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ORIGINAL ARTICLE

Power aggregation operators based on Yager t-norm and t-conorm for complex q-rung orthopair fuzzy information and their application in decision-making problems

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Abstract
The complex q-rung orthopair fuzzy (CQ-ROF) set can describe the complex uncertain information. In this manuscript, we develop the Yager operational laws based on the CQ-ROF information and Yager t-norm and t-conorm. Furthermore, in aggregating the CQ-ROF values, the power, averaging, and geometric aggregation operators have played a very essential and critical role in the environment of fuzzy set. Inspired from the discussed operators, we propose the CQ-ROF power Yager averaging (CQ-ROFPYA), CQ-ROF power Yager ordered averaging (CQ-ROFPYOA), CQ-ROF power Yager geometric (CQ-ROFPYG), and CQ-ROF power Yager ordered geometric (CQ-ROFPYOG) operators. These operators are the modified version of the Power, Yager, averaging, geometric, and the combination of these all based on fuzzy set (FS), intuitionistic FS, Pythagorean FS, q-rung orthopair FS, complex FS, complex intuitionistic FS, and complex Pythagorean FS. Moreover, we also discuss the main properties of the proposed operators. Additionally, we develop a multi-attribute decision-making (MADM) method based on the developed operators. To show the supremacy and validity of the proposed method, the comparison between the proposed method and some existing methods is done by some examples, and results show that the proposed method is better than the others in terms of generality and effectiveness.

Keywords Complex q-rung orthopair fuzzy sets · Power aggregation operators · Yager t-norm and t-conorm · Multi-attribute decision-making (MADM)

Abbreviations

CQ-ROF	Complex q-rung orthopair fuzzy
CQ-ROFPYA	CQ-ROF power Yager averaging
CQ-ROFPYOA	CQ-ROF power Yager ordered averaging
CQ-ROFPYG	CQ-ROF power Yager geometric
CQ-ROFPYOG	CQ-ROF power Yager ordered geometric
MADM	Multi-attribute decision-making
FS	Fuzzy sets
IFS	Intuitionistic FS
PFS	Pythagorean FS
Q-ROFS	Q-rung orthopair FS
CFS	Complex fuzzy sets
CIFS	Complex intuitionistic FS
CPFS	Complex Pythagorean FS
CQ-ROFS	Complex q-rung orthopair FS
PA	Power aggregation
AOs	Aggregation operators

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Introduction

The MADM can be considered as a problem-evaluating procedure to get the best one from the family of alternatives. In traditional MADM, the attribute values can only be expressed by real numbers, to describe the uncertain information for the attribute values, Zadeh [1] proposed FS to describe fuzzy information. Further, Atanassov [2] improved the FS and proposed the IFS by adding one extra term based on the FS, called falsity grade with a characteristic

$0 \leq \overline{\overline{\Delta_{\Im}}}(x) + \overline{\overline{\Xi_{\Im}}}(x) \leq 1$. Moreover, Yager [3, 4] modified the IFSs and proposed the PFS and Q-ROFS with two new characteristics: $0 \leq \left(\overline{\overline{\Delta_{\Im}}}(x)\right)^2 + \left(\overline{\overline{\Xi_{\Im}}}(x)\right)^2 \leq 1$ and $0 \leq \left(\overline{\overline{\Delta_{\Im}}}(x)\right)^{\Lambda} + \left(\overline{\overline{\Xi_{\Im}}}(x)\right)^{\Lambda} \leq 1,\ \Lambda \geq 1$. The Q-ROFS is a generalization of Fs, IFS, and PFS. When $\Lambda = 2$, Q-ROFS is reduced to PFS; When $\Lambda = 1$, Q-ROFS is reduced to IFS; and when $\Lambda = 1$ and $\overline{\overline{\Xi_{\Im}}}(x) = 0$, Q-ROFS is reduced to FS. Now, the Q-ROFS, PFS, IFS, and FS have received a lot of attention from various individuals because of their structure and characteristics.

Because the Q-ROFS, PFS, IFS, and FS can only express one-dimensional information, in some real decision problems, for example, to describe the amplitude and phase of a variable, they cannot describe this problem. So, it is necessary to add another dimensional information based on them. Therefore, Ramot et al. [5] extended the range of FS by utilizing the unit disc instead of the unit interval and proposed the CFS. The truth grade in CFS is expressed by a complex number where the values of real and imaginary parts are contained in the unit interval. The FS is a special case of the CFS. Furthermore, Alkouri and Salleh [6] proposed the CIFS which is massively modified from the FSs, and IFSs. In CFSs, they meet the characteristics $0 \leq \overline{\overline{\Delta_{\widetilde{rp}}}}(x)+\overline{\overline{\Xi_{\widetilde{rp}}}}(x) \leq 1$ and $0 \leq \overline{\overline{\Delta_{\widetilde{ip}}}}(x)+\overline{\overline{\Xi_{\widetilde{ip}}}}(x) \leq 1$. Further, Ullah et al. [7] and Liu et al. [8, 9] modified the CIFS and proposed the CPFS and CQ-ROFS with the characteristics: $0 \leq \overline{\overline{\Delta_{\widetilde{rp}}}}^2(x) + \overline{\overline{\Xi_{\widetilde{rp}}}}^2(x) \leq 1,\ 0 \leq \overline{\overline{\Delta_{\widetilde{ip}}}}^2(x) + \overline{\overline{\Xi_{\widetilde{ip}}}}^2(x) \leq 1$ and $0 \leq \overline{\overline{\Delta_{\widetilde{rp}}}}^{\Lambda}(x)+\overline{\overline{\Xi_{\widetilde{rp}}}}^{\Lambda}(x) \leq 1,\ 0 \leq \overline{\overline{\Delta_{\widetilde{ip}}}}^{\Lambda}(x)+\overline{\overline{\Xi_{\widetilde{ip}}}}^{\Lambda}(x) \leq 1$, $\Lambda \geq 1$. The FSs, IFSs, PFSs, Q-ROFSs, CFSs, CIFSs, and CPFSs are the special case of the CQ-ROFS.

Literature review

The PA operator was proposed by Yager [10] in 2001, which is the combination of two different structures such as power weighting and aggregation operators, to aggregate the finite family of attribute values into a singleton value. In addition, Yager [11] proposed Yager t-norm and t-conorm, which is a very famous and reliable operational laws. Furthermore, Xu [12] developed the simple averaging AOs for IFS. Xu and Yager [13] presented the geometric AOs for IFS. Xu [14] developed the PA operators for IFS. Rahman et al. [15] proposed the geometric AOs for PFS. Wei and Lu [16] developed the PA operators for PFS. Shahzadi and Akram [17] proposed the Yager aggregation operators for PFS. Liu and Wang [18] developed simple AOs for Q-ROFS. Riaz et al. [19] derived the geometric AOs for Q-ROFSs. Akram and Shahzadi [20] developed the Yager aggregation operators for Q-ROFS. Garg and Rani [21, 22] developed the AOs for CIFS and the geometric AOs for CIFSs. Rani and Garg [23] examined the PA operators based on the CIFS. Moreover, Akram et al. [24] developed the novel Yager AOs based on the CPFSs. Garg et al. [25] proposed the PA operators based on the CQ-ROF information. Further, some scholars developed some new and different types of operators based on prevailing concepts, for example, Senapati et al. [26] developed the Aczel-Alsina geometric AOs under the IFS. Furthermore, Garg et al. [27] examined the novel combination of Schweizer-Sklar and prioritized AOs for IFSs. Azeem et al. [28] evaluated the Einstein AOs for the complex IFSs. Mahmood and Ali [29] combined the Aczel-Alsina and power aggregation operators based on the complex IFSs, Senapati et al. [30] developed the power Aczel-Alsina AOs for IFS. Moreover, Jin et al. [31] proposed Aczel-Alsina AOs for the complex PFS and Liu et al. [32] exposed the Archimedean AOs for complex PFSs. A brief analysis of some FSs is shown in Table 1.

Two types of symbols are used in Table 1, such as $\surd$ and $\times$, which is represented the yes and no. From the data in Table 1, it is clear that the CQ-ROFS is a very complete and reliable structure because it contained the truth grade, falsity grade, phase term, and power conditions as well. Therefore, based on the CQ-ROFS, we develop some powerful aggregation operators.

Motivation and advantages

Now, we observed that the PA operators based on Yager t-norm and t-conorm for CQ-ROF information was not researched. However, it is very clear that the CQ-ROFSs are more general than some existing fuzzy sets, such as FSs, IFSs, PFSs, Q-ROFSs, CFSs, CIFSs, and CPFSs, and the PA operators based on Yager t-norm and t-conorm are more general than some existing averaging and geometric AOs, it is necessary to propose some new PAs for CQ-ROFSs based on Yager t-norm and t-conorm, and then to develop the MADM methods based on the proposed operators to solve real-life decision problems. The major advantages of the proposed operators are explained as

1. When $\Lambda = 2$ in CQ-ROFSs, then all the proposed operators are converted into the ones based on the CPFSs.
2. When $\Lambda = 1$ in CQ-ROFSs, then all the proposed operators are converted into the ones based on the CIFSs.
3. When $\overline{\overline{\Delta_{ip-z}}} = \overline{\overline{\Xi_{\widetilde{ip-z}}}} = 0$ in CQ-ROFSs, then all the proposed operators are converted into the ones based on the Q-ROFSs.
4. When $\overline{\overline{\Delta_{ip-z}}} = \overline{\overline{\Xi_{\widetilde{ip-z}}}} = 0$ and $\Lambda = 2$ in CQ-ROFSs, then all the proposed operators are converted into the ones based on the PFSs.

Table 1 The characteristics for some FSs

Some Fuzzy sets	Truth grade	Falsity grade	Phase term	Power full condition
FSs	√	×	×	×
IFSs	√	√	×	×
PFSs	√	√	×	×
Q-ROFSs	√	√	×	√
CFSs	√	×	√	×
CIFSs	√	√	√	×
CPFSs	√	√	√	×
CQ-ROFSs	√	√	√	√

5. When $\overline{\overline{\Delta_{ip-z}}} = \overline{\overline{\Xi_{\widetilde{ip-z}}}} = 0$ and $\Lambda = 1$ in CQ-ROFSs, then all the proposed operators are converted into the ones based on the IFSs.

Furthermore, the power averaging\geometric aggregation operators, averaging/geometric aggregation operators, and Yager averaging\geometric aggregation operators based on the FSs, IFSs, PFSs, Q-ROFSs, CFSs, CIFSs, CPFSs, and CQ-ROFSs are the particular cases of the developed operators, which can give a big and wide space to make a valuable decision for decision-makers.

Main contributions

The main contributions of this study are shown as follows:

1. We develop the main operational laws for the CQ-ROF values based on the Yager t-norm and t-conorm such as Yager operational laws.
2. We propose the CQ-ROFPYA operator and CQ-ROFPYOA operator based on the CQ-ROF information and power Yager averaging operators.
3. We propose the CQ-ROFPYG operator and CQ-ROFPYOG operator based on the CQ-ROF information and power Yager geometric operators.
4. We also explore the major properties of the proposed operators such as idempotency, monotonicity, and boundedness.
5. We develop the MADM method based on the proposed operators with the CQ-ROF information.
6. Compare the proposed operators with some existing operators and show the effectiveness of the developed approaches.
   The main contributes of the proposed works are stated in Fig. 1.

The main construct of this paper is shown as: In "Preliminaries", we introduce the basic concepts and theories such as the PA operator, CQ-ROFS and their operational laws, and Yager t-norm and t-conorm. In "Power Yager aggregation operators for CQ-ROFNs" section, we propose the Yager operational laws for CQ-ROFNs. Moreover, we develop the CQ-ROFPYA, CQ-ROFPYOA, CQ-ROFPYG, and CQ-ROFPYOG operators, and discuss their fundamental properties of them. In "MADM methods based on the proposed operators" section, we propose a MADM method based on the proposed operators for CQ-ROF information. In "Comparative analysis" section, we compare the derived operators with various existing operators, and some practical examples are used to show the effectiveness of the proposed approaches. Concluding remarks are given in "Conclusion" section.

Preliminaries

For convenience, the parameters and variables used in this manuscript are briefly explained in Table 2.

Definition 1 [10] For the collection of positive integers $\widetilde{\beth}_z$, $z = 1, 2, \ldots, p$, the PA operator is given by:

$$PA(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p) = \frac{\sum_{z=1}^{p}(1 +^{\circ} \mathrm{F}(\widetilde{\beth}_z))\widetilde{\beth}_z}{\sum_{z=1}^{p}(1 +^{\circ} \mathrm{F}(\widetilde{\beth}_z))}, \tag{1}$$

Where $(\widetilde{\beth}_z) = \sum_{\substack{z=1\\ z\neq y}}^{p} SuP(\widetilde{\beth}_z, \widetilde{\beth}_y)$, which should hold the following axioms:

$$SuP(\widetilde{\beth}_z, \widetilde{\beth}_y) \in [0, 1], \tag{2}$$

$$SuP(\widetilde{\beth}_z, \widetilde{\beth}_y) = SuP(\widetilde{\beth}_y, \widetilde{\beth}_z), \tag{3}$$

$$SuP(\widetilde{\beth}_z, \widetilde{\beth}_y) \geq SuP(\widetilde{\beth}_a, \widetilde{\beth}_b)\text{If}\left|\widetilde{\beth}_z - \widetilde{\beth}_y\right| \leq \left|\widetilde{\beth}_a - \widetilde{\beth}_b\right|. \tag{4}$$

Definition 2 [9] The CQ-ROF set $\widetilde{\beth}$ based on universal set X is defined by:

$$\widetilde{\beth} = \left\{\left(\overline{\overline{\Delta_{\widetilde{\beth}}}}(x), \overline{\overline{\Xi_{\widetilde{\beth}}}}(x)\right) : x \in X\right\}. \tag{5}$$

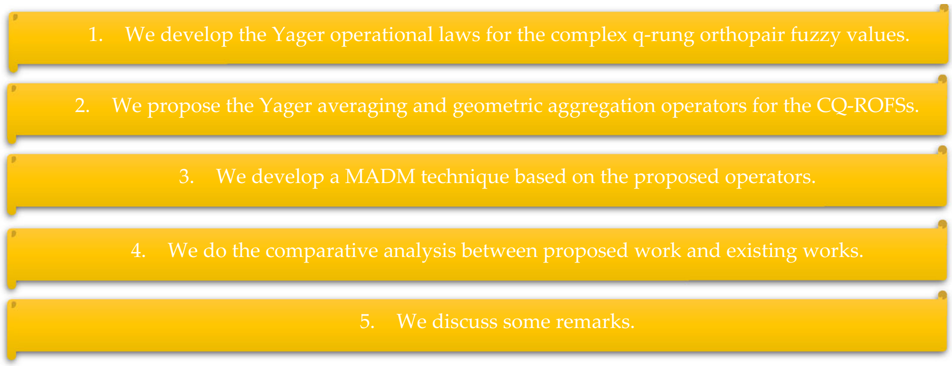

Fig. 1 The main contributes of the proposed works

Table 2 The meanings of all contained parameters and variables

Symbol	Meaning	Symbol	Meaning	Symbol	Meaning
$\tilde{\beth}_z$	Positive integers	X	Universal set	$\Lambda \geq 1$	Positive number
$\tilde{\beth}$	CQ-ROF set	x	Element of the universal set	∇_{SV}	Score values
$\overline{\overline{\Delta}}_{\tilde{\beth}}$	Complex truth grade	$\overline{\overline{\Delta}}_{\widetilde{rp}}$	The real part of truth grade	$\overline{\overline{\Delta}}_{\widetilde{ip}}$	The imaginary part of truth grade
$\overline{\overline{\Xi}}_{\tilde{\beth}}$	Complex falsity grade	$\overline{\overline{\Xi}}_{\widetilde{rp}}$	The real part of falsity grade	$\overline{\overline{\Xi}}_{\widetilde{ip}}$	The imaginary part of falsity grade
Π_{rg}	Complex neutral grade	Π_{rp}	The real part of the neutral grade	Π_{ip}	The imaginary part of the neutral grade
∇_{AV}	Accuracy values	φ	Positive number		

The $\overline{\overline{\Delta}}_{\tilde{\beth}}(x) = \left(\overline{\overline{\Delta}}_{\widetilde{rp}}(x), \overline{\overline{\Delta}}_{ip}(x)\right)$ and $\overline{\overline{\Xi}}_{\tilde{\beth}}(x) = \left(\overline{\overline{\Xi}}_{\widetilde{rp}}(x), \overline{\overline{\Xi}}_{\widetilde{ip}}(x)\right)$ are called a truth or positive and falsity or negative grades with $0 \leq \overline{\overline{\Delta}}_{\widetilde{rp}}^{\Lambda}(x) + \overline{\overline{\Xi}}_{\widetilde{rp}}^{\Lambda}(x) \leq 1$ and $0 \leq \overline{\overline{\Delta}}_{\widetilde{ip}}^{\Lambda}(x) + \overline{\overline{\Xi}}_{\widetilde{ip}}^{\Lambda}(x) \leq 1$, $\Lambda \geq 1$. The simplest form of refusal or neutral grade is expressed by: $\Pi_{rg}(x) = \left(\Pi_{rp}(x), \Pi_{ip}(x)\right) = \left(\left(1 - \left(\overline{\overline{\Delta}}_{\widetilde{rp}}^{\Lambda}(x) + \overline{\overline{\Xi}}_{\widetilde{rp}}^{\Lambda}(x)\right)\right)^{\frac{1}{\Lambda}}, \left(1 - \left(\overline{\overline{\Delta}}_{\widetilde{ip}}^{\Lambda}(x) + \overline{\overline{\Xi}}_{\widetilde{ip}}^{\Lambda}(x)\right)\right)^{\frac{1}{\Lambda}}\right)$. Moreover, a CQ-ROFN is expressed by $\tilde{\beth}_z = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2, \ldots, p$.

Definition 3 [8] For any two CQ-ROFNs $\tilde{\beth}_z = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2$, the Algebraic operational laws of them are defined by:

$$\tilde{\beth}_1 \oplus \tilde{\beth}_2 = \left(\begin{array}{c} \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-1}}^{\Lambda} + \overline{\overline{\Delta}}_{\widetilde{rp-2}}^{\Lambda} - \overline{\overline{\Delta}}_{\widetilde{rp-1}}^{\Lambda}\overline{\overline{\Delta}}_{\widetilde{rp-2}}^{\Lambda}\right)^{\frac{1}{\Lambda}}, \right. \\ \left. \left(\overline{\overline{\Delta}}_{\widetilde{ip-1}}^{\Lambda} + \overline{\overline{\Delta}}_{\widetilde{ip-2}}^{\Lambda} - \overline{\overline{\Delta}}_{\widetilde{ip-1}}^{\Lambda}\overline{\overline{\Delta}}_{\widetilde{ip-2}}^{\Lambda}\right)^{\frac{1}{\Lambda}}\right), \\ \left(\overline{\overline{\Xi_{\widetilde{rp-1}}\Xi_{\widetilde{rp-2}}}}, \overline{\overline{\Xi_{\widetilde{ip-1}}\Xi_{\widetilde{ip-2}}}}\right) \end{array}\right), \tag{6}$$

$$\tilde{\beth}_1 \otimes \tilde{\beth}_2 = \left(\begin{array}{c} \left(\overline{\overline{\Delta_{\widetilde{rp-1}}\Delta_{\widetilde{rp-2}}}}, \overline{\overline{\Delta_{\widetilde{ip-1}}\Delta_{\widetilde{ip-2}}}}\right), \\ \left(\left(\overline{\overline{\Xi}}_{\widetilde{rp-1}}^{\Lambda} + \overline{\overline{\Xi}}_{\widetilde{rp-2}}^{\Lambda} - \overline{\overline{\Xi}}_{\widetilde{rp-1}}^{\Lambda}\overline{\overline{\Xi}}_{\widetilde{rp-2}}^{\Lambda}\right)^{\frac{1}{\Lambda}}, \right. \\ \left. \left(\overline{\overline{\Xi}}_{\widetilde{ip-1}}^{\Lambda} + \overline{\overline{\Xi}}_{\widetilde{ip-2}}^{\Lambda} - \overline{\overline{\Xi}}_{\widetilde{ip-1}}^{\Lambda}\overline{\overline{\Xi}}_{\widetilde{ip-2}}^{\Lambda}\right)^{\frac{1}{\Lambda}}\right) \end{array}\right), \tag{7}$$

$$\varphi\tilde{\beth}_1 = \left(\begin{array}{c} \left(\left(1 - \left(1 - \overline{\overline{\Delta}}_{\widetilde{rp-1}}^{\Lambda}\right)^{\varphi}\right)^{\frac{1}{\Lambda}}, \right. \\ \left. \left(1 - \left(1 - \overline{\overline{\Delta}}_{\widetilde{ip-1}}^{\Lambda}\right)^{\varphi}\right)^{\frac{1}{\Lambda}}\right), \\ \left(\overline{\overline{\Xi}}_{\widetilde{rp-1}}^{\varphi}, \overline{\overline{\Xi}}_{\widetilde{ip-1}}^{\varphi}\right) \end{array}\right), \tag{8}$$

$$\tilde{\beth}_1^{\varphi} = \left(\begin{array}{c} \left(\overline{\overline{\Delta}}_{\widetilde{rp-1}}^{\varphi}, \overline{\overline{\Delta}}_{\widetilde{ip-1}}^{\varphi}\right), \\ \left(\left(1 - \left(1 - \overline{\overline{\Xi}}_{\widetilde{rp-1}}^{\Lambda}\right)^{\varphi}\right)^{\frac{1}{\Lambda}}, \left(1 - \left(1 - \overline{\overline{\Xi}}_{\widetilde{ip-1}}^{\Lambda}\right)^{\varphi}\right)^{\frac{1}{\Lambda}}\right) \end{array}\right). \tag{9}$$

Definition 4 [8] For any two CQ-ROFNs $\tilde{\beth}_z = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2$, the score and accuracy values of them are defined as

$$\nabla_{SV}\left(\tilde{\beth}_z\right) = \frac{1}{2}\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}^{\Lambda} + \overline{\overline{\Delta}}_{ip-z}^{\Lambda} - \overline{\overline{\Xi}}_{\widetilde{rp-z}}^{\Lambda} - \overline{\overline{\Xi}}_{\widetilde{ip-z}}^{\Lambda}\right) \in [-1, 1], \tag{10}$$

$$\nabla_{AV}\left(\tilde{\beth}_z\right) = \frac{1}{2}\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}^{\Lambda} + \overline{\overline{\Delta}}_{ip-z}^{\Lambda} + \overline{\overline{\Xi}}_{\widetilde{rp-z}}^{\Lambda} + \overline{\overline{\Xi}}_{\widetilde{ip-z}}^{\Lambda}\right)$$

$$\in [0,\ 1]. \tag{11}$$

For Eq. (10) and Eq. (11), we can get the comparison method for any two CQ-ROFNs

(1) when $\nabla_{SV}\left(\widetilde{\gimel}_1\right) > \nabla_{SV}\left(\widetilde{\gimel}_2\right) \Rightarrow \widetilde{\gimel}_1 > \widetilde{\gimel}_2$;
(2) when $\nabla_{SV}\left(\widetilde{\gimel}_1\right) < \nabla_{SV}\left(\widetilde{\gimel}_2\right) \Rightarrow \widetilde{\gimel}_1 < \widetilde{\gimel}_2$;
(3) when $\nabla_{SV}\left(\widetilde{\gimel}_1\right) = \nabla_{SV}\left(\widetilde{\gimel}_2\right) \Rightarrow$ when $\nabla_{AV}\left(\widetilde{\gimel}_1\right) > \nabla_{AV}\left(\widetilde{\gimel}_2\right) \Rightarrow \widetilde{\gimel}_1 > \widetilde{\gimel}_2$; when $\nabla_{AV}\left(\widetilde{\gimel}_1\right) < \nabla_{AV}\left(\widetilde{\gimel}_2\right) \Rightarrow \widetilde{\gimel}_1 < \widetilde{\gimel}_2$.

Further, we introduce Yager's t-norm and t-conorm (proposed by Yager [11]) which are stated as

$$x \oplus y = \min\left(1,\ \left(x^{\Pi} + y^{\Pi}\right)^{\frac{1}{\Pi}}\right),\ \Pi \geq 1, \tag{12}$$

$$x \otimes y = 1 - \min\left(1,\ \left((1-x)^{\Pi} + (1-y)^{\Pi}\right)^{\frac{1}{\Pi}}\right),\ \Pi \geq 1. \tag{13}$$

Power Yager aggregation operators for CQ-ROFNs

In this subsection, we will develop the CQ-ROFPYA, CQ-ROFPYOA, CQ-ROFPYG, and CQ-ROFPYOG operators, and discuss their fundamental properties.

Definition 5 For any family of CQ-ROFNs $\widetilde{\gimel}_z = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2, \ldots, p$, and $\varphi > 0$, Yager's operational laws of them are defined as

$$\widetilde{\gimel}_1 \oplus \widetilde{\gimel}_2 = \begin{pmatrix} \begin{pmatrix} \left(\min\left(1,\ \left(\overline{\overline{\Delta}}_{\widetilde{rp-1}}^{\Lambda\Pi} + \overline{\overline{\Delta}}_{\widetilde{rp-2}}^{\Lambda\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1,\ \left(\overline{\overline{\Delta}}_{\widetilde{ip-1}}^{\Lambda\Pi} + \overline{\overline{\Delta}}_{\widetilde{ip-2}}^{\Lambda\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{pmatrix}, \\ \begin{pmatrix} \left(1 - \min\left(1,\ \left(\left(1 - \overline{\overline{\Xi}}_{\widetilde{rp-1}}^{\Lambda}\right)^{\Pi} + \left(1 - \overline{\overline{\Xi}}_{\widetilde{rp-2}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1 - \min\left(1,\ \left(\left(1 - \overline{\overline{\Xi}}_{\widetilde{ip-1}}^{\Lambda}\right)^{\Pi} + \left(1 - \overline{\overline{\Xi}}_{\widetilde{ip-2}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{pmatrix} \end{pmatrix}, \tag{14}$$

$$\widetilde{\gimel}_1 \otimes \widetilde{\gimel}_2 = \begin{pmatrix} \begin{pmatrix} \left(1 - \min\left(1,\ \left(\left(1 - \overline{\overline{\Delta}}_{\widetilde{rp-1}}^{\Lambda}\right)^{\Pi} + \left(1 - \overline{\overline{\Delta}}_{\widetilde{rp-2}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1 - \min\left(1,\ \left(\left(1 - \overline{\overline{\Delta}}_{\widetilde{ip-1}}^{\Lambda}\right)^{\Pi} + \left(1 - \overline{\overline{\Delta}}_{\widetilde{ip-2}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{pmatrix}, \\ \begin{pmatrix} \left(\min\left(1,\ \left(\overline{\overline{\Xi}}_{\widetilde{rp-1}}^{\Lambda\Pi} + \overline{\overline{\Xi}}_{\widetilde{rp-2}}^{\Lambda\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1,\ \left(\overline{\overline{\Xi}}_{\widetilde{ip-1}}^{\Lambda\Pi} + \overline{\overline{\Xi}}_{\widetilde{ip-2}}^{\Lambda\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{pmatrix} \end{pmatrix} \tag{15}$$

$$\varphi\widetilde{\gimel}_1 = \begin{pmatrix} \begin{pmatrix} \left(\min\left(1,\ \left(\varphi\left(\overline{\overline{\Delta}}_{\widetilde{rp-1}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1,\ \left(\varphi\left(\overline{\overline{\Delta}}_{\widetilde{ip-1}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{pmatrix}, \\ \begin{pmatrix} \left(1 - \min\left(1,\ \left(\varphi\left(1 - \overline{\overline{\Xi}}_{\widetilde{rp-1}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1 - \min\left(1,\ \left(\varphi\left(1 - \overline{\overline{\Xi}}_{\widetilde{ip-1}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{pmatrix} \end{pmatrix}, \tag{16}$$

$$\widetilde{\gimel}_1^{\varphi} = \begin{pmatrix} \begin{pmatrix} \left(1 - \min\left(1,\ \left(\varphi\left(1 - \overline{\overline{\Delta}}_{\widetilde{rp-1}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1 - \min\left(1,\ \left(\varphi\left(1 - \overline{\overline{\Delta}}_{\widetilde{ip-1}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{pmatrix}, \\ \begin{pmatrix} \left(\min\left(1,\ \left(\varphi\left(\overline{\overline{\Xi}}_{\widetilde{rp-1}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1,\ \left(\varphi\left(\overline{\overline{\Xi}}_{\widetilde{ip-1}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{pmatrix} \end{pmatrix}. \tag{17}$$

Definition 6 For the collection of CQ-ROFNs $\widetilde{\gimel}_z = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2, \ldots, p$, the CQ-ROFPYA operator is defined by:

$$CQ-ROFPYA\left(\widetilde{\gimel}_1, \widetilde{\gimel}_2, \ldots, \widetilde{\gimel}_p\right) = \overline{\overline{\aleph}}_1\widetilde{\gimel}_1 \oplus \overline{\overline{\aleph}}_2\widetilde{\gimel}_2 \oplus, \ldots, \oplus\overline{\overline{\aleph}}_p\widetilde{\gimel}_p = \oplus_{z=1}^{p}\overline{\overline{\aleph}}_z\widetilde{\gimel}_z, \tag{18}$$

where

$$\overline{\overline{\aleph}}_z = \frac{\left(1 +^{\circ} \mathrm{F}\left(\widetilde{\gimel}_z\right)\right)}{\sum_{z=1}^{p}\left(1 +^{\circ} \mathrm{F}\left(\widetilde{\gimel}_z\right)\right)}, \tag{19}$$

where ${}^{\circ}\mathrm{F}\left(\widetilde{\gimel}_z\right) = \sum_{z=1, z\neq y}^{p} SuP\left(\widetilde{\gimel}_z, \widetilde{\gimel}_y\right)$, $SuP\left(\widetilde{\gimel}_z, \widetilde{\gimel}_y\right) = 1 - Dis\left(\widetilde{\gimel}_z, \widetilde{\gimel}_y\right)$, and

$$Dis\left(\widetilde{\gimel}_z, \widetilde{\gimel}_y\right) = \frac{1}{4}\left(\left|\overline{\overline{\Delta}}_{\widetilde{rp-z}} - \overline{\overline{\Delta}}_{\widetilde{rp-y}}\right| + \left|\overline{\overline{\Delta}}_{\widetilde{ip-z}} - \overline{\overline{\Delta}}_{\widetilde{ip-y}}\right| + \left|\overline{\overline{\Xi}}_{\widetilde{rp-z}} - \overline{\overline{\Xi}}_{\widetilde{rp-y}}\right| + \left|\overline{\overline{\Xi}}_{\widetilde{ip-z}} - \overline{\overline{\Xi}}_{\widetilde{ip-y}}\right|\right), \tag{20}$$

which should hold the following axioms:

$$SuP\left(\widetilde{\gimel}_z, \widetilde{\gimel}_y\right) \in [0,\ 1], \tag{21}$$

$$SuP\left(\widetilde{\gimel}_z, \widetilde{\gimel}_y\right) = SuP\left(\widetilde{\gimel}_y, \widetilde{\gimel}_z\right), \tag{22}$$

$$SuP\left(\tilde{\beth}_z, \tilde{\beth}_y\right) \geq SuP\left(\tilde{\beth}_a, \tilde{\beth}_b\right) \text{If} \left|\tilde{\beth}_z - \tilde{\beth}_y\right| \leq \left|\tilde{\beth}_a - \tilde{\beth}_b\right|. \tag{23}$$

Theorem 1 *For the collection of CQ-ROFNs* $\tilde{\beth}_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \overline{\overline{\Delta_{ip-z}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right), z = 1, 2, \ldots, p$, *the final value of Eq.* (18) *is also a CQ-ROFN, such as:*

$$CQ-ROFPYA\left(\tilde{\beth}_1, \tilde{\beth}_2, \ldots, \tilde{\beth}_p\right) = \begin{pmatrix} \left(\left(\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{ip-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{rp-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(1-\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{ip-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix}. \tag{24}$$

Obviously, Eq. (24) *is the modified version of the power, Yager, and power Yager aggregation operators for FSs, IFSs, PFSs, Q-ROFSs, CFSs, CIFSs, CPFSs, and CQ-ROFSs.*

Proof We can prove it by mathematical induction for p.

(1) When $p = 2$ in Eq. (24), we have

$$\overline{\overline{\aleph}}_1\tilde{\beth}_1 = \begin{pmatrix} \left(\left(\min\left(1, \left(\overline{\overline{\aleph}}_1\left(\overline{\overline{\Delta_{\widetilde{rp-1}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(\min\left(1, \left(\overline{\overline{\aleph}}_1\left(\overline{\overline{\Delta_{\widetilde{ip-1}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\overline{\overline{\aleph}}_1\left(1-\overline{\overline{\Xi_{\widetilde{rp-1}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(1-\min\left(1, \left(\overline{\overline{\aleph}}_1\left(1-\overline{\overline{\Xi_{\widetilde{ip-1}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix},$$

$$\overline{\overline{\aleph}}_2\tilde{\beth}_2 = \begin{pmatrix} \left(\left(\min\left(1, \left(\overline{\overline{\aleph}}_2\left(\overline{\overline{\Delta_{\widetilde{rp-2}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(\min\left(1, \left(\overline{\overline{\aleph}}_2\left(\overline{\overline{\Delta_{\widetilde{ip-2}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\overline{\overline{\aleph}}_2\left(1-\overline{\overline{\Xi_{\widetilde{rp-2}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(1-\min\left(1, \left(\overline{\overline{\aleph}}_2\left(1-\overline{\overline{\Xi_{\widetilde{ip-2}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix}$$

$$CQ-ROFPYA\left(\tilde{\beth}_1, \tilde{\beth}_2\right) = \oplus_{z=1}^{2}\overline{\overline{\aleph}}_z\tilde{\beth}_z = \overline{\overline{\aleph}}_1\tilde{\beth}_1 \oplus \overline{\overline{\aleph}}_2\tilde{\beth}_2$$

$$= \begin{pmatrix} \left(\left(\min\left(1, \left(\overline{\overline{\aleph}}_1\left(\overline{\overline{\Delta_{\widetilde{rp-1}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(\min\left(1, \left(\overline{\overline{\aleph}}_1\left(\overline{\overline{\Delta_{\widetilde{ip-1}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\overline{\overline{\aleph}}_1\left(1-\overline{\overline{\Xi_{\widetilde{rp-1}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(1-\min\left(1, \left(\overline{\overline{\aleph}}_1\left(1-\overline{\overline{\Xi_{\widetilde{ip-1}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix}$$

$$\oplus \begin{pmatrix} \left(\left(\min\left(1, \left(\overline{\overline{\aleph}}_2\left(\overline{\overline{\Delta_{\widetilde{rp-2}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(\min\left(1, \left(\overline{\overline{\aleph}}_2\left(\overline{\overline{\Delta_{\widetilde{ip-2}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\overline{\overline{\aleph}}_2\left(1-\overline{\overline{\Xi_{\widetilde{rp-2}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(1-\min\left(1, \left(\overline{\overline{\aleph}}_2\left(1-\overline{\overline{\Xi_{\widetilde{ip-2}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \left(\left(\min\left(1, \left(\sum_{z=1}^{2} \overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(\min\left(1, \left(\sum_{z=1}^{2} \overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{ip-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\sum_{z=1}^{2} \overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{rp-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(1-\min\left(1, \left(\sum_{z=1}^{2} \overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{ip-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix},$$

i.e., Eq. (24) is held when $p = 2$.

(2) Suppose when $p = k$, Eq. (24) is held, then we have:

$$CQ-ROFPYA\left(\tilde{\beth}_1, \tilde{\beth}_2, \ldots, \tilde{\beth}_k\right) = \begin{pmatrix} \left(\left(\min\left(1, \left(\sum_{z=1}^{k} \overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(\min\left(1, \left(\sum_{z=1}^{k} \overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{ip-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\sum_{z=1}^{k} \overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{rp-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \left(1-\min\left(1, \left(\sum_{z=1}^{k} \overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{ip-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix}.$$

Then, when $p = k + 1$, we have:

$$CQ-ROFPYA\left(\tilde{\beth}_1, \tilde{\beth}_2, \ldots, \tilde{\beth}_{k+1}\right) = \overline{\overline{\aleph}}_1\tilde{\beth}_1 \oplus \overline{\overline{\aleph}}_2\tilde{\beth}_2 \oplus, \ldots, \oplus\overline{\overline{\aleph}}_k\tilde{\beth}_k \oplus \overline{\overline{\aleph}}_{k+1}\tilde{\beth}_{k+1}$$

$$= \oplus_{z=1}^{k}\overline{\overline{\aleph}}_z\widetilde{\beth}_{\boldsymbol{z}} \oplus \overline{\overline{\aleph}}_{k+1}\widetilde{\beth}_{\boldsymbol{k}+1}$$

$$= \begin{pmatrix} \left(\left(\min\left(1, \left(\sum_{z=1}^{k}\overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1, \left(\sum_{z=1}^{k}\overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{ip-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\sum_{z=1}^{k}\overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{rp-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1-\min\left(1, \left(\sum_{z=1}^{k}\overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{ip-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix}$$

$$\oplus \overline{\overline{\aleph}}_{k+1}\widetilde{\beth}_{\boldsymbol{k}+1}$$

$$= \begin{pmatrix} \left(\left(\min\left(1, \left(\sum_{z=1}^{k}\overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1, \left(\sum_{z=1}^{k}\overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{ip-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\sum_{z=1}^{k}\overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{rp-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1-\min\left(1, \left(\sum_{z=1}^{k}\overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{ip-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix}$$

$$\oplus \begin{pmatrix} \left(\left(\min\left(1, \left(\overline{\overline{\aleph}}_{k+1}\left(\overline{\overline{\Delta_{\widetilde{rp-k+1}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1, \left(\overline{\overline{\aleph}}_{k+1}\left(\overline{\overline{\Delta_{\widetilde{ip-k+1}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\overline{\overline{\aleph}}_{k+1}\left(1-\overline{\overline{\Xi_{\widetilde{rp-k+1}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1-\min\left(1, \left(\overline{\overline{\aleph}}_{k+1}\left(1-\overline{\overline{\Xi_{\widetilde{ip-k+1}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \left(\left(\min\left(1, \left(\sum_{z=1}^{k+1}\overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1, \left(\sum_{z=1}^{k+1}\overline{\overline{\aleph}}_z\left(\overline{\overline{\Delta_{\widetilde{ip-z}}}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right), \\ \left(\left(1-\min\left(1, \left(\sum_{z=1}^{k+1}\overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{rp-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1-\min\left(1, \left(\sum_{z=1}^{k+1}\overline{\overline{\aleph}}_z\left(1-\overline{\overline{\Xi_{\widetilde{ip-z}}}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}\right) \end{pmatrix},$$

i.e., when $p = k + 1$, Eq. (24) is also held.

(3) Based on (1) and (2), we can get the conclusion when p is any positive value, Eq. (24) is held.

Furthermore, we discuss their valuable and feasible properties such as idempotency, monotonicity, and boundedness.

Property 1 *(*idempotency*):* When $\widetilde{\beth}_z = \widetilde{\beth} = \left(\left(\overline{\overline{\Delta_{\widetilde{rp}}}}, \overline{\overline{\Delta_{ip}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp}}}}, \overline{\overline{\Xi_{\widetilde{ip}}}}\right)\right)$, $z = 1, 2, \ldots, p$, then

$$CQ-ROFPYA\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_{\boldsymbol{p}}\right) = \widetilde{\beth}. \tag{25}$$

Property 2 *(*monotonicity*):* When $\widetilde{\beth}_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \overline{\overline{\Delta_{ip-z}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right) \leq \widetilde{\beth}''_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}'', \overline{\overline{\Delta_{ip-z}}}''\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}'', \overline{\overline{\Xi_{\widetilde{ip-z}}}}''\right)\right)$, $z = 1, 2, \ldots, p$, then

$$\begin{aligned} &CQ-ROFPYA\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_{\boldsymbol{p}}\right) \\ &\quad \leq CQ-ROFPYA\left(\widetilde{\beth}''_1, \widetilde{\beth}''_2, \ldots, \widetilde{\beth}''_{\boldsymbol{p}}\right). \end{aligned} \tag{26}$$

Property 3 *(*boundedness*):* When $\widetilde{\beth}^-_z = \left(\left(\min_z\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \min_z\overline{\overline{\Delta_{ip-z}}}\right), \left(\max_z\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \max_z\overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right)$ and $\widetilde{\beth}^+_z = \left(\left(\max_z\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \max_z\overline{\overline{\Delta_{ip-z}}}\right), \max_z\overline{\overline{\Delta_{ip-z}}}\right), \left(\min_z\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \min_z\overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right)$, $z = 1, 2, \ldots, p$, then

$$\widetilde{\beth}^-_z \leq CQ-ROFPYA\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_{\boldsymbol{p}}\right) \leq \widetilde{\beth}^+_z. \tag{27}$$

Definition 7 For the collection of CQ-ROFNs $\widetilde{\beth}_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \overline{\overline{\Delta_{ip-z}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right)$, $z = 1, 2, \ldots, p$, the CQ-ROFPYOA operator is defined by:

$$\begin{aligned} &CQ-ROFPYOA\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_{\boldsymbol{p}}\right) \\ &= \overline{\overline{\aleph}}_1\widetilde{\beth}_{\boldsymbol{o}(1)} \oplus \overline{\overline{\aleph}}_2\widetilde{\beth}_{\boldsymbol{o}(2)} \oplus, \ldots, \oplus\overline{\overline{\aleph}}_p\widetilde{\beth}_{\boldsymbol{o}(\boldsymbol{p})} \\ &= \oplus_{z=1}^{p}\overline{\overline{\aleph}}_z\widetilde{\beth}_{\boldsymbol{o}(z)}. \end{aligned} \tag{28}$$

With $o(z) \leq o(z-1)$, where

$$\overline{\overline{\aleph}}_z = \frac{\left(1 +^{\circ} \mathrm{F}\left(\widetilde{\beth}_{\boldsymbol{z}}\right)\right)}{\sum_{z=1}^{p}\left(1 +^{\circ} \mathrm{F}\left(\widetilde{\beth}_{\boldsymbol{z}}\right)\right)}, \tag{29}$$

where ${}^{\circ}\mathrm{F}\left(\widetilde{\beth}_z\right) = \sum_{\substack{z=1 \\ z\neq y}}^{p} SuP\left(\widetilde{\beth}_{\boldsymbol{z}}, \widetilde{\beth}_{\boldsymbol{y}}\right)$, $SuP\left(\widetilde{\beth}_{\boldsymbol{z}}, \widetilde{\beth}_{\boldsymbol{y}}\right) = 1 - Dis\left(\widetilde{\beth}_{\boldsymbol{z}}, \widetilde{\beth}_{\boldsymbol{y}}\right)$, and

$$\begin{aligned} Dis\left(\widetilde{\beth}_{\boldsymbol{z}}, \widetilde{\beth}_{\boldsymbol{y}}\right) = \frac{1}{4}\Big(&\left|\overline{\overline{\Delta_{\widetilde{rp-z}}}} - \overline{\overline{\Delta_{\widetilde{rp-y}}}}\right| + \left|\overline{\overline{\Delta_{\widetilde{ip-z}}}} - \overline{\overline{\Delta_{\widetilde{ip-y}}}}\right| \\ &+ \left|\overline{\overline{\Xi_{\widetilde{rp-z}}}} - \overline{\overline{\Xi_{\widetilde{rp-y}}}}\right| + \left|\overline{\overline{\Xi_{\widetilde{ip-z}}}} - \overline{\overline{\Xi_{\widetilde{ip-y}}}}\right|\Big), \end{aligned} \tag{30}$$

which should hold the following axioms:

$$SuP\left(\widetilde{\beth}_{\boldsymbol{z}}, \widetilde{\beth}_{\boldsymbol{y}}\right) \in [0, 1], \tag{31}$$

$$SuP\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right) = SuP\left(\widetilde{\beth}_y, \widetilde{\beth}_z\right), \tag{32}$$

$$SuP\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right) \geq SuP\left(\widetilde{\beth}_a, \widetilde{\beth}_b\right)\text{If}\left|\widetilde{\beth}_z - \widetilde{\beth}_y\right| \leq \left|\widetilde{\beth}_a - \widetilde{\beth}_b\right|. \tag{33}$$

Theorem 2 *For the collection of CQ-ROFNs* $\widetilde{\beth}_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \overline{\overline{\Delta_{ip-z}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right)$, $z = 1, 2, \ldots, p$, *the final value of Eq.* (28) *is also a CQ-ROFN, such as*:

$$CQ - ROFPYOA\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p\right) = \begin{pmatrix} \left(\left(\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(\overline{\overline{\Delta_{\widetilde{rp-o(z)}}}}^{\wedge\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\wedge}}, \\ \left(\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(\overline{\overline{\Delta_{\widetilde{ip-o(z)}}}}^{\wedge\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\wedge}}\right), \\ \left(\left(1 - \min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(1 - \overline{\overline{\Xi_{\widetilde{rp-o(z)}}}}^{\wedge}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\wedge}}, \\ \left(1 - \min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(1 - \overline{\overline{\Xi_{\widetilde{ip-o(z)}}}}^{\wedge}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\wedge}}\right) \end{pmatrix}. \tag{34}$$

Obviously, Eq. (34) is the modified version of the power, Yager, and power Yager aggregation operators for FSs, IFSs, PFSs, Q-ROFSs, CFSs, CIFSs, CPFSs, and CQ-ROFSs. Furthermore, we discuss their valuable and feasible properties such as idempotency, monotonicity, and boundedness.

Property 4 *(*idempotency*)*: When $\widetilde{\beth}_z = \widetilde{\beth} = \left(\left(\overline{\overline{\Delta_{\widetilde{rp}}}}, \overline{\overline{\Delta_{ip}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp}}}}, \overline{\overline{\Xi_{\widetilde{ip}}}}\right)\right)$, $z = 1, 2, \ldots, p$, then

$$CQ - ROFPYOA\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p\right) = \widetilde{\beth}. \tag{35}$$

Property 5 (monotonicity)*:* When $\widetilde{\beth}_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \overline{\overline{\Delta_{ip-z}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right) \leq \widetilde{\beth}''_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}'', \overline{\overline{\Delta_{ip-z}}}''\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}'', \overline{\overline{\Xi_{\widetilde{ip-z}}}}''\right)\right)$, $z = 1, 2, \ldots, p$, then

$$CQ - ROFPYOA\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p\right) \leq CQ - ROFPYOA\left(\widetilde{\beth}''_1, \widetilde{\beth}''_2, \ldots, \widetilde{\beth}''_p\right). \tag{36}$$

Property 6 *(*boundedness*)*: When $\widetilde{\beth}^-_z = \left(\left(\min_z \overline{\overline{\Delta_{\widetilde{rp-z}}}}, \min_z \overline{\overline{\Delta_{ip-z}}}\right), \left(\max_z \overline{\overline{\Xi_{\widetilde{rp-z}}}}, \max_z \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right)$ and $\widetilde{\beth}^+_z = \left(\left(\max_z \overline{\overline{\Delta_{\widetilde{rp-z}}}}, \max_z \overline{\overline{\Delta_{ip-z}}}\right), \left(\min_z \overline{\overline{\Xi_{\widetilde{rp-z}}}}, \min_z \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right)$, $z = 1, 2, \ldots, p$, then

$$\widetilde{\beth}^-_z \leq CQ - ROFPYOA\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p\right) \leq \widetilde{\beth}^+_z. \tag{37}$$

Definition 8 For the collection of CQ-ROFNs $\widetilde{\beth}_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \overline{\overline{\Delta_{ip-z}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right)$, $z = 1, 2, \ldots, p$, the CQ-ROFPYG operator is defined by:

$$CQ - ROFPYG\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p\right) = \widetilde{\beth}_1^{\overline{\overline{\aleph}}_1} \otimes \widetilde{\beth}_2^{\overline{\overline{\aleph}}_2} \otimes, \ldots, \otimes \widetilde{\beth}_p^{\overline{\overline{\aleph}}_p} = \otimes_{z=1}^{p} \widetilde{\beth}_z^{\overline{\overline{\aleph}}_z}, \tag{38}$$

where $\overline{\overline{\aleph}}_z = \frac{(1+{}^{\circ}\mathrm{F}(\widetilde{\beth}_z))}{\sum_{z=1}^{p}(1+{}^{\circ}\mathrm{F}(\widetilde{\beth}_z))}$, ${}^{\circ}\mathrm{F}\left(\widetilde{\beth}_z\right) = \sum_{\substack{z=1 \\ z \neq y}}^{p} SuP\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right)$, $SuP\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right) = 1 - Dis\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right)$, and $Dis\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right) = \frac{1}{4}\left(\left|\overline{\overline{\Delta_{\widetilde{rp-z}}}} - \overline{\overline{\Delta_{\widetilde{rp-y}}}}\right| + \left|\overline{\overline{\Delta_{\widetilde{ip-z}}}} - \overline{\overline{\Delta_{\widetilde{ip-y}}}}\right| + \left|\overline{\overline{\Xi_{\widetilde{rp-z}}}} - \overline{\overline{\Xi_{\widetilde{rp-y}}}}\right| + \left|\overline{\overline{\Xi_{\widetilde{ip-z}}}} - \overline{\overline{\Xi_{\widetilde{ip-y}}}}\right|\right)$, which should hold the following axioms: (1) $SuP\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right) \in [0, 1]$, (2) $SuP\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right) = SuP\left(\widetilde{\beth}_y, \widetilde{\beth}_z\right)$ and (3) $SuP\left(\widetilde{\beth}_z, \widetilde{\beth}_y\right) \geq SuP\left(\widetilde{\beth}_a, \widetilde{\beth}_b\right)\text{If}\left|\widetilde{\beth}_z - \widetilde{\beth}_y\right| \leq \left|\widetilde{\beth}_a - \widetilde{\beth}_b\right|$.

Theorem 2 *For the collection of CQ-ROFNs* $\widetilde{\beth}_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \overline{\overline{\Delta_{ip-z}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right)$, $z = 1, 2, \ldots, p$, *the final value of Eq.* (38) *is also a CQ-ROFN, such as*:

$$CQ - ROFPYG\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p\right) = \begin{pmatrix} \left(\left(1 - \min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(1 - \overline{\overline{\Delta_{\widetilde{rp-z}}}}^{\wedge}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\wedge}}, \\ \left(1 - \min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(1 - \overline{\overline{\Delta_{\widetilde{ip-z}}}}^{\wedge}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\wedge}}\right), \\ \left(\left(\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}^{\wedge\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\wedge}}, \\ \left(\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(\overline{\overline{\Xi_{\widetilde{ip-z}}}}^{\wedge\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\wedge}}\right) \end{pmatrix}. \tag{39}$$

Obviously, Eq. (39) is the modified version of the power, Yager, and power Yager aggregation operators for FSs, IFSs, PFSs, Q-ROFSs, CFSs, CIFSs, CPFSs, and CQ-ROFSs. Furthermore, we discuss their valuable and feasible properties such as idempotency, monotonicity, and boundedness.

Property 7 *(*idempotency*)*: When $\widetilde{\beth}_z = \widetilde{\beth} = \left(\left(\overline{\overline{\Delta_{\widetilde{rp}}}}, \overline{\overline{\Delta_{ip}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp}}}}, \overline{\overline{\Xi_{\widetilde{ip}}}}\right)\right)$, $z = 1, 2, \ldots, p$, then

$$CQ - ROFPYG\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p\right) = \widetilde{\beth}. \tag{40}$$

Property 8 *(*monotonicity*)*: When $\widetilde{\beth}_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}, \overline{\overline{\Delta_{ip-z}}}\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}, \overline{\overline{\Xi_{\widetilde{ip-z}}}}\right)\right) \leq \widetilde{\beth}''_z = \left(\left(\overline{\overline{\Delta_{\widetilde{rp-z}}}}'', \overline{\overline{\Delta_{ip-z}}}''\right), \left(\overline{\overline{\Xi_{\widetilde{rp-z}}}}'', \overline{\overline{\Xi_{\widetilde{ip-z}}}}''\right)\right)$, $z = 1, 2, \ldots, p$, then

$$CQ - ROFPYG\left(\widetilde{\beth}_1, \widetilde{\beth}_2, \ldots, \widetilde{\beth}_p\right)$$

$$\leq CQ-ROFPYG\left(\widetilde{\gimel}_1'', \widetilde{\gimel}_2'', \ldots, \widetilde{\gimel}_{\boldsymbol{p}}''\right). \tag{41}$$

Property 9 (boundedness)*:* When $\widetilde{\gimel}_z^- = \left(\left(\min_z \overline{\overline{\Delta}}_{\widetilde{rp-z}}, \min_z \overline{\overline{\Delta}}_{ip-z}\right), \left(\max_z \overline{\overline{\Xi}}_{\widetilde{rp-z}}, \max_z \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$ and $\widetilde{\gimel}_z^+ = \left(\left(\max_z \overline{\overline{\Delta}}_{\widetilde{rp-z}}, \max_z \overline{\overline{\Delta}}_{ip-z}\right), \left(\min_z \overline{\overline{\Xi}}_{\widetilde{rp-z}}, \min_z \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2, \ldots, p$, then

$$\widetilde{\gimel}_z^- \leq CQ-ROFPYG\left(\widetilde{\gimel}_1, \widetilde{\gimel}_2, \ldots, \widetilde{\gimel}_{\boldsymbol{p}}\right) \leq \widetilde{\gimel}_z^+. \tag{42}$$

Definition 9 For the collection of CQ-ROFNs $\widetilde{\gimel}_z = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2, \ldots, p$, the CQ-ROFPYOG operator is defined by:

$$\begin{aligned} &CQ-ROFPYOG\left(\widetilde{\gimel}_1, \widetilde{\gimel}_2, \ldots, \widetilde{\gimel}_{\boldsymbol{p}}\right) \\ &= \widetilde{\gimel}_{\boldsymbol{o}(1)}^{\overline{\overline{\aleph}}_1} \otimes \widetilde{\gimel}_{\boldsymbol{o}(2)}^{\overline{\overline{\aleph}}_2} \otimes, \ldots, \otimes \widetilde{\gimel}_{\boldsymbol{o}(\boldsymbol{p})}^{\overline{\overline{\aleph}}_p} \\ &= \otimes_{z=1}^{p} \widetilde{\gimel}_{\boldsymbol{o}(\boldsymbol{z})}^{\overline{\overline{\aleph}}_z}. \end{aligned} \tag{43}$$

With $o(z) \leq o(z-1)$, where $\overline{\overline{\aleph}}_z =$, $^\circ\mathrm{F}\left(\widetilde{\gimel}_{\boldsymbol{z}}\right) = \sum_{\substack{z=1\\ z\neq y}}^{p} SuP\left(\widetilde{\gimel}_{\boldsymbol{z}}, \widetilde{\gimel}_{\boldsymbol{y}}\right)$, $SuP\left(\widetilde{\gimel}_{\boldsymbol{z}}, \widetilde{\gimel}_{\boldsymbol{y}}\right) = 1 - Dis\left(\widetilde{\gimel}_{\boldsymbol{z}}, \widetilde{\gimel}_{\boldsymbol{y}}\right)$, and $Dis\left(\widetilde{\gimel}_{\boldsymbol{z}}, \widetilde{\gimel}_{\boldsymbol{y}}\right) = \frac{1}{4}\left(\left|\overline{\overline{\Delta}}_{\widetilde{rp-z}} - \overline{\overline{\Delta}}_{\widetilde{rp-y}}\right| + \left|\overline{\overline{\Delta}}_{\widetilde{ip-z}} - \overline{\overline{\Delta}}_{\widetilde{ip-y}}\right| + \left|\overline{\overline{\Xi}}_{\widetilde{rp-z}} - \overline{\overline{\Xi}}_{\widetilde{rp-y}}\right| + \left|\overline{\overline{\Xi}}_{\widetilde{ip-z}} - \overline{\overline{\Xi}}_{\widetilde{ip-y}}\right|\right)$, which should hold the following axioms:$SuP\left(\widetilde{\gimel}_{\boldsymbol{z}}, \widetilde{\gimel}_{\boldsymbol{y}}\right) \in [0, 1]$,$SuP\left(\widetilde{\gimel}_{\boldsymbol{z}}, \widetilde{\gimel}_{\boldsymbol{y}}\right) = SuP\left(\widetilde{\gimel}_{\boldsymbol{y}}, \widetilde{\gimel}_{\boldsymbol{z}}\right)$ and $SuP\left(\widetilde{\gimel}_{\boldsymbol{z}}, \widetilde{\gimel}_{\boldsymbol{y}}\right) \geq SuP\left(\widetilde{\gimel}_{\boldsymbol{a}}, \widetilde{\gimel}_{\boldsymbol{b}}\right)If\left|\widetilde{\gimel}_{\boldsymbol{z}} - \widetilde{\gimel}_{\boldsymbol{y}}\right| \leq \left|\widetilde{\gimel}_{\boldsymbol{a}} - \widetilde{\gimel}_{\boldsymbol{b}}\right|$.

Theorem 4 *For the collection of CQ-ROFNs* $\widetilde{\gimel}_z = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2, \ldots, p$, *the final value of Eq.* (43) *is also a CQ-ROFN, such as*:

$$CQ-ROFPYOG\left(\widetilde{\gimel}_1, \widetilde{\gimel}_2, \ldots, \widetilde{\gimel}_{\boldsymbol{p}}\right) = \left(\begin{array}{c} \left(\begin{array}{c} \left(1-\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(1-\overline{\overline{\Delta}}_{\widetilde{rp-o(z)}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(1-\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(1-\overline{\overline{\Delta}}_{\widetilde{ip-o(z)}}^{\Lambda}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{array}\right), \\ \left(\begin{array}{c} \left(\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(\overline{\overline{\Xi}}_{\widetilde{rp-o(z)}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}}, \\ \left(\min\left(1, \left(\sum_{z=1}^{p} \overline{\overline{\aleph}}_z \left(\overline{\overline{\Xi}}_{\widetilde{ip-o(z)}}^{\Lambda\Pi}\right)\right)^{\frac{1}{\Pi}}\right)\right)^{\frac{1}{\Lambda}} \end{array}\right) \end{array}\right). \tag{44}$$

Obviously, Eq. (44) is the modified version of the power, Yager, and power Yager aggregation operators for FSs, IFSs, PFSs, Q-ROFSs, CFSs, CIFSs, CPFSs, and CQ-ROFSs. Furthermore, we discuss their valuable and feasible properties such as idempotency, monotonicity, and boundedness.

Property 10 (idempotency)*:* When $\widetilde{\gimel}_z = \widetilde{\gimel} = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp}}, \overline{\overline{\Delta}}_{ip}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp}}, \overline{\overline{\Xi}}_{\widetilde{ip}}\right)\right)$, $z = 1, 2, \ldots, p$, then

$$CQ-ROFPYOG\left(\widetilde{\gimel}_1, \widetilde{\gimel}_2, \ldots, \widetilde{\gimel}_{\boldsymbol{p}}\right) = \widetilde{\gimel}. \tag{45}$$

Property 11 (monotonicity)**:** When $\widetilde{\gimel}_z = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right) \leq \widetilde{\gimel}_z'' = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}'', \overline{\overline{\Delta}}_{ip-z}''\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}'', \overline{\overline{\Xi}}_{\widetilde{ip-z}}''\right)\right)$, $z = 1, 2, \ldots, p$, then

$$\begin{aligned} &CQ-ROFPYOG\left(\widetilde{\gimel}_1, \widetilde{\gimel}_2, \ldots, \widetilde{\gimel}_{\boldsymbol{p}}\right) \\ &\leq CQ-ROFPYOG\left(\widetilde{\gimel}_1'', \widetilde{\gimel}_2'', \ldots, \widetilde{\gimel}_{\boldsymbol{p}}''\right). \end{aligned} \tag{46}$$

Property 12 (boundedness)**:** When $\widetilde{\gimel}_z^- = \left(\left(\min_z \overline{\overline{\Delta}}_{\widetilde{rp-z}}, \min_z \overline{\overline{\Delta}}_{ip-z}\right), \left(\max_z \overline{\overline{\Xi}}_{\widetilde{rp-z}}, \max_z \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$ and $\widetilde{\gimel}_z^+ = \left(\left(\max_z \overline{\overline{\Delta}}_{\widetilde{rp-z}}, \max_z \overline{\overline{\Delta}}_{ip-z}\right), \left(\min_z \overline{\overline{\Xi}}_{\widetilde{rp-z}}, \min_z \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right)$, $z = 1, 2, \ldots, p$, then

$$\widetilde{\gimel}_z^- \leq CQ-ROFPYOG\left(\widetilde{\gimel}_1, \widetilde{\gimel}_2, \ldots, \widetilde{\gimel}_{\boldsymbol{p}}\right) \leq \widetilde{\gimel}_z^+. \tag{47}$$

MADM methods based on the proposed operators

To get the best choice from the family of preferences, we need to develop the MADM methods based on the proposed operators. In this section, we propose the MADM methods with CQ-ROFNs based on the presented operators.

For a MADM problem, $\widetilde{\gimel}_1, \widetilde{\gimel}_2, \ldots, \widetilde{\gimel}_{\boldsymbol{p}}$ are the finite family of alternatives, and for every alternative, it is evaluated by the finite attributes such as $C_1, C_2, \ldots, C_n$, and the evaluation value $\widetilde{\gimel}_{\boldsymbol{zj}}$ for alternative $\widetilde{\gimel}_{\boldsymbol{z}}$ under attribute C_j is expressed by a CQ-ROFN as $\widetilde{\gimel}_{zj} = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-zj}}, \overline{\overline{\Delta}}_{ip-zj}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-zj}}, \overline{\overline{\Xi}}_{\widetilde{ip-zj}}\right)\right)$, $z = 1, 2, \ldots, p$; $j = 1, 2, \ldots, n$. Where $\left(\overline{\overline{\Delta}}_{\widetilde{rp-zj}}, \overline{\overline{\Delta}}_{ip-zj}\right)$ and $\left(\overline{\overline{\Xi}}_{\widetilde{rp-zj}}, \overline{\overline{\Xi}}_{\widetilde{ip-zj}}\right)$ express a truth and falsity grades with $0 \leq \overline{\overline{\Delta}}_{\widetilde{rp-zj}}^{\Lambda} + \overline{\overline{\Xi}}_{\widetilde{rp-zj}}^{\Lambda} \leq 1$ and $0 \leq \overline{\overline{\Delta}}_{ip-zj}^{\Lambda} + \overline{\overline{\Xi}}_{\widetilde{ip-zj}}^{\Lambda} \leq 1$, $\Lambda \geq 1$. In the following, we give the decision-making steps based on the proposed operators:

Step 1: Normalize the decision matrix

First, we need to obtain the decision matrix in which the element is expressed by a CQ-ROFN as $\widetilde{\gimel}_{zj} = \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-zj}}, \overline{\overline{\Delta}}_{ip-zj}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-zj}}, \overline{\overline{\Xi}}_{\widetilde{ip-zj}}\right)\right)$, $z = 1, 2, \ldots, p$;

Table 3 The decision matrix in CQ-ROFNs

	C_1	C_2	C_3	C_4
$\widetilde{\beth}_1$	((0.8, 0.8), (0.3, 0.6))	((0.81, 0.81), (0.31, 0.61))	((0.82, 0.82), (0.32, 0.62))	((0.83, 0.83), (0.33, 0.63))
$\widetilde{\beth}_2$	((0.7, 0.6), (0.5, 0.6))	((0.71, 0.61), (0.51, 0.61))	((0.72, 0.62), (0.52, 0.62))	((0.73, 0.63), (0.53, 0.63))
$\widetilde{\beth}_3$	((0.8, 0.2), (0.8, 0.7))	((0.81, 0.21), (0.81, 0.71))	((0.82, 0.22), (0.82, 0.72))	((0.83, 0.23), (0.83, 0.73))
$\widetilde{\beth}_4$	((0.8, 0.7), (0.8, 0.8))	((0.81, 0.71), (0.81, 0.81))	((0.82, 0.72), (0.82, 0.82))	((0.83, 0.73), (0.83, 0.83))
$\widetilde{\beth}_5$	((0.7, 0.5), (0.4, 0.5))	((0.71, 0.51), (0.41, 0.51))	((0.72, 0.52), (0.42, 0.52))	((0.73, 0.53), (0.43, 0.53))

Table 4 The aggregated results by the proposed operators

Comprehensive values for the different alternatives	$CQ-ROFPYA$	$CQ-ROFPYG$
$\widetilde{\beth}_1$	((0.8164, 0.8164), (0.000001, 0.0008))	((0.0139, 0.0139), (0.3186, 0.6169))
$\widetilde{\beth}_2$	((0.7166, 0.6169), (0.0001, 0.0008))	((0.036, 0.0008), (0.5173, 0.6169))
$\widetilde{\beth}_3$	((0.8164, 0.2199), (0.0139, 0.0036))	((0.0139, 0.0002), (0.8164, 0.7166))
$\widetilde{\beth}_4$	((0.8164, 0.2199), (0.0139, 0.0139))	((0.0139, 0.000002), (0.8164, 0.8164))
$\widetilde{\beth}_5$	((0.7166, 0.5173), (0.0.00004, 0.0001))	((0.0036, 0.0001), (0.4178, 0.5173))

$j = 1, 2, \ldots, n,$ then we need to convert the information in cost type to benefit type by

$$N = \begin{cases} \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-zj}}, \overline{\overline{\Delta}}_{ip-zj}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-zj}}, \overline{\overline{\Xi}}_{\widetilde{ip-zj}}\right)\right)_{p\times n} & \text{for benefit type} \\ \left(\left(\overline{\overline{\Xi}}_{\widetilde{rp-zj}}, \overline{\overline{\Xi}}_{\widetilde{ip-zj}}\right), \left(\overline{\overline{\Delta}}_{\widetilde{rp-zj}}, \overline{\overline{\Delta}}_{ip-zj}\right)\right)_{p\times n} & \text{for cost type} \end{cases}.$$

Step 2: Aggregate all attribute values for each alternative based on the CQ-ROFPYA operator and CQ-ROFPYG operator, and get the comprehensive value $\widetilde{\beth}_z$ by

$$\begin{aligned}\widetilde{\beth}_z &= \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right) \\ &= CQ-ROFPYA\left(\widetilde{\beth}_{z1}, \widetilde{\beth}_{z2}, \ldots, \widetilde{\beth}_{zn}\right),\end{aligned}$$

and

$$\begin{aligned}\widetilde{\beth}_z &= \left(\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}, \overline{\overline{\Delta}}_{ip-z}\right), \left(\overline{\overline{\Xi}}_{\widetilde{rp-z}}, \overline{\overline{\Xi}}_{\widetilde{ip-z}}\right)\right) \\ &= CQ-ROFPYG\left(\widetilde{\beth}_{z1}, \widetilde{\beth}_{z2}, \ldots, \widetilde{\beth}_{zn}\right).\end{aligned}$$

Step 3: Calculate the score value for comprehensive value $\widetilde{\beth}_z$ by

$$\nabla_{SV}\left(\widetilde{\beth}_z\right) = \frac{1}{2}\left(\overline{\overline{\Delta}}_{\widetilde{rp-z}}^{\Lambda} + \overline{\overline{\Delta}}_{ip-z}^{\Lambda} - \overline{\overline{\Xi}}_{\widetilde{rp-z}}^{\Lambda} - \overline{\overline{\Xi}}_{\widetilde{ip-z}}^{\Lambda}\right).$$

Step 4: Get the ranking results according to the score values and give the best choice from the collection of finite alternatives.

In the following, we give some examples to show the decision steps.

Example 1 A university wants to select a new faculty member for Fuzzy mathematics in the department of mathematics and statistics, university of ABC, Islamabad, Pakistan. For the position of fuzzy mathematics, five candidates are applied who are expressed by $\widetilde{\beth}_1$, $\widetilde{\beth}_2$, $\widetilde{\beth}_3$, $\widetilde{\beth}_4$ and $\widetilde{\beth}_5$ and are regarded as the family of alternatives. Further, they are evaluated by the following features C_1: fifteen-year experience, C_2: fifteen impact factor publications, C_3: age of the candidate, C_4: and personality and extra skills of the candidate. Based on the above MADM methods, we can solve this problem, and the decision steps are shown as follows.

Table 5 The score values for the different alternatives

Score Values	$CQ-ROFPYA$	$CQ-ROFPYG$
$\widetilde{\beth}_1$	0.816	− 0.4539
$\widetilde{\beth}_2$	0.6663	− 0.5649
$\widetilde{\beth}_3$	0.5095	− 0.7596
$\widetilde{\beth}_4$	0.7527	− 0.8077
$\widetilde{\beth}_5$	0.6169	− 0.4656

Step 1: Normalize the decision matrix.

First, we get the decision matrix in which the element is expressed by the CQ-ROFNs shown in Table 3, then we normalize them. Because all the evaluation values are benefit type, they do not need to be normalized.

Step 2: Aggregate all attribute values of each alternative by the CQ-ROFPYA operator and CQ-ROFPYG operator, and the comprehensive values are shown in Table 4.

Step 3: Obtain the score values of aggregated values, which are shown in Table 5.

Step 4: Get the ranking results according to the score values which are shown in Table 6.

Table 6 The ranking results

Methods	Rankingvalues
CQ − ROFPYAoperator	$\widetilde{\beth}_1 > \widetilde{\beth}_4 > \widetilde{\beth}_2 > \widetilde{\beth}_5 > \widetilde{\beth}_3$
CQ − ROFPYGoperator	$\widetilde{\beth}_1 > \widetilde{\beth}_5 > \widetilde{\beth}_2 > \widetilde{\beth}_3 > \widetilde{\beth}_4$

So, we get the same best choice is $\widetilde{\beth}_1$ according to the CQ-ROFPYA operator and the CQ-ROFPYG operator. Further, we use the proposed method to solve some decision problems with different uncertain information.

Example 2. Considering a decision problem with the CPF information and the decision matrix is shown in Table 7.

Further, we aggregate all attribute values of each alternative by the CQ-ROFPYA operator and CQ-ROFPYG operator, and the comprehensive values are shown in Table 8.

Obtain the score values of aggregated values, which are shown in Table 9.

Get the ranking results according to the score values which are shown in Table 10.

We get the same ranking result by two operators, of course, the best choice is $\widetilde{\beth}_4$.

Example 3 Considering a decision problem with the CIF information and the decision matrix is shown in Table 11.

We aggregate all attribute values of each alternative by the CQ-ROFPYA operator and CQ-ROFPYG operator, and the comprehensive values are shown in Table 12.

Obtain the score values of aggregated values, which are shown in Table 13.

Table 9 The score values with CPFNs

Score values	CQ − ROFPYA	CQ − ROFPYG
$\widetilde{\beth}_1$	0.4881	− 0.3092
$\widetilde{\beth}_2$	0.4382	− 0.3539
$\widetilde{\beth}_3$	0.6947	− 0.005
$\widetilde{\beth}_4$	0.728	0.0948
$\widetilde{\beth}_5$	0.4988	− 0.2436

Table 10 The ranking results with CPFNs

Methods	Rankingvalues
CQ − ROFPYAoperator	$\widetilde{\beth}_4 > \widetilde{\beth}_3 > \widetilde{\beth}_5 > \widetilde{\beth}_1 > \widetilde{\beth}_2$
CQ − ROFPYGoperator	$\widetilde{\beth}_4 > \widetilde{\beth}_3 > \widetilde{\beth}_5 > \widetilde{\beth}_1 > \widetilde{\beth}_2$

Get the ranking results according to the score values which are shown in Table 14.

We get the same ranking result by two operators, of course, the best choice is $\widetilde{\beth}_5$.

Examples 2 and 3 can show the effectiveness of the proposed methods.

Further, using the information in Tables 3, 7, and 11, we compare the proposed methods with some famous existing methods.

Comparative analysis

To further show the advantages of the proposed methods in this manuscript, we compare them with some existing

Table 7 The decision matrix with CPFNs

	C_1	C_2	C_3	C_4
$\widetilde{\beth}_1$	((0.6, 0.7), (0.5, 0.6))	((0.61, 0.71), (0.51, 0.61))	((0.62, 0.72), (0.52, 0.62))	((0.63, 0.73), (0.53, 0.63))
$\widetilde{\beth}_2$	((0.6, 0.6), (0.6, 0.5))	((0.61, 0.61), (0.61, 0.51))	((0.62, 0.62), (0.62, 0.52))	((0.63, 0.63), (0.63, 0.53))
$\widetilde{\beth}_3$	((0.8, 0.7), (0.3, 0.4))	((0.81, 0.71), (0.31, 0.41))	((0.82, 0.72), (0.32, 0.42))	((0.83, 0.73), (0.33, 0.43))
$\widetilde{\beth}_4$	((0.7, 0.8), (0.2, 0.3))	((0.71, 0.81), (0.21, 0.31))	((0.72, 0.82), (0.22, 0.32))	((0.73, 0.83), (0.23, 0.33))
$\widetilde{\beth}_5$	((0.7, 0.5), (0.4, 0.5))	((0.71, 0.51), (0.41, 0.51))	((0.72, 0.52), (0.42, 0.52))	((0.73, 0.53), (0.43, 0.53))

Table 8 The aggregated values with CPFNs

comprehensive values for the different alternatives	$CQ - ROFPYA$	$CQ - ROFPYG$
$\widetilde{\beth}_1$	((0.6153, 0.7153), (0.1428, 0.2115))	((0.2115, 0.3008), (0.5154, 0.6153))
$\widetilde{\beth}_2$	((0.6153, 0.6153), (0.2115, 0.1428))	((0.2115, 0.2115), (0.6153, 0.5154))
$\widetilde{\beth}_3$	((0.8152, 0.7153), (0.051, 0.0902))	((0.4202, 0.3008), (0.3156, 0.4155))
$\widetilde{\beth}_4$	((0.7153, 0.8152), (0.0234, 0.051))	((0.3008, 0.4202), (0.2159, 0.3156))
$\widetilde{\beth}_5$	((0.7153, 0.5154), (0.0902, 0.1428))	((0.3008, 0.1428), (0.4155, 0.5154))

Table 11 The decision matrix with CIFNs

	C_1	C_2	C_3	C_4
$\widetilde{\beth}_1$	((0.5, 0.4), (0.1, 0.2))	((0.51, 0.41), (0.11, 0.21))	((0.52, 0.42), (0.12, 0.22))	((0.53, 0.43), (0.13, 0.23))
$\widetilde{\beth}_2$	((0.6, 0.2), (0.1, 0.2))	((0.61, 0.21), (0.11, 0.21))	((0.62, 0.22), (0.12, 0.22))	((0.63, 0.23), (0.13, 0.23))
$\widetilde{\beth}_3$	((0.7, 0.1), (0.1, 0.2))	((0.71, 0.11), (0.11, 0.21))	((0.72, 0.12), (0.12, 0.22))	((0.73, 0.13), (0.13, 0.23))
$\widetilde{\beth}_4$	((0.6, 0.6), (0.2, 0.2))	((0.61, 0.61), (0.21, 0.21))	((0.62, 0.62), (0.22, 0.22))	((0.63, 0.63), (0.23, 0.23))
$\widetilde{\beth}_5$	((0.7, 0.5), (0.1, 0.3))	((0.71, 0.51), (0.11, 0.31))	((0.72, 0.52), (0.12, 0.32))	((0.73, 0.53), (0.13, 0.33))

Table 12 The aggregated matrix with CIFNs

comprehensive values for the different alternatives	$CQ-ROFPYA$	$CQ-ROFPYG$
$\widetilde{\beth}_1$	((0.5151, 0.5142), (0.1149, 0.2149))	((0.5149, 0.4149), (0.1155, 0.2153))
$\widetilde{\beth}_2$	((0.6151, 0.2153), (0.1149, 0.2149))	((0.6148, 0.2149), (0.1155, 0.2153))
$\widetilde{\beth}_3$	((0.7151, 0.1155), (0.1149, 0.2149))	((0.7148, 0.1149), (0.1155, 0.2153))
$\widetilde{\beth}_4$	((0.6151, 0.6151), (0.2149, 0.2149))	((0.6148, 0.6148), (0.2153, 0.2153))
$\widetilde{\beth}_5$	((0.7151, 0.5151), (0.1149, 0.3149))	((0.7148, 0.5149), (0.1155, 0.3152))

Table 13 The score values with CIFNs

Score Values	$CQ-ROFPYA$	$CQ-ROFPYG$
$\widetilde{\beth}_1$	0.3002	0.2995
$\widetilde{\beth}_2$	0.2503	0.2495
$\widetilde{\beth}_3$	0.2504	0.2494
$\widetilde{\beth}_4$	0.4002	0.3995
$\widetilde{\beth}_5$	0.4002	0.3995

Table 14 The ranking results with CIFNs

Methods	Rankingvalues
CQ − ROFPYAoperator	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_3 > \widetilde{\beth}_2$
CQ − ROFPYGoperator	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_3 > \widetilde{\beth}_2$

method based on some operators [12–25], including the averaging aggregation operators (AOs) for IFS from Xu [12], the geometric AOs for IFS from Xu and Yager [13], PA operators for IFS from Xu [14], the geometric AOs for PFS by Rahman et al. [15]. The PA operators for PFS by Wei and Lu [16], the Yager aggregation operators for PFS by Shahzadi and Akram [17], the simple AOs for Q-ROFS by Liu and Wang [18], the geometric AOs for Q-ROFSs from Riaz et al. [19], the Yager aggregation operators for Q-ROFS by Akram and Shahzadi [20], AOs for CIFS by Garg and Rani [21], geometric AOs for CIFSs by Garg and Rani [22], the power AOs for CIFS by Rani and Garg [23], the Yager AOs for CPFSs by Akram et al. [24], and the PA operators for CQ-ROFS by Garg et al. [25]. The comparative analysis based on the information in Table 3 is shown in Table 15.

From Table 15, we can know the best choice is $\widetilde{\beth}_1$ based on the CQ-ROFPYA operator, the CQ-ROFPYG operator, and the methods from Garg et al. [25]. They get the same best result. However, the operators in [12–24] can only process the uncertain information from FSs, IFSs, CIFSs, PFSs, Q-ROFS, and CPFSs, and they cannot deal with the information in CQ-ROFS, our methods can deal with all mentioned information types. In addition, compared with the methods in [25], our methods used the operational laws based on Yager t-norm and t-conorm, and [25] used the traditional operational laws. [25] is a special case of our methods. Our methods are more general than the methods in [12–25].

Further, the comparative analysis based on the information on CPFSs in Table 7 is shown in Table 16.

From Table 16, we can know that the best choice is $\widetilde{\beth}_4$ based on the CQ-ROFPYA operator, the CQ-ROFPYG operator, Yager AOs for CPFSs [24], and the PA operators for CQ-ROFS [25]. Of course, they also produce the same ranking result, this can explain the effectiveness of the proposed methods. However, the operators in [12–23] can only process the uncertain information from FSs, IFSs, CIFSs, PFSs, and Q-ROFSs, and they cannot deal with the information in CPFSs and CQ-ROFS, our methods can deal with all mentioned information types. In addition, compared with the methods in [24], our methods used Power operators, and [25] used traditional operators and CPFs. Our methods are more general than the methods in [12–25].

Moreover, the comparative analysis based on the information on CIFSs in Table 11 is shown in Table 17.

Table 15 Comparative analysis based on the information in Table 3

Methods	Scorevalues	Rankingvalues
Xu [12]	Restricted/Limited	Restricted/Limited
Xu and Yager [13]	Restricted/Limited	Restricted/Limited
Xu [14]	Restricted/Limited	Restricted/Limited
Rahman et al. [15]	Restricted/Limited	Restricted/Limited
Wei and Lu [16]	Restricted/Limited	Restricted/Limited
Shahzadi and Akram [17]	Restricted/Limited	Restricted/Limited
Liu and Wang [18]	Restricted/Limited	Restricted/Limited
Riaz et al. [19]	Restricted/Limited	Restricted/Limited
Akram and Shahzadi [20]	Restricted/Limited	Restricted/Limited
Garg and Rani [21]	Restricted/Limited	Restricted/Limited
Garg and Rani [22]	Restricted/Limited	Restricted/Limited
Rani and Garg [23]	Restricted/Limited	Restricted/Limited
Akram et al. [24]	Restricted/Limited	Restricted/Limited
Garg et al. [25]	0.351,0.101,− 0.2483,− 0.0491,0.1511	$\tilde{\beth}_1 > \tilde{\beth}_5 > \tilde{\beth}_2 > \tilde{\beth}_4 > \tilde{\beth}_3$
CQ-ROFPYA operator	0.816,0.6663,0.5095,0.7527,0.6169	$\tilde{\beth}_1 > \tilde{\beth}_4 > \tilde{\beth}_2 > \tilde{\beth}_5 > \tilde{\beth}_3$
CQ-ROFPYG operator	− 0.4539,− 0.5649,− 0.7596,− 0.8077,− 0.4656	$\tilde{\beth}_1 > \tilde{\beth}_5 > \tilde{\beth}_2 > \tilde{\beth}_3 > \tilde{\beth}_4$

Table 16 Comparative analysis based on the information in Table 7

Methods	Score values	Ranking values
Xu [12]	Restricted/Limited	Restricted/Limited
Xu and Yager [13]	Restricted/Limited	Restricted/Limited
Xu [14]	Restricted/Limited	Restricted/Limited
Rahman et al. [15]	Restricted/Limited	Restricted/Limited
Wei and Lu [16]	Restricted/Limited	Restricted/Limited
Shahzadi and Akram [17]	Restricted/Limited	Restricted/Limited
Liu and Wang [18]	Restricted/Limited	Restricted/Limited
Riaz et al. [19]	Restricted/Limited	Restricted/Limited
Akram and Shahzadi [20]	Restricted/Limited	Restricted/Limited
Garg and Rani [21]	Restricted/Limited	Restricted/Limited
Garg and Rani [22]	Restricted/Limited	Restricted/Limited
Rani and Garg [23]	Restricted/Limited	Restricted/Limited
Akram et al. [24]	0.4862,0.4362,0.6909,0.7236,0.4959	$\tilde{\beth}_4 > \tilde{\beth}_3 > \tilde{\beth}_5 > \tilde{\beth}_1 > \tilde{\beth}_2$
Garg et al. [25]	0.1004,0.0503,0.4005,0.5006,0.1504	$\tilde{\beth}_4 > \tilde{\beth}_3 > \tilde{\beth}_5 > \tilde{\beth}_1 > \tilde{\beth}_2$
CQ-ROFPYA operator	0.4881,0.4382,0.6947,0.728,0.4988	$\tilde{\beth}_4 > \tilde{\beth}_3 > \tilde{\beth}_5 > \tilde{\beth}_1 > \tilde{\beth}_2$
CQ-ROFPYG operator	− 0.3092,− 0.3539,− 0.005,0.0948,− 0.2436	$\tilde{\beth}_4 > \tilde{\beth}_3 > \tilde{\beth}_5 > \tilde{\beth}_1 > \tilde{\beth}_2$

From Table 17, we can know that the best choice is $\tilde{\beth}_4$ based on the CQ-ROFPYA operator, the CQ-ROFPYG operator, AOs for CIFS [21], geometric AOs for CIFSs [22], the power AOs for CIFS [23] Yager AOs for CPFSs [24], and the PA operators for CQ-ROFS [25]. Our method in the CQ-ROFPYA operator produced the same ranking result as the operators in [21, 23], and [24], while our method in the CQ-ROFPYG operator produced the same ranking result as the operators in [22] because they used the geometric AOs. Compared with [21], the operators in [21] only can deal with CIFS, and only used the traditional AOs; Compared with [22], the operators in [22] only can deal with CIFS, and only used the traditional geometric AOs; Compared with [23], the operators in [23] only can deal with CIFS, and only used the traditional operational laws. Our methods are more general than theirs.

Table 17 Comparative analysis based on the information in Table 11

Methods	Score values	Ranking values
Xu [12]	Restricted/Limited	Restricted/Limited
Xu and Yager [13]	Restricted/Limited	Restricted/Limited
Xu [14]	Restricted/Limited	Restricted/Limited
Rahman et al. [15]	Restricted/Limited	Restricted/Limited
Wei and Lu [16]	Restricted/Limited	Restricted/Limited
Shahzadi and Akram [17]	Restricted/Limited	Restricted/Limited
Liu and Wang [18]	Restricted/Limited	Restricted/Limited
Riaz et al. [19]	Restricted/Limited	Restricted/Limited
Akram and Shahzadi [20]	Restricted/Limited	Restricted/Limited
Garg and Rani [21]	0.3004,0.2504,0.2504,0.4003,0.4004	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_3 > \widetilde{\beth}_2$
Garg and Rani [22]	0.2998,0.2498,0.2497,0.3999,0.3999	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_2 > \widetilde{\beth}_3$
Rani and Garg [23]	0.3005,0.2505,0.2506,0.4005,0.4005	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_3 > \widetilde{\beth}_2$
Akram et al. [24]	0.3002,0.2502,0.2503,0.4001,0.4001	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_3 > \widetilde{\beth}_2$
Garg et al. [25]	0.3005,0.2505,0.2506,0.4005,0.4005	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_3 > \widetilde{\beth}_2$
CQ-ROFPYA operator	0.3002,0.2503,0.2504,0.4002,0.4002	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_3 > \widetilde{\beth}_2$
CQ-ROFPYG operator	0.2995,0.2495,0.2494,0.3995,0.3995	$\widetilde{\beth}_5 > \widetilde{\beth}_4 > \widetilde{\beth}_1 > \widetilde{\beth}_2 > \widetilde{\beth}_3$

Our proposed operators are based on CQ-ROFS, PA, and the Yager operational laws, and they are novel and interesting to cope with unreliable and awkward information in decision-making problems.

In addition, no special software and hardware are required due to the small amount of computation, and Excel can meet this requirement.

Conclusion

According to the PA operators and Yager t-norm and t-conorm under the consideration of CQ-ROF information, we derived the following ideas:

1. We developed Yager operational laws for CQ-ROF values.
2. We proposed the famous CQ-ROFPYA, CQ-ROFPYOA, CQ-ROFPYG, and CQ-ROFPYOG operators.
3. We discussed some properties such as idempotency, monotonicity, and boundedness.
4. We proposed a MADM methods based on the proposed operators for CQ-ROF information.
5. We compared the proposed operators with some existing operators to show the effectiveness of the proposed methods.

In the future, we study the fuzzy multi-criteria decision-making problems based on the assessment of hydroponic vertical farming [33], investigation of the fuzzy Dombi EDAS technique [34], analysis of type-2 Gaussian fuzzy information [35], analysis of fuzzy secure filtering [36], interval-type-2 fuzzy non-homogeneous information [37], and fuzzy fault detection under the presence of the Markov jump systems [38], then we try to utilize them to artificial intelligence, machine learning, game theory, clustering analysis, pattern recognition, and medical diagnosis.

Availability of data and materials Not applicable.

Declarations

Conflict of interest The authors declare they have no conflicts of interest.

Ethical approval and consent to participate We declare that we do have no commercial or associative interests that represent a conflict of interests in connection with this manuscript. There are no professional or other personal interests that can inappropriately influence our submitted work.

Consent for publication All authors agree to publish this paper.

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