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Privacy-preserving bipartite output consensus for continuous-time heterogeneous multi-agent systems via event-triggered impulsive control

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Abstract
This paper studies the problem of differentially private bipartite output consensus in continuous-time heterogeneous multi-agent systems (MASs) characterized by antagonistic interactions. A novel hybrid privacy-preserving event-triggered impulsive consensus protocol is introduced to protect the agent's initial information from disclosure, which involves a discrete-time information transmission based on an event-triggering mechanism. Using stochastic Lyapunov method, sufficient conditions have been obtained to achieve mean square bipartite output consensus with a guaranteed level of privacy. Furthermore, the differential privacy of competitive agent pairs is exclusively secured by the proposed control scheme by injecting Laplace noise. The protocol also effectively prevents Zeno behavior by imposing a lower bound for impulsive intervals under all event-triggered conditions. A simulation example is provided to validate the effectiveness of the theoretical result.

Keywords Differential privacy · Bipartite output consensus · Heterogeneous multi-agent systems · Event-triggered impulsive control

Introduction

With recent advancements in networking technology and wireless communication, the consensus problem of multi-agent systems (MASs) has received tremendous research interest from various fields, such as mobile-robot systems [1], wide-area networks [2], autonomous underwater vehicles [3], etc. In practice, agents often exhibit both antagonistic and cooperative relationships in many multi-agent networks, which can be represented in terms of signed graphs. In this case, the problem of bipartite consensus was introduced as an important issue in the field of MASs, and several research works have been obtained on this topic [4, 5]. It should be noted that previous results on bipartite consensus have focused on homogeneous agents. However, the dynamics of real-world systems are often different, and the bipartite output consensus problem of heterogeneous MASs deserves further attention. In this regard, a unified framework for bipartite output synchronization of linear heterogeneous agents was investigated in [6]. In [7], the bipartite output consensus of heterogeneous linear MASs was studied through the integration of event-triggered and adaptive control strategies, under both fixed and switching topologies. Further, to improve the convergence rate, finite-time event-triggered bipartite output consensus for heterogeneous linear MASs was discussed in [8]. In [9], a distributed reduced-dimensional observer was designed to estimate leader states and system matrices, and an edge-based adaptive protocol was proposed to ensure bipartite output synchronization. A bipartite output formation control problem was considered in [10], where an adaptive distributed dynamic event-triggered controller was proposed without requiring any global information about the network. The above research has made great contributions to the development of bipartite output consensus, but they do not take into account the issue of privacy leakage within these problems.

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Information sharing among agents to reach agreement throughout the network is an essential element in solving consensus problems. However, directly sharing agent states can compromise privacy and lead to the leakage of sensitive information. As a result, privacy-preserving problems have become crucial in consensus algorithm design. In recent years, several techniques, including homomorphic encryption [11], state decomposition [12], output mask [13], differential privacy [14] and monodirectional information exchange [15], have been employed to ensure privacy in consensus problems. Among these methods, differential privacy is particularly notable due to its strong mathematical guarantees and no assumptions regarding background knowledge of the adversary [16, 17]. Numerous studies have applied differential privacy to consensus problems in MASs. A differential private average consensus algorithm with an event-triggered scheme is designed in [18] for discrete-time MASs. In [19], the problem of resilient consensus with faulty agents over digraphs is considered by employing a differentially private mean-subsequence-reduced(MSR) method. In [20], the author addressed the differentially private consensus problem for general multivariable discrete-time MASs. A differentially private consensus problem for cooperative–competitive MASs was investigated in [21] where privacy only works for competitive agents. By using Rényi divergence, a distributed differentially private algorithm based on Gaussian noises was developed in [22] to solve the output consensus problem. In addition, the differentially private consensus problem has been further developed in [23] for discrete-time MASs over switching networks. The author first addressed the differentially private average consensus of MASs with positive agents in [24] using non-decaying positive Gaussian noises. Very recently, by injecting time-varying noises, a differentially private bipartite consensus was investigated in [25]. However, little attention has been paid to the privacy-preserving bipartite output consensus problem of heterogeneous MASs so far.

It is noteworthy that the majority of existing literature on differential private consensus has primarily focused on discrete-time models, with less focus on continuous-time MASs. Impulsive control [26], a method for converting continuous dynamics into discrete form, has been extensively studied [27, 28]. However, these studies typically depend on fixed or predetermined impulse sequences, which can be conservative. Recent work [29] of differential private average output consensus in continuous-time MASs combining periodic sampling with hybrid control to reduce communication loads, yet this discrete-time interaction scheme still requires time-triggering, which may result in a high communication burden among agents. To overcome this issue, the event-triggered impulsive control (ETIC) method was proposed. The primary advantage of ETIC is intelligent sampling and transmission inside the communication network since the impulsive sequence is determined by a well-designed event-triggered mechanism. Leaderless quasi-synchronization of heterogeneous MASs was investigated in [30] using centralized and distributed ETIC control strategies. In the case of leader-following consensus problems, quasi-consensus with random packet loss has been examined in [31] based on Lyapunov theory. Building upon the ETIC method, the leader-following problem for unknown nonlinear MASs was considered in [32] with an improved event-triggering function to reduce resource utilization. Moreover, leader-following mean square consensus for stochastic MASs with randomly occurring uncertainties and nonlinearities was discussed in [33]. A general ETIC method was used to solve the bipartite tracking output consensus problem for MASs with a dynamic leader in [34] under switching topology and nonzero control inputs. A two-layer distributed control scheme, which includes an ETIC method and a fault-tolerant controller, was provided in [35] to solve the containment control problem of linear MASs with deception attacks and actuator faults. To the best of our knowledge, the problem of differentially private bipartite consensus for continuous-time MASs under the ETIC method has not yet been addressed.

Based on a hierarchical hybrid framework, this paper investigates the bipartite output consensus of continuous-time heterogeneous MASs with differential privacy requirements. The main contributions of this work are as follows. First, we design a novel privacy-preserving algorithm for output average consensus in heterogeneous cooperative-competitive MASs. Compared with [25, 36] only considering homogeneous MASs, we consider the heterogeneous MASs. Second, the development and implementation of an impulsive controller that employs an event-triggered mechanism is introduced. This novel method performs very efficiently in terms of lower transmission and computation costs. Moreover, a fixed lower bound on impulsive intervals was proposed to prevent Zeno behavior. Compared with [22, 29], we design a control scheme that is a combination of impulsive control and event-triggered control, which can effectively save resources and generalize continuous-time systems to discrete-time systems. Additionally, the communication topologies are generalized from unsigned graphs [14, 19, 20] to signed graphs in our study. Finally, we demonstrate that our algorithm ensures mean square convergence, with privacy preservation only applied to competitive agents.

The structure of this article is as follows. The preliminaries and problem formulation are given in "Preliminaries" and "Problem formulation" sections, respectively. The main results are given in "Main results" section, which includes the design of the hybrid controller, and the analysis of consensus and privacy. The simulation result is given in "Numerical examples" and "Conclusion" sections concludes this article.

Preliminaries

Notations

Denote the set of real numbers by $\mathbb{R}$, the set of n- dimensional real vectors by $\mathbb{R}^n$, and the set of $n \times n$ real matrices by $\mathbb{R}^{n\times n}$. $I_n \in \mathbb{R}^{n\times n}$ is identity matrix. Denote $\otimes$ as the Kronecker product. diag$(\cdot)$ represents a diagonal matrix. col$(\cdot)$ represents a concatenation. sign$(\cdot)$ denotes a sign function. $||x||_1$ and $||x||$ denote 1-norm and 2-norm of vector x, respectively. For matrix A, $||A||$ denotes the 2-norm of A, $\rho(A)$ denotes the spectral radius of A. For a random variable $X \in \mathbb{R}$, $\mathbb{E}[X]$ and $Var[X]$ denotes the expectation and variance. We denote by $X \sim \mathrm{Lap}(0, b)$ a zero mean random variable with Laplace distribution and variance $2b^2$, and its probability density function is given by $\mathcal{L}(x; b) = \frac{1}{2b}e^{-\frac{|x|}{b}}$. For a random vector $Y \in \mathbb{R}^n$, $cov(Y)$ represents the covariance matrix.$\mathcal{R}(H)$ denotes the range space of function H.For a set Ω, $\mathcal{B}(\Omega)$ denotes the Borel σ-algebra of Ω. For a subset P of real numbers, inf(P) denotes the greatest real number that is less than or equal to all numbers in P.

Graph theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a signed digraph, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of vertices representing agents. $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of edges, and $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix of G, where $a_{ij} \neq 0 \Leftrightarrow (v_i, v_j) \in \mathcal{E}$ and $a_{ij} = 0 \Leftrightarrow (v_i, v_j) \notin \mathcal{E}$. Specifically, the interactions between agent i and j is cooperative if $a_{ij} > 0$ and competitive if $a_{ij} < 0$. It is assumed that the digraph with no self-loops, i.e., $a_{ii} = 0, i \in \{1 \ldots, n\}$ and satisfies the digon sign-symmetry property $a_{ij}a_{ji} \geq 0$. Let $\mathcal{N}_i$ denote the neighbor set of the node i, that is, $\mathcal{N}_i = \{j|(v_j, v_i \in \mathcal{E})\}$. The cardinality of $\mathcal{N}_i$ is $|\mathcal{N}_i|$. A directed path from agent i to agent j is a sequence of ordered edges $\{(v_i, v_{i_1}), (v_{i_1}, v_{i_2}), \ldots, (v_{i_l}, v_j)\}$. A signed directed graph is said to contain a directed spanning tree if there is at least one agent, which has a directed path to every other agent. The in-degree and out-degree of agent i can be separately defined as $d^i_{in} = \sum_{j\in\mathcal{N}_i} |a_{ij}|$ and $d^i_{\mathrm{out}} = \sum_{j\in\mathcal{N}_i} |a_{ji}|$. A signed digraph $\mathcal{G}$ is balanced if $d^i_{in} = d^i_{\mathrm{out}} = d_i$ for $i \in \{1 \ldots, N\}$. For a balanced digraph, we denote the smallest degree as $d_{\min} = \min\{d_i, i \in \mathcal{V}\}$. A signed Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $L = D - A$, where $D = \mathrm{diag}(d^1_{in}, \ldots, d^N_{in})$.

Definition 1 A signed digraph is structurally balanced if there exists a bipartition of nodes $\mathcal{V}_1$ and $\mathcal{V}_2$ satisfying $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, such that $a_{ij} > 0, i, j \in \mathcal{V}_q (q \in 1, 2)$ and $a_{ij} < 0, i \in \mathcal{V}_q, j \in \mathcal{V}_{3-q}$.

Related lemmas

Lemma 1 [37] *The signed digraph is structurally balanced if and only if there exists a gauge matrix $S = \mathrm{diag}(s_1, \ldots, s_N)$ with $s_i \in \{\pm 1\}$ such that $\mathcal{A}_s = S\mathcal{A}S$, where $\mathcal{A}_s = [|a_{ij}|] \in \mathbb{R}^{N\times N}$ is a non-negative matrix and all off-diagonal entries of $L_s = SLS$ are non-positive.*

Lemma 2 [38] *If the symmetry matrix $B \in \mathbb{R}^{n\otimes n}$ is positive definite or positive semi-definite, then for an arbitrary matrix $A \in \mathbb{R}^{n\otimes m}$, the following inequality holds*

$$tr(A^T BA) \leq tr(A^T A)tr(B). \tag{1}$$

Problem formulation

Consider a MAS consisting of N continuous-time heterogeneous agents with the following dynamics:

$$\begin{cases} \dot{x_i}(t) = A_i x_i(t) + B_i u_i(t), \\ y_i(t) = C_i x_i(t). \end{cases} \tag{2}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^l$ represent the system state, the control input, and the measurement output, respectively. A_i, B_i, and C_i are system matrices with compatible dimensions.

Definition 2 The heterogeneous MASs (2) can achieve mean square bipartite output consensus if there exists a random variable y^* such that

$$\lim_{t\to\infty} \mathbb{E}\left\|y_i(t) - s_i y^*\right\|^2 = 0, s_i \in \{\pm 1\}, \forall i \in \mathcal{V}. \tag{3}$$

The objective of this article is to develop a distributed controller that enables the system (2) to achieve mean square bipartite output consensus while guaranteeing differential privacy.

In what follows, we introduce the definitions of differential privacy used in this article.

Definition 3 [39] For any given vector $\sigma \in \mathbb{R}^l$, two initial states $y(0) = \mathrm{col}(y_1(0), y_2(0), \ldots, y_N(0)) \in \mathbb{R}^{Nl}$ and $y^{'}(0) = \mathrm{col}(y^{'}_1(0), y^{'}_2(0), \ldots, y^{'}_N(0)) \in \mathbb{R}^{Nl}$ are said to be σ-adjacent if there exists an integer $i_0 \in \mathcal{V}$ such that

$$||y_i(0) - y^{'}_i(0)||_1 \leq \begin{cases} ||\sigma||_1, & i = i_0, \\ 0, & i \neq i_0. \end{cases} \tag{4}$$

Definition 4 [20] (Differential privacy) Given $\epsilon > 0, \sigma > 0$ and any two σ-adjacent initial states $y(0)$, $y^{'}(0)$, a randomized mechanism $\mathbb{R}^{Nl} \rightarrow \mathcal{R}(\mathcal{M})$ is ϵ-differentially private for any $\mathcal{O} \in \mathcal{B}(\mathcal{R}(\mathcal{M}))$, if

$$\mathbb{P}[\mathcal{M}(y(0)) \in \mathcal{O}] \leq e^{\epsilon}\mathbb{P}[\mathcal{M}(y^{'}(0)) \in \mathcal{O}], \tag{5}$$

where ϵ is the privacy level.

Throughout this paper, the following assumptions should be satisfied.

Assumption 1 The directed signed graph $\mathcal{G}$ is structurally balanced and contains a directed spanning tree.

Assumption 2 All the matrices $(A_i, B_i), i = 1, \ldots, N$, are stabilizable.

Assumption 3 All the matrices $(A_i, C_i), i = 1, \ldots, N$ are detectable.

Assumption 4 For $i \in \mathcal{V}$, there exists solution (Π_i, Γ_i), satisfying the following equations:

$$\begin{cases} \mathbf{0} = A_i\Pi_i + B_i\Gamma_i, \\ C_i\Pi_i = I_{l\times l}. \end{cases} \tag{6}$$

Main results

Hybrid event-triggered impulsive controller design

To achieve bipartite output consensus of (2), we design a hybrid controller based on the approach in [29, 40] as follows:

$$\begin{cases} \dot{\zeta}_i(t) = \sum_{k=1}^{\infty} \gamma \sum_{j\in\mathcal{N}_i} a_{ij}(\varphi_j(t) \\ \quad -\mathrm{sign}(a_{ij})\zeta_i(t))\delta(t-t_k), \\ u_i(t) = K_{1i}x_i(t) + K_{2i}\zeta_i(t), \end{cases} \tag{7}$$

with

$$\begin{cases} \varphi_j(t_k) = \zeta_j(t_k) + \frac{1-\mathrm{sign}(a_{ij})}{2}\eta_j(t_k), \\ \eta_{j,z}(t_k) \sim \mathrm{Lap}(0, c_j q_j^k), z \in (1, \ldots, l), \end{cases} \tag{8}$$

where $\zeta_i(t) \in \mathbb{R}^l$ is the reference state with $\zeta_i(0) = y_i(0)$. γ is the impulsive gain, K_{1i} and K_{2i} and are the control parameters to be designed. $\delta(\cdot)$ is the Dirac function, and t_k are the event-triggered impulsive instants for $k = 1, 2, \ldots,$ which satisfy $0 < t_1 < t_2 < \cdots <$ with $\lim_{k\to\infty} t_k = +\infty$. $\eta_j(t_k) \in \mathbb{R}^l$ is a random vector with each entry to be independent and identically distributed (i.i.d.) Laplace distribution.

The MASs under the event-triggered impulsive control strategy can be rewritten as

$$\begin{cases} u_i(t) = K_{1i}x_i(t) + K_{2i}\zeta_i(t), \\ \dot{\zeta}_i(t) = 0, t_k \in (t_k, t_{k+1}), \\ \zeta_i(t_k) = \zeta_i(t_k^-) + \gamma \sum_{j\in\mathcal{N}_i} a_{ij}(\varphi_j(t_k^-) \\ \quad -\mathrm{sign}(a_{ij})\zeta_i(t_k^-)). \end{cases} \tag{9}$$

Remark 1 It is evident that each agent only receives information from its neighbors at the impulsive time instant. This implies that information transfer between agents only occurs at these discrete trigger times.

Denote $A = \mathrm{diag}(A_1, \ldots, A_N)$, $B = \mathrm{diag}(B_1, \ldots, B_N)$, $K_1 = \mathrm{diag}(K_{11}, \ldots, K_{1N})$, $\Pi = \mathrm{diag}(\Pi_1, \ldots, \Pi_N)$, $\Gamma = \mathrm{diag}(\Gamma_1, \ldots, \Gamma_N)$. Then, define the consensus error and the following error as $\bar{\zeta}_i(t) = \zeta_i(t) - \frac{1}{N}\sum_{i=1}^{N} s_i s_j \zeta_j(t)$, $\bar{x}_i(t) = x_i(t) - \Pi_i\zeta_i(t)$. Moreover, define $\bar{\zeta}(t) = \mathrm{col}(\bar{\zeta}_1(t), \ldots, \bar{\zeta}_N(t))$, $\bar{x}(t) = \mathrm{col}(\bar{x}_1(t), \ldots, \bar{x}_N(t))$ and $\eta(t) = \mathrm{col}(\eta_1(t), \ldots, \eta_N(t))$. Substituting (7) and (8) in (2), for $t \in (t_k, t_{k+1})$, we have

$$\dot{\bar{\zeta}}(t) = 0. \tag{10}$$

Under Assumption 4 and $K_{2i} = \Gamma_i - K_{1i}\Pi_i$, it holds that

$$\begin{aligned} \dot{\bar{x}}(t) &= \dot{x}(t) - \Pi\dot{\zeta}(t) \\ &= Ax(t) + Bu(t) - \mathbf{0} \\ &= (A + BK_1)x(t) - B(\Gamma - K_1\Pi)\zeta(t) \\ &= (A + BK_1)\bar{x}(t). \end{aligned} \tag{11}$$

Let $M = I_N - \frac{1}{N}ss^T$, for $t = t_k$, from Eq. (9), we have

$$\begin{aligned} \bar{\zeta}(t_k) &= (M \otimes I_l)\zeta(t_k) \\ &= [(I_N - \gamma L) \otimes I_l]\bar{\zeta}(t_k^-) \\ &\quad + \frac{\gamma}{2}[M(\mathcal{A} - \mathcal{A}_s) \otimes I_l]\eta(t_k^-) \end{aligned} \tag{12}$$

and

$$\begin{aligned} \bar{x}(t_k) &= x(t_k) - \Pi\zeta(t_k) \\ &= \bar{x}(t_k^-) + \gamma\Pi(L \otimes I_l)\bar{\zeta}(t_k^-) \\ &\quad - \frac{\gamma}{2}[(\mathcal{A} - \mathcal{A}_s) \otimes I_l]\eta(t_k^-). \end{aligned} \tag{13}$$

From (10)–(13), the closed-loop system with respect to $\bar{\zeta}(t)$ and $\bar{x}(t)$ can be rewritten as

$$\begin{cases} \dot{\zeta}_i(t) = 0, t \in (t_k, t_{k+1}), \\ \bar{\zeta}(t_k) = [(I_N - \gamma L) \otimes I_l]\bar{\zeta}(t_k^-) \\ +\frac{\gamma}{2}[M(\mathcal{A} - \mathcal{A}_s) \otimes I_l]\eta(t_k^-), \\ \dot{\bar{x}}(t) = (A + BK_1)\bar{x}(t), \\ \bar{x}(t_k) = \bar{x}(t_k^-) + \gamma\Pi(L \otimes I_l)\bar{\zeta}(t_k^-) \\ -\frac{\gamma}{2}[(\mathcal{A} - \mathcal{A}_s) \otimes I_l]\eta(t_k^-). \end{cases} \tag{14}$$

Denote $\hat{A} = A + BK_1$, $\mathcal{A}^* = \mathcal{A} - \mathcal{A}_s$, it follows from (14) that $\bar{\zeta}(t_{k+1}^-) = \bar{\zeta}(t_k)$ and $\bar{x}(t_{k+1}^-) = e^{\hat{A}(t_{k+1}^- - t_k)}\bar{x}(t_k)$, then

$$\begin{cases} \bar{\zeta}(t_{k+1}) = [(I_N - \gamma L) \otimes I_l]\bar{\zeta}(t_k) + \frac{\gamma}{2}(M\mathcal{A}^* \\ \otimes I_l)\eta(t_{k+1}^-), \\ \bar{x}(t_{k+1}) = e^{\hat{A}(t_{k+1}^- - t_k)}\bar{x}(t_k) + \gamma\Pi(L \otimes I_l)\bar{\zeta}(t_k) \\ -\frac{\gamma}{2}\Pi(\mathcal{A}^* \otimes I_l)\eta(t_{k+1}^-). \end{cases} \tag{15}$$

Consensus analysis

In this subsection, we first prove that the output of each heterogeneous system converges to the average of the reference states in the mean square. Then, we prove that heterogeneous MASs are able to achieve mean square bipartite output average consensus.

Theorem 1 *Under Assumptions 1–4, suppose there exists positive definite matrix $P \in \mathbb{R}^{Nl \times Nl}$, positive definite matrix $Q \in \mathbb{R}^{n^* \times n^*}$ with $n^* = \sum_{i=1}^N n_i$ such that the following inequalities*

$$\begin{pmatrix} P & [(I_N - \gamma L) \otimes I_l]^T P \\ P[(I_N - \gamma L) \otimes I_l] & P \end{pmatrix} > 0, \tag{16}$$

$$\begin{pmatrix} Q & (e^{\hat{A}\Delta t})^T Q \\ Qe^{\hat{A}\Delta t} & Q \end{pmatrix} > 0 \tag{17}$$

hold, then

$$\lim_{t\to\infty} \mathbb{E}\left\| y(t) - \frac{1}{N}(ss^T \otimes I_l)\zeta(t)\right\|^2 = 0 \tag{18}$$

and the event-triggered condition is designed as:

$$\begin{aligned} t_{k+1} = \min\{&\inf\{t | t > t_k + \Delta t, \bar{X}_\zeta^T(t_k)W^T(t) \\ &UW(t)\bar{X}_\zeta(t_k) > \beta\bar{X}_\zeta^T(t_k)U\bar{X}_\zeta(t_k)\}, t_k + T_u\}, \end{aligned} \tag{19}$$

where $\bar{X}_\zeta^T(t) = \left[\bar{\zeta}^T(t)\ \bar{x}^T(t)\right]^T$, $W(t) = \begin{bmatrix} W_1 & 0 \\ W_2 & W_3(t) \end{bmatrix}$, $W_1 = (I_N - \gamma L) \otimes I_l$, $W_2 = \gamma\Pi(L \otimes I_l)$, $W_3(t_{k+1}^-) = e^{\hat{A}(t_{k+1}^- - t_k)}$, $T_u > 0$ is an upper bound of impulsive interval and can be chosen large enough, $\Delta t > 0$ is a constant, $0 < \beta < 1$, $U \in \mathbb{R}^{N^* \times N^*}$ with $N^* = \sum_{i=1}^N n_i + Nl$ is a symmetric positive definite matrix such that $W^T(t + \Delta t)UW(t + \Delta t) - U < 0$.

Proof From (15), we have:

$$\begin{cases} \bar{\zeta}(t_{k+1}) = W_1\bar{\zeta}(t_k) + R_1\eta(t_{k+1}^-), \\ \bar{x}(t_{k+1}) = W_2\bar{\zeta}(t_k) + W_3(t_{k+1}^-)\bar{x}(t_k) + R_2\eta(t_{k+1}^-), \end{cases} \tag{20}$$

and the compact form of the above system can be described by

$$\bar{X}_\zeta(t_{k+1}) = W(t_{k+1}^-)\bar{X}_\zeta(t_k) + R\eta(t_{k+1}^-), \tag{21}$$

where $R = \left[R_1\ R_2\right]^T$, $R_1 = \frac{\gamma}{2}(M\mathcal{A}^* \otimes I_l)$, $R_2 = -\frac{\gamma}{2}\Pi(\mathcal{A}^* \otimes I_l)$.

Since conditions (16) and (17) hold, $\rho(W_1) < 1$ and $\rho(W_3(t_k + \Delta t)) < 1$, which means that $\rho(W(t_k + \Delta t)) < 1$ and there must exist the symmetric positive definite matrix U such that $W^T(t_k + \Delta t)UW(t_k + \Delta t) - U < 0$.

Let $V(k) = \bar{X}_\zeta^T(t_k)U\bar{X}_\zeta(t_k)$, then we have $\Delta V(k) = V(k+1) - V(k)$, and the expectation of $\Delta V(k)$ is

$$\begin{aligned} \mathbb{E}[\Delta V(k)] &= \mathbb{E}[\bar{X}_\zeta^T(t_{k+1})U\bar{X}_\zeta(t_{k+1}) - \bar{X}_\zeta^T(t_k)U\bar{X}_\zeta(t_k)] \\ &= \mathbb{E}[\bar{X}_\zeta^T(t_k)W^T(t_{k+1}^-)UW(t_{k+1}^-)\bar{X}_\zeta(t_k) \\ &\quad - \bar{X}_\zeta^T(t_k)U\bar{X}_\zeta(t_k) \\ &\quad + \eta^T(t_{k+1}^-)R^TUR\eta^T(t_{k+1}^-)] \\ &= \mathbb{E}[\bar{X}_\zeta^T(t_k)(W^T(t_{k+1}^-)UW(t_{k+1}^-) - U)\bar{X}_\zeta(t_k)] \\ &\quad + tr(R^TUR)\mathbb{E}[\eta^T(t_{k+1}^-)\eta(t_{k+1}^-)]. \end{aligned} \tag{22}$$

Let $\Delta V_1(k) = \bar{X}_\zeta^T(t_k)(W^T(t_{k+1}^-)UW(t_{k+1}^-) - U)\bar{X}_\zeta(t_k)$ and according to Lemma 2, we have

$$\begin{aligned} \mathbb{E}[\Delta V(k)] &= \mathbb{E}[\Delta V_1(k)] + tr(R^TUR)\mathbb{E} \\ &\quad \left[\sum_{i=1}^N \sum_{z=1}^l \eta_{i,z}^T(t_{k+1}^-)\eta_{i,z}(t_{k+1}^-)\right] \\ &\le \mathbb{E}[\Delta V_1(k)] + tr(R^TR)tr(U)\mathbb{E} \\ &\quad \left[\sum_{i=1}^N \sum_{z=1}^l \eta_{i,z}^T(t_{k+1}^-)\eta_{i,z}(t_{k+1}^-)\right]. \end{aligned} \tag{23}$$

Then, enumerating and calculating the sums of both sides in Eq. (23), we have as $k \to \infty$

$$\begin{aligned} &\mathbb{E}[V(k)] - \mathbb{E}[V(0)] \le \mathbb{E}\left[\sum_{t=0}^{k-1} \Delta V_1(t)\right] \\ &+ tr(R^TR)tr(U)\sum_{i=1}^N \frac{2c_i^2}{1 - q_i^2} I_l. \end{aligned} \tag{24}$$

Using Schur complement, according to condition (16), condition (17) and event-triggered condition (19), we have $\Delta V_1(k) < 0$ when $t_{k+1} - t_k > \Delta t$. Then, we have $\mathbb{E}[\Delta V_1(k)] < 0$, $\mathbb{E}[V(k)] \ge 0$. From (24), we can further get that $\mathbb{E}[V(k)]$ and $\mathbb{E}[\Delta V_1(k)]$ are both bounded. Consequently, there must exist a positive real number $0 < r < 1$

such that

$$\mathbb{E}[V(k+1)] \le \mathbb{E}\left[(1-r)V(k) + tr(R^T R)tr(U)\sum_{i=1}^{N} \times \sum_{z=1}^{l} \eta_{i,z}^T(t_{k+1}^-)\eta_{i,z}(t_{k+1}^-)\right]. \tag{25}$$

As $k \to \infty$, the contribution of the first term in (25) converge to 0, and in the second term we have $\sum_{i=1}^{N}\sum_{z=1}^{l} E[\eta_{i,z}^T (t_{k+1}^-)\eta_{i,z}(t_{k+1}^-)] = 2\sum_{i=1}^{N} c_i^2 q_i^{2k} I_l$ which also converge to 0. Therefore, we conclude that $\mathbb{E}[V(k)] = 0$ as $k \to \infty$.

Since $U > 0$ and $\mathbb{E}[V(k)] = 0$, we have $\lim_{k\to\infty}\mathbb{E}[\bar{X}_\zeta(t_k)] = 0$, which means that $\lim_{k\to\infty}\mathbb{E}[\bar{\zeta}(t_k)] = 0$ and $\lim_{k\to\infty}\mathbb{E}[\bar{x}(t_k)] = 0$. Then,

$$\begin{aligned}
&\lim_{t\to\infty}\mathbb{E}\left\|y(t) - \frac{1}{N}(ss^T \otimes I_l)\zeta(t)\right\|^2 \\
&= \lim_{t\to\infty}\mathbb{E}\left\|Cx(t) - C\Phi\zeta(t) + C\Phi\zeta(t) - \frac{1}{N}(ss^T \otimes I_l)\zeta(t)\right\|^2 \\
&\le 2\lim_{t\to\infty}\mathbb{E}\|Cx(t) - C\Phi\zeta(t)\|^2 + 2\lim_{t\to\infty}\mathbb{E}\left\|C\Phi\zeta(t) - \frac{1}{N}(ss^T \otimes I_l)\zeta(t)\right\|^2 \\
&= 2\lim_{t\to\infty}\mathbb{E}\|C\bar{x}(t)\|^2 + 2\lim_{t\to\infty}\mathbb{E}\left\|\bar{\zeta}(t)\right\|^2 \\
&= 0.
\end{aligned} \tag{26}$$

The proof is completed. □

Theorem 2 *Suppose Assumptions 1–4 hold and $A_i + B_i K_{1i}$ is Hurwitz, the heterogeneous multi-agent system (2) will achieve mean square bipartite output average consensus and $\lim_{t\to\infty}\mathbb{E}||y_i(t) - s_i y^*||^2 = 0$, where y^* is a random vector, $\mathbb{E}[y^*] = \frac{1}{N}(\mathbf{1}_N^T S \otimes I_l)y(t_0)$.*

Proof Based on Assumption 1, we have $\mathbf{1}_N^T SL = \mathbf{0}$. Accordingly, it is derived from (9) that

$$\begin{aligned}
(\mathbf{1}_N^T S \otimes I_l)\zeta(t_k) &= (\mathbf{1}_N^T S(I_N - \gamma L) \otimes I_l)\zeta(t_k^-) \\
&\quad + \frac{\gamma}{2}(\mathbf{1}_N^T S\mathcal{A}^* \otimes I_l)\eta(t_k^-) \\
&= (\mathbf{1}_N^T S \otimes I_l)\zeta(t_k^-) \\
&\quad + \frac{\gamma}{2}(\mathbf{1}_N^T S\mathcal{A}^* \otimes I_l)\eta(t_k^-) \\
&= (\mathbf{1}_N^T S \otimes I_l)\zeta(t_{k-1}) \\
&\quad + \frac{\gamma}{2}(\mathbf{1}_N^T S\mathcal{A}^* \otimes I_l)\eta(t_k^-).
\end{aligned} \tag{27}$$

By taking iterations, we can obtain that

$$\begin{aligned}
(\mathbf{1}_N^T S \otimes I_l)\xi(t_k) &= \sum_{i\in\mathcal{V}} s_i\zeta_i(t_0) \\
&\quad + \frac{\gamma}{2}\sum_{p=1}^{k-1}(\mathbf{1}_N^T S\mathcal{A}^* \otimes I_l)\eta_i(t_p^-)
\end{aligned} \tag{28}$$

which

$$\begin{aligned}
&\lim_{k\to\infty}(\mathbf{1}_N^T S \otimes I_l)\xi(t) \\
&= \sum_{i\in\mathcal{V}} s_i\xi_i(t_0) + \frac{\gamma}{2}\sum_{p=1}^{\infty}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}(a_{ij}s_j - a_{ij}s_i)\eta_i(t_p^-) \\
&= \sum_{i\in\mathcal{V}} s_i\xi_i(t_0) + \frac{\gamma}{2}\sum_{p=1}^{\infty}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}(s_i - s_j)|a_{ij}|\eta_i(t_p^-).
\end{aligned} \tag{29}$$

According to Theorem 1, we have

$$\begin{aligned}
&\lim_{t\to\infty}\mathbb{E}||y_i(t) - s_i y^*||^2 \\
&\le \lim_{t\to\infty}\mathbb{E}||y_i(t) - \frac{1}{N}(\mathbf{1}_N^T S \otimes I_l)\xi(t)||^2 \\
&\quad + \lim_{t\to\infty}\mathbb{E}||\frac{1}{N}(\mathbf{1}_N^T S \otimes I_l)\xi(t) - y^*||^2 = 0
\end{aligned} \tag{30}$$

with $y^* = \frac{1}{N}\sum_{i\in\mathcal{V}} s_i\xi_i(t_0) + \frac{\gamma}{2N}\sum_{p=1}^{\infty}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\tilde{a}_{ij}\eta_i(t_p^-)$, $\tilde{a}_{ij} = (s_i - s_j)|a_{ij}|$. By the fact that $\eta_i(t_h^-)$ are i.i.d., we have

$$\begin{aligned}
\mathbb{E}[y^*] &= \mathbb{E}\left[\frac{1}{N}\sum_{i\in\mathcal{V}} s_i\xi_i(t_0) + \frac{\gamma}{2N}\sum_{h=1}^{\infty}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\tilde{a}_{ij}\eta_i(t_h^-)\right] \\
&= \frac{1}{N}\sum_{i\in\mathcal{V}} s_i\xi_i(t_0) = \frac{1}{N}\sum_{i\in\mathcal{V}} s_i y_i(t_0).
\end{aligned} \tag{31}$$

□

Remark 2 Because in Theorem 1 there exists a constant $\Delta t > 0$ such that $t_{k+1} - t_k > \Delta t$, then the Zeno behavior can be naturally excluded.

Remark 3 Compared to the work in [15], which proposed an alternating direction method of multiplier (ADMM) to solve the consensus problem of reflection coefficients, the event-triggered impulsive control strategy proposed in this paper can mitigate limited communications resources. In addition, compared with the monodirectional information exchange method to safeguard information privacy, the implementation of differential privacy in this paper is independent of a specific network structure or communication model, making it more flexible to different systems.

Privacy analysis

In this subsection, we analyze the ϵ_i privacy of the proposed algorithm. First, we consider the initial states of an individual agent's output $y(0) = \mathrm{col}(y_1(0), y_2(0), \ldots, y_N(0))$ as the private data. We then denote the sequence of reference states $\chi(y(0), O) = \{\zeta_i(t_k), i \in \mathcal{V}, k = 0, 1, \ldots\}$ as the trajectory of the system, and the sequence of transmitted states $O = \{\varphi_i(t_k), i \in \mathcal{V}, k = 0, 1, \ldots\}$ as the observation of the system. Define the randomized mechanism $\mathcal{M}(\cdot)$ as a stochastic map from the private data $y(0)$ to an observation sequence O.

Theorem 3 *If the agent and its neighbors are in a competitive relationship, the proposed algorithm can preserve ϵ_i-differential privacy for agent $i \in \mathcal{V}$ with*

$$\epsilon_i = \frac{||\delta||_1(1-\gamma d_i)}{c_i(q_i - 1 + \gamma d_i)}. \tag{32}$$

Proof Because the observations $O = \{\varphi_i(t_k^-), i \in \mathcal{V}, k = 1, 2, \ldots\}$ for $y(0)$ and $y^{'}(0)$ are the same, based on Eq. (9), we have

$$\zeta_i^{y(0),O}(t_{k+1}^-) = (1-\gamma d_i)\zeta_i^{y(0),O}(t_k^-) + \gamma \sum_{j \in \mathcal{N}_i} a_{ij}\varphi_j(t_k^-) \tag{33}$$

and

$$\zeta_i^{y^{'}(0),O}(t_{k+1}^-) = (1-\gamma d_i)\zeta_i^{y^{'}(0),O}(t_k^-) + \gamma \sum_{j \in \mathcal{N}_i} a_{ij}\varphi_j(t_k^-). \tag{34}$$

Therefore,

$$\begin{aligned} &\zeta_i^{y^{'}(0),O}(t_{k+1}^-) - \zeta_i^{y(0),O}(t_{k+1}^-) \\ &\quad = \zeta_i^{y^{'}(0),O}(t_k) - \zeta_i^{y(0),O}(t_k) \\ &\quad = (1-\gamma d_i)(\zeta_i^{y^{'}(0),O}(t_k^-) - \zeta_i^{y(0),O}(t_k^-)) \\ &\quad = (1-\gamma d_i)^k(\zeta_i^{y^{'}(0),O}(t_0) - \zeta_i^{y(0),O}(t_0)). \end{aligned} \tag{35}$$

Define $R = \{\chi(y(0), O), O \in \mathcal{O}\}$ and $R^{'} = \{\chi(y^{'}(0), O), O \in \mathcal{O}\}$ are the set of possible in the observation set $\mathcal{O}$. Meanwhile, let $f(y(0), \chi(y(0), O))$ and $f(y^{'}(0), \chi(y^{'}(0), O))$ as the probability density function of the trajectories, respectively. Given initial states $y(0)$, the observation sequence $O = \{\varphi_i(t_k^-), i \in \mathcal{V}, k = 1, 2, \ldots\}$ is unique defined by the noise sequence $\{\eta_i(t_k^-), i \in \mathcal{V}, k = 0, 1, \ldots\}$ according to (9). Then, the probability density function of observation sequence O is given as follows

$$\begin{aligned} &f(y(0), \chi(y(0), O)) \\ &\quad = \prod_{i=1}^{N}\prod_{p=1}^{k} f(y(0), \chi(y(0), O)_i(t_p^-)) \\ &\quad = \prod_{i=1}^{N}\prod_{z=1}^{l}\prod_{p=1}^{k} \mathcal{L}(O_{i,z}(t_p^-) - \chi(y(0), O)_{i,z}(t_p^-); b_i(t_p^-)). \end{aligned} \tag{36}$$

For a pair of private datasets, since they have the same observation up to time T, there exists a bijection $h(\cdot) : R \to R^{'}$ such that for $\chi(y(0), O) \in R$, $\chi(y^{'}(0), O) \in R^{'}$, it has $h(\chi(y(0), O)) = \chi(y^{'}(0), O)$. Then, from (37) and using the bijection $h(\cdot)$, one can get

$$\begin{aligned} &\frac{f(y(0), \chi(y(0), O))}{f(y^{'}(0), \chi(y^{'}(0), O))} \\ &\quad = \frac{\prod_{i=1}^{N}\prod_{z=1}^{l}\prod_{p=1}^{k} \mathcal{L}(O_{i,z}(t_p^-) - \chi(y(0), O)_{i,z}(t_p^-); b_i(t_p^-))}{\prod_{i=1}^{N}\prod_{z=1}^{l}\prod_{p=1}^{k} \mathcal{L}(O_{i,z}(t_p^-) - h(\chi(y(0), O)_{i,z}(t_p^-); b_i(t_p^-)))} \\ &\quad = \frac{\prod_{z=1}^{l}\prod_{p=1}^{k} \mathcal{L}(O_{i_0,z}(t_p^-) - \chi(y(0), O)_{i_0,z}(t_p^-); b_{i_0}(t_p^-))}{\prod_{z=1}^{l}\prod_{p=1}^{k} \mathcal{L}(O_{i_0,z}(t_p^-) - h(\chi(y(0), O))_{i_0,z}(t_p^-); b_{i_0}(t_p^-))} \\ &\quad = \prod_{p=1}^{k} e^{\frac{||(O_{i_0}(t_p^-) - \chi(y(0),O)_{i_0}(t_p^-))||_1 - ||(O_{i_0}(t_p^-) - h(\chi(y(0),O))_{i_0}(t_p^-))||_1}{b_{i_0}(t_p^-)}} \\ &\quad \le \prod_{p=1}^{k} e^{\frac{||(O_{i_0}(t_p^-) - \chi(y(0),O)_{i_0}(t_p^-)) - ||(O_{i_0}(t_p^-) - h(\chi(y(0),O))_{i_0}(t_p^-))||_1}{b_{i_0}(t_p^-)}} \\ &\quad \le e^{\sum_{p=1}^{k} \frac{||\zeta_{i_0}^{'}(t_p^-) - \zeta_{i_0}(t_p^-)||}{c_{i_0}q_{i_0}^p}} \le e^{\sum_{p=1}^{k} \frac{||\delta||_1(1-\gamma d_{i_0})^p}{c_{i_0}q_{i_0}^p}}. \end{aligned} \tag{37}$$

Subsequently, integrating both sides of (37) and letting $k \to \infty$ yields

$$\begin{aligned} \mathbb{P}[\mathcal{M}(y(0)) \in \mathcal{O}] &\le e^{\frac{||\delta||_1(1-\gamma d_{i_0})}{c_{i_0}(q_{i_0} - 1 + \gamma d_{i_0})}} \mathbb{P}[\mathcal{M}(y^{'}(0)) \in \mathcal{O}], \\ & i_0 \in \mathcal{V}, \end{aligned} \tag{38}$$

where i_0 can be any agent in the network. The proof is now complete. □

Remark 4 Theorem 3 reveals that the privacy level ϵ_i is related to the parameter c_i, q_i, and the degree d_i. According to (32), we find that greater c_i, q_i, and d_i give a smaller ϵ_i, which may lead to better privacy protection.

Numerical examples

Example 1 Consider a MAS with six agents over a signed graph $\mathcal{G}$ (see Fig. 1), where the red dashed lines and the

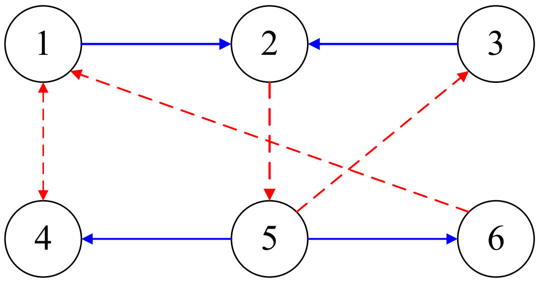

Fig. 1 Structurally balanced signed graph $\mathcal{G}$ containing a spanning tree

blue solid lines separately indicate the antagonistic and cooperative interactions. Since $\mathcal{G}$ is structural balanced, $\mathcal{V}_1 = \{v_1, v_2, v_3\}$ and $\mathcal{V}_2 = \{v_4, v_5, v_6\}$ are two competitive groups. The dynamics of (2) are provided as follows:

$$A_1 = A_3 = A_5 = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, B_1 = B_3 = B_5 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C_1 = C_3 = C_5 = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$A_2 = A_4 = A_6 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix}, B_2 = B_4 = B_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$
$$C_2 = C_4 = C_6 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}.$$

Based on (6), we can get the solution of that $\Pi_1 = \Pi_3 = \Pi_5 = [-1, 1]^T$, $\Gamma_1 = \Gamma_3 = \Gamma_5 = 1$ and $\Pi_2 = \Pi_4 = \Pi_6 = [1, 0, 0]^T$, $\Gamma_2 = \Gamma_4 = \Gamma_6 = -1$. Then, we select the feedback gain matrices $K_{11} = [4.3, -1.32]$, $K_{12} = [5.7, -3.4]$, $K_{13} = [14, -35]$, $K_{14} = [37.2, 35.94, 8.5]$, $K_{15} = [106, 68.7, 12.3]$, $K_{16} = [176.4, 97.6, 15.2]$ such that $A_i + B_i K_{1i}$ are Hurwitz matrices.

The initial states are randomly selected from the interval [0, 10] and let $c_i = 0.8$, $q_i = 0.95$, $\Delta t = 0.02$, $\gamma = 0.15$, $\beta = 0.7$. The corresponding results are presented in Figs. 2, 3, 4 and 5. Figure 2 shows the output trajectories $y_i(t)$ of all agents, demonstrating that outputs $y_1(t)$, $y_2(t)$ and $y_3(t)$ in subgroup $\mathcal{V}_1$ converge to consensus, while outputs $y_4(t)$, $y_5(t)$ and $y_6(t)$ in subgroup $\mathcal{V}_2$ converge to the opposite value. Figure 3 depicts the triggered instants for agents 1-6. We can observe that the triggering instants of each agent are finite, which verifies the exclusion of Zeno behavior. As depicted in Fig. 4, the output errors of all agents asymptotically converge to zero, thereby confirming that the proposed control strategy achieves bipartite output consensus for the heterogeneous MASs. Figure 5 reveals the relation between privacy level ϵ_i and the parameter set (c_i, q_i, d_i) for $\gamma = 0.15$. It shows that ϵ_i decreases as c_i, q_i, and d_i increase, which implies stronger privacy protection for agents with more neighbors.

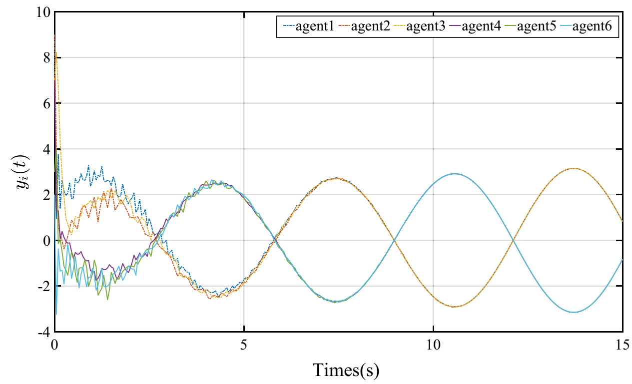

Fig. 2 The output trajectories $y_i(t)$ of six agents

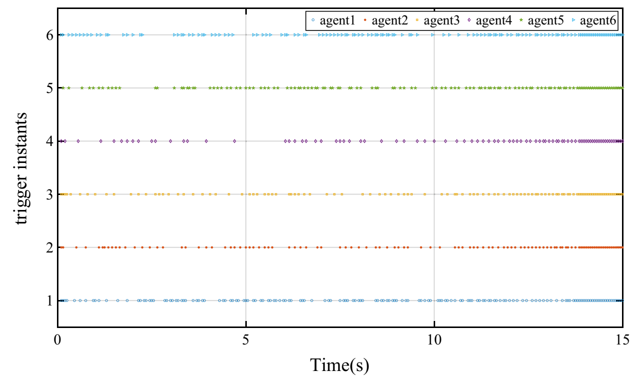

Fig. 3 Triggering instants under the proposed controller (7)

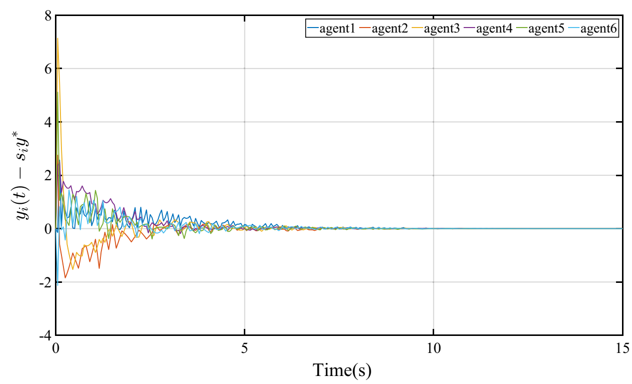

Fig. 4 The bipartite output consensus errors of six agents

Example 2 Consider a MAS with four agents over a signed graph $\mathcal{G}$ (see Fig. 6), where the red dashed lines and the blue solid lines separately indicate the antagonistic and cooperative interactions. Since $\mathcal{G}$ is structural balanced, $\mathcal{V}_1 = \{v_1, v_2\}$ and $\mathcal{V}_2 = \{v_3, v_4\}$ are two competitive groups. The dynamics of (2) are provided as follows:

$$A_1 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0.5 & -1 \\ 0 & 1 \end{bmatrix},$$

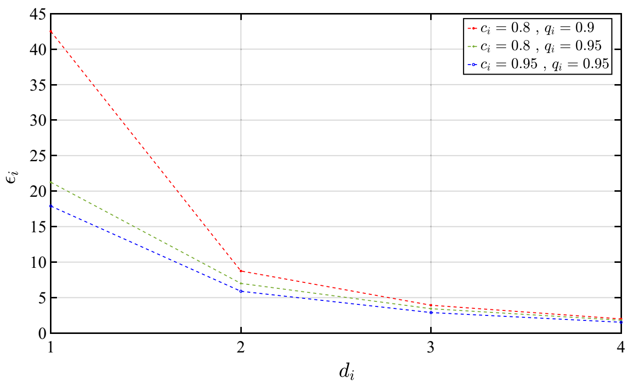

Fig. 5 The change of privacy level ϵ_i with respect to the parameter set (c_i, q_i, d_i) when $\gamma = 0.15$

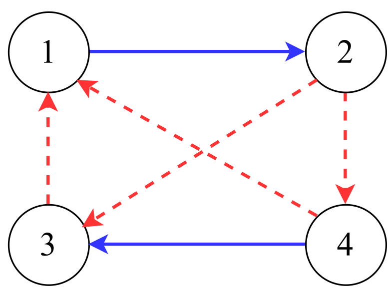

Fig. 6 Structurally balanced signed graph $\mathcal{G}$ containing a spanning tree

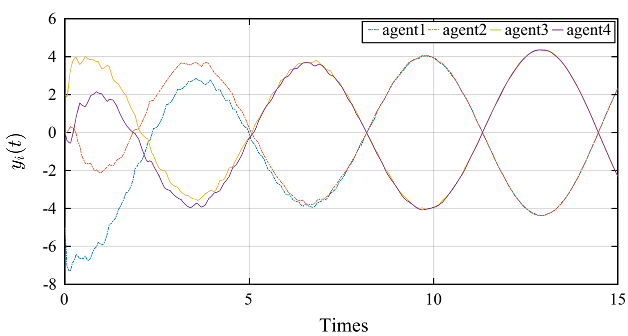

Fig. 7 The output trajectories $y_i(t)$ of four agents

$$A_3 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_3 = \begin{bmatrix} 2 & 1 \end{bmatrix}, C_4 = \begin{bmatrix} 1 & 0.5 \end{bmatrix}. \quad (39)$$

Based on (6), we can get the solution of that $\Pi_1 = [1, 0]^T$, $\Pi_2 = [1, 0]^T$, $\Pi_3 = [1, -1]^T$, $\Pi_4 = [0, 2]^T$ and $\Gamma_1 = -1$, $\Gamma_2 = -0.5$, $\Gamma_3 = -2$, $\Gamma_= -2$. Then, we select the feedback gain matrices $K_{11} = [6.3, 9.92]$, $K_{12} = [7.2, 13.8]$, $K_{13} = [7.6, 12.95]$, $K_{14} = [10.25, -3.65]$ such that $A_i + B_i K_{1i}$ are Hurwitz matrices.

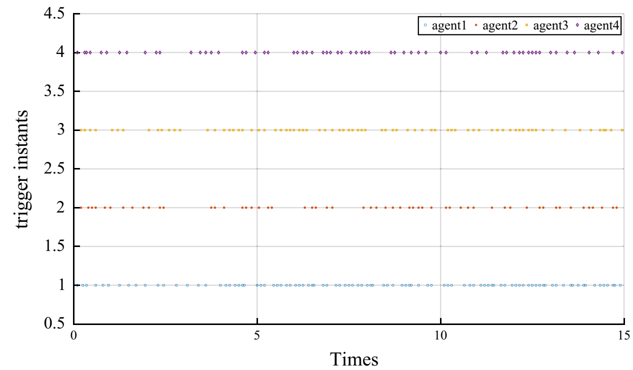

Fig. 8 Triggering instants under the proposed controller (7)

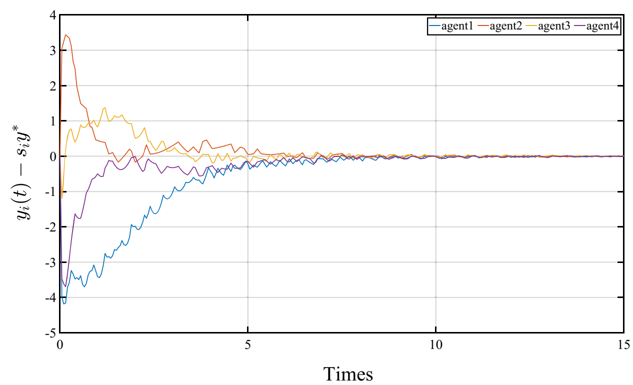

Fig. 9 The bipartite output consensus errors of four agents

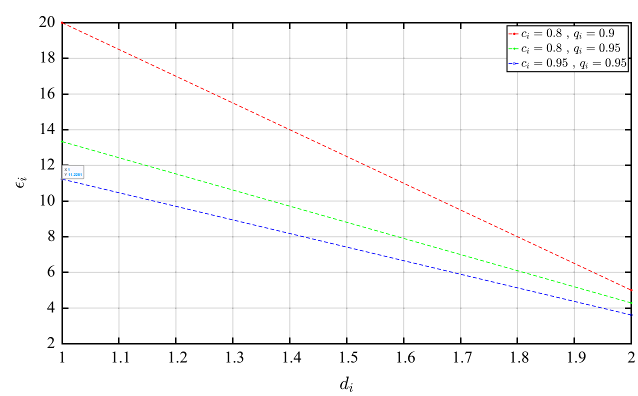

Fig. 10 The change of privacy level ϵ_i with respect to the parameter set (c_i, q_i, d_i) when $\gamma = 0.2$

The initial states are randomly selected from the interval $[-5, 5]$ and let $c_i = 0.8, q_i = 0.95, \Delta t = 0.02, \gamma = 0.2, \beta = 0.5$. The corresponding results are presented in Figs. 7, 8 and 9. Figure 7 shows the output trajectories $y_i(t)$ of all agents, demonstrating that outputs $y_1(t)$, $y_2(t)$ in subgroup $\mathcal{V}_1$ converge to consensus, while outputs $y_3(t)$, $y_4(t)$ in subgroup $\mathcal{V}_2$ converge to the opposite value. Figure 8 depicts the triggered instants for agents 1–4. We can observe that the triggering instants of each agent are finite, which verifies the exclusion of Zeno behavior. As depicted in Fig. 9, the output errors of all agents asymptotically converge to zero, thereby confirming that the proposed control strategy achieves bipartite output consensus for the heterogeneous MASs. Figure 10 reveals the relation between privacy level ϵ_i and the parameter set (c_i, q_i, d_i) for $\gamma = 0.2$. It shows that ϵ_i decreases as c_i, q_i, and d_i increase, which implies stronger privacy protection for agents with more neighbors.

Conclusion

In this paper, a differentially private bipartite output consensus problem for continuous-time heterogeneous MASs over signed topology is addressed. To handle this problem, a novel hybrid impulsive controller is designed, where the event-triggered mechanism is taken into account. In contrast to the current time-triggered approach, a significant reduction in control accuracy and communication costs can be achieved. Besides, the proposed control strategy avoids Zeno behavior by enforcing a fixed lower bound on time intervals. A formal analysis of the ϵ_i-differential privacy of the system is also provided. Future work will focus on differentially private bipartite output consensus with dynamic event-triggered control, switching topology, and privacy preservation of cell-free networks. Moreover, privacy preservation in practice, including protecting the privacy of traffic analysis and health research, will be further studied to extend our proposed control scheme.

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Declarations

Conflict of interest No potential conflict of interest was reported by the authors.

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