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A Review of *How to Teach Mathematics* 3rd Edition, Steven Krantz. (2015) American Mathematical Society. ISBN: 978-1-4704-2552-4

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"How to Teach Mathematics" is an excellent resource for anyone who teaches post-secondary mathematics. While the author, Steven G. Krantz, is a self-proclaimed traditionalist and much of the book is focused on lecturing, even the most ardent active learning proponent will find useful nuggets. A number of the author's recommendations are supported by the recent study conducted by the Mathematical Association of America (MAA), which found that calculus students' attitudes toward mathematics were positively affected by a factor labeled "good teaching" (Sonnert, Sadler, Sadler, and Bressoud 2015). Good teaching comprised 22 student survey items such as "My calculus instructor made me feel comfortable asking questions during class" and "My calculus instructor discussed applications of calculus." There is substantial overlap between these 22 items and the practices Krantz recommends.

The book is also a useful read for those who conduct research in undergraduate mathematics education because it provides insights and perspectives from a reflective mathematician. Further, the author provides numerous experience-based ideas about teaching that may represent interesting researchable questions. So far, studies focused on attributes of good lectures that Krantz recommends (e.g., Lew, Fukawa-Connelly, Mejía-Ramos, and Weber 2016) have not demonstrated that these attributes impact student learning in the intended ways. However, the overlap between Krantz's recommendations and the "good teaching" factor from the MAA study suggests that studies focused on these recommendations and student *affect* could be fruitful. While the main purpose of teaching mathematics is to support students' learning, it would be a mistake to dismiss the importance of affect, especially given its likely significant role in STEM retention.

In summary, I feel comfortable recommending the book to anyone interested in mathematics teaching, especially at the undergraduate level. My goal for the balance of

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my review is *not* to support this recommendation (although I may accidentally do so). Instead I critically address two aspects of the book from my own perspective and that of the current mathematical education research literature. First I discuss the author's arguments, both direct and indirect, in support of lecturing. Then I discuss the author's perspectives on what it means to learn and teach mathematics. I hope that this discussion will spark readers' curiosity regarding "How to Teach Mathematics" and prepare them for a reading experience that is sometimes frustrating but often interesting, entertaining, and enlightening.

On Lecturing. Krantz begins his section on lectures with a vignette featuring a violin sitting in an empty room. The first two individuals to enter and "play" the violin produce screeches and off-key notes, while the third produces heavenly sounds. This third individual is the famous violinist Isaac Stern and it turns out that the violin was a Stradivarius. This story is used to make the point that statistics indicating that students are not learning calculus well (e.g., Bressoud, Carlson, Mesa, and Rasmussen 2013) should not be used as evidence that lecturing does not work. Instead, Krantz argues that, "lecture doesn't work very well because most of us are not very good at it." (p. 7). Krantz does make a valid point here that should be acknowledged by educational researchers. Just as research into curriculum effectiveness needs to take into account implementation fidelity, research aimed at investigating lecture effectiveness should take into account the nature and quality of the lecturing.

However, as I read the vignette, I could not help but see it as a context for an argument *against* lecturing. If I imagine myself watching these three individuals play the violin, I doubt that I would find Mr. Stern's performance any more helpful than the other two if asked to try my own hand at playing the Stradivarius. If one thinks of mathematics as something that humans do rather than a collection of facts and procedures, it seems unlikely that effective instruction would emphasize watching an expert rather than engaging in mathematical activity oneself. Thus, while it my be true that many instructors do a poor job lecturing, research has so far not established that good lectures are significantly better than bad lectures in terms of supporting learning. Certainly the finding of Lew et al. (2016), that students do a poor job of picking out the main points of a lecture, provides reason to be skeptical. Krantz's argument is also severely challenged by a recent meta-analysis of 225 studies from several STEM disciplines that found that student outcomes are significantly better when some kind of "active learning" is used (Freeman et al. 2014). This suggests either that lecturing is less effective than active learning approaches *or* that instructors tend to do a better job of implementing active learning approaches. Given the greater experience that most instructors have with lecture, the second explanation seems unlikely.

Krantz also argues that, "[l]ectures have been used to good effect for more than 3000 years" and that the lecture, "is a powerful teaching device that has stood the test of time" (p. 7). Krantz offers no evidence that lecture has been used to good effect and no support for the claim that it is a powerful teaching device. Certainly human history provides many examples of things that were done for a very long time but subsequently replaced by better ways of doing things. Interestingly, the author follows these assertions about lecturing by admitting that mathematics instruction in the United States is "not, overall, a great success" (p. 7). He attributes this to the "dreary reality" of underpaid, overworked faculty teaching students who are taking mathematics only because their major requires it. While this discouraging description may ring true to many readers, it also makes it difficult to accept the author's claims that lecturing is a proven technique.

So far I have painted a rather negative picture of Krantz's support for lecturing. However, I do find in the book creditable arguments in favor of lecturing in some contexts and for some purposes. The author notes that a lecture "can have wit, erudition, and sparkle" and that it can "arouse curiosity, inform, and amuse". (p. 7). He also insightfully observes that the traditional lecture approach to teaching assumes that the students are actively engaged in doing mathematics *outside of the classroom* and the expectation is that *that* is where the important learning happens.

I think there is a strong argument to be made in favor of this aspect of the traditional approach in cases where the students have developed to the point of knowing what it means to do mathematics. It makes sense to put the students in charge of this work and to have them do it in an environment that may be more suitable than a classroom session (e.g., a leisurely block of time on a couch with a cup of coffee and a cuddly cat). When I teach graduate level abstract algebra I begin the year using a lot of active learning with the goal of helping students learn to engage in mathematical inquiry. But then I transition to a mode where I use a lot of lecturing and explicitly expect the students to engage in the hard mathematical work outside of class. I also believe that a witty, erudite lecture can be used to arouse curiosity, inform, and amuse in ways that can inspire students to enthusiastically engage and succeed in such endeavors. However, in most situations, I would argue that there are better uses of the precious time that students have to interact with an expert, and the research literature mentioned above tends to support this argument.

The author's perspective on teaching and learning. Krantz makes a number of statements that reveal his perspective on what it means to learn and teach mathematics. For the most part, these are exactly what one might expect from an advocate of lecturing. In the preface, Krantz states that, "[i]f you are not transmitting knowledge, then you are not teaching." (p. xix). In the main text he states that, "the purpose of a class is to transmit knowledge and information" (p. 44) and speaks of, "getting your mathematics across to the students." (p. 59). These statements indicate a perspective that mathematics is a body of knowledge (owned by the expert teacher) that the teacher puts into the students' brains. These ways of talking about mathematics, learning, and teaching are at odds with almost all of the theoretical perspectives (e.g., constructivism) that underlie modern research in mathematics education.

This difference of perspectives can make it hard for a reader accustomed to the mathematics education literature to come to terms with some of what Krantz has to say about teaching. For example, at one point he makes that statement that "[a]n adequate instructor records the material accurately on the blackboard and then goes home" (p. 69). It is only from a "transmission of information" perspective of teaching mathematics that one could consider such instruction adequate. However, taking the book as a whole, Krantz's perspective is more nuanced than some of the quotes above suggest. We see this in his statement that, "[a] truly dynamic instructor interacts with the students, excites their intellectual curiosity, and helps them to discover ideas for themselves" (p. 69). Here and elsewhere, we see that Krantz does see learning as more than just receiving information and teaching as more than just transmitting it. One important theme that he develops along these lines is focused on scholarly discourse. He states that, "[p]art of your job as a teacher is to help students learn to engage in scholarly discourse" and he suggests that the teacher "try to create an atmosphere in which you and the students are co-explorers" (p. 85).

I would argue that it is largely because of this nuanced (some might say inconsistent) perspective that when it comes to teaching practice, Krantz's recommendations are often consistent with the educational research literature. When Krantz describes good lectures, he paints a picture of highly engaging and interactive experiences. He seems aware that most of the work of learning happens when the students are doing mathematics. Perhaps the clearest evidence that Krantz's nuanced view of teaching and learning informs his recommendations is seen in his discussion of flipped classrooms. So often when I hear instructors talk about flipped classrooms the focus is on the videos that the students are supposed to view outside of class. Krantz, however, recognizes that the "main point" (p. 99) of a flipped course is what happens when students are in class doing problems and interacting with one another and the instructor. To me this indicates a perspective that acknowledges the need for learners to be actively involved in their own learning rather than passive receivers of knowledge.

Conclusions. At first blush, "How to Teach Mathematics", with its focus on lecturing, is somewhat at odds with the current literature in mathematics education. The evidence against the value of lecturing continues to grow, and much of the author's language reflects a transmission model of learning that is nearly extinct in the research literature. However, a careful read reveals large areas of agreement between Krantz's views and the mathematics education literature. As such, I think this book could play a helpful role in the ongoing conversations between mathematicians and mathematics educators regarding undergraduate mathematics instruction.

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