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University Teachers' Resources Systems and Documents

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Abstract Mathematics teachers interact with resources of various kinds in their work in and out of class. The documentational approach of didactics is a theoretical approach that has been elaborated to investigate these interactions and their consequences in terms of teachers' practice and teachers' beliefs in particular. According to this approach, using resources teachers develop documents, encompassing these resources and a scheme of use; they also develop structured documents systems and resources systems. I interviewed six university teachers in France and collected the resources they used and designed. Drawing on these data I analyzed their documents and their resources systems, focusing on the features of resources use that appear specific to the university (rather than secondary school) context. I observed that lecturers can develop important agency in the design and use of teaching resources. On the other hand, PhD students starting to teach seem to conform their teaching to ready-made resources; nevertheless, their personal resources and beliefs intervened in their documentation work. Resources and beliefs deriving from the lecturers' research can also influence the development of documents and thus contribute to shaping the mathematics taught.

Keywords Documentation work · Documents · Resources · Resources systems · Teachers' beliefs · Teachers' practice

Introduction: Interactions Between Teachers and Resources at University

The study presented here belongs to the growing body of research concerning teachers' practice at university (Nardi et al. 2005; Jaworski et al. 2009). The starting point of my work is the following hypothesis: in all aspects of their professional activity, university

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teachers interact with a variety of resources (Adler 2000). These interactions influence teachers' practices and professional beliefs (Rezat 2010); conversely, the use of resources by teachers is influenced by their practices and beliefs. The aim of the work presented here is to investigate this hypothesis, in order to contribute to the understanding of teaching practices at university and of the factors governing these practices.

The interactions between teachers and resources at university and their consequences have not been much studied by research yet. Pinto (2013) observed two instructors who built two very different lessons, drawing on the same lesson plan. Grenier-Boley (2014) investigated linear algebra tutorials; he evidenced that the teacher's interventions and the students' activity in class were linked with the kind of mathematical tasks chosen by the teacher in a list of exercises. While neither of these two works focuses on resources use, we can interpret them this way. The two instructors followed by Pinto (2013) drew on the same resource, but built from it very different lessons; Pinto evidences that these differences are consequences of their backgrounds and personal intentions. The work by Grenier-Boley (2014) stipulated that the content of the mathematical tasks chosen by a teacher influenced his interventions in class. Mesa and Griffiths (2012) worked explicitly on resources use by faculties, analysing their use of textbooks. Using Rabardel's instrumental approach, they observed three kinds of mediations, in particular mediations between the teachers and the object of their activity, namely the design of instruction. Most of these mediations were pragmatic: textbooks were used to design lecture notes, homework, and assessment texts, for example. Some of these mediations were epistemic (transforming the subject him/herself), when the teachers reflected on the lecture notes they wrote drawing on a textbook and on their possible reception by students. Studying the use of textbooks by university teachers, González-Martín (2015) shows that the courses of five teachers he followed in Québec (on series of real numbers) closely adhered to the textbook. The study I present here is closely connected with the works by Mesa and Griffiths and by González-Martín, but considers all kinds of resources for the teachers including textbooks.

Previous research from Kindergarten to end of secondary school levels (Gueudet and Trouche 2009; Gueudet et al. 2013a, b) have evidenced that teachers in their work in class and out of class interact with many resources of different kinds. Teachers are not passive users of these resources, but active designers of their own teaching resources and of the curriculum actually proposed to their students. Their use of resources depends on many factors: teachers' professional activity takes place in institutions (Chevallard 2006) which shape their *praxis,* including their use of resources, at different levels of didactic determination (Winsløw 2015). Teachers' working environment (in terms of available material and physical space as well as students and colleagues, Ruthven 2009) is also an important factor influencing teachers' transactions with resources. Teachers' professional beliefs also play a central role in the use (or non-use) of resources at these levels. I hypothesize that similar phenomena take place at university level. In a previous study (presented in Gueudet et al. 2014) I analyzed the work of a mathematics teacher in a technological institute with such a perspective. The study presented here extends and deepens this previous research.

In this paper, I first introduce a specific theoretical framework, the documentational approach and, in section 2, the research questions investigated here. In section 3 I present the context, the data collection and methodology. In section 4 I analyze three contrasting cases of what is called the "resources system" for a teacher, at the level of a given teaching unit. In section 5 I analyze two case studies, presenting the documents developed by teachers for their given mathematical contents. In the conclusion I discuss implications of this study and more generally the perspectives offered by the approach retained.

Theoretical Frame: The Documentational Approach

The documentational approach (Gueudet and Trouche 2009) has been developed in order to study the interactions between teachers and resources and their consequences, in a context where an abundance of teaching resources is available, on the Internet in particular. I present here the main concepts of this approach used in this study.

Origin: Instrumental Approach and Schemes

The documentational approach is rooted in the instrumental approach developed by Rabardel (1995/2002). According to Rabardel, a subject engaged in a goal-oriented activity interacts with *artefacts*: products of the human activity, designed for a goal-oriented activity. Along with these interactions, the subject develops a personal construct called an *instrument*. The instrument associates the artefacts (or part(s) of these artefacts) and a scheme of use. The definition of scheme retained in the instrumental approach is given by Vergnaud (1998). A scheme has four parts:

– An *aim*;
– *Rules of action:* regular ways of acting for the same aim;
– *Operational invariants* of two kinds: *theorems-in-actions* (propositions considered as true by the subject) and *concepts-in-action* (concepts considered as relevant for the subject);
– Possibilities of *inferences*: the subject can adapt his/her activity to the special features of a given situation corresponding to the same aim.

The instrumental approach draws itself on activity theory (Leont'ev 1978; Engeström 2001). The aim, in Vergnaud's definition of scheme, can correspond with the goal in activity theory (the goal of an individual, engaged in a collective object-oriented activity). Nevertheless, the goal of an activity can be very general: for example, in the case of teachers, a goal of their activity can be to design instruction (Mesa and Griffiths 2012).

According to Vergnaud, a scheme developed by a subject is associated with a *class of situations* (the term "class" is here used as a synonym of "set"). In the case of professional situations, a class of situations gathers all professional situations corresponding to the same aim of the activity. For teachers, their whole activity could be seen as a single class of situations, corresponding to the aim, "design instruction", and associated with a "big scheme". Nevertheless, I contend that such level of generality would not be helpful to render precisely the different aspects of this activity. Thus the teachers' aims I consider, and the corresponding situations classes are more restricted:

for example, “preparing and setting up an assessment” is a possible aim, and thus also corresponds to a class of situations. The subject is aware of the aim of his/her activity. This activity can also be linked with less explicit objectives. These objectives can belong to the operational invariants: for example “At least two-thirds of the students must succeed in the exam” can be an operational invariant associated with the class of situations “preparing and setting up an assessment”. It is not an aim, in particular because the teacher is probably not aware of it.

Documents, Documents Systems, Resources Systems

The documentational approach prolongs the instrumental approach. Referring to the work of Adler (2000), it focuses on teachers and considers resources, defined as anything likely to re-source the teacher’s practice. Adler considers material resources, but also socio-cultural and human resources. Artefacts can be resources, if they re-source the teacher’s practice; but a resource can also be a puzzled expression on the face of a student, which typically is not an artefact.

Teachers look for resources, though sometimes they meet resources that they were not looking for (discussions with a colleague at the coffee machine, for example). They associate these resources, modify them, conceive their own resources and use them with students. All this activity is called *the documentation work* of the teacher (Gueudet and Trouche 2009). Documentation work holds a central place in the teachers’ professional activity, and it is the part of this activity that I study. During this documentation work, teachers develop a *document*: the association of resources and of a scheme of use (Vergnaud 1998) of these resources. For teachers, the operational invariants in the scheme are professional beliefs (Rezat 2010). For example, in Gueudet et al. (2014), we studied the case of a teacher, Peter, working in a technological institute. Peter teaches mathematics for computer science students. For his tutorials on the Gauss method, he uses Scilab (a numerical computation software) because he considers that the best way for his students to understand this method is to write an algorithm and implement it with Scilab. Within the frame of the documentational approach, I interpret this in terms of document. During his documentation work within this technological institute Peter developed a document. This document comprises several resources: Scilab and mathematics texts about the Gauss method in particular; a scheme of use of these resources, encompassing an aim, “preparing and setting up a tutorial on the Gauss method”; and rules of actions, like: “after a presentation of the Gauss method in the lecture, the students write the corresponding algorithm, implement it on Scilab and test it on examples during the tutorial”. These rules of actions are governed by an operational invariant, here a theorem-in-action: “*writing an algorithm is an efficient way for the students to understand and learn a method*”.

The process of development of a document is called a *documentational genesis*. Multiple documentational geneses occur along with the teacher’s work for various goals; they contribute to produce the *documents system* of the teacher, which is the structured set of all the documents he/she developed (Gueudet et al. 2012). We also introduced the concept of *resources system* of a teacher (Ruthven 2009; Gueudet et al. 2012), which can be considered as the “resources” part of his/her documents system: the set of all the resources used by the teacher, structured according to the aims of his/her activity. The documents system of a teacher is very complex; it can be considered at different scales, more or less general; it comprises many resources, associated with rules of actions and operational invariants. The same operational invariant (for example, "*writing an algorithm is an efficient way for the students to understand and learn a method*" like in Peter's case evoked above) can be associated with different aims, and thus intervene in different documents. In order to reduce this complexity, I have made here several choices. Firstly I will not investigate documents systems, but only resources systems on the one hand and documents on the other hand.

The structure of the documents systems and resources systems corresponds to the structure of the teacher's activity, which means, referring to Vergnaud (1998), to different classes of professional situations. Since a class of situation, according to Vergnaud, is associated with a scheme, it is possible to consider that for a given class of situations, the teacher develops a single document. Nevertheless I made a different choice here, in order to combine analyses at different levels of generality. I consider that aims like "preparing an assessment", which do not depend on the mathematical content involved characterize several documents, corresponding to a given class of situations; and that aims like "preparing an assessment on linear algebra" (which can be considered as a sub-aim of the previous one) depending on the mathematical content correspond to a single document. Other choices are possible; this particular choice is grounded in my intention to observe the organisation of the resources system (through classes of situation) at a general level; and to analyse documents, in particular operational invariants linked with the mathematical content. Hence one class of situation, defined by a general aim, is associated with several documents, corresponding to sub-aims.

Research Questions

The documentational approach leads me to claim that the interactions between teachers and resources (mathematical texts, software, etc.) generate documents and resources systems, and that investigating these documents and resources systems is helpful to understand the teachers' practice and the mathematics taught at university. The research questions studied here can be formulated as follows:

(1) What are the features of the resources systems developed by university teachers, and how do these features inform us about the teachers' practices at university?
(2) What are the features of the documents developed by university teachers, and how do these features inform us about the teachers' practices at university?

I try to answer these questions, focusing on the features which appear specific to the university institution (differing from the secondary school in particular). Lecturers at university are both teachers and researchers. The documentational approach only focuses on the teaching activity; we can assume that lecturers also develop a resources system for research, but this is not investigated here. Nevertheless I am interested in the influence of research on lecturers' documentational geneses for teaching.

I present in the next section the data and methods used to study these questions. Research question (1) is investigated in section 4 (about resources systems); research question (2) is investigated in section 5 (about documents). Section 6 presents a discussion combining analyses of resources systems and of documents to investigate teachers' practices, section 7 presents the conclusion and directions for future research.

Context and Methodology of the Study

Investigating the development of documents by teachers requires collecting much information for each case studied. A quantitative study, based on questionnaires, can provide information about the resources used, but not about the schemes developed. For this reason the documentational approach uses case studies. When inquiring about the documents of a given teacher, I collected both data given by the teacher, corresponding to his/her view on his/her documents, and data corresponding to the teacher's actual work, in particular his/her material resources. The combination of both kinds of data is a central tool in the documentational approach.

Data Collection

During the academic year 2013–2014 I met six university teachers working in France in the same, middle-sized university (which I will call University U). In this university in 2013–2014, around 270 students were enrolled in a 2-year program entitled "Mathematics, Computer science, Engineering, Electronics and Economics" (MCEEE in what follows). The teaching was organised in "teaching units", 6 or 5 in each semester. Amongst these 6 or 5 teaching units, between 1 and 4 concern mathematics, depending on the students' orientation. For each teaching unit in mathematics the students attend between 4 and 6 h per week, half of it as lectures, half of it as tutorials (meaning here, work in groups of less than 40 students on exercises).

The teachers were chosen to represent a variety of conditions likely to influence their documentation work: experience, research domain, studies in France or abroad, position, and gender. They also teach in a variety of "teaching units", concerning calculus, linear algebra, number theory, probabilities, numerical analysis or formal computation in the first or second year of university (I did not address in the interviews their teaching after the second year). Two (Mary and John) were PhD students in charge of tutorials only; the remaining four were lecturers and they were in charge of lectures and tutorials. Table 1 presents the profile of each teacher interviewed, and the teaching unit chosen for the interview.

Another important aspect of the context of my study was a change to the upper secondary school official curriculum: students entering the university in 2013–2014 (the year when data were collected) had followed the new curriculum, which is significantly different from the previous one. For example, this new curriculum added algorithms, expanded probability and statistics, reduced geometry, and eliminated differential equations.

I met each of these colleagues for an individual interview lasting approximately one hour (see the interview guidelines in Appendix). This interview took place in their office, where they prepare their courses. During the interview, I also requested "a guided tour of their resources system": I asked them to show me their books and other paper resources, their files and folders in their computer, and explain how they are classified, when and for which reasons they are modified, etc. I collected as far as

Table 1 Profiles of the teachers participating in the study and teaching units considered in the interviews

	Experience	Country of the studies position	Research domain	Teaching unit (for the interview)
Bob (M)	7 years	France Lecturer	Numerical analysis	Numerical methods for math majors year 2
Doris (F)	17 years	France Lecturer	Symbolic computation	Formal computation for math majors years 1 and 2
Nadia (F)	24 years	Italy Lecturer	Partial differential equations	Calculus for all MCEEE students year 1
Bill (M)	13 years	Germany and UK Lecturer	Geometric theory of groups	Linear algebra for all MCEEE students year 1
Mary (F)	2 years	France PhD student	Geometric theory of groups	Linear algebra for all MCEEE students year 1
John (M)	1 year	France PhD student	Spectral theory	Calculus for physics students year 2

possible all the resources mentioned (for books I only noted their references). The interviews were recorded and transcribed. Translations into English provided here are my own.

Analysing the Data: Building "Documents Tables"

Investigating documents and documentational geneses means to observe schemes of use of resources, with all their four elements: aim, rules of action, operational invariants and inferences. The data I collected were not sufficient for a complete investigation of schemes: a follow-up of the teachers over a longer period is needed to identify how teachers adapt their schemes to new situations, and thus the inferences component of the scheme. Thus instead of looking for schemes, I analyzed the data to identify aims of the activity, associated resources, stable ways to use these resources (rules of action) and discourses justifying these stable ways (operational invariants).

Firstly I tracked in each interview the aim of the activity mentioned by the teacher ("preparing a tutorial"; "writing an assessment text", etc.). Some categories were cited in the interview guidelines: lectures, tutorials, computer session, assessment. As would be expected they were present in the aims mentioned by the teacher (I formulated these aims as "Preparing and setting up a lecture" etc.); others appeared in the interview. I coded a new aim when the interview comprised a sentence such as: "I use this resource (e.g. book, e-mail, etc.) for …". For each aim I added the resources teachers used. Some of these resources were explicitly mentioned as such by the teacher; these are mostly material resources, so I also coded other elements as resources in the interview when they modified the teacher's practice (e.g. "international experience", "experience as student", etc.). The next step was to identify stable elements in the way these resources were used (rules of action). Concerning stability, I relied on the teachers' declarations (e.g., "for preparing a tutorial, I always start by solving myself all the exercises of the exercise list"). Finally I noted which beliefs were expressed about mathematics or about pedagogical issues (this corresponds to statements in the interview such as: "I do this way… because I think that …"). In a second step, I compared these teachers' declarations with the resources I collected (files proposed to students on a Moodle platform, books, etc.), made some adjustments and also inferred possible beliefs not expressed by the teacher.

With these elements, I constituted for each teacher a first tentative list of documents. In Table 2 below, I present an extract of such a list, stemming from the data collected for Bob. I presented the complete table to Bob, who corrected and complemented the table when needed.

In such a table, a given line describes four elements belonging to the same document: resources, and three elements of the associated scheme of use, namely: the aim, the rules of action, and the operational invariants. I do not claim to give a complete description of the scheme of use: in the table I only mention elements that are observed or inferred from the data. I exemplify here how to read the first line of the table. In his interview, Bob mentioned a particular aim: "preparing the first lecture on numerical methods". Since this lecture is the first at semester 2 of year 2, he uses the curriculum of year 1 and semester 1 of year 2 in order to make his mind on the students' previous knowledge. Then he writes a summary of this previous knowledge, which will be presented at the beginning of the lecture: this is a rule of action (according to him, he does this each time he has to prepare the first lecture of a given teaching unit). I infer from these observations an operational invariant, here a theorem-in-action which can be formulated as: *The new lecture must be connected with the students' previous knowledge.* Bob also uses his notes of the previous year and the notes of the previous teacher to write his lecture notes. He declares that he wants in this teaching unit to make clear for the students that "*the results they know are connected: mean value theorem, Taylor*

Table 2 Extract of Bob's documents table for the teaching unit on numerical methods

Aims	Resources used	Rules of actions (Way to use the resources)	Operational invariants (Reasons for using them this way)
Preparing the first lecture on numerical methods	Curriculum of year 1 and year 2 semester 1 Written notes (his notes and the notes of the previous teacher)	The first lecture starts by recalling previous knowledge which will be used The first lecture also introduces what is new in the perspective of the course	The new lecture must be connected with the students' previous knowledge. "The students must learn that the results they know are connected: mean value theorem, Taylor formula, etc.".
Preparing and setting up exercises for a tutorial on interpolation	Book (Crouzeix and Mignot 1984) Written notes (his notes) Previous exam texts Scilab Tables of values of functions, including an old table of a Bessel function	Some exercises are chosen from the book and adapted to the students' level Some exercises are chosen from the notes of the previous year Choose tables of values of functions and values which are not in the table.	The students must know how to use interpolation to produce approximate values of a function "Using links with the history of mathematics increases students' motivation"

formula, etc.". I contend that this is an operational invariant, which influences the Bob's design of this first lecture on numerical methods.

The complete table from which I extracted the two lines in Table 2 can be considered as a representation of the teachers' documents system, at the level of his/her whole teaching activity, if the table has been constituted for this level (or similarly at the level of a given teaching unit, or at the level of a mathematical theme). Nevertheless the table corresponding to the whole activity, or even to the activity at the level of a teaching unit, is very complex. For example Table 2 above corresponds only to the example of two documents, linked with precise mathematical contents. These documents are associated with classes of situations, corresponding to more general aims: "preparing and setting up a lecture", and "preparing and setting up a tutorial". A representation of Bob's documents system needs to encompass these classes of situations and all the corresponding documents; this would be very complex. Thus I have chosen a focus only on resources systems, which means on classes of situations (corresponding to general aims, independent of the mathematical content involved) and on the resources involved in documents corresponding to these classes.

Resources Systems for a Teaching Unit

I examine here the level of a teaching unit, which can encompass several mathematical themes: "complex numbers" and "limits and continuity", for example. Amongst the six teachers I followed, I observed three different kinds of resources systems at this level. A first distinction depends on the kind of teaching unit: "traditional" teaching units on the one hand, and teaching units with computer sessions on the other. A second distinction comes from the experience of the teacher: newcomers, here PhD students on the one hand and experienced lecturers on the other. In the next sub-section I present and compare the resources systems for a traditional teaching unit of an experienced lecturer and of a PhD student. I have chosen a teaching unit on linear algebra: I have indeed for this teaching unit data concerning both an experienced lecturer (Bill) and a PhD student (Mary). In the following sub-section I present the resources system of an experienced lecturer (Doris) for a teaching unit with computer sessions. I do not claim that these three kinds represent all the possibilities: they represent the teachers I interviewed. For example there is certainly a type of resources system of a novice teacher for teaching units with computers, but I did not interview such a teacher. One consequence of the choices presented above is that the resources system of John is not presented in this paper; it is in fact very similar to Mary's resources system.

Resources Systems for Teaching in a "Traditional" Teaching Unit: The Example of Linear Algebra

In University U, linear algebra is taught during the second semester of year 1, in a teaching unit entitled "Linear Algebra 1" (henceforth LA1 here). This teaching unit has existed for more than 20 years, although its content has been changed over the years to take into account evolutions in the secondary school curriculum; for this reason I consider it "traditional". As in the other "traditional" teaching units, calculators can be used during the tutorials, but they are forbidden during the assessments. Maple is recommended as a tool for the students to check, for example, calculations on matrices, but there are no computer sessions. The main resources given to the students in LA1 (as in other “traditional” teaching units) are the *polycopie*, which corresponds more or less to typed lecture notes. In France no textbooks are used at university; in many teaching units a *polycopie* is given to the students and constitutes the shared reference for the text of the lecture, the definitions, theorems, etc. In the LA1 teaching unit, the *polycopie* was written in 2006 by a group of lecturers. The students also receive a list of exercises (for the whole teaching unit) and have access on the web to the text of previous assessments. In University U in 2013–2014, the *polycopie*, exercise sheet, and texts of previous assessments were the material resources given to students in all “traditional” teaching units. Two of the teachers I interviewed taught in LA1: Bill, an experienced lecturer and Mary, a student in the second year of her PhD.

Table 3 below presents Bill’s and Mary’s resources systems for their teaching in LA1, which means: the different aims of their activity, characterizing different classes of situations, and the resources associated with these classes.

Concerning classes of situations, while the three first in the table (concerning lectures, tutorials, and assessments) were mentioned in the interview, the fourth

Table 3 Resources systems for the LA1 teaching unit, Bill and by Mary

Situations classes (Aims)	Resources used by Bill	Resources used by Mary
Preparing and setting up a lecture	Polycopie (with notes) A US book (Bretscher 2005) His own international experience Discussions with students at the end of lectures, evaluation of the teaching unit by students	*Not present for Mary*
Preparing and setting up a tutorial	Polycopie (with notes) Exercise sheets Previous assessment texts Discussions with the students, students’ productions (homework and assessments)	Polycopie Exercise sheets Previous assessment texts E-mails from lecturer, his/her webpage Discussions with the students, students’ productions (homework and assessments) Her own experience as student Summary of the lecture
Preparing and setting up an assessment	Exercise sheets Previous assessment texts e-mail with colleagues Students productions	Exercise sheets Previous assessment texts e-mail with colleagues Students productions
Communicating with students and colleagues	Discussions and e-mails with students and colleagues website of the university, of the maths department Files on his webpage to inform colleagues and students Discussions on specific students with colleagues.	Discussions and e-mails with students and colleagues website of the university, of the maths department Files on the lecturer’s webpage

one “communicating with students and colleagues” has been identified through the interviews. Its aim concerned the organisation of the whole teaching unit: deciding the dates of the assessments, choosing who is responsible for writing a first version of each assessment, following students who encounter difficulties, students with special needs etc. About the resources involved, it can be observed that some of them intervened for several classes of situations and thus held a pivotal role in the teacher’s documentation work. For the lecturer, the polycopie connected the “lecture” and “tutorial” classes of situations; while the “exercise sheets” and the “previous assessment texts” connected for both Bill and Mary the “tutorial” and “assessment” classes.

In a perspective of comparison between Bill’s and Mary’s resources systems, I observe that for Mary, who only taught tutorials, the documentation work could be limited to reading each week the lecturer’s e-mail which described the content of the lecture and recommended the exercises to choose; working on these exercises in class with her students, eventually choosing other complementary exercises. Nevertheless Mary did additional documentation work. Because she was a student in the same university, she knew that some students do not attend the lecture, so she prepared a summary of the lecture which she presented during the tutorial each time new content is addressed. Mary was also involved in the collective elaboration of one of the assessments and in the correction of students’ sheets—and for assessments, her role was the same than Bill’s role.

PhD students in University U teach in tutorials, but not in lectures. They can only develop limited agency in their documentation work (much less than a novice teacher at secondary school, for example). They do not participate in the design of resources given to the students; rather, they are more or less expected to align with the recommendations of the lecturer. Their use of the common exercise sheets is likely to influence their views on the teaching of mathematics and contribute to the development of a perspective shared with more experienced colleagues. I observed it as presented above for the case of Mary, and similarly for the case of John. Their more personal views are likely to develop from other kinds of resources: in the case of Mary, this was her own experience as a student.

In his documentation work for linear algebra lectures, Bill used the polycopie and followed in particular the order of presentation of the notions (in this teaching unit there are two lecturers, who must coordinate their lectures because of the regular assessments for all the students). But he also drew on a book from the US that he first met during a post-doc position in Vancouver. Bill also taught calculus in another teaching unit; to prepare his lectures in this teaching unit he used a German book. His international experience (he was at school in Germany, did his PhD in the UK and held a post-doc in Canada before obtaining a permanent position in France) was an important resource for his documentation work. He also participated in the initial elaboration and further modifications of the exercises list.

I also note here that neither Mary nor Bill used resources (such as lesson plans or exercises) downloaded from the Internet. This is a notable difference with the resources used by secondary school teachers, as observed in previous research (e.g., Gueudet and Trouche 2009).

Resources Systems for a Teaching Unit with Computers: The Example of Formal Computation

A teaching unit on formal computation is provided at University U in the second semester of the first year (entitled "Formal Methods 1", henceforth FM1 here) and the first semester of the second year (called "Formal Methods 2", FM2). FM1 lasts 30 h over 10 weeks: 10 h of lectures, 10 h of tutorials and 10 h of computer sessions. It is addressed only to maths majors and is dedicated to algorithms and their implementation in Maple (replaced in 2014–2015 by a new software, Sage). Around 70 students followed it in 2013–2014. Doris gave the lecture (for all the students); she taught the tutorials for one group (amongst three parallel groups), and computer sessions for two sub-groups (each tutorial group is divided in two halves for the computer sessions).

Doris designed the unit by herself 2 years earlier. Other colleagues who assisted in the tutorials or in the computer sessions were PhD students, but they gave no input into the design of the content, whether for the lecture, tutorials or computer sessions. Doris designed all these resources herself, using mainly books for higher levels (sometimes for graduate students) and creating her own exercises.

Doris decided to use Moodle in order to offer students the possibility of uploading their Maple programs. Now she also uses this platform to transmit resources to the students and to her colleagues. She does not distribute a polycopie at the beginning of the semester, but uploads on Moodle typed lecture notes at the end of the week. Similarly she gives each week (or every 2 weeks) an exercise sheet for the tutorial and another one for the computer sessions.

I compare here Doris' resources system for FM1 with Bill's resources system for LA1. Naturally, for Doris there is an additional class of situations, associated with the aim "Preparing and setting up a computer session". But other differences appear: in particular the Moodle platform is a central resource, used for all classes of situations. Like Bill, Doris uses exercise sheets and previous assessment texts both in the "tutorial" and "assessment" classes of situations. Moreover in her case, Maple is also used in three classes of situations, thus for three different aims, namely: "Preparing and setting up a lecture" (Doris video-projected Maple files during the lecture), "Preparing and setting up an assessment" (the final assessment always comprises questions on Maple) and naturally "Preparing and setting up a computer session".

Concerning Doris' documentation work, a first point to be noted here is her role as designer of her teaching. She conceived the content of the teaching unit and all the resources given to the students: her lecture notes as well as the exercises for the tutorial or the computer sessions. She built these exercises drawing on her own knowledge of Maple and on a book for year 3. At secondary school in France, the teachers we followed in our research on documentation work (e.g., Gueudet and Trouche 2009) almost never wrote their own exercises or problems, but found them in textbooks or in Internet resources. By contrast, Doris, who never searched for Internet resources to design her course, considered that no Internet resource could meet her precise needs. As a consequence, she cannot contrast her choices with other colleagues' choices and opinions; her only way to improve her course is to test it with the students, observe difficulties and change it the following year.

A second point to note is that Doris also uses Maple as a tool in her own research, as well as her teaching. This point surfaced during the interview when I asked about the teacher's links with research, in terms of resources used in particular (see question 5 in the Appendix). Similarly Bob, who teaches in the other teaching unit with computer sessions (about numerical analysis) uses Scilab for his teaching (Table 2) and for his research. By contrast, the other four teachers I interviewed, who only participate in "traditional" teaching units, use different resources in their documentation work for teaching from those they use for research. In the next section I will substantiate these initial statements through the analysis of specific documents.

Documents at University: Two Case Studies

In section 4 I have analysed resources systems at the level of a teaching unit, and chosen not to present the teachers' beliefs involved (for the sake of clarity, and to avoid the presentation of very general beliefs that do not inform us enough about the mathematics taught). In section 5, the analyses concern the level of a mathematical theme; I focus on documents with their four components: aims, resources, rules of actions and beliefs. Attention is given to two contrasting case studies: Doris, already mentioned in the previous section, and Nadia, who is also an experienced lecturer. I focus in particular on the features of their documents likely to be specific at the university level.

Doris and Her Teaching of the Euclidean Algorithm

Table 4 above presents the classes of situation and the resources used by Doris for the FM1 teaching unit. In FM1 she teaches in particular formal computation for number theory (number theory has been taught by other colleagues during semester 1 in another teaching unit called AR1), including the Euclidean algorithm, which is the first chapter of the course.

Table 5 below is linked with Table 4, but offers a different view. Table 4 represents a resources system at the level of the whole teaching unit: it only comprises the general aims of Doris' activity and the resources she uses for each aim. Table 5, on the other hand, is a "documents" table; it focuses on the teaching of the Euclidean algorithm, and presents documents with 4 components: aim and resources as well as rules of action and operational invariants.

A central belief cited several times by Doris in the interview and shaping her documentation work for the aims mentioned in Table 5 is that the students be convinced that the mathematics is useful and that they must "apply" the mathematics learned in the course. Moreover, a good way to "apply" them is to program methods using, for example, Euclidean algorithm. Another way (in a second, more elaborate step) is to compare the efficiency of two methods in terms of cost. These beliefs are strongly connected with Doris' own research in formal computation: she spends a lot of time programming and evaluating the cost of methods.

The Euclidean algorithm and its proof have been presented to the students during the first semester. Most of the students have in fact already met Euclidean algorithm at secondary school, if they followed the "math specialty" in grade 12. In grade 12, and during the first semester, the students see the proof of Euclidean algorithm presented by the teacher. But they are not expected to be able to do this proof themselves. In grade

Table 4 Resources system for the FM1, Doris

Situations classes (Aims)	Resources used by Doris
Preparing and setting up a lecture	Her own lecture notes (from the previous year, then adapted) Book (year 3 level) Moodle platform, Maple, Computer and video-projector Students' evaluations from previous year
Preparing and setting up a tutorial	Exercise sheets Assessment texts from previous years Moodle platform Students' productions (homework and assessments)
Preparing and setting up a computer session	Computer session exercise sheets Computers, Maple, Moodle platform Students' productions (Maple programs)
Preparing and setting up an assessment	Exercise sheets Assessment texts from previous years Maple E-mails with the colleagues who teach the tutorial and computer session
Communicating with students and colleagues	e-mails with students and colleagues website of the university, of the maths department Moodle platform The files uploaded on Moodle inform both students and colleagues

12, the students have also learned to write and program algorithms; some of them might have already programmed the Euclidean algorithm, but not with Maple.

In the lecture Doris presents the algorithm once again, but she does not give the proof. She presents some numerical applications, and then starts the work on the cost of Euclidean algorithm. The following theorem is presented during the first lecture (Fig. 1):

The lecture finishes this way (I know this from the resources collected, since Doris uploads her lecture notes after the course), the proof of the theorem is not presented, because Doris wants to propose to her students an exercise about this proof. Hence the following exercise is proposed during the first tutorial (Fig. 2):

1. *Prove that for* $1 \leq i \leq N\text{-}1$, $r_{i-1} \geq 2r_{i+1}$, *then that* $r_0\, r_1 \geq 2^{N-1} r_{N-1} r_N$
2. *Deduce that the number of iterations in the Euclidean algorithm applied to a and b is smaller than* $2log_2(a)$.*)*

This exercise constitutes a first work about the cost of Euclidean algorithm. Question 1 in Exercise 4 asks for two proofs; the first one is similar to a proof given in the lecture, but for the second one the students must find a method by themselves. Question 2 uses the function log_2, which has been introduced in the first lecture and is not familiar to students, who must use in particular the property $log_2(2) = 1$ but who can get confused with the *ln* function. So this exercise prepares the following computer session, as revealed by Doris in the interview: in this computer session the students must program Euclidean algorithm with Maple, then write a program displaying the number of iterations in Euclidean algorithm and observe its result for several input values. But

Table 5 Doris' documents table concerning Euclidean algorithm (main rules of action and operational invariants)

Aims	Resources	Rules of action	Operational invariants
Preparing and setting up a lecture on Euclidean algorithm	Her own lecture notes from the previous year Book (year 3) Moodle platform Maple Computer and video-projector Students' evaluations from previous year	Doris writes the course on the blackboard She sometimes projects Maple sheets. After the course Doris uploads her lecture notes on the Moodle platform She compares a "naïve" method and Euclidean algorithm	"The objective of the teaching is programming." "They must evaluate the efficiency of an algorithm." "The content is not new, the idea is to show them that the maths they learned during the first semester are useful."
Preparing and setting up the tutorials on Euclidean algorithm	Exercise sheets; Assessment texts from previous years; Moodle platform; Students' productions	Doris suggests to her colleagues the exercises to choose in the exercise sheet.	"The exercises are chosen to prepare the computer session."
Preparing and setting up the computer sessions on Euclidean algorithm	Computer session exercise sheets Maple Computers Moodle platform Students' productions (Maple programs)	The students implement with Maple the algorithms met in the course. There is always a basic exercise, then some more elaborate exercises. The students are asked to upload their programs on the Moodle platform. Doris reads their work, then uploads the correction of the computer session.	"The texts I have chosen lead them to really do applications [of the course]" "Some of the students are very good and go fast." "The machine must evidence that the computation is very long with a naïve method and short with an optimized method". "Asking them to upload their work makes them feel that it is important".
Preparing and setting up an assessment on the Euclidean algorithm	Assessment texts from previous years E-mails with the colleagues who teach the tutorial and computer session	Make short assessments (10 min) at the beginning of each course The calculator is forbidden for the assessment.	"The students must learn the course, only regular assessments drive them to do that." The assessment corresponding to the Euclidean algorithm chapter comprises an exercise about the application of this algorithm to two given integers.

Théorème 3 *Supposons* $a>b>0$ *et notons* $\varphi = \frac{1+\sqrt{5}}{2}$. *Alors le nombre d'itérations N dans l'algorithme d'Euclide appliqué à a et b vérifie*

$$N \leq \frac{\ln(a)}{\ln(\varphi)} < 1.441 \log_2(a).$$

Fig. 1 Theorem about the cost of Euclidean algorithm, Doris lecture notes. (*Translation: Suppose* $a > b > 0$ *and* $\varphi = \frac{1+\sqrt{5}}{2}$. *Then the number N of iterations in Euclidean algorithm applied to a and b is such that:…)*

at the same time this exercise is clearly linked with a work on the proofs given in the lecture: for question 1 students must reproduce a proof presented a few days before in the lecture; while question 2 prepares them for a proof that will be given in the next lecture.

This precise analysis confirms that a belief that can be formulated as "*I want to show that the mathematics they learned during the first semester is useful*" holds an important place in Doris' documents. It is connected for her with the use of software. At secondary school in France, showing that mathematics is useful is presented as an objective of the teaching in the official curriculum, and the software should contribute to it. The usefulness of mathematics is evidenced through inquiry-based teaching; the software is a tool for the students in their inquiry. In Doris' teaching, the link between the software and the usefulness of mathematics is different. Programming with Maple leads the students to apply the mathematics they know (or should know) to use them as a tool in particular to compare the efficiency of methods. This articulation between mathematics and software use is completely new for the students; it is directly linked with Doris' research activity.

Another aspect to be emphasized is that, even if Doris does not mention proof as an objective of this teaching unit, it holds an important place in it. Students have to write proofs in exercises; they are supposed to be able to use proofs given in the lecture as tools that they can adapt for similar properties presented as exercises.

I will now study a different case, and return later to the above statements in a comparative analysis.

Nadia and Her Teaching of Complex Numbers

Nadia is an experienced lecturer who has taught mathematics at university for 24 years. I focus here on a teaching unit called "analysis 1" (AN1; 6 h each week over 12 weeks), which is addressed to all first-year students (9 groups of around 30 students each). This teaching unit is considered a "transition" teaching unit: its content is strongly connected with mathematics taught at secondary school, and the students stay in small groups with

Exercice 4 : Soient a et b des entiers tels que $a>b>0$. Soit $(r_i)_{0\leq i\leq N+1}$ la suite des restes dans l'algorithme d'Euclide appliqué à a et b. Elle est définie par $r_0=a, r_1=b$ et pour $1\leq i\leq N, r_{i+1}=reste(r_{i-1}, r_i)$ avec $r_i\neq 0$ et $r_{N+1}=0$.

1. Montrer que pour $1\leq i\leq N-1$, $r_{i-1}\geq 2r_{i+1}$ puis que $r_0 r_1 \geq 2^{N-1} r_{N-1} r_N$.
2. En déduire que le nombre d'itérations dans l'algorithme d'Euclide appliqué à a et b est majoré par $2\log_2(a)$.

Fig. 2 Exercise about the cost of Euclidean algorithm, first tutorial. (*Translation: Let a and b be integers such that* $a > b > 0$. *Let* $(r_i)_{1\leq i\leq N+1}$ *the sequence of rests in the Euclidean algorithm applied to a and b. It is defined by* $r_0 = a, r_1 = b$ *and for* $1 \leq i \leq N$, $r_{i+1} = rest(r_{i-1}, r_i)$ *with* $r_i \neq 0$ *and* $r_{N+1} = 0$

the same teacher for the lecture and for the tutorial. The topics studied in it are: complex numbers, functions, limits, integration, and differential equations. Nadia has taught this teaching unit for more than 10 years. I examined the documents she developed for this teaching unit, with a focus on her teaching of complex numbers, in the first week of year 1 (Table 6).

In her interview Nadia expressed a strong belief which guides the design of her course: for her the objective of the AN1 teaching unit is to "bring the students up to the standards". What are these reference standards for her? Her explanation is: "they know less mathematics than 10 or 15 years ago". Nadia considers that the objective of this teaching unit is to teach the students what was taught at secondary school when she started teaching at university. Another important belief is that, given that this teaching unit concerns all the first-year students, and that most of them will not do maths later, she does not teach proof. She considers that teaching proof is under the responsibility of colleagues in the teaching unit on number theory (AR1, mentioned above).

She does not use any kind of software in class, nor the calculator, which is forbidden in exams. She knows that calculators can do all the technical work which is central in this teaching (like solving second degree equations in **C**, see below). Asked about resources coming from her research, she reported that she does not use such resources, but emphasizes two important aspects. The first concerns the use of maths books: Nadia regrets that this year no book was used as a shared reference for the AN1 teaching unit. For her, learning to use a mathematics book should be an important objective of university teaching, and she recommends the students use a calculus book available at the university library. The second aspect is that she supports a "research attitude" from her students: keep attempting an exercise, even if they do not know how to solve it. This can seem contradictory to her previous claims, which sounded like "students only learn to apply techniques".

Concerning her teaching of complex numbers, she mentions different aspects that she considers important: the students must know how to represent a complex number (given by an algebraic, trigonometric or exponential form) by a point on the plane. They must also know how to solve a second degree equation with complex coefficients. Let us consider the type of task (Chevallard 2006): "solving a second degree equation with complex coefficients".

At secondary school, the students meet complex numbers in grade 12, and they learn to solve in **C** a second degree equation with real coefficients. For a polynomial P defined by $P(z) = az^2 + bz + c$, the method presented in all grade 12 textbooks to solve $P(z) = 0$ is to start by computing $\Delta = b^2\text{-}4\,ac$. Three cases are then possible:

– For $\Delta > 0$, the two roots are $\frac{-b\pm\sqrt{\Delta}}{2a}$;
– If $\Delta = 0$, there is a double root $\frac{-b}{2a}$
– For $\Delta < 0$, the two roots are $\frac{-b\pm i\sqrt{-\Delta}}{2a}$

In this technique, the students use the square root symbol, and compute the square root from a positive real number. What is new, compared with solving the same equation in IR, is that the equation also has two solutions when Δ is negative, and that the two square roots of a negative real number Δ are $\pm i\sqrt{-\Delta}$.

Table 6 Nadia's documents table for complex numbers in the AN1 teaching unit (main rules of action and operational invariants)

Aims	Resources	Rules of action	Operational invariants
Preparing and setting up a lecture on complex numbers	AN1 polycopie Books Old polycopie Her own course notes from previous years Students' answers to her questions	Nadia presents her lecture on the blackboard, sometimes using only her memory, sometimes her notes, and sometimes the old polycopie. Nadia does not present proofs. At the beginning of the lecture, Nadia tries to assess the students' prerequisites by asking simple questions. She works with the students to understand the formulas in the AN1 polycopie. She asks the students to use also a book, available at the library.	"The course is not centred on proofs, the objective is to bring the students up to the standard, most of them will not do maths later." "They must learn to write coherent sentences and reasoning." "The students must learn to use mathematics books" "At the beginning of the course, some students are not able to solve a second degree equation with coefficients in IR." "The students must learn to solve second degree equations with complex coefficients."
Preparing and setting up a tutorial on complex numbers	Exercise sheets Assessment texts from previous years	Nadia chooses exercises in the exercise sheets and they are solved during the tutorial. She sends students to the blackboard to solve the exercises. She tries to follow each student individually. The students are not allowed to use their calculators.	"The students must learn to apply the formulas which are in the polycopie." "The students must try things and search even if they do not know the solution." The students must develop calculation skills. "Now the calculators can directly solve the second degree equations, so they cannot be allowed."
Preparing and setting up an assessment on complex numbers	Assessment texts from previous years E-mails with colleagues	Two teachers prepare the first version of the assessment text, then it is discussed by e-mail in the team. Each teacher corrects his/her own students, except for the final exam.	"The assessment text must be very similar to the exercises done during the tutorial." The assessment text comprise on exercise asking to solve a second degree equation in C.

At university, solving the equation starts similarly by computing $\Delta = b^2\text{-}4\ ac$, but computing the two square roots of Δ is a new task. How is this new task presented in the resources used by Nadia?

Nadia uses for her own lecture the presentation given in an old polycopie, written in 1995. This text presents how to compute the two square roots of a complex number (solve the equation $\omega^2 = z$) with two methods that I summarize briefly here. Firstly if z is written in an algebraic form: $z = a + ib$, and $\omega = \alpha + i\beta$, the identification of the real and imaginary parts gives $\alpha^2\text{-}\beta^2 = a$ and $2\alpha\beta = b$. With the equality of the modulus we also have $\alpha^2 + \beta^2 = \sqrt{a^2 + b^2}$. Using these three equations, it is possible to compute α and β.

The second method proposed in the 1995 polycopie is: if z is written in an exponential form: $z = re^{i\alpha}$, then $\omega = \pm\sqrt{r}e^{i\frac{\alpha}{2}}$.

The methods are followed (in the old polycopie) by an example: computing, with the two methods, the square roots of $z = 2 + 2i$. After this example, two remarks are added: one about the choice of the most appropriate method, according to z (this remark is linked with the assumption that the students can easily use different representations of a complex number, although research on this topic – e.g., Panaoura et al. 2006 – proved the opposite); the other stating that "it is forbidden in **C** to use the square root sign, because √ is not a function on **C**". Nadia also presents the example and the two remarks in her course (according to her interview statements and lecture notes).

A first statement, considering Nadia's course, is that the technique used to solve second degree equations with complex coefficients is much more difficult than the technique learned at secondary school for equations with real coefficients, because of this additional step (solve the equation $\omega^2 = \Delta$). Two methods are possible and the students need to choose the more appropriate one. If they use the method with the algebraic form, they need to solve a system of three equations. Moreover it is now forbidden to use the square root sign, which was part of the method seen in grade 12; and the explanation "because √ is not a function on **C**" refers to a theoretical definition of function which is not seen at secondary school.

The second statement concerns the activity mentioned by Nadia as "reading the polycopie with the students". In the AN1 polycopie, the method is presented as follows (Fig. 3):

This presentation is very different from the properties met at secondary school—and also different from the presentation retained by Nadia. For example, it is not written that

Proposition 3.2 *Soit $\Delta = X + iY$ un complexe non nul. L'équation $\delta^2 = \Delta$ possède exactement deux solutions : $\delta_1 = x+iy$ et $\delta_2 = -\delta_1$. On a* $x^2+y^2=\sqrt{X^2+Y^2}$, $x^2\text{-}y^2=X$ *et* $2xy=Y$.
Par conséquent, x^2 *et* $-y^2$ *sont solutions de l'équation*

$$Z^2 - XZ - \frac{Y^2}{4} = 0.$$

On a $x^2 = \frac{1}{2}(\sqrt{X^2+Y^2}+X),\ y^2 = \frac{1}{2}(\sqrt{X^2+Y^2}-X)$.

De plus x et y sont de même signe si $Y \geq 0$ *et ils sont de signes opposés si* $Y < 0$.

Fig. 3 Property and method, computing the square roots of a complex number. (*Translation: Let $\Delta = X + iY$ be a non-zero complex number. The equation $\delta^2 = \Delta$ has exactly two solutions $\delta_1 = x + iy$ and $\delta_2 = -\delta_1$. We have: $x^2 + y^2 = \sqrt{(X^2 + Y^2)}$, $x^2\text{-}y^2 = X$ and $2xy = Y$. Hence x^2 and $-y^2$ are solutions of the equation $Z^2\text{-}XZ\text{-}Y^2/4 = 0$. We have … Moreover, x and y have the same sign if $Y \geq 0$ and have opposed signs if $Y \leq 0$*)

in the equation Z^2-XZ-Y^2/4 = 0, Z is the unknown. Moreover, it is important to understand that this equation concerns only real numbers (otherwise it can seem much more complicated than the initial equation!), and this is not stated in the text. The students are probably not able, from reading the polycopie, to understand this property and to connect it with Nadia's course. Hence Nadia's rule of action, "We work with the students to understand the formulas in the polycopie", corresponds to a real necessity.

During the tutorial, the students have to solve the three equations:

$$\text{(a) } 2z^2-6z+5=0 \quad \text{(b) } 5z^2+(9-7i)\,z+2-6i=0 \quad \text{(c) } z^2-(3+4i)\,z-1+5i=0$$

At the final exam, the following exercise is proposed (Fig. 4):

This exercise is simpler than (b) and (c) above. Computing the square roots of Δ is presented as a first question; students should be able to use the results of this first question in the second one. Nevertheless, the computation (which must be done by hand, calculators being forbidden) is quite difficult, in spite of the hint given at the beginning: √289 = 17.

From this study of Nadia's documents concerning complex numbers I note in particular the following statements. The belief "*they must learn to use mathematics books*" is certainly not present at secondary school – in the studies we conducted at this level (e.g., Gueudet and Trouche 2009) the only books are textbooks, which the students are not asked to read: the textbooks are used by the teacher to give exercises. I contend that it comes from Nadia's research activity: she spends a lot of time reading mathematics which have been written by colleagues. This is a typical consequence of a documentational genesis. Nadia considers reading an important part of mathematical activity (according to her interview statements). I claim that this belief comes from her own mathematical activity: Nadia reads mathematics in books and articles. The teachers at secondary school in France do not read mathematics books or articles (according to our previous studies, e.g. Gueudet and Trouche 2009); some of them read professional newspapers, but the mathematics in them remains quite simple. Nadia is very experienced and she knows that reading mathematics, even in the polycopie, must be taught to the students. This belief is associated with another: "*the students must learn to write coherent mathematical sentences*". This reading/writing articulation is central for Nadia.

Another important feature is linked with the belief that "*the students must develop computation skills*". Nadia never uses the calculator or any other kind of software; she wants the students to compute by hand. At secondary school, in direct contrast, the use of calculators and various kinds of software is an explicit objective of the official curriculum. Nadia declared in her interview that she tried to use Maple 10 years ago

Exercice 1

On rappelle que $\sqrt{289}=17$.

1. Calculer les racines carrées du nombre complexe -15+8*i*.
2. Déterminer les racines réelles ou complexes du polynôme $P(z)=z^2+(3+2i)z+5+i$

Fig. 4 Exercise proposed at the final exam. (*Translation: We recall that √289 = 17. 1. Compute the square roots of the complex number −15 + 8i. 2. Determine the real or complex roots of the polynomial P(z) = z^2 + (3 + 2i)z + 5 + i*) with the AN1 students, but stopped because "the students already have difficulties, for a simple exercise, to write the solution as coherent sentences with a coherent reasoning… Maple cannot help with that". Over the years, students' productions have constituted resources for Nadia, and these resources have produced a modification of her practice: she stopped using Maple in this teaching unit.

Contrasting the Two Cases

I have chosen to examine the cases of Nadia and Doris because they teach in very different teaching units for first-year students at University U. In the first instance, the differences observed above can be interpreted as consequences of these different conditions. Doris herself created the main resources of the FM1 teaching unit. Nadia participated over the years with colleagues in elaborating the main resources of the AN1 teaching unit (polycopie and exercise sheets), but her documentation work consists more now in the use of existing resources. Doris uses several digital resources, mainly Maple and a Moodle platform, while Nadia does not use digital resources for this teaching and has chosen in particular to avoid using Maple. This difference can be interpreted as a consequence of both the content of the teaching unit (learning to use Maple is one of the FM1 teaching unit's objectives) and the kind of students concerned (AN1 is for all the students, FM1 is only for Math majors).

Another important difference in Doris' and Nadia's cases is that, while Nadia considers that she must avoid proofs in her teaching, Doris dedicates a very important part to proofs in her lectures and in the exercises proposed. Analysis of the resources she designs clearly indicates, for example, that she expects the proofs of theorems given in the course to constitute resources for the students when they solve exercises. This difference could also be seen as a simple consequence of the different students; nevertheless, I contend that it is more fully interpreted in terms of resources and associated professional beliefs. Indeed, research in higher education (e.g., Farah 2015) has evidenced that teachers in France expect students to read all the proofs given in the course and use these proofs as examples of possible reasoning that they can adapt to other situations. But many students, even Maths majors, do not include reading proofs in their personal work. Thus Doris' expectations concerning proofs are unlikely to correspond to the actual personal work of students. Nadia declared in her interview that she follows each student individually: students' productions are central resources in her documentation work. Thus I infer from the data collected for Nadia that, by working for many years with these resources, she developed an operational invariant such as, "*first-year students do not use the proofs given in the course as resources to solve exercises and build their own reasoning*".

In spite of the very different contexts, I also observed commonalities in the two cases. Some of these common points concern excluding the use of some resources: Nadia and Doris do not search for Internet resources. Nadia reported that she already has all the resources she needs, while Doris considered it impossible to find on the Internet resources addressing her precise needs. They do not use secondary school textbooks either, even though the recent changes in the secondary school curriculum are linked with the contents they teach. In fact both Nadia and Doris believe that they cannot draw on students' previous knowledge, and prefer to introduce such content as something completely new.

Another common point for Nadia and Doris is the presence of resources, and more interestingly beliefs, resulting from their own research experience. Nadia praised the use of mathematics books and the development of reading/writing skills in mathematics. Moreover she supports the adoption of a "research attitude" by the students. Doris tries to convince the students of the usefulness of mathematics through the use of mathematics to compare the efficiency of algorithms which is central in her own research. These beliefs are present in documents developed by Nadia and by Doris, and contribute to shape the mathematics they teach.

Resources Systems, Documents and Mathematics Teachers' Practices at University

In sections 4 and 5, through the analysis of resources systems on the one hand and of documents on the other hand, I already presented several results concerning mathematics teachers' practices at university. In section 6, I combine results coming from the resources systems analyses and from documents analyses, not only concerning the cases presented above but all the 6 teachers I interviewed. I focus here only on the most important statements stemming from these analyses; I also try to formulate more general hypotheses, not limited to the cases studied.

Alignment, Agency, and Novice Teachers

An important issue, when studying documentation work, is the balance between "alignment with the content of teaching resources" and "agency in the creation and use of original teaching resources". I emphasize several results concerning this balance.

Novice teachers, here PhD students (Mary and John), seem to be expected to align non-creatively with given resources. This can be considered an implicit form of professional development, in a context where no explicit teacher education is proposed. The years spent by novice teachers teaching only in tutorials, using ready-made exercise sheets, certainly can be interpreted as a way for them to enter a community sharing a common practice (Biza et al. 2014). Nevertheless the documentational approach demonstrates that personal beliefs always contribute to shaping the teachers' practice, and the case of Mary reported here confirms that even newcomers can and do include personal dimensions in their documentational work. Mary (section 4 above) prepares summaries of the lecture because she knows from her own experience that some students do not attend it. Most probably, for novice teachers, personal dimensions coming from their own experience as students, still quite recent, play an important role. In Gueudet et al. (2014), I noted that the teacher I followed, Peter (who was already experienced) continued using a 20-years old book that he had as student for preparing his courses.

For teachers (in particular novice teachers, but also more experienced teachers, like Bill and Nadia) who had a previous teaching experience in another university (possibly abroad with a very different culture), this experience is also likely to constitute a resource; hence the documents developed are also likely to include this personal dimension. I have also observed that some lecturers develop significant agency in the creation and use of original teaching resources for some specific contents (Doris for formal computation; the same holds for Bob, who teaches numerical analysis).

Use of Digital Resources

The documentational approach has been developed in the context of an abundance of available resources, digital resources in particular. In this university-based study, digital resources seem to occupy a restricted role. In the specific context of my study, software can be very important in the work of teachers for teaching units with computer sessions (Doris and Bob). In other teaching units, however, no specific mathematical software is used and calculators are forbidden for first-year students. This can be a local situation at this university; I do not claim that most university teachers (in France or in other countries) do not use software for their teaching of mathematics. Concerning digital resources, I also maintain that none of the teachers I met searched the Internet for resources (lesson plans, exercises) designed by colleagues. They do not seem to consider that searching for Internet resources takes too long, but either that they do not need additional resources, or that they cannot find on the Internet a resource corresponding to their precise didactical intentions. I claim that this situation is likely to concern all mathematics teachers at university in France; in particular, many national websites are dedicated to sharing teaching resources for secondary school teachers, and only a few for university teachers. The Internet seems to only serve the purpose of communication between teachers or between teachers and students, which is an important issue, but it is not used in France for sharing or collectively designing resources, beyond exam texts.

Research and Teaching

Another issue I highlight here is the presence, in the teachers' documentation work for teaching, of resources and beliefs coming from their research activity. The link between research and teaching activities for university teachers has already been considered by research in university mathematics education; Winsløw (2012) calls it "the teaching-research nexus". A documentational approach perspective underscores in a specific manner the interplay between research and teaching at university, by evidencing that (at least some) university teachers expect a use of resources by their students similar to their own resources use for their research activity. For example the importance of reading and writing mathematical texts increases at university, compared with secondary school; I contend that this can be a consequence of teachers' beliefs developed within the research activity. These beliefs have probably been developed by most, if not all, lecturers; other beliefs, however, are more personal, like the usefulness of mathematics evidenced through the cost of algorithms (see Doris' case). The influence of these more personal beliefs resulting from research also explains the specific agency of teachers in their documentation work at university.

Conclusion

The research questions studied here were:

(1) What are the features of the resources systems developed by university teachers, and how do these features inform us about the teachers' practices at university?
(2) What are the features of the documents developed by university teachers, and how do these features inform us about the teachers' practices at university?

Through the analysis of the data I collected with 6 teachers with different profiles, I observed different kinds of resources systems. The differences where linked on the one hand with the responsibilities given to the teacher; these responsibilities indeed change the possible aims of the teacher's activity: teaching lectures or not, and this is connected with the teacher's experience, in the university where my study took place. The differences between resources systems were linked on the other hand with the kind of teaching unit concerned: with or without computer sessions; this naturally changes the possible aims of the activity, but also seems to change the resources used, since digital resources were only used (in the cases I studied) for teaching units with computer sessions. Concerning the resources used for tutorials by the teachers followed, the (traditional) exercises sheets and previous assessment texts, seem to still play an important role. The assessments are known to shape the didactic contract (the implicit rules followed by teachers and students, Brousseau 1997; for the link with assessments see e.g. Lebaud 2009). I contend that this can be interpreted within the frame of the documentational approach: the previous assessment texts are resources that shape teachers' and students' activity, and lead to a stability of the didactic contract across the years. Nevertheless, other kinds of resources intervene (not only material resources), and these resources can contribute to evolutions in the teaching practices: for example the international experience, for the teachers who had one such experience, also the discussions with students.

Concerning documents, I focused on two case studies, linked with particular mathematical contents: the Euclidean algorithm, on the one hand, and complex numbers on the other. I observed precise teachers' beliefs (theorems-in-action, referring to Vergnaud 1998) of different kinds. These beliefs can concern the actual students' competencies, like "novice students are able to use different representations of complex numbers", or what students should learn at university, concerning precise contents like "students must learn to compare the efficiency of algorithms" or more general mathematical practices, like "students must learn to use mathematics books"; or teaching strategies "the idea is to show students that the mathematics they learned are useful". These beliefs shape the interactions between teachers and resources: proposing a mathematics book as a useful resource to students, choosing in a polycopie (written 20 years before) an example using different representations of complex numbers, designing computer sessions where the students work on programming the methods they learned in a previous course on number theory, for example. At the same time, these beliefs were developed through interactions with resources, in the teaching activity but also in other activities: learning (as student several years before) or researching in particular. A comparison between these beliefs and secondary school teachers' beliefs could be an interesting further step, enlightening some gaps experienced by students during the secondary-tertiary transition.

I also note more generally that this study exemplifies the analysis of university teachers' documents and resources systems, deepening the preliminary study presented in Gueudet et al. (2014). Such an investigation highlights teachers' practice, including their out of class practice and how it is grounded in teachers' professional beliefs. I did not have the opportunity to observe in class the six university teachers interviewed because they consented to the interviews at the end of their courses when they had more free time. Further investigation of university teachers' documents naturally requires taking this new step, including direct observation in the data collection in order to be able to combine the teachers' testimonies not only with their actual resources but also with several kinds of data. It is also necessary to study how interactions with students become resources for the teachers; this could lead to a renewed perspective on teachers' pedagogical development (Nardi et al. 2005). Another observation stemming from the cases studied here is that the documentation work of teachers at university seems to be rarely a collective work; and that potentially interesting resources designed by teachers are not shared with colleagues. Supporting a more collective documentational work at university is also an important perspective to be considered in future research, to enable the resources designed to be collectively improved (Pepin et al. 2015). I consider these perspectives an important direction for further research on University Mathematics Education, because in higher education, alike other levels, documentation work is central in teachers' work.

Appendix: Interview Guidelines

This interview concerns resources (mostly material resources), intervening in your teaching for the first or second university year. My aim is to understand which resources you use, which resources you design for your students etc.

During the interview, the researcher also makes a "*visit of the resources system*". *The interview takes place in the teacher's office. The teacher shows the resources used: books, exercises sheets, how they are organized; his/her computer, the organization of his/her files in folders* etc.

Years of experience in teaching: Research domain:

1) Let us consider a teaching you did this year, for example « linear algebra in year 1 ». Which resources did you use, and design, for this teaching? For the lectures, if you gave lectures; For the tutorials or computer sessions; For the preparation of the intermediate assessments and exams texts. How do you choose a resource (for example, a given exercise for the tutorial)? If you modify resources, for which reasons, and how? How are your resources classified, when do you suppress a resource?
2) About digital resources: do you use a professional webpage, a virtual learning environment, specific software? Do you use online resources to prepare your courses, do you project slides during your courses?
3) About collective work: do you work with colleagues to prepare your teaching? Which kind of work do you make for your teaching with colleagues?

4) A. For experienced teachers: which evolutions do you retain in the last 10 years, concerning the resources you use and design for your courses?
   B. For novice teachers: do intend to modify your teaching next year, how and why?
5) Link with research: are there resources that you use both for your research and for your teaching? Or other links, between your teaching and your research?
6) Did we forget to mention important resources, or something else that you consider important concerning your teaching?

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