Fostering Spatial Ability Through Computer-Aided Design: a Case Study

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Abstract

Spatial ability is considered a major factor of intelligence and is increasingly important in times of digitization. This article explores the fostering of spatial ability through computer-aided design software. Different notions of spatial ability will be discussed, and, finally, a concept consisting of five aspects will be described. In addition, literature reviews on the connection between the use of computers and the fostering of spatial ability, as well as on the use of 3D printing technology in mathematics education, are given. Building on this, a case study is presented which examines the work of two middle-school students using computer-aided design software within a workshop at the University of Siegen. From the data material, basic possible actions within such software are derived. These are, based on theory, connected with the five aspects of the specific concept of spatial ability used. The results show various perspectives for the fostering of spatial ability with computer-aided design software.

Keywords Case Study · Computer-Aided Design · Spatial Ability · Spatial Geometry · 3D Printing

Introduction

Spatial ability is an important intelligence factor, a major and central ability that has a lasting influence on our perceptions and conceptions of our environment and on the way we interact with it. (Maier, 1994, p. 123; *authors' translation*)

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This quotation shows the importance of spatial ability as an aspect of intelligence. It influences everyday life, as well as the learning at school in different subjects. The development of the spatial ability of students is a central but challenging goal of mathematics teaching.

In times of digitization, students increasingly have to deal with virtual, three-dimensional representations. On the one hand, spatial ability is a pre-condition for working with such representations. On the other, the use of 3D software has the potential to enhance students' spatial ability. In this article, empirical evidence will be provided for the relationship between 3D software and spatial ability using the example of computer-aided design software. The 3D printing technology and hence computer-aided design software are relatively new tools for the teaching of mathematics but have become of high interest in recent years.

Our focus is on the following research question: "Which actions are performed by students while constructing a model in a computer-aided design software and how are those related to aspects of spatial ability?". It was investigated by considering two middle-school students operating with computer-aided design software within a case study approach. Categories such as "Changing position of solids," "Rotating solids," or "Duplicating solids" could be formed on the basis of the data material. These were connected to Maier's (1994) five central aspects of spatial ability, "spatial perception," "visualization," "mental rotation," "spatial relation," and "spatial orientation." The results show potential for the fostering of spatial ability in such settings and, thus, provide perspectives for the use of computer-aided design software in mathematics education.

Spatial Ability and the Use of Computers

In the literature, there is no generally accepted definition of the term *spatial ability* because there is no consensus on the nature of the phenomenon. Consequently, we refer to Lohman (1993), who gives a reasonable definition:

Spatial ability may be defined as the ability to generate, retain, retrieve, and transform well-structured visual images. It is not a unitary construct. There are, in fact, several spatial abilities, each emphasizing different aspects of the process of image generation, storage, retrieval, and transformation. (p. 3)

More generally, Mc Gee (1979) claims that spatial ability is the ability to formulate mental images and to manipulate them in the mind. According to both people, spatial ability can be defined as a combination of different sub-skills, creating these images and generating information about them. Moreover, it is not just about storing and retrieving images in memory but also about the ability *to deal actively with these images*.

Mathematics education research on spatial ability ties in with psychological approaches and theories that regard this ability as an essential component of intelligence. The main lines of this research use factor analysis as a statistical method to clarify the structure of human intelligence. In this article, we apply the model of spatial intelligence according to Maier (1994), who distinguishes the following five elements of spatial intelligence: *spatial perception*, *visualization*, *mental rotation*, *spatial relation*, and *spatial orientation*. Maier developed these categories by analyzing and

summarizing several important structural concepts of spatial ability. In this process, he paid special attention to the category systems of Thurstone (1950) and Linn and Peterson (1985). The five elements which he elaborates on can be described as follows:

1. *Spatial perception* tests require the location of the horizontal or the vertical, in spite of distracting information.
2. *Visualization* comprises the ability to visualize a configuration in which there is movement or displacement among (internal) parts of the configuration.
3. *Mental rotation* involves the ability to rotate a 2D or 3D figure rapidly and accurately.
4. *Spatial relation* mean the ability to comprehend the spatial configuration of objects or parts of an object and their relation to each other.
5. *Spatial orientation* is the ability to orient oneself physically or mentally in space.

In tasks requiring the first element (*spatial perception*), the individual's own spatial position is not part of the problem, because the person is outside of the situation. Spatial perception tasks require the location of the horizontal or the vertical in spite of distracting information. Generally, these are *static* mental processes, which means that the relation of the subject to the objects changes, but the spatial relations between the objects themselves do not change. The second element (*visualization*) includes tasks that require mainly *dynamic* mental processes, which means that spatial relations between the objects are changed. This refers to mental activities such as moving, folding, and cutting an object. Here, the person's own spatial position is also not part of the task. Similar to *visualization*, the mental processes of the third element (*mental rotation*) are generally *dynamic*, and the person's own spatial position is not part of this task, too. The mental processes of the fourth element (*spatial relation*) are always *static*. It is the ability to comprehend the spatial configuration of objects or parts of an object and their relation to each other. Again, the person's own spatial position is not an essential part of the problem. The fifth element (*spatial orientation*) is the ability to orient oneself physically or mentally in space. In consequence, the person's own spatial position is necessarily an essential part of the task, and the mental processes are generally *dynamic*. This sub-component contains factor k (left–right discrimination). Explicit exercises alongside exemplary illustrations to clarify the elements of spatial ability are published in Weigand et al. (2018) and Maier (1994, 1998).

In school, spatial skills can be used in specific ways for many mathematical tasks. But, obviously, they are used in a wider range than just for solving geometrical exercises. For example, in arithmetic, spatial abilities can be helpful to clarify and internalize the basic operations (addition as a progression and subtraction as a regression). Thus, operations are represented by spatial relations in students' minds (cf. Maier, 1994).

Training of Spatial Ability and Its Relation to Mathematical Thinking

Bishop (1980) wrote in his review, “it is my experience that the mathematics education community does not fully recognize the possibility or the desirability of [...] spatial training” (p. 265). Maier (1998) agreed with this statement, “It was shown that in many grades most of the elements occur hardly or not at all” (p. 70). But, as mentioned at the beginning, cultural

requirements necessarily demand the training of these five elements. Hence, Maier's following thesis emerges as an approach for teaching spatial geometry: "Based on psychological research findings, all five elements of spatial ability have to be specifically trained" (p. 70).

Maier (1999) reports on several studies that prove that spatial ability can be trained in people of different ages. For example, Ben-Chaim, Lappan, and Houang (1989) conducted a training program with cube buildings (cube multiples composed of small wooden cubes), which were to be constructed and isometrically represented on the basis of elevations, and vice versa. The results convincingly demonstrated that middle-school students, grades 6 through 8, dramatically improved their performance after 3 weeks of instruction in spatial visualization activities. The gains of the training program were still measurable even after 1 year. Battista, Wheatley, and Talsma (1982) developed a training program for students of mathematics education on the symmetry of polygons and the construction of polyhedra. The students built and modified concrete models, made sketches, folded paper, and studied tiling and tangram figures. The test results show a significant gain in spatial ability.

In a recent and systematic meta-analysis, Uttal, Meadow, Tipton, Hand, Alden, Warren, and Newcombe (2013) investigated the connection between spatial training and the development of spatial skill. Their results clearly attest the malleability of spatial skills. Even a small amount of training can improve the spatial ability of individuals. This applies both to male and female, as well as young and adult, individuals. Hence, they conclude that spatial training plays an important role in the education and improvement of spatial skills, and of mathematics and science in general.

The relationship between the spatial ability of an individual and its performance in mathematics has been investigated in various studies. In a current review, Young, Levine, and Mix (2018a) describe how spatial thinking relates to mathematical thinking. They conclude that children's numerical and spatial abilities are related at the level of shared underlying processes across development and remain functionally distinct on each occasion. In addition, they mention several studies which show that spatial skills are correlated with performance in mathematics and that they can be improved via interventions in the classroom, above all in early education like primary school and pre-school contexts (e.g., Cheng & Mix, 2014; Levine, Ratliff, Huttenlocher & Cannon, 2012; Lowrie, Logan & Ramful, 2017; Nath & Szűcs, 2014; Witt, 2011). Young et al. (2018a) provide evidence that "forms of early intervention, which help to get children's spatial and numerical skills on track early, are especially important to closing later gaps in achievement across STEM areas" (p.139). Furthermore, Young, Levine, and Mix (2018b) note that spatial ability contributes to performances in mathematics. This leads to the assumption that spatial skills could be a basis of mathematical skills.

The focus of the research on the connection between spatial and mathematical skills is on the aspect of mental rotation. Connections between mental rotation as an aspect of spatial ability and skills in arithmetic (e.g., Carr, Steiner, Kyser & Biddlecomb, 2008; Geary, Sauls, Liu & Hoard, 2000), geometry (e.g., Casey, Nuttall & Pezaris, 1997; Delgado & Prieto, 2004), problem-solving (e.g., Hegarty & Kozhevnikov, 1999), and general mathematical skills (e.g., Kytälä, Aunio, Lehto, Van Luit & Hautamäki, 2003) were found. For this reason, Grüßing (2012) argues that the mental rotation component is particularly decisive for the differences in mathematical achievement. Young et al. (2018b) also show that the various sub-skills of mental rotation can meaningfully be linked with aspects of

mathematical skills; hence they argue for experimental studies based on such cognitive process models.

Connections between other aspects of spatial ability and mathematical skills have also been identified in empirical studies. For example, Hegarty and Waller (2005) note that visual perception and spatial ability are connected to mathematical thinking and that these components are especially important for mathematical problem-solving. In a meta-analysis, Mix and Cheng (2011) found a positive correlation between mathematical and spatial abilities in general, across ages and tasks.

The research shows that there is a connection between spatial ability and mathematical thinking. The aim of this article is not to discuss this connection in detail. However, the results nonetheless show the necessity of promoting spatial ability and its central importance for mathematical performance. Thus, the training of spatial ability should be a central concern of mathematics education. In this article, we want to deal with the fostering of spatial ability in the context of digital tools, especially computer-aided design software.

Spatial Ability and the Use of Digital Tools in Mathematics Education

In regard to the growing use of digital tools in mathematics education, the computer is a helpful tool to perform experiments with virtual solids using *dynamic spatial geometry software* (DSGS). One can dynamically change different parameters of a representation, so that connections between different projections become apparent. Thus, DSGS has many similarities with the specific computer-aided design software considered in this study, so that the results can, to a significant extent, be transferred. However, computer-aided design software focuses on the direct handling of solids (changing parameters, moving and rotating, combining) and has therefore been selected for this study.

By means of the computer, an intermediate level of concretization or abstraction can be located between the relatively abstract (static) drawing on paper and the concrete handling with a physical model of a solid (Weigand & Weth, 2002). DSGS can be seen as an enrichment and reinforcement of geometry education, offering the possibility to create complex mathematical constructions. The greatest advantage over conventional methods lies in the dynamics itself. Students can act in a virtual space and focus on this way of demonstrating the effects of movements and changes to existing configurations. Weigand et al. (2018) write that operating with representational body models (enactive representation level) is an essential basis for mathematical work in virtual space.

Schumann (2007) shows how the *use of spatial geometry tools* (respectively, DSGS) is connected with two important cognitive ability areas (*spatial geometric knowledge* and *spatial ability*), which are to be developed in geometry education (see Fig. 1). He writes that *spatial abilities*, especially the elements *visualization* and *mental rotation*, are necessary pre-conditions for *spatial geometric knowledge* (knowledge about the calculation of parameters like volume or surface area) and working with DSGS, such as *Cabri-3D* or *GeoGebra*. Furthermore, the use of DSGS fosters, on the one hand, the acquisition, understanding, and application of *spatial geometric knowledge* and, on the other, the *spatial ability* by acting in virtual space.

According to Hattermann, Kadunz, Rezat, and Sträßer (2015), scientific investigations on the application of DSGS in school are always limited to a small number of

Fig. 1 Relations among digital interactive space geometry tools, spatial geometric knowledge, and spatial ability (based on Schumann, 2007, p. 17)

participants. They refer to the study of Hattermann (2011) who investigated students studying mathematics for education in dealing with *Archimedes Geo-3D* and *Cabri-3D*. This study indicates that the use of DSGS cannot be based on a simple, intuitive generalization of experiences from plane geometry into spatial geometry. Learners have difficulty transferring their experiences.

With regard to the fostering of spatial ability, Hattermann et al. (2015) mention a study (Luig & Sträßer, 2009) that shows a possible fostering of selected factors of spatial ability. Ten students in ninth grade participated in this investigation by working with *Cabri-3D*. The results of this study demonstrate that especially students who previously had a weak spatial ability showed significant increases in this respect. Particularly striking improvements were seen in the factors *visualization* and *spatial relation*. According to Luig and Sträßer, it was generally identified that the students improved most noticeably in the factors that required dynamic thinking.

Computer-Aided Design and 3D Printing in Mathematics Education

In recent years, digital media have been becoming increasingly important in education and, especially, in mathematics teaching. The 3D printing technology is a relatively new digital tool with, in our view, high potential for mathematical learning. It enables the production of three-dimensional solid objects on the basis of virtual 3D models. These are created using computer-aided design software (hereafter, CAD software) and converted into control commands for the printer using slicer software. The 3D printer builds the object layer by layer from liquid plastic. Thus, the 3D printing technology consists of a software component (CAD and slicer software) and a hardware component (3D printer) (cf. Dilling & Witzke, 2019).

In this article, we want to deal in particular with the CAD software *Tinkercad*. It is an application that enables so-called direct modeling. This means that geometric, basic solids (cubes, cylinders, etc.) can be moved on the work plane and directly changed by dragging individual elements (points, surfaces, edges). With the help of Boolean operators, the sections and unities of individual basic solids can be formed, and, in this way, even complex solids can be constructed. The use of the direct modeling method is quite intuitive, because changes to the body become visible immediately. This allows spontaneous changes of the objects and a kind of experimental approach. *Tinkercad* is a free browser-based application provided by Autodesk. The basic functionalities of the software are illustrated in Fig. 2.

A number of different applications of the 3D printing technology have been developed to enhance mathematics learning. From the field of geometry, Dilling and Witzke (2020a) investigated its use to enhance the dealing with tessellations. Hoffart and Pielsticker (2018) considered the development of models of cubes in primary school. In the field of algebra, Pielsticker (2018) examined processes of material-related justification of the binomial formulae. Dilling (2019), Dilling and Witzke (2018), and Dilling and Witzke (2020b) developed various settings using 3D printing technology for the learning of concepts of differential and integral calculus. Ng and Sinclair (2018) considered tangents and graphs produced with a 3D pen. In stochastics, Pielsticker (2020) investigated negotiation processes in the context of loaded dices.

Alongside the development and evaluation of exemplary applications, the potential and the characteristics of this new technology have been investigated in several empirical studies, too. Ng (2017) explains its benefits for learning about the volume of solids. In a qualitative analysis, it could be observed that the students developed a deep understanding of the relationship between changes of the linear dimensions of the objects and their volumes through the use of *Tinkercad* and 3D printers. Dilling and Witzke (2020b) have qualitatively examined the effects of a specific learning environment from the field of calculus with the use of 3D printing technology on the understanding of concepts of differential calculus. It turned out that the students developed ideas about covariation and the slope of a tangent of a graph.

A study conducted by Panorkou and Pratt (2016) examined the relationship between the use of CAD software and the development of ideas about dimensions. Students

Fig. 2 Screenshot and description of the basic functionalities of Tinkercad

were able to gain first insights into the topic by working with the parametric CAD software *Google SketchUp* and the integrated dimensional tool. The students interpreted the term *dimension* in two ways—as the freedom to move and as the capacity to house. These ideas can be used as a basis for the development of further mathematical concepts, such as the concept of vector space. Since there are many similarities in 3D modeling between *SketchUp* and *Tinkercad*, which is the foundation for this article, most of the results of that study can be transferred.

A Case Study

The literature review above has shown the importance of spatial ability for the learning of mathematics. A few studies have also explored the relationship between the use of 3D software and the spatial ability of students (see, e.g., Luig & Sträßer, 2009; Hattermann et al., 2015). The 3D printing technology is a relatively new tool in mathematics education. The relationship between the use of CAD software and spatial ability is investigated in this study by focusing on the following research question:

Which actions are performed by students while constructing a model in a computer-aided design software and how are those related to aspects of spatial ability? In order to explicate the relationship between CAD software and spatial ability, a case study approach according to Yin (2014) was chosen. Two male students (at the age of 14 and 15) created a pantograph together using 3D printing technology as part of an after-school workshop. A pantograph is a mechanical instrument that is used to draw an identical, enlarged, or miniaturized copy of a two-dimensional figure. The construction process of the students working with *Tinkercad* was videotaped using the screen-recording function, while the conversation was recorded with audio for a deeper scientific investigation. The observed situation took 35 min and occurred on the second day of the workshop, so that the students already had had some experience with the software.

The analysis of the data was performed in two consecutive steps. In the first step, basic actions of the students in the construction process were identified and categorized according to the method of structuring qualitative content analysis (Mayring, 2000). For this purpose, the data material was first described in detail and the unit of analysis was determined. In the present case, the analyzed data included the 35-min screen record. The unit of analysis, the smallest analyzed part, was each action of the students in the CAD software that results in changes of the 3D model itself or the view of the 3D model. Afterwards, the actions of the students were paraphrased, reoccurring actions combined and finally described in a system of categories. It contains the definition of a category and an example from the data material (in this case, in the form of a written description and a screenshot). In the second step of the analysis, the explicated actions were assigned to the aspects of spatial ability according to Maier (1994). For this purpose, each individual action was examined with regard to its aspects.

Results and Interpretation

The analysis of the construction process of the two students with the use of qualitative content analysis produced a total of eight inductively formed categories that describe

general actions performed by the students while working with the CAD software. The complete system of categories can be found in Table 1.

Category 1 describes the selection of a basic solid from a number of pre-defined solids. The students initially chose a cube for their construction, but they also worked with a cylinder later on in the analyzed situation. In order to select the appropriate bodies, the students must already have had an idea of the composition of the final product. Thus, there seems to be a relation between the selection of a basic solid and the aspect *spatial relation* of the concept of spatial ability denoted by Maier (1994).

The changing of parameters of solids by dragging elements of the object or entering values of the parameters (e.g., the length, width, and height of a cube so that it became a flat, elongated cuboid) is part of category 2. This action is linked to the aspect of *spatial perception*, as the different dimensions have to be distinguished correctly from each other and also to the aspect of *visualization*, because it is visualized how parts of a solid change by changing parameters. Category 3 includes the changing of the position of solids by dragging or entering of values (e.g., moving a cuboid to the corner of the work plane (x - y plane) or lifting it up along the z -axis). Thus, this action is also related to the aspects of *spatial perception* and *visualization*.

Category 4 comprises the rotation of objects. The individual objects can be rotated in any dimension through any angle. To do this correctly, the dimensions have to be distinguished (*spatial perception*), and there has to be an underlying, foreseen idea of the rotated object (*mental rotation*). The duplicating of solids is part of category 5. This action is linked to the aspect of *spatial relation*, because there has to be the idea that the same object is needed several times for the construction.

Category 6 describes the connecting of solids by Boolean operators. For this purpose, the single objects can be set in two modes: “solid” and “hole.” They can then be combined with the function “grouping”. This action is quite clearly linked to the aspect of *spatial relation*, since it deals with the composition of more complex objects from basic solids. In addition, the action can be assigned to the aspect *visualization*, too, since the sections and unities change the internal structure of the single objects.

The zooming in and out of the display by scrolling the mouse wheel or using the touchpad is part of category 7. Category 8 describes the rotating of the total view with the computer mouse. Both actions are related to the aspect of *spatial orientation*, because they concern the examination of an object from different perspectives and distances. Category 8 can additionally be assigned to the aspect of *mental rotation*, since the rotation of the entire screen can be seen as the simultaneous rotation of many solids.

The relations between the explicated actions of the students in the CAD program and the aspects of spatial ability according to Maier (1994) are illustrated in Fig. 3.

Discussion

By analyzing the work of two students with *Tinkercad*, eight basic actions with CAD software could be identified. These can be linked to the five aspects of spatial ability (*spatial perception*, *visualization*, *mental rotation*, *spatial relation*, *spatial orientation*) according to Maier (1994). The results show that CAD software has the potential to address all five aspects. Even in a construction process with a comparatively short

Table 1 System of categories with actions of students using CAD software

<p>Category 1: Selecting basic solids Definition: Students select basic solids.</p> <p>Example: The students selected a cube from the available basic solids.</p>	<p>Category 2: Changing parameters of solids Definition: Students change parameters (length, width, height) of solids.</p> <p>Example: The students changed the length, width and height of a cuboid.</p>
<p>Category 3: Changing position of solids Definition: Students change the position of solids.</p> <p>Example: The students moved a cuboid to a corner of the work plane (x-y-plane).</p>	<p>Category 4: Rotating solids Definition: Students rotate solids.</p> <p>Example: The students rotated a cuboid in the x-y-plane.</p>
<p>Category 5: Duplicating solids Definition: Students duplicate solids.</p> <p>Example: The students duplicated a cuboid.</p>	<p>Category 6: Connecting solids Definition: Students connect solids by Boolean operators.</p> <p>Example: The students formed the set difference of a cuboid and a cylinder.</p>
<p>Category 7: Zooming in and out Definition: Students zoom in and out.</p> <p>Example: The students zoomed out the total view.</p>	<p>Category 8: Rotating the total view Definition: Students rotate the total view.</p> <p>Example: The students rotated the total view counterclockwise.</p>

Fig. 3 Relation between the explicated actions within the CAD software and the aspects of spatial ability

duration of 35 min, a large number of connections could be detected. Each of the actions occurred multiple times during the analyzed period of time, with some actions being identified as occurring considerably more frequently (e.g., rotating solids) than others (e.g., duplicating solids). The students performed actions with the software intuitively and had no difficulty in executing them. In general, the approach of the students was very goal-oriented. The fostering of spatial ability was not the focus of the learning situation, but nevertheless took place within the design process.

All five aspects of spatial ability could be addressed in this way. The aspect of *spatial perception* is addressed by changing parameters of solids, changing the position of solids, and rotating solids. The actions of changing parameters of solids, changing position of solids, as well as connecting solids are related to the *visualization* aspect. *Mental rotation* is especially addressed by rotating solids and rotating the total view, while *spatial relations* are addressed by selecting basic solids, as well as duplicating and connecting solids. Finally, zooming in and out and rotating the field of view refers to the aspect of *spatial orientation*.

The fostering of the different aspects of spatial ability is a central goal of mathematics education. It also forms a basis for mathematical thinking in general, as described in the theoretical background. The aspect of mental rotation plays a role of particular interest and is demonstrably linked to skills from different domains of mathematics. The aspect of mental rotation, as well as the other aspects of spatial ability, seems to be specifically addressed by actions in the CAD software *Tinkercad*.

The case study was able to provide first insights into the possibilities of promoting spatial ability through CAD software, but cannot make any statements about the actual effectiveness. For this purpose, further controlled investigations with a larger sample and standardized tests for the numerical determination of spatial ability should be employed. It should also be examined, if a larger corpus of qualitative data (including more students or over a longer period of time) be able to support the given claims. These further studies can be based on the qualitative results of this study.

Conclusion

In this article, the connection between the use of CAD software and the development of spatial ability was examined. The importance of spatial ability as a main factor of intelligence could be demonstrated. Furthermore, different concepts of spatial ability were discussed. Finally, this concept according to Maier (1994) was presented, which comprises the five aspects *spatial perception*, *visualization*, *mental rotation*, *spatial relation*, and *spatial orientation*. A literature review on the connection between digital technologies and spatial ability considered initial results from several empirical studies. The basic principles of 3D printing technology, especially CAD software, were explained with the example of *Tinkercad*. Several studies on the use of 3D printing technology in mathematics teaching from recent years can provide initial insights into the research topic of this article.

The following research question was examined in a case study: “Which actions perform students while using computer-aided design software and how are those related to aspects of spatial ability?”. The systematic identification of basic actions with CAD software and the theory-based allocation to the five aspects of spatial ability allowed essential potentials for the fostering of spatial ability to be developed. All five aspects were addressed even in the short, observed situation.

In total, important insights were gained concerning the connection between CAD software and spatial ability. The results can be transferred to a certain extend to other digital tools (e.g., dynamic spatial geometry software). A large-scale study can be conducted to investigate the effectiveness of the use of CAD in the development of spatial ability using the results of this case study.

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