 1992). To follow the formal definition of mathematical concepts in undergraduate courses advanced mathematical thinking is required (Edwards et al. 2005) to allow students to connect their previously learned concepts with the new concept definition (De Guzmán et al. 1998; Gueudet 2008).

The literature also emphasizes differences in *generating mathematical evidence*. Although school mathematics curricula include reasoning and proofs (e.g., CCSSI 2010; KMK 2012), school mathematics focuses on reasoning and convincing rather than on rigorous deductive argumentations and formal proof requiring advanced mathematical thinking (e.g., Tall 1991; Hoyles et al. 2001). In contrast, in university mathematics, proofs take an essential role in generating scientific evidence through elemental logical deductions from formal definitions, properties, or statements; and, as such, go far beyond the level of school mathematics (De Guzmán et al. 1998; Healy and Hoyles 1998; Tall 1992).

Finally, there are differences in how *mathematics is presented*. In school, mathematical concepts are typically presented by examples, which are often based on visual representations or informal situations. These examples allow students to create a mental image and try to avoid reinforcing misconceptions typically held by students (Tall 1992; Törner et al. 2014). Based on such examples, a formal description of properties (sometimes even a formal definition) might be introduced or developed. In contrast, mathematics in STEM studies is typically presented in a dense, formal and systematic structure known as “definition-theorem-proof” (Leviatan 2008; Almeida 2000). This structure does not reflect the process of how mathematical knowledge is generated. Instead, students are only presented with the final product of mathematical thinking (Tall 1991).

In summary, much research has already emphasized the differences between school mathematics and university mathematics. Major differences refer to (a) which mathematical activities are emphasized; (b) the way mathematical terms are introduced; (c) how mathematical evidence is generated; and (d) how mathematics is presented. From a mathematics education perspective, such differences between school and university mathematics may cause a misfit between persons and environment, when new undergraduate STEM students leave school and start their studies at university.

Person-Environment-(Mis-)Fit

In fact, the transition to university mathematics courses in STEM disciplines is reported to be problematic (Engelbrecht 2010; Gueudet 2008), often resulting in high drop-out rates (e.g., Heublein et al. 2014; National Audit Office 2007; Rylands and Coady 2009). With respect to the theory of person-environment fit (e.g., Edwards and Shipp 2007), this transition problem can be viewed as a misfit between university undergraduates and the new academic learning environment (see Etzel and Nagy 2016). Person-environment fit, in general, describes “the congruence, match, similarity, or correspondence between the

person and the environment” (Edwards and Shipp 2007, p. 211), and has been studied in organizational and educational psychology. For educational settings, particularly higher education settings, Etzel and Nagy (2016) emphasized the importance of so-called needs-supplies (N-S), demands-abilities (D-A), and interest-major (I-M) fit. All these types of fit are based on the assumption that an individual brings abilities and interests and that the environment carries ability requirements and a reinforcer system. If abilities and ability requirements (D-A fit) as well as interests and reinforcer systems (N-S fit) correspond with each other, person and environment are fitting, which again results in satisfactoriness and satisfaction (Etzel and Nagy 2016; Lubinski and Benbow 2000). When applying person-environment fit to higher education settings, a correspondence between the students’ interest and the students’ major (I-M fit, though closely related to N-S fit) is of additional importance (Etzel and Nagy 2016).

The before mentioned findings concerning the high dropout rates in STEM degree programs suggest a missing congruence or fit between students and universities with respect to mathematics in STEM programs. Approaches for a better alignment between students and the academic environment at the universities include: (a) the facilitation of choices of study degree programs congruent to students’ interests and abilities (see Lubinski and Benbow 2000), (b) mathematical transition courses that support mathematical abilities of new STEM students (e.g., Leviatan 2008; Griesse 2017), and (c) better communication and coordination between schools and universities (recommended by Thomas et al. 2015). All such approaches for better alignment require transparent information on both the person and environment side.

The Person-Side: Mathematical Abilities and Interests of First Semester STEM Students

Generally speaking, in many countries high school graduates – even those choosing advanced mathematics courses – seem to not meet an intermediate benchmark of mathematics achievement, that is they are struggling to demonstrate knowledge of concepts and procedures in algebra, calculus and geometry to solve routine problems (Mullis et al. 2009; Mullis et al. 2016). In more detail, new STEM undergraduates seem to employ surface learning and understanding, to be inflexible in switching between the embodied, symbolic and formal worlds, to focus on an exclusive process view and to apply only intuitive thinking (e.g., Godfrey and Thomas 2008 for abstract algebra; Thomas and Stewart 2011 for linear algebra; Carlson et al. 2015 for calculus and analysis). For instance, students were reported to have a limited concept of equality that is not based on a formal understanding of equivalence relation and its properties (Godfrey and Thomas 2008) and to view functions as a picture of an event and recipe to get an answer instead of two quantities changing together and a mapping of input values of the function’s domain to output values in the function’s range (summarized by Carlson et al. 2015). Students were also found to have an unstable conceptualization of slope preventing covariational reasoning (Nagle et al. 2013; Thompson 1994).

With respect to mathematical skills, students were found to struggle with algebraic manipulation, numerate skills, the application of mathematical knowledge to solve multi-step problems and the ability to reason mathematically (Hourigan and O’Donoghue 2007; Kajander and Lovric 2005). Students’ fragmented and surface concept understanding may pose challenges when solving complex problems (e.g.,

Carlson and Bloom 2005) or when applying formal mathematical reasoning (e.g., Harel 2008; Engelbrecht 2010). Finally, students also seem to have inadequate learning strategies and perceptions about the learning of mathematics at university (e.g., Kajander and Lovric 2005; Engelbrecht 2010).

The Environment-Side: Mathematical Requirements of First Year Mathematics Courses in STEM Degree Programs

Given the differences between school mathematics and university mathematics (see above), new undergraduates can be assumed to face specific requirements of university mathematics courses. Concerning the mathematical activities, students are required

- (a) to think and work formally (e.g., proving, precisely defining) rather than intuitively
- (b) to follow a formal introduction to mathematical terms
- (c) to follow formal proofs when generating new mathematical knowledge, and
- (d) to cope with a definition-theorem-proof-like presentation of mathematics.

There are also some studies providing empirical insights into which mathematical knowledge and skills are required for successful studies. For instance, conceptual knowledge of function has been found to have a high significance for success in university calculus (e.g., Carlson, Madison, and West 2015). Additionally, introducing students to the nature of university mathematics with its characterizing activities, formal concepts and tools (e.g., Leviatan 2008) as well as facilitating students' confidence (e.g., Carmichael and Taylor 2005) have been reported to ease the transition to university mathematics in context of bridging courses. With respect to mathematical processes, research suggests the importance of problem solving skills (Carlson and Bloom 2005) and experiences with proofs (Balacheff 2008; Hanna and Barbeau 2008; Harel 2008) and (formal) definitions (Harel 2008) as requirements for mathematics courses at universities. Additionally, adequate expectations regarding university mathematics (Hoyle et al. 2001; Nardi 1996; De Guzmán et al. 1998; Thompson 1994) and the learning of mathematics at the universities (Kajander and Lovric 2005; Inglis et al. 2012; Thompson 1994), as well as appropriate interest (e.g., Marsh et al. 2005; Carlson 1999), motivation, beliefs and self-regulation (e.g., Carlson and Bloom 2005; Hailikari et al. 2008) and persistence with regard to complex mathematical problem-solving (Carlson 1999) seem to be important for the transition to university mathematics.

All these studies provided important insights into selected requirements of the universities. However, it is unclear, if, taken together, these selected requirements provide a comprehensive list of minimal requirements for the STEM studies. Also, it is unclear, how these selected aspects, which basically stem from analyses of successful and unsuccessful students, correspond with university instructors' expectations of prerequisites for first semester students in STEM programs – expectations, which influence the requirements of the learning environment at universities. In fact, there are only few single studies investigating the instructors' views (e.g., Sutherland and Dewhurst 1999; Konegen-Grenier 2001). Overall, these studies revealed aspects of mathematical content knowledge, abilities to perform specific mathematical processes, and general personal characteristics as necessary prerequisites from the perspective of university instructors. However, the findings differed across the studies. In addition, the expectations of the university

instructors or departments may vary across different disciplines (Sutherland and Dewhurst 1999; Conley et al. 2011). The studies also come with methodological issues. First, most of the studies are rather dated (e.g., Sutherland and Dewhurst 1999; Konegen-Grenier 2001). Second, Sutherland and Dewhurst (1999), for example, investigated data based on official school documents without the possibility to include supplemental requirements. Third, studies employing questionnaires with given categories (e.g., Sutherland and Dewhurst 1999; Conley et al. 2011) may have biased the university instructors' perspective. Finally, the studies may have underlined the importance of mathematics skills and knowledge to university success in general without specifying specific concepts or skills (e.g., Konegen-Grenier 2001).

The Present Study

Viewing the transition from high school to university STEM education through the lens of person-environment-fit theory, it is important to take into account both the person and the environment side. Prior research has already provided insights into students' abilities and interests. There are also some studies identifying requirements according to the universities' learning environment. However, some studies focus on only particular aspects that are required, others take a broader perspective but have methodological issues. Yet, a systematic investigation of university instructors' expectations – which can reasonably be assumed to strongly influence the environment side and accordingly supplement existing research findings – is still missing so far. The present study therefore addresses the following research questions:

- (1) To what extent do university mathematics instructors of different STEM disciplines agree on a common catalogue of necessary mathematical prerequisites for new undergraduates?
- (2) Which mathematical abilities and interests of students (requirements of the environment side) are regarded as minimal prerequisites for first semester STEM degree courses by university mathematics instructors?

Methods

To approach the research questions, we conducted a Delphi study with university instructors involved in the mathematics education of new STEM undergraduates. Investigations employing the Delphi technique consist of an iteration of expert surveys with feedback to the participants after every survey round (Linstone and Turoff 1975). The iteration process is continued until stability in the responses – not necessarily consensus – is reached (Rowe et al. 1991). Delphi studies are characterized by the anonymity of participants, the collection of group opinion and the possibility for the participants to refine their responses during the iteration process (e.g., Linstone and Turoff 1975). This method reduces bias due to group-effects such as the influence of a dominant individual or group pressure (Rowe et al. 1991).

For data collection, we focused on one country (Germany) and aimed at realizing a comprehensive sample by approaching all mathematics instructors from all universities which recently offered a first-year mathematics course in a STEM program. Germany is

a federal republic and each of the 16 federal states is accountable for its educational system (i.e., the school system and curricula). The curricula are aligned with national educational standards valid for the whole country. With respect to the transition from high school to university in Germany, it is important to note, that the high school final examination (“Abitur”) in one federal state is valid as general university admission eligibility for all universities in the whole country and that there are no additional university entrance tests for STEM programs. Higher education institutions in Germany are predominantly public (private universities are the exception) and there are only two types of universities, universities and universities of applied sciences, which may somewhat differ in their level of educational requirements in the first semester. Though universities in Germany clearly differ in their research output and level, the mathematics courses in bachelor STEM programs are quite similar within each university type. Hence, for examining school-university transition from the perspective of person-environment fit theory, the clearly structured and rather homogeneous situation in Germany provides very good conditions – especially when compared to a more international sampling where additional culture-specific aspects must be taken into account.

Sample

In order to describe the expected necessary mathematical prerequisites for new undergraduates in STEM degree courses, we invited university instructors who lecture in first-year mathematics courses in STEM degree programs to participate in our expert survey. This sample possesses the necessary mathematical knowledge and skills, is responsible for the implemented curricula and its requirements, has experience in accompanying the transition of new university undergraduates, and is able to effectively communicate about mathematical aspects of university readiness. To recruit experts, we searched the module descriptions and course catalogs (available online) of all 399 German universities and universities of applied sciences for those instructors who taught first-year mathematics courses for new undergraduates in STEM degree programs in the last five years (i.e., October 2010 until September 2015). In the context of this study, STEM covers programs of mathematics (e.g., mathematical statistics, economical mathematics), sciences (e.g., physics, chemistry, biology, geography, computer sciences) and engineering (e.g., mining, process engineering, traffic engineering, electrical engineering, construction engineering, industrial engineering). Following the national interpretation of STEM (in German: MINT = Mathematik, Informatik, Naturwissenschaft, Technik) degree programs in context of architecture, medical sciences and psychology were not included. The online search resulted in a population of 2233 instructors of whom 2138 could be reached by email. To generate this list of experts, we only used information publicly available. The participants were informed by mail and e-mail about the goals and the procedure of the online survey which emphasized that participation was anonymous and voluntary. If an expert decided to not participate, he or she did not encounter any negative consequences.

Procedure

The present Delphi study included three survey rounds.

Round 1

Following Rowe et al. (1991) the main purpose of the first Delphi round was to generate a framework of potential prerequisites in a bottom-up analysis of the data provided by a small subsample of university instructors. To this end, we employed three narrative open-ended questions. First, participants were asked to describe the characteristics of mathematical university readiness or rather what mathematical knowledge and skills new STEM undergraduates should acquire before starting the first semester. Then, the participants indicated which mathematical aspects they would include in a mathematical entrance test for STEM programs. Finally, they were asked about the differences between successful and unsuccessful new STEM undergraduates with a special focus on the mathematical prerequisites the students acquired before entering the university. To adequately represent the identified population of university instructors, we invited a subsample of 82 instructors balancing the STEM disciplines taught, the institution type and the federal state of the university. In this first round 36 university instructors accepted our invitation and participated.

In order to generate a framework of expected learning prerequisites, the collected data were analyzed by means of qualitative content analysis (Mayring 2014) in a four step procedure. First, we pooled all responses to the three open-ended questions and compartmentalized them into smaller units based on separators in the responses of participants (e.g., “skill to abstract thinking, perseverance” was split into the two units “skill to abstract thinking” and “perseverance”). Second, we summarized those units representing similar learning prerequisites (e.g., “calculations including absolute values and inequalities,” “solving simple inequalities fluently,” and “solving linear equations and inequalities in one variable” were put together in one prerequisite). Third, we aggregated all the prerequisites similar to each other to form a sub-category (e.g., “linear and quadratic inequalities,” “absolute value equalities,” “linear and quadratic equalities,” “existence and uniqueness of solutions,” and “linear equation systems including up to three variables (without matrix representation)” were aggregated into one sub-category). Finally, we successively aggregated these sub-categories into categories (e.g., “calculus” and “linear algebra and analytical geometry”) which were then structured into four main categories (e.g., “mathematical content” and “mathematical processes”). This four-step, inductive bottom-up generation of categories was employed to minimize the bias when structuring the collected responses. As a result, we revealed a list of 152 mathematical learning prerequisites which were summarized in four main categories: Learning prerequisites addressing (1) mathematical content (e.g., “definition/concept of function” and “component representation of vectors in $\mathbb{R}^3$ ”), (2) mathematical processes (e.g., “fluently and proficiently using general heuristic principles,” “development and formulation of mathematical proofs to a given statement”), (3) views about the nature of mathematics (e.g., “proof is a central task of mathematics”), and (4) personal characteristics (e.g., “willingness to solve requiring and abstract mathematical problems”).

In order to verify the assignment of single learning prerequisites to the main categories and thus to check whether the distinction of the four main categories representing different characteristics made sense, we performed an inter-rater reliability analysis. Two scholars assigned the learning prerequisites to the four main categories and reached an acceptable to very good agreement (average pairwise agreement between 85% and 97%; Cohen’s κ between .60 and .94). To further confirm this framework, that is the 152 learning

prerequisites structured in the sub-categories of four main categories (1–4), these first results were compared to the original responses of the participants with focus on completeness and then looped back to the experts in Round 2 of the Delphi study.

Round 2

In Round 2, the identified 152 learning prerequisites, structured into sub-categories and main categories, were presented to all 2138 university instructors, of whom 952 participants responded. For each mathematical learning prerequisite, the participants were asked if and on which level the prerequisite is regarded necessary for new STEM undergraduates. For mathematical content aspects the university instructors were asked to choose between either “not necessary,” “Level 1,” or “Level 2.” Level 1 was defined as:

“Basic knowledge with regards to mathematical content, algorithms or routines. These can be described or executed. Level 1 corresponds e.g. to task requirements like executing, recognizing, reconstructing, converting, calculating or knowing.”

Level 2 was defined as:

“Flexible and strongly linked knowledge for a creative application in order to generate new knowledge or problem solutions based on heuristic processes, connections or generalizations. Level 2 corresponds e.g. to task requirements such as transferring, interpreting, evaluating, analyzing, proving and generalizing.”

For mathematical processes, we did not differentiate in levels of understanding in Round 2 and the participants chose between “not necessary” and “necessary.” For the views about the nature of mathematics, again participants chose from “not necessary,” “Level 1,” “Level 2.” Here, Level 1 was defined as: “The particular view about mathematics is available at the start of first semester as abstract meta-knowledge (that is the learners have been told this information)” and Level 2 as: “The learners have made experiences with the particular characteristic of mathematics, e.g. by observing, reflecting or performing the mathematical activity.”

For personal characteristics, participants were asked to rate the necessity of the single prerequisites on a 4-point Likert scale (from “unimportant” to “important”). For all 152 prerequisites, participants were asked to provide comments if they felt the given prerequisites needed more specification, if they wanted to add new aspects, or if they felt the structure of the categories was not appropriate or valid.

For data analysis in Round 2, we specified consensus criteria which go beyond a simple majority criterion: A mathematical learning prerequisite was assumed necessary if (a) more than $2/3$ of all participants *and* (b) the majority in each of the three subject-groups (participants teaching mathematics, (i) only in mathematics programs, (ii) only in STE programs without mathematics, and (iii) in both mathematics and STE programs) *and* (c) the majority in each institution type (university and university of applied sciences) agreed to the necessity. To assume a prerequisite was not necessary we applied criteria that were even more conservative: Only if more than $3/4$ of all university instructors and more than $2/3$ of the instructors of each subject-group and each institution type rated the aspect as not necessary then we assumed a rejection of

the aspect. If we found a consensus for a particular mathematical content aspect or view of mathematics we assigned it to the level which was voted for by at least 50% of the population. If neither level was voted for by at least 50% we again asked specifically for the level of necessity in the following survey round.

As a result of Round 2, we found a consensus for 105 mathematical prerequisites to be necessary and 3 to be not necessary. There was no consensus for 44 prerequisites. We received 1302 (partly conflicting) comments suggesting specifications for the given prerequisites or adding new aspects. No objections or comments were made on the structure, the denomination of the sub-categories and categories, or the assignment of single learning prerequisites to the categories. This finding substantiated and again validated our categorization procedure in Round 1. If an additional aspect was stated by at least 3 of the 952 university instructors, we assumed it as significant and included it in the next round. Based on this criterion, 14 mathematical prerequisites were specified. Moreover, 4 prerequisites were split up and 23 were newly added to the survey, so that the number of mathematical prerequisites increased from 152 to 179.

Additionally, several participants suggested to introduce different levels for prerequisites in the main category “Mathematical processes.” Based on these comments, we introduced Level 1 (“The particular mathematical process can be reproduced in familiar situations (e.g. solving proof problems analogously to a known type of task) and applied to unfamiliar situations addressing simple content of middle school mathematics”) and Level 2 (“The particular mathematical process can additionally be performed in unfamiliar situations addressing content of high school mathematics”).

Round 3

To consolidate and finalize the catalogue of mathematical prerequisites, the above results were once more fed back to the full sample, of whom 664 participated. A comparison of the participants from Rounds 2 and 3 concerning university type, discipline taught, and teaching experience revealed that the subsamples were comparable. In Round 3, for the newly added prerequisites and the prerequisites without consensus in Round 2 we asked again for the options: not necessary, Level 1, and Level 2. For the aspects confirmed in Round 2 according to our consensus criteria, we asked for a final confirmation (6-point Likert scale: 1 = “total disagreement” to 6 = “total agreement”). The results showed that our criteria for a consensus were accepted by the sample (mean agreement rate: 92.4%; minimum agreement rate: 87.1%; with agreement defined as a rating of 4 to 6 on the Likert scale).

Results

Over the three Delphi rounds, 179 prerequisites regarding four main categories were proposed: (1) mathematical content, (2) mathematical processes, (3) views about the nature of mathematics, and (4) personal characteristics (see Table 1). In the following, we elaborate on the degree of consensus we found and provide a brief overview of these main categories (for a full list and detailed description of each prerequisite, see the [online supplement](#) of this article).

Table 1 Identified mathematical prerequisites by category: Overall number of prerequisites proposed by university instructors and number of prerequisites considered as necessary based on a consensus among university instructors

		Number of proposed prerequisites	Number of necessary prerequisites (consensus)
(1) Mathematical content	Basics	50	46
	Calculus	30	20
	Vectors and matrices	16	7
	Statistical and general content	10	4
(2) Mathematical processes	Basic skills	9	9
	Reasoning and proof	9	8
	Mathematical communication	5	5
	Mathematical definition	4	3
	Problem-solving	8	7
	Mathematical modelling	6	4
	Mathematical information literacy	1	1
(3) Views about the nature of mathematics		9	7
(4) Personal characteristics	Attitudes	11	11
	Cognitive characteristics	7	5
	Social skills	4	3
Total		179	140

Degree of Consensus

Out of the 179 proposed mathematical prerequisites, we found a consensus of 144 (80.4%). 140 aspects were agreed to be necessary mathematical requirements (see Table 1) and 4 aspects (rather formal and abstract mathematical concepts like formal-axiomatic definition of vector space) were agreed not to be necessary. Concerning 35 (19.6%) of the proposed mathematical requirements there was no consensus among the university instructors. With regard to 27 of these aspects we found no consensus for the whole sample and even no consensus within each subsample (i.e., with respect to the subject-groups or types of university). Concerning the remaining 8 aspects there was no consensus for the whole sample but for at least one subsample.

Overall, this result indicates a consensus among the university instructors concerning the necessity of most of the mathematical requirements identified in the study. When we found a lack of consensus it seemed to be due to disagreement within the whole sample and within single subsamples respectively rather than between the subsamples. This conclusion is substantiated by the result that there is not a single prerequisite which was rated necessary for one group of students and not necessary for another. As a result, we may assume that the 140 identified prerequisites, in fact, can be considered a common consensus description of mathematical requirements for all STEM disciplines and different institutions.

Identified Mathematical Prerequisites

The list of mathematical prerequisites was generated inductively from the original statements provided by the university instructors. Mathematics university instructors in Germany typically are experts on disciplinary mathematics or any other STE discipline, but they usually did not receive a special training in didactics of mathematics. From a mathematics education perspective, the identified aspects sometimes address rather well-defined concepts, processes, views and characteristics, and sometimes are rather broad. Nevertheless, the named prerequisites are precise enough to both be viewed as a comprehensive description of requirements for the environment side and to be grouped into larger categories (on inter-rater reliability of this grouping see above).

Mathematical Content

The majority of necessary mathematical requirements refer to mathematical content. University mathematics instructors expect new STEM undergraduates to be knowledgeable of the basic content aspects that correspond to middle school mathematics (e.g., fractions, proportionality, equations and systems of equations, geometric figures, elemental functions). For example, students need to have a basic knowledge (Level 1) of the existence and uniqueness of mathematical solutions of equations and systems of equations and concerning the concept of functions. A flexible and advanced knowledge for a creative application in problem-solving tasks (Level 2) is exclusively required concerning six basic content prerequisites (e.g., fractions, percentages and proportionality, linear and quadratic equations and functions). Additionally, basic knowledge (Level 1) of calculus (e.g., concerning an intuitive concept of limits without the explicit use of sequences and procedures to calculate limits; concerning the fundamental theorem of calculus) as well as about vectors (e.g., vector representations of points, line or plane; scalar product) is required as prerequisites. Finally, the participants rated necessary prerequisites regarding combinatorics and probability (permutations, variations, combinations, counting principles, binomial distribution and normal distribution) as well as general mathematical content (e.g., propositional logic).

Mathematical Processes

University instructors also expected new STEM undergraduates to employ typical mathematical processes. Students should be able to perform basic mathematical processes in unfamiliar situations (Level 2) (e.g., fluently and precisely performing mathematical standard procedures without electronic tools; proficiently handling and transferring between mathematical representations; understanding written mathematical formulations including technical terms and mathematical symbols and notations; comprehending and explaining of given definitions; fluently and proficiently using general heuristic principles). Additionally, in familiar situations (Level 1) students need to be able to execute mathematical processes of reasoning and proving (e.g., understanding and verifying a given proof; developing and formulating mathematical hypotheses and supporting arguments of plausibility), communicating about mathematics (e.g., using precise mathematical notation to represent mathematics), defining (e.g., adequately formulating of mathematical definitions of known concepts), problem solving (e.g., deducing

problem-solving strategies for mathematical problems based on given solutions) and mathematical modeling (e.g., solving of real-world problems applying mathematical tools; evaluating different mathematical models for the same real-world situation). Finally, the ability to collect information from mathematical textbooks, the internet, or other sources (incl. The critical reflection of the sources) in terms of a mathematical information literacy was regarded as necessary by the participants.

Views about the Nature of Mathematics

Adequate views about the nature of mathematics were also regarded as prerequisites for new STEM undergraduates. From the perspective of university instructors, students should possess a meta-knowledge (Level 1) about, for example, the necessity of mathematical precision with respect to definitions and argumentations, the central role of proving when generating mathematical evidence, and the idea that university mathematics should be seen as an open system that includes more and qualitatively different aspects than discussed in school mathematics.

Personal Characteristics

Finally, the university instructors agreed on the importance of specific personal attributes. In this regard, it was considered important that students have, for example, curiosity and interest concerning mathematics, the skill and willingness to work independently on mathematical tasks, and perseverance with demanding mathematical problems. Additionally, general cognitive characteristics (e.g., ability to work in a focused manner; creativity and imagination; precise abstract and logical thinking), and social skills (e.g., willingness to communicate with students and instructors; ability to work in teams) were regarded as important.

Discussion

International research showed that many new STEM undergraduates fail to cope with the challenges related to the secondary-tertiary transition in mathematics. Based on the theory of person-environment fit it may be assumed that one main reason for dropout is the insufficient fit between the abilities and interests of the students, on the one hand, and the requirements of the mathematics courses at the universities, on the other. This study aimed at an empirical description of the requirements based on the expectations of stakeholders, which influence the demands of the environment side (i.e., university instructors for first-year mathematics courses in STEM programs). To this end, we conducted a Delphi study which led to a consensus set of 140 required prerequisites concerning aspects of (1) mathematical content, (2) mathematical processes, (3) views about the nature of mathematics, and (4) personal characteristics. Against the background of person-environment fit theory, all identified aspects describe the (ability) requirements of the environment side (though they certainly have corresponding parts on the person side). With respect to the three types of fit important for higher education (need-supplies (N-S), demands-abilities (D-A), and interest-major (I-M) fit), prerequisites for mathematical content (1) and processes (2) are relevant for D-A fit and

prerequisites regarding the views about the nature of mathematics (3) as well as personal characteristics (4) are relevant for N-S and I-M fit.

Overall, the prerequisites identified by consensus address the level of school mathematics rather than that of university mathematics education. With respect to *mathematics content*, the university instructors identified aspects which are highly congruent with content included in school mathematics. In particular, most of the content aspects were regarded necessary on a basic level, only six aspects were assumed necessary as flexible and highly linked knowledge. Furthermore, aspects from high school mathematics (e.g., concepts of limit and continuity) were identified necessary as intuitive conceptions but not as formal conceptions. Similarly, the *mathematical processes* identified by consensus are also largely fitting mathematics education standards for middle and high schools in several countries (for Germany: KMK, 2012). Again, basic processes are regarded necessary (and necessary to be employed to known and unknown mathematical content); in contrast, there was no consensus on the necessity to perform more formal processes (such as developing formal definitions or proofs). This corresponds to our finding, that STEM students are expected to have meta-knowledge of the *nature of mathematics*, but no experiences by observing, reflecting or performing disciplinary mathematics on their own. However, it is important to note that school mathematics standards and curricula typically do not explicitly include such aspects of the nature of mathematics. Likewise, *student characteristics* (such as interest to engage with mathematics) are regarded important aims of school mathematics, and are thus in line with the expectations named by the university instructors. But again, such characteristics typically come as a by-product of school mathematics education in favor of promoting an understanding of mathematics content and mathematical processes. In summary, axiomatic-formal concepts and other aspects from the level of university mathematics are regarded as less important prerequisites in favor of a firm understanding of basic concepts, comprehension of definitions and proofs as well as adequate perceptions and attributes. That is, school mathematics should provide a sound mathematical basis for formal mathematical thinking taking place at the university.

Implications for Research on the Transition between School and University

One finding of the present study is that university instructors expect prerequisites regarding not only mathematical content aspects but also mathematical processes, views about the nature of mathematics, and personal attributes. This result corresponds to the recommendations of Conley et al. (2011) and differs from the exclusive focus on mathematical content and skills as employed in other studies and recommendations (e.g., Sutherland and Dewhurst 1999; COSH 2014). When investigating the transition between school and university, the set of prerequisites identified in this study and provided in the [online supplement](#) of this article may therefore help to establish content validity.

Another intriguing result is the fact that we found such broad consensus given that we included experts teaching in so many different STEM disciplines and from different types of universities. In contrast to the results of the studies of Sutherland and Dewhurst (1999) and Conley et al. (2011) there is a high congruency of the university instructors on the mathematical requirements. There were tendencies to a higher variance in the participants' ratings only with respect to requirements associated with formalism, the application of mathematics to real-world phenomena and the adequate conceptions of

mathematics. Thus, when modeling and investigating the transition from school to university STEM education, common models may be assumed when focusing on mathematics and similar, if not the same, instruments assessing students' abilities, skills, beliefs, and attributes related to mathematics may be employed for all STEM disciplines.

Many of the mathematical prerequisites defined by the university instructors reflect mathematical aspects that are discussed in recent research on the transition between school and university and the abilities of students. For example, with regard to mathematical content aspects, students are expected to know about the properties of equivalence relation and implication. Such knowledge is viewed as a foundation to interact fully with the equation sign (Godfrey and Thomas 2008). In line with considerations of Carlson et al. (2015), the university instructors expect that students have a concept image of mathematical functions that corresponds with the concept definition and thus enables covariational reasoning. Additionally, students are required to comprehend and verify given proofs and generate mathematical hypotheses and support arguments of plausibility as well as to comprehend mathematical definitions and formulate their own definitions of known concepts (see Harel 2008; Hanna and Barbeau 2008). Regarding problem solving, the university instructors in our study rated necessary conceptual knowledge, heuristics and affects (as described by Carlson and Bloom 2005). Interestingly, we found no consensus view that students should necessarily be able to control their own thinking in mathematical situations, which is regarded as an important requirement in problem solving (e.g., Carlson 1999; Carlson and Bloom 2005). Besides those metacognitive skills all other aspects that are regarded important in the context of problem solving like motivational and self-regulative learning characteristics (e.g., Carlson 1999) are reflected in prerequisites expected by university instructors such as: perseverance, mathematical confidence and adequate perceptions of mathematics. Moreover, the prerequisites identified in the Delphi study include adequate views of the nature of university mathematics as it is discussed by Kajander and Lovric (2005), Engelbrecht (2010) and Luk (2005). In summary, there has already been research on which aspects of students' mathematical thinking, practices and dispositions are helpful to decrease student attrition in STEM, and how such thinking, practices and dispositions would need to be addressed in education to ease the transition from school to university mathematics. Basically all these investigated aspects relate well to our findings of university instructors' expectations. However, our present work provides a much larger scope of expectations; the presented catalogue may thus help to identify further aspects of mathematical thinking, practices and dispositions to be investigated by further research on the transition from school to university mathematics.

Finally, the identified consensus prerequisites do not correspond to the formal and abstract level of university but rather to the concrete and informal level of school mathematics education. Given the high drop-out rates in STEM degree courses, this calls for further investigations, for example, which of the prerequisites predict study success, or which abilities, skills, beliefs, and attributes new STEM undergraduates, in fact, have developed during schooling. As presented above, findings from many studies examining the person side indicated that new undergraduates in STEM programs lack basic mathematical competencies and miss central goals of school mathematics (e.g., Lawson 1997; Hawkes and Savage 2000; Faulkner et al. 2010). Also, future research should investigate how well the expected prerequisites are aligned with the normative settings given by (high) school curricula.

Limitations

Although this study is based on a systematic empirical investigation, the following limitations should be considered. First, the presented mathematical prerequisites only reflect the expectations reported by the university instructors. To investigate whether the university instructors really require only these prerequisites in their courses and whether the identified prerequisites have a significant influence on students' success, further studies are needed. Second, this study is based on a bottom-up analysis of university instructors' responses regarding mathematical prerequisites for STEM degree programs. Consequently, for the description of the prerequisites and the structuring categories original responses of the instructors have been used. This approach partly resulted in prerequisites and structures that differ from existing concepts and structures in mathematics education research (e.g., concerning the separation of content and processes) and led to some descriptions of prerequisites, which are rather coarse-grained. Additionally, this approach is based on the written responses of the participating university instructors. In order to ensure that the instructors all have the same understanding of a single expectation, a more fine-grained operationalization would be necessary (see implications below). Third, the survey of university instructors focused on mathematical requirements of the environment side (students' knowledge, skills, views and attributes that are required by university STEM programs from the perspective of university instructors). Employing such a research approach does not provide insights into the actual abilities and interests of students or how students need to be educated in prior education to meet these requirements (person side). Finally, our study only reflects the German situation. Studies taking into account other countries may lead to different results. However, our results are in line with the few international studies available (e.g., Conley et al. 2011; Sutherland and Dewhurst 1999), many aspects of the four main categories – mathematical content, mathematical processes, views about the nature of mathematics, and personal attributes – identified in this study have also been addressed in prior studies from other countries, and the mathematical content aspects in particular show a large overlap to what is suggested in prior studies as well.

Implications for Educational Practice

Besides research, this study also informs different stakeholders in education. Stakeholders from educational administration and politics may use our findings for identifying and recalibrating goals of secondary mathematics education. Schools and universities could employ them for the development and improvement of supporting programs like focus courses for students interested in STEM (schools) or bridging courses (universities). Additionally, the results of this study may support students to choose study degree programs congruent with their interests and abilities by increasing the transparency of mathematical requirements for STEM degree programs. Finally, the identified mathematical requirements could be employed to initiate a negotiation between schools and universities about commonly accepted requirements that ensure transparency for all stakeholders and future undergraduates (see Thomas et al. 2015). Such negotiations would also help to create a common meaning of the expected prerequisites among the university instructors and the school representatives. This would certainly include a more fine-grained description of the aspects and require operationalizations, e.g. by typical sample

tasks and expected solutions. In fact, as a result of the study presented here, the Ministry of Education, Science and Cultural Affairs in the German federal state of Schleswig-Holstein funded an initiative to bring together stakeholders from schools, universities and educational administration, which are currently developing a pool of tasks representing the expected requirements (based on the catalogue of aspects presented). This pool of tasks will not only help to inform teachers and university instructors, but also to provide a self-test for students interested in studying in the STEM field. This initiative may serve as an example for other stakeholders in other regions and countries as well.

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Compliance with Ethical Standards

Conflict of Interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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