Hesitant T-spherical Dombi fuzzy aggregation operators and their applications in multiple criteria group decision-making

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Abstract

A hesitant fuzzy (HF) set is an extension of the fuzzy sets and a T-spherical fuzzy set (T-SFS) is a generalization of the spherical fuzzy set (SFS). HF set has a significant role for modelling disagreements of the decision-makers over membership degree of an element. Also, T-SFS is quite effective in the modelling of the uncertainty for decision-making (DM) problems. In this paper, we define the concept of hesitant T-spherical fuzzy (HT-SF) set (HT-SFS) by combining concepts of HF set and T-SFS, and present some set-theoretical operations of HT-SFSs. We also develop the Dombi operations on HT-SFSs. We present some aggregation operators based on Dombi operators, including hesitant T-spherical Dombi fuzzy weighted arithmetic averaging operator, hesitant T-spherical Dombi fuzzy weighted geometric averaging operator, hesitant T-spherical Dombi fuzzy ordered weighted arithmetic averaging operator, and hesitant T-spherical Dombi fuzzy ordered weighted geometric averaging operator, and investigate some properties of them. In addition, we give a multi-criteria group decision-making method and algorithm of the proposed method under the hesitant T-spherical fuzzy environment. To show the process of proposed method, we present an example related to the selection of the most suitable person for the assistant professorship position in a university. Besides this, we present a comparative analysis with existing operators to reveal the advantages and authenticity of our technique.

Keywords Hesitant fuzzy set · Spherical fuzzy set · T-spherical fuzzy set · Dombi operators · Decision-making

Introduction

The notion of fuzzy set (FS) was defined by Zadeh [1] in 1965 to model some problems involving uncertainty. The FS has found application in many different fields such as computer science, medical science, clustering, robotic, optimization, and data mining. For example, Gadekallu and Gao [2] proposed a model using an approach based on rough sets for reducing the attributes and fuzzy logic system for classification for prediction of the Heart and Diabetes diseases; Sankaran et al. [3] proposed a method to provide a multi-path and multi-constraint Qualify of Service based on Reliable Fuzzy and Heuristic Concurrent Ant Colony Optimization. These studies are two example made in 2021 related to application of fuzzy logic and sets.

Zadeh [1] characterize an FS by the membership function of which codomain is interval $[0, 1]$. In an FS, if the membership degree (MD) of an element is μ , then its non-membership degree (NMD) is $1 - \mu$. Namely, in an FS, hesitation degree of an element is accepted as “0”. However, this perspective has some constraints. To overcome with this constraints, Atanassov [4] introduced the concept of intuitionistic FS (IFS) as a generalization of FSs. An IFS is defined by assigning two values from the range $[0, 1]$, named MD μ and NMD ν , under the condition $\mu + \nu \leq 1$ for all elements of the working universe. However, this set is not useful when $\mu + \nu > 1$. Therefore, Yager [5,6] defined the Pythagorean FS (PyFS) as an extension of IFS under condition $\mu^2 + \nu^2 \leq 1$. Another extension of IFS is Picture FS (PFS) defined by Cuong [7,8]. PFS is a useful tool for representing human opinion, because a PFS can model judgments about an object or idea using degrees of yes, abstention, no, and rejection. A PFS is identified three degrees of an element, called MD (μ), abstinence degree (AD) or neutral degree (γ), and NMD (ν) with the condition $0 \leq \mu + \gamma + \nu \leq 1$. Although PFS has wide applications in some field such as decision-making (DM) [9–15], similarity measure [16–20],

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correlation coefficient [21,22], and clustering [23,24], it is not sufficient in modelling some problems when $\mu + \gamma + \nu > 1$. For this reason, Gungogdu and Kahraman [25,26] inaugurated the design of spherical FS (SFS) which is an extension of PFS satisfying the condition $0 \leq \mu^2 + \gamma^2 + \nu^2 \leq 1$. They also studied on SFS operations and applications of this set in DM problems. Kahraman et al. [25] proposed a DM method by integrating the SFS and TOPSIS method, and gave an application of the proposed method in the selection of hospital location. Mahmood et al. [27] defined the T-spherical FS (T-SFS) as an extension of the SFS with condition $0 \leq \mu^q + \gamma^q + \nu^q \leq 1$ and gave some applications in medical diagnosis and decision-making problems of T-SFS and SFS. Ullah et al. [28] introduced the similarity measures for T-SFSs and presented an application in pattern recognition. Garg et al. [29] presented improved interactive aggregation operators for T-SFSs and studied on operational laws of these operators. Ullah et al. [30] described some ordered weighted geometric (OWG) and hybrid geometric (HG) operators, and gave a numerical example involving multi-attribute decision-making (MADM) problem. Ullah et al. [31] introduced the concept of interval-valued T-SFS (IVT-SFS) and basic operations of them. They also defined two aggregation operators including weighted averaging and weighted geometric operators for IVT-SFS, and presented an MCDM method. Liu et al. [32] pointed out some limitations in operational laws of SFS and T-SFS, and suggested some novel operational laws for SFS and T-SFS. They also introduced Power Muirhead Mean Operator for T-SFS by combining power average operator with Muirhead Mean operator and presented an MAGDM method based on proposed operators. Recently, T-SFS has gained attention of researchers working on MCDM methods, MCGDM methods, and aggregation operators. For example, divergence measure of T-SFSs [33], immediate probabilistic Interactive averaging aggregation operators of T-SFSs [34], T-SF soft sets and their aggregation operators [35], generalized T-SF weighted aggregation operators on neutrosophic sets [36], T-SF Einstein Hybrid Aggregation operators [37], correlation coefficients for T-SFSs [38], T-SF Hamacher aggregation operators [39], complex T-SF aggregation operators [40], and T-spherical Type-2 fuzzy sets [41] are some of them.

The hesitant FS (HFS) is another extension of the FS for modelling the problems in which decision-makers have different opinions about an alternative or element in considered universe. The HFS was defined by Torra and Narukawa in [42,43]. To explain the basic idea behind of the concept of the hesitant fuzzy set, we give an example: two decision-makers discuss the membership grade of an element to a set, and while one of them assigns membership grade 0.7 for the element, the other may assign 0.3. In such cases, making a common decision is difficult. In a such case, the HFS is a useful tool. Because of advantages of HFS, many

researchers have been developed multiple decision-making methods and they have presented their applications under HF environment [44–49]. Xia et al. [50] described some HF aggregation operators and developed a group decision-making method. Chen et al. [51] interpreted the idea of interval-valued hesitant fuzzy sets (IvHFSs) which is a generalization of HFS. Peng et al. [52] investigated the continuous HF aggregation operators with the aid of continuous OWA operator, and they defined the C-HFOWA operator and C-HFOWG operator with their essential properties. They also extended these operators interval-valued HFS. Mu et al. [53] introduced a novel aggregation principle for HF elements (HFE). Amin et al [54] defined some aggregation operators for triangular cubic linguistic hesitant fuzzy sets. Fahmi et al. [55] defined some new operation laws for trapezoidal cubic hesitant fuzzy (TrCHF) numbers and introduced some new aggregation operators. Jiang et al. [56] defined the concept of interval-valued dual HFS, and described aggregation operators under interval-valued dual HF environment based on Hamacher t-norm and t-conorm. Liu et al. [57] introduced the Dombi aggregation operators of interval-valued hesitant fuzzy set based on Dombi t-norm and t-conorm. Some studies related to aggregation operator of HFS, extension of HFS, and decision-making can be found [58–72].

It has been mentioned above that HFSs are an important set structure in modelling problems involving multiple decision-makers, in terms of revealing the ideas of decision-makers. In an HFS, the HFEs are subsets of the interval [0,1]. In other words, the elements of an HFE express their degree of membership. It is insufficient to express non-membership and neutral status. As mentioned above, TSFS is defined as a generalization of FS, IFS, PyFS, PFS, qROFS, and SFS, and finds application in decision-making problems. Some generalizations of HFSs are available in the literature. To use the advantages of HFSs and T-SFSs together, in this article, we define a novel concept called the hesitant T-spherical fuzzy set (HT-SFS) by combining concepts of HF and T-SFS. In HT-SFS, more than one T-spherical fuzzy value can be assigned to elements of the set containing the elements to be evaluated. T-SFS theory deals only one T-spherical fuzzy value for an element. Therefore, it does not suffice to model problems including disagreements of the opinion of decision-makers about an element or object. On the other hand, an HT-SFS can handle such situation. We also introduce some aggregation operators based on Dombi operators, including hesitant T-spherical Dombi fuzzy weighted arithmetic averaging (HTSDFWAA) operator, hesitant T-spherical Dombi fuzzy weighted geometric averaging (HTSDFWGA) operator, hesitant T-spherical Dombi fuzzy ordered weighted arithmetic averaging (HTSDFOWAA) operator, and hesitant T-spherical Dombi fuzzy ordered weighted geometric averaging (HTSDFOWGA) operator, and obtain some properties of them. Furthermore, we give a multi-criteria group

decision-making (MCGDM) method and algorithm of the proposed method under the HT-SF environment. To show the process of proposed method, we present an example related to the selection of most suitable person for the assistant professorship position in a university. Besides this, we have presented a comparison of the proposed methods with each other and a comparison table of the proposed clusters with other extensions of the FS.

Preliminaries

This section provides some basic definitions and operations that will be needed in the next sections. First, for the reader's convenience, a table of notations by chapter is given in Table 1.

Definition 1 [25] An SFS $\mathbb{S}$ on a universe $\mathfrak{X}$ is represented as follows:

$$\mathbb{S} = \{(x, s(x), i(x), d(x)) : x \in \mathfrak{X}\},$$

where $s(x), i(x), d(x) \in [0, 1]$, $0 \leq s^2(x) + i^2(x) + d^2(x) \leq 1$ for all $x \in \mathfrak{X}$. We consider the triplet (s, i, d) as SF number (SFN). Here, s , i , and d are the membership degree (MD), abstinence degree (AD), and non-membership degree (NMD) of $x \in \mathbb{S}$, respectively. Further $\pi_{\mathbb{S}}(x) = \sqrt{1 - (s^2(x) + i^2(x) + d^2(x))}$ is the hesitancy degree of x in $\mathbb{S}$.

Definition 2 [27] A T-SF Set (T-SFS) on $\mathfrak{X}$ is defined as

$$T = \{(x, s(x), i(x), d(x)) : x \in \mathfrak{X}\},$$

where $s, i, d : \mathfrak{X} \rightarrow [0, 1]$ are membership, abstinence, and non-membership functions with a condition that

$$0 \leq s^n(x) + i^n(x) + d^n(x) \leq 1.$$

The refusal degree of T-SFS is defined as

$$r(x) = \sqrt[n]{1 - (s^n(x) + i^n(x) + d^n(x))},$$

and the triplet (s, i, d) is called the T-SF number (T-SFN).

Definition 3 [42,43] Let $\mathfrak{X}$ be a fixed set, a hesitant fuzzy set (HFS) on $\mathfrak{X}$ is in terms of a function that applied to $\mathfrak{X}$ returns of $[0, 1]$. The mathematical symbol of HFS

$$A = \{< x, h_A(x) > : x \in \mathfrak{X}\},$$

where $h_A(x)$ is a set of some values in $[0, 1]$, denoting the possible membership degrees of the element $x \in \mathfrak{X}$ to the set A . $h = h_A(x)$ is called an HF element (HFE).

From now on, set of all T-spherical fuzzy numbers is denoted by Υ .

Hesitant T-spherical fuzzy sets

In this part, we define the concept of hesitant T-Spherical fuzzy sets and their set-theoretical operations.

Definition 4 Let $\mathfrak{X}$ be a nonempty set. A hesitant T-SFS (HT-SFS) over $\mathfrak{X}$ denoted by $\mathbb{T}_H$ is defined as follows:

$$\mathbb{T}_H = \{(x, \mathfrak{h}(x)) : \mathfrak{h}(x) \subseteq \Upsilon, x \in \mathfrak{X}\}.$$

Here, $\mathfrak{h}(x) = \mathfrak{h}$ is collection of T-SFNs and $\mathfrak{h}$ is called HT-SF element (HT-SFE). The number of elements of an HT-SFE is called length of HT-SFE $\mathfrak{h}$ and denoted by $\ell_{\mathfrak{h}}$.

In other words, an HT-SFS is collection of HT-SFEs.

Example 1 Let us consider a set $\mathfrak{X} = \{x_1, x_2, x_3, x_4\}$. Then, for $t = 3$, we can write an HT-SFS $\mathbb{T}$ as follows:

$$\mathbb{T} = \left\{ (x_1, \{(0.6, 0.8, 0.4), (0.4, 0.5, 0.9), (0.6, 0.2, 0.7)\}), \right. \\ (x_2, \{(0.3, 0.9, 0.5), (0.2, 0.4, 0.7)\}), \\ (x_3, \{(0.9, 0.3, 0.2), (0.6, 0.6, 0.6), (0.4, 0.3, 0.8), \\ \left. (0.5, 0.7, 0.6)\}), (x_4, \{(0.7, 0.2, 0.2)\}) \right\}.$$

Definition 5 Let $\mathfrak{h}$ be an HT-SFE. Then, score value of HT-SFE $\mathfrak{h}$ denoted by $SV(\mathfrak{h})$ is defined as

$$SV(\mathfrak{h}) = \frac{1}{\ell_{\mathfrak{h}}} \sum_{k=1}^{\ell_{\mathfrak{h}}} (s_k^n - d_k^n) \quad (1)$$

for some positive integer n . Here, $SV(\mathfrak{h}) \in [-1, 1]$.

Definition 6 Let $\mathfrak{h}$ be an HT-SFE. Then, accuracy value of HT-SFE $\mathfrak{h}$ denoted by $AV(\mathfrak{h})$ is defined as

$$AV(\mathfrak{h}) = \frac{1}{\ell_{\mathfrak{h}}} \sum_{k=1}^{\ell_{\mathfrak{h}}} (s_k^n + i_k^n + d_k^n)$$

for the positive integer n . Here, $AV(\mathfrak{h}) \in [0, 1]$.

Definition 7 Let $\mathfrak{h}_1$ and $\mathfrak{h}_2$ be two HT-SFEs, $SV(\mathfrak{h}_1)$ and $SV(\mathfrak{h}_2)$ are the score values of $\mathfrak{h}_1$ and $\mathfrak{h}_2$, respectively, and $AV(\mathfrak{h}_1)$ and $AV(\mathfrak{h}_2)$ are the accuracy values of $\mathfrak{h}_1$ and $\mathfrak{h}_2$, respectively. Then

1. If $SV(\mathfrak{h}_1) < SV(\mathfrak{h}_2)$ then $\mathfrak{h}_1 < \mathfrak{h}_2$
2. If $SV(\mathfrak{h}_1) > SV(\mathfrak{h}_2)$ then $\mathfrak{h}_1 > \mathfrak{h}_2$
3. If $SV(\mathfrak{h}_1) = SV(\mathfrak{h}_2)$, there are three cases

Table 1 Frequently used notations in “Preliminaries”, “Hesitant T-spherical fuzzy sets”, and “Hesitant T-spherical Dombi fuzzy aggregation operators”

FS	Fuzzy set
SFS	Spherical fuzzy set
SFN	Spherical fuzzy number
T-SFS	T-spherical fuzzy set
$\mathfrak{X}$	Initial universe
$s(x)$ (or s)	Membership degree (MD)
$i(x)$ (or i)	Abstinence degree (AD)
$d(x)$ (or d)	Non-membership degree (NMD)
HT-SFS	Hesitant T-spherical fuzzy set
HT-SFE	Hesitant T-spherical fuzzy element
SV(h)	Score value of hesitant T-spherical fuzzy element h
AV(h)	Accuracy value of hesitant T-spherical fuzzy element h
ℓ_h	Length of hesitant T-spherical fuzzy element h
h^-	Lower bound of hesitant T-spherical fuzzy element h
h^+	Upper bound of hesitant T-spherical fuzzy element h
$\triangleleft$	Hesitant T-spherical fuzzy subset
$\cup$	Union operation of HT-SFSs
$\cap$	Intersection operation of HT-SFSs
$\mathbb{T}^c$	Complement of HT-SFSs $\mathbb{T}$
$f \otimes g$	Dombi t-norm of f and g
$f \oplus g$	Dombi t-conorm of f and g
γ	Parameter in Dombi operator
$\omega = (w_1, w_2, \dots, w_m)$	Weight vector with m component
HTSDFWAA	Hesitant T-spherical Dombi fuzzy weighted arithmetic averaging
HTSDFWGA	Hesitant T-spherical Dombi fuzzy weighted geometric averaging
HTSDFOWAA	Hesitant T-spherical Dombi fuzzy ordered weighted arithmetic averaging
HTSDFOWGA	Hesitant T-spherical Dombi fuzzy ordered weighted geometric averaging

- (a) If $AV(h_1) < AV(h_2)$, then $h_1 < h_2$
 (b) If $AV(h_1) > AV(h_2)$, then $h_1 > h_2$
 (c) If $AV(h_1) = AV(h_2)$, then $h_1 = h_2$.

Example 2 Let us consider HT-SFEs $h_1 = \{(0.6, 0.8, 0.4), (0.4, 0.5, 0.9), (0.6, 0.2, 0.7)\}$ and $h_2 = \{(0.3, 0.9, 0.5), (0.2, 0.4, 0.7)\}$ of HT-SFS $\mathbb{T}$ given in Example 1, $n = 3$; then

$$\begin{aligned}
 SV(h_1) &= \frac{1}{\ell_{h_1}} \left[(0.6^3 - 0.4^3) + (0.4^3 - 0.9^3) \right. \\
 &\quad \left. + (0.6^3 - 0.7^3) \right] \\
 &= -0.2133, \\
 SV(h_2) &= \frac{1}{\ell_{h_2}} [(0.3^3 - 0.5^3) + (0.2^3 - 0.7^3)] \\
 &= -0.2165.
 \end{aligned}$$

Definition 8 Let $\mathbb{T}_1 = \{(x, h_1(x)) : x \in \mathfrak{X}\}$ and $\mathbb{T}_2 = \{(x, h_2(x)) : x \in \mathfrak{X}\}$ be two HT-SFSs over a common uni-

verse $\mathfrak{X}$. If, for all $x \in \mathfrak{X}$ $SV(h_1(x)) \leq SV(h_2(x))$, then it is said that $\mathbb{T}_1$ is an HT-SF subset of $\mathbb{T}_2$, and denoted by $\mathbb{T}_1 \triangleleft \mathbb{T}_2$.

Example 3 Let $\mathbb{T}_1$ and $\mathbb{T}_2$ be two HT-SFSs over $\mathfrak{X} = \{x_1, x_2, x_3\}$ for $n = 3$ given as follows:

$$\begin{aligned}
 \mathbb{T}_1 &= \left\{ (x_1, \{(0.5, 0.4, 0.3), (0.6, 0.5, 0.4), (0.7, 0.2, 0.6)\}), \right. \\
 &\quad (x_2, \{(0.6, 0.2, 0.4), (0.7, 0.1, 0.6), (0.5, 0.5, 0.2)\}), \\
 &\quad \left. (x_3, \{(0.9, 0.6, 0.1), (0.4, 0.4, 0.4), (0.6, 0.2, 0.5)\}) \right\} \\
 \mathbb{T}_2 &= \left\{ (x_1, \{(0.7, 0.6, 0.5), (0.7, 0.5, 0.5), (0.8, 0.3, 0.6)\}), \right. \\
 &\quad (x_2, \{(0.8, 0.3, 0.3), (0.6, 0.2, 0.2), (0.5, 0.7, 0.1)\}), \\
 &\quad \left. (x_3, \{(0.7, 0.3, 0.3), (0.9, 0.2, 0.2), (0.5, 0.1, 0.3)\}) \right\}.
 \end{aligned}$$

Then, using Eq. (1), for $x_i \in \mathfrak{X}$ ($i = 1, 2, 3$), SVs of HT-SFEs are obtained as follows:

From the table, it is clear that $\mathbb{T}_1 \triangleleft \mathbb{T}_2$.

	x_1	x_2	x_3
$SV(h_1)$	0.126	0.132	0.273
$SV(h_2)$	0.244	0.270	0.378

Set-theoretical operations of HT-SFSs

In this section union, intersection and complement of an HT-SFS are defined with their examples.

Definition 9 Let $h = \{(s_t, i_t, d_t) : 1 \leq t \leq \ell_h\}$ be a T-SFE over $\mathfrak{X}$. Then, lower and upper bounds of h are defined as follows:

$$h^- = \min_t (s_t^n - d_t^n)$$

$$h^+ = \max_t (s_t^n - d_t^n),$$

respectively.

The following example can be given to explain lower and upper bound of h .

Let $h = \{(0.5, 0.4, 0.3), (0.6, 0.5, 0.4), (0.7, 0.2, 0.5)\}$ be an HT-FS element, $n = 3$, $\ell_h = 3$

$$\begin{aligned} h^- &= \min\{(0.5^3 - 0.3^3), (0.6^3 - 0.4^3), (0.7^3 - 0.5^3)\} \\ &= \min\{0.116, 0.152, 0.218\} \\ &= 0.116, \end{aligned}$$

$$\begin{aligned} h^+ &= \max\{0.116, 0.152, 0.218\} \\ &= 0.218. \end{aligned}$$

Definition 10 Let $\mathbb{T}_1$ and $\mathbb{T}_2$ be two HT-SFSs over $\mathfrak{X}$ and let h_1 and h_2 be HT-SFEs of $\mathbb{T}_1$ and $\mathbb{T}_2$ for all $x \in \mathfrak{X}$. Then, based on HT-SFEs, set-theoretical operations between $\mathbb{T}_1$ and $\mathbb{T}_2$ are defined as follows:

1. Union:

$$\mathbb{T}_1 \cup \mathbb{T}_2 = \bigcup_{x \in \mathfrak{X}} \left\{ \left(x, \bigcup_{\substack{(s_1, i_1, d_1) \in h_1 \\ (s_2, i_2, d_2) \in h_2}} \{(s_k, i_k, d_k) : s_k^n - d_k^n = \max\{s_1^n - d_1^n, s_2^n - d_2^n\}, k = 1, 2\} \right) \right\}$$

2. Intersection:

$$\mathbb{T}_1 \cap \mathbb{T}_2 = \bigcup_{x \in \mathfrak{X}} \left\{ \left(x, \bigcup_{\substack{(s_1, i_1, d_1) \in h_1 \\ (s_2, i_2, d_2) \in h_2}} \{(s_k, i_k, d_k) : s_k^n - d_k^n = \min\{s_1^n - d_1^n, s_2^n - d_2^n\}, k = 1, 2\} \right) \right\}$$

3. Complement:

$$\mathbb{T}_1^c = \bigcup_{x \in \mathfrak{X}} \left\{ \left(x, \bigcup_{(s_1, i_1, d_1) \in h_1} \{(d_1, i_1, s_1)\} \right) \right\}.$$

Example 4 Let $\mathbb{T}_1$ and $\mathbb{T}_2$ be two HT-SFSs over $\mathfrak{X} = \{x_1, x_2, x_3\}$ for $n = 4$ given as follows:

$$\mathbb{T}_1 = \left\{ (x_1, \{(0.5, 0.7, 0.9), (0.3, 0.2, 0.7), (0.6, 0.5, 0.4)\}), (x_2, \{(0.4, 0.6, 0.3), (0.2, 0.5, 0.9)\}), (x_3, \{(0.8, 0.4, 0.3)\}) \right\}$$

$$\mathbb{T}_2 = \left\{ (x_1, \{(0.4, 0.8, 0.5), (0.5, 0.7, 0.6)\}), (x_2, \{(0.2, 0.7, 0.6), (0.4, 0.3, 0.8)\}), (x_3, \{(0.5, 0.4, 0.8), (0.6, 0.7, 0.2), (0.9, 0.2, 0.4)\}) \right\}.$$

Then, using Definition 10, union, intersection, and complement of HT-SFEs are $\mathbb{T}_1$ and $\mathbb{T}_2$ are obtained as follows:

$$\begin{aligned} \mathbb{T}_1 \cup \mathbb{T}_2 &= \left\{ (x_1, \{(0.4, 0.8, 0.5), (0.5, 0.7, 0.6), (0.6, 0.5, 0.4)\}), (x_2, \{(0.4, 0.6, 0.3), (0.2, 0.7, 0.6), (0.4, 0.3, 0.8)\}), (x_3, \{(0.8, 0.4, 0.3), (0.9, 0.2, 0.4)\}) \right\}, \end{aligned}$$

$$\begin{aligned} \mathbb{T}_1 \cap \mathbb{T}_2 &= \left\{ (x_1, \{(0.5, 0.7, 0.9), (0.3, 0.2, 0.7), (0.4, 0.8, 0.5), (0.5, 0.7, 0.6)\}), (x_2, \{(0.2, 0.7, 0.6), (0.4, 0.3, 0.8), (0.2, 0.5, 0.9)\}), (x_3, \{(0.5, 0.4, 0.8), (0.6, 0.7, 0.2), (0.8, 0.4, 0.3)\}) \right\}, \end{aligned}$$

and

$$\begin{aligned} \mathbb{T}_1^c &= \left\{ (x_1, \{(0.9, 0.7, 0.5), (0.7, 0.2, 0.3), (0.4, 0.5, 0.6)\}), (x_2, \{(0.3, 0.6, 0.4), (0.9, 0.5, 0.2)\}), (x_3, \{(0.3, 0.4, 0.8)\}) \right\}, \end{aligned}$$

respectively.

Hesitant T-spherical Dombi fuzzy aggregation operators

Aggregation operators are an important tool for obtaining a single value from many values. In this section, first, we define Dombi operators for two HT-SFEs based on Dombi t-norm and t-conorm. An HT-SFE is a collection of T-SFEs which have three components called MD, AD, and NMD. In summation ($\oplus$) operation between HT-SFEs, we use Dombi t-conorm for MDs, Dombi t-norm for AD, and Dombi t-norm for NMD. In product ($\otimes$) operation between HT-SFEs, we use Dombi t-conorm for MDs, Dombi t-norm for AD, and Dombi t-norm for NMD. Then, we define hesitant T-spherical Dombi fuzzy weighted arithmetic averaging operators and hesitant T-spherical Dombi fuzzy weighted geometric averaging operators as a generalization of the Dombi operators for two HT-SFEs. In these operations, only HT-SFEs are weighted and ignored the importance of the ordered position of HT-SFEs. This is a drawback, and to avoid this drawback, we introduce hesitant T-spherical Dombi fuzzy ordered weighted arithmetic (geometric) averaging operators. In these operators, we consider both the weights of the elements and the importance degrees of the hesitant fuzzy elements according to the score functions. Namely, First, we sort the HT-SFEs according to their score values using the score function we defined, then upon this ordering, we discard the weight vectors given at the beginning without changing the order. The basic idea related to the ordered weighted averaging (OWA) is presented in [73].

Dombi t-norm and t-conorm

Dombi product and Dombi sum which are specific types of triangular norms and conorms given in [74] as follows:

Definition 11 [74] Let f and g be two real numbers in the interval $[0, 1]$. Then, Dombi t-norm is given by

$$f \otimes g = \frac{1}{1 + \left(\left(\frac{1-f}{f} \right)^\gamma + \left(\frac{1-g}{g} \right)^\gamma \right)^{\frac{1}{\gamma}}}, \quad \gamma > 0.$$

Dombi t-conorm is given by

$$f \oplus g = 1 - \frac{1}{1 + \left(\left(\frac{1-f}{f} \right)^{-\gamma} + \left(\frac{1-g}{g} \right)^{-\gamma} \right)^{\frac{1}{\gamma}}}, \quad \gamma > 0,$$

respectively.

Dombi operations of HT-SFEs

In this subsection, we define some Dombi operations between HT-SFEs.

Definition 12 Let $h_1 = \{(s_{1t}, i_{1t}, d_{1t}) : 1 \leq t \leq \ell_{h_1}\}$ and $h_2 = \{(s_{2r}, i_{2r}, d_{2r}) : 1 \leq r \leq \ell_{h_2}\}$ be two HT-SFEs and $\gamma > 0$, and the Dombi operations for HT-SF elements are defined as follows:

- $$h_1 \oplus h_2 = \bigcup_{\substack{(s_{1t}, i_{1t}, d_{1t}) \in h_1 \\ (s_{2r}, i_{2r}, d_{2r}) \in h_2}} \left\{ \left(\sqrt[n]{1 - \frac{1}{1 + \left\{ \left(\frac{s_{1t}^n}{1-s_{1t}^n} \right)^\gamma + \left(\frac{s_{2r}^n}{1-s_{2r}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left\{ \left(\frac{1-i_{1t}^n}{i_{1t}^n} \right)^\gamma + \left(\frac{1-i_{2r}^n}{i_{2r}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left\{ \left(\frac{1-d_{1t}^n}{d_{1t}^n} \right)^\gamma + \left(\frac{1-d_{2r}^n}{d_{2r}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \right) \right\}$$
- $$h_1 \otimes h_2 = \bigcup_{\substack{(s_{1t}, i_{1t}, d_{1t}) \in h_1 \\ (s_{2r}, i_{2r}, d_{2r}) \in h_2}} \left\{ \left(\sqrt[n]{\frac{1}{1 + \left\{ \left(\frac{1-s_{1t}^n}{s_{1t}^n} \right)^\gamma + \left(\frac{1-s_{2r}^n}{s_{2r}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left\{ \left(\frac{1-i_{1t}^n}{i_{1t}^n} \right)^\gamma + \left(\frac{1-i_{2r}^n}{i_{2r}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \sqrt[n]{1 - \frac{1}{1 + \left\{ \left(\frac{d_{1t}^n}{1-d_{1t}^n} \right)^\gamma + \left(\frac{d_{2r}^n}{1-d_{2r}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \right) \right\}$$
- $$\lambda h_1 = \bigcup_{(s_{1t}, i_{1t}, d_{1t}) \in h_1} \left\{ \left(\sqrt[n]{1 - \frac{1}{1 + \left\{ \lambda \left(\frac{s_{1t}^n}{1-s_{1t}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left\{ \lambda \left(\frac{1-i_{1t}^n}{i_{1t}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left\{ \lambda \left(\frac{1-d_{1t}^n}{d_{1t}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \right) \right\}$$

$$4. \quad h_1^\lambda = \bigcup_{(s_{1l}, i_{1l}, d_{1l}) \in h_1} \left\{ \left(\sqrt[n]{\frac{1}{1 + \left\{ \lambda \left(\frac{1-s_{1l}^n}{s_{1l}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left\{ \lambda \left(\frac{1-i_{1l}^n}{i_{1l}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left\{ \lambda \left(\frac{1-d_{1l}^n}{d_{1l}^n} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}} \right\}.$$

Example 5 Let us consider $h(x_1) = h_1 = \{(0.6, 0.8, 0.4), (0.4, 0.5, 0.9), (0.6, 0.2, 0.7)\}$ and $h(x_2) = h_2 = \{(0.3, 0.9, 0.5), (0.2, 0.4, 0.7)\}$ in Example 1. For $n = 3$, $\gamma = 1$, and $\lambda = 2$

$$\begin{aligned} h_1 \oplus h_2 &= \{(0.6151, 0.7549, 0.3536), \\ &\quad (0.6045, 0.3922, 0.3849), (0.4443, 0.4925, 0.4925), \\ &\quad (0.4141, 0.3536, 0.6726), (0.6151, 0.1998, 0.4655), \\ &\quad (0.6045, 0.1928, 0.5915)\} \end{aligned}$$

$$\begin{aligned} h_1 \otimes h_2 &= \{(0.2908, 0.7549, 0.5587), \\ &\quad (0.1981, 0.3922, 0.7187), (0.2685, 0.4925, 0.9041), \\ &\quad (0.1928, 0.3536, 0.9136), (0.2908, 0.1998, 0.7364), \\ &\quad (0.1981, 0.1928, 0.7994)\} \end{aligned}$$

$$\begin{aligned} 2h_1 &= \{(0.7082, 0.7007, 0.3209), \\ &\quad (0.4937, 0.4055, 0.8309), (0.7082, 0.1590, 0.5915)\} \end{aligned}$$

$$\begin{aligned} h_1^2 &= \{(0.4947, 0.7007, 0.4937), \\ &\quad (0.3209, 0.4055, 0.9448), (0.4947, 0.1590, 0.7994)\}. \end{aligned}$$

Hesitant T-spherical Dombi fuzzy weighted arithmetic averaging operator

Definition 13 Let $\mathcal{H}^m = \{h_k = \{(s_{kj}, i_{kj}, d_{kj}) : 1 \leq j \leq \ell_{h_k}, k = 1, 2, \dots, m\}$ be an m dimensional collection of HT-SFEs. A hesitant T-spherical Dombi fuzzy weighted averaging (HTSDFWAA) operator is defined by a function HTSDFWAA: $\mathcal{H}^m \rightarrow \mathcal{H}$ as follows:

$$\begin{aligned} \text{HTSDFWAA}(h_1, h_2, h_3, \dots, h_m) &= \bigoplus_{z=1}^m (w_z h_z) \\ &= (w_1 h_1) \oplus (w_2 h_2) \oplus \dots \oplus (w_m h_m), \end{aligned}$$

where w_z is weight of $h_z (z = 1, 2, \dots, m)$, $0 \leq w_z \leq 1$ and $\sum_{z=1}^m w_z = 1$.

We get the following theorem that follows the Dombi operations on HT-SFEs.

Theorem 1 Let $h_k \in \mathcal{H}^m$. Then

$$\text{HTSDFWAA}(h_1, h_2, \dots, h_m)$$

$$= \bigoplus_{z=1}^m (w_z h_z)$$

$$= \bigcup_{\substack{(s_1, i_1, d_1) \in h_1 \\ (s_2, i_2, d_2) \in h_2 \\ \dots \\ (s_m, i_m, d_m) \in h_m}} \left(\sqrt[n]{\frac{1 - \frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1-i_z^n}{i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1 - \frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1-i_z^n}{i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1 - \frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1-d_z^n}{d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}} \right),$$

where $\omega = (w_1, w_2, \dots, w_m)$ be the m weight vector of $h_k (k = 1, 2, \dots, m)$, such that $\omega_k > 0$ and $\sum_{k=1}^m w_k = 1$.

Proof The theorem can be proved by the method of mathematical induction as follows:

(i) When $m = 2$, based on Dombi operations on HT-SFEs, we obtain the following results:

$$\begin{aligned} w_1 h_1 &= \bigcup_{(s_1, i_1, d_1) \in h_1} \left(\sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_1 \left(\frac{s_1^n}{1-s_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_1 \left(\frac{1-i_1^n}{i_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_1 \left(\frac{s_1^n}{1-s_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_1 \left(\frac{1-i_1^n}{i_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_1 \left(\frac{s_1^n}{1-s_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_1 \left(\frac{1-d_1^n}{d_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}} \right) \\ w_2 h_2 &= \bigcup_{(s_2, i_2, d_2) \in h_2} \left(\sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_2 \left(\frac{s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_2 \left(\frac{1-i_2^n}{i_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_2 \left(\frac{s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_2 \left(\frac{1-i_2^n}{i_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_2 \left(\frac{s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_2 \left(\frac{1-d_2^n}{d_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}} \right) \\ w_1 h_1 \oplus w_2 h_2 &= \left(\sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_1 \left(\frac{s_1^n}{1-s_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}} + \left(w_2 \left(\frac{s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_1 \left(\frac{1-i_1^n}{i_1^n} \right)^\gamma + w_2 \left(\frac{1-i_2^n}{i_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_1 \left(\frac{s_1^n}{1-s_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}} + \left(w_2 \left(\frac{s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_1 \left(\frac{1-i_1^n}{i_1^n} \right)^\gamma + w_2 \left(\frac{1-i_2^n}{i_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1 - \frac{1}{1 + \left(w_1 \left(\frac{s_1^n}{1-s_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}} + \left(w_2 \left(\frac{s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}}{1 + \left(w_1 \left(\frac{1-d_1^n}{d_1^n} \right)^\gamma + w_2 \left(\frac{1-d_2^n}{d_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}} \right) \end{aligned}$$

$$\begin{aligned}
&= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2}} \left(\sqrt[n]{1 - \frac{1}{1 + \left(w_1 \left(\frac{s_1^n}{1-s_1^n} \right)^\gamma + w_2 \left(\frac{s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\
&\quad \sqrt[n]{1 - \frac{1}{1 + \left(w_1 \left(\frac{i_1^n}{1-i_1^n} \right)^\gamma + w_2 \left(\frac{i_2^n}{1-i_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\
&\quad \left. \sqrt[n]{1 - \frac{1}{1 + \left(w_1 \left(\frac{d_1^n}{1-d_1^n} \right)^\gamma + w_2 \left(\frac{d_2^n}{1-d_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}} \right) \\
&\quad w_1 \mathfrak{h}_1 \oplus w_2 \mathfrak{h}_2 \\
&= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2}} \left(\sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^2 w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\
&\quad \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^2 w_z \left(\frac{i_z^n}{1-i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\
&\quad \left. \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^2 w_z \left(\frac{d_z^n}{1-d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}} \right) \\
&\quad \bigcup_{(s_{k+1}, i_{k+1}, d_{k+1}) \in \mathfrak{h}_{k+1}} \left(\sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\
&\quad \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{i_z^n}{1-i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\
&\quad \left. \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{d_z^n}{1-d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}} \right) \\
&\quad \oplus_{z=1}^{k+1} (w_z \mathfrak{h}_z)
\end{aligned}$$

Then, the theorem holds for $m = 2$

- (ii) Suppose the theorem holds when $z = k$ that is

$$\begin{aligned}
&\oplus_{z=1}^k (w_z \mathfrak{h}_z) \\
&= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_k, i_k, d_k) \in \mathfrak{h}_k}} \left(\sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\
&\quad \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{i_z^n}{1-i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\
&\quad \left. \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{d_z^n}{1-d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}} \right) \\
&= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_{k+1}, i_{k+1}, d_{k+1}) \in \mathfrak{h}_{k+1}}} \left(\sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^{k+1} w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\
&\quad \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^{k+1} w_z \left(\frac{i_z^n}{1-i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\
&\quad \left. \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^{k+1} w_z \left(\frac{d_z^n}{1-d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}} \right)
\end{aligned}$$

Then, the theorem holds for $z = k+1$. Hence, the theorem is proved for all $z \in \mathbb{N}$. $\square$

When $z = k+1$

$$\begin{aligned}
&\oplus_{z=1}^k (w_z \mathfrak{h}_z) \oplus w_{k+1} \mathfrak{h}_{k+1} \\
&= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_k, i_k, d_k) \in \mathfrak{h}_k}} \left(\sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\
&\quad \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{i_z^n}{1-i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\
&\quad \left. \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{d_z^n}{1-d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}} \right) \\
&\quad \oplus w_{k+1} \mathfrak{h}_{k+1} \\
&\quad \oplus_{z=1}^k (w_z \mathfrak{h}_z) \oplus w_{k+1} \mathfrak{h}_{k+1}
\end{aligned}$$

Example 6 Let us consider $\mathfrak{h}_1 = \{(0.6, 0.8, 0.4), (0.4, 0.5, 0.9), (0.6, 0.2, 0.7)\}$, $\mathfrak{h}_2 = \{(0.3, 0.9, 0.5), (0.2, 0.4, 0.7)\}$, and $\mathfrak{h}_3 = \{(0.5, 0.4, 0.3)\}$ for $n = 3$. When $\gamma = 1$, with weighted vector $\omega = (0.5, 0.3, 0.2)$, we get

$$\begin{aligned}
&\text{HTSDFWAA}(\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3) = \oplus_{z=1}^3 (w_z \mathfrak{h}_z) \\
&= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ (s_3, i_3, d_3) \in \mathfrak{h}_3}} \left(\sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\
&\quad \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{i_z^n}{1-i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\
&\quad \left. \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{d_z^n}{1-d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}} \right)
\end{aligned}$$

$$\begin{aligned} & \text{HTSDFWAA}(\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3) \\ &= \{(0.5298, 0.6051, 0.3843), (0.5246, 0.4846, 0.3961), \\ & \quad (0.4049, 0.5100, 0.4568), (0.3941, 0.4391, 0.4813), \\ & \quad (0.5298, 0.2474, 0.4461), (0.5246, 0.2423, 0.4683)\}. \end{aligned}$$

Theorem 2 (Idempotency) Let $\mathfrak{h}_k$ ($k = 1, 2, \dots, m$) be a number of HT-SFNs. Then, $\mathfrak{h}_k = (s_k, i_k, d_k)$ ($k = 1, 2, \dots, m$) be a number of HT-SFEs are all equal, i.e., $\mathfrak{h}_k = \mathfrak{h}$ for all k , then $\text{HTSDFWAA}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_m) = \mathfrak{h}$.

Hesitant T-spherical Dombi fuzzy weighted geometric averaging (HTSDFWGA) operator

Definition 14 Let $\mathcal{H}^m = \{\mathfrak{h}_k = \{(s_{kj}, i_{kj}, d_{kj}) : 1 \leq j \leq \ell_{\mathfrak{h}_k}, k = 1, 2, \dots, m\}$ be an m -dimensional collection of HT-SFEs. An HTSDFWGA operator is defined by a function HTSDFWGA: $\mathcal{H}^m \rightarrow \mathcal{H}$ as follows:

$$\begin{aligned} & \text{HTSDFWGA}(\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3, \dots, \mathfrak{h}_m) \\ &= \bigotimes_{z=1}^m (\mathfrak{h}_z^{w_z}) \\ &= (\mathfrak{h}_1^{w_1}) \otimes (\mathfrak{h}_2^{w_2}) \otimes \dots \otimes (\mathfrak{h}_m^{w_m}), \end{aligned}$$

where w_z is weight of $\mathfrak{h}_z$ ($z = 1, 2, \dots, m$), $0 \leq w_z \leq 1$ and $\sum_{z=1}^m w_z = 1$.

Theorem 3 Let $\mathfrak{h}_k \in \mathcal{H}^m$. Then

$$\begin{aligned} & \text{HTSDFWGA}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_m) = \bigotimes_{z=1}^m (\mathfrak{h}_z^{w_z}) \\ &= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_m, i_m, d_m) \in \mathfrak{h}_m}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1-s_z^n}{s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ & \quad \left. \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1-i_z^n}{i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ & \quad \left. \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1-d_z^n}{d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right), \quad (2) \end{aligned}$$

where $\omega = (w_1, w_2, \dots, w_m)$ be the m weighted vector of $\mathfrak{h}_k$ ($k = 1, 2, \dots, m$), such that $w_k > 0$ and $\sum_{k=1}^m w_k = 1$.

Proof The theorem can be proved by the mathematical induction as follows:

(i) When $m = 2$, we have

$$\mathfrak{h}_1^{w_1} =$$

$$\begin{aligned} & \bigcup_{(s_1, i_1, d_1) \in \mathfrak{h}_1} \left(\sqrt[n]{\frac{1}{1 + \left(w_1 \left(\frac{1-s_1^n}{1-s_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left(w_1 \left(\frac{1-i_1^n}{1-i_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ & \quad \left. \sqrt[n]{\frac{1}{1 + \left(w_1 \left(\frac{1-d_1^n}{1-d_1^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right) \\ & \mathfrak{h}_2^{w_2} = \\ & \bigcup_{(s_2, i_2, d_2) \in \mathfrak{h}_2} \left(\sqrt[n]{\frac{1}{1 + \left(w_2 \left(\frac{1-s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left(w_2 \left(\frac{1-i_2^n}{1-i_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ & \quad \left. \sqrt[n]{\frac{1}{1 + \left(w_2 \left(\frac{1-d_2^n}{1-d_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right) \end{aligned}$$

and

$$\begin{aligned} & \mathfrak{h}_1^{w_1} \otimes \mathfrak{h}_2^{w_2} \\ &= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2}} \left(\sqrt[n]{\frac{1}{1 + \left(w_1 \left(\frac{1-s_1^n}{1-s_1^n} \right)^\gamma + w_2 \left(\frac{1-s_2^n}{1-s_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ & \quad \sqrt[n]{\frac{1}{1 + \left(w_1 \left(\frac{1-i_1^n}{1-i_1^n} \right)^\gamma + w_2 \left(\frac{1-i_2^n}{1-i_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\ & \quad \left. \sqrt[n]{\frac{1}{1 + \left(w_1 \left(\frac{1-d_1^n}{1-d_1^n} \right)^\gamma + w_2 \left(\frac{1-d_2^n}{1-d_2^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right) \\ & \mathfrak{h}_1^{w_1} \otimes \mathfrak{h}_2^{w_2} \\ &= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^2 w_z \left(\frac{1-s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ & \quad \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^2 w_z \left(\frac{1-i_z^n}{1-i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\ & \quad \left. \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^2 w_z \left(\frac{1-d_z^n}{1-d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right). \end{aligned}$$

Then, the theorem holds for $m = 2$.

(ii) Suppose that the theorem holds for $z = k$, that is

$$\begin{aligned} & \bigotimes_{z=1}^k (\mathfrak{h}_z^{w_z}) \\ &= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_k, i_k, d_k) \in \mathfrak{h}_k}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{1-s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ & \quad \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{1-i_z^n}{1-i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \\ & \quad \left. \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{1-d_z^n}{1-d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right). \end{aligned}$$

We prove that equation is true for $z = k + 1$

$$\begin{aligned}
 & \otimes_{z=1}^k (\mathfrak{h}_z^{w_z}) \otimes \mathfrak{h}_{k+1}^{w_{k+1}} \\
 &= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_k, i_k, d_k) \in \mathfrak{h}_k}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{1-i_z^n}{i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{1-d_z^n}{d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right) \\
 & \otimes \mathfrak{h}_{k+1}^{w_{k+1}} \otimes_{z=1}^k (\mathfrak{h}_z^{w_z}) \otimes \mathfrak{h}_{k+1}^{w_{k+1}} \\
 &= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_k, i_k, d_k) \in \mathfrak{h}_k}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{1-i_z^n}{i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^k w_z \left(\frac{1-d_z^n}{d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right) \otimes \\
 & \bigcup_{(s_{k+1}, i_{k+1}, d_{k+1}) \in \mathfrak{h}_{k+1}} \left(\sqrt[n]{\frac{1}{1 + \left(w_{k+1} \left(\frac{s_{k+1}^n}{1-s_{k+1}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left(w_{k+1} \left(\frac{1-i_{k+1}^n}{i_{k+1}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{1 - \frac{1}{1 + \left(w_{k+1} \left(\frac{1-d_{k+1}^n}{d_{k+1}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right) \\
 & \otimes_{z=1}^{k+1} (\mathfrak{h}_z^{w_z}) \\
 &= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_{k+1}, i_{k+1}, d_{k+1}) \in \mathfrak{h}_{k+1}}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^{k+1} w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^{k+1} w_z \left(\frac{1-i_z^n}{i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^{k+1} w_z \left(\frac{1-d_z^n}{d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right).
 \end{aligned}$$

Then, the theorem holds for $z = k + 1$. Hence, the proof is completed. $\square$

Example 7 Let us consider HT-SFEs given in Example 6. Then, we get

$$\text{HTSDFWGA}(\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3) = \otimes_{z=1}^3 (\mathfrak{h}_z^{w_z})$$

$$= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ (s_3, i_3, d_3) \in \mathfrak{h}_3}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{s_z^n}{1-s_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{1-i_z^n}{i_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{1-d_z^n}{d_z^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right)$$

HTSDFWGA($\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3$)

$$= \{(0.4053, 0.6051, 0.4241), (0.2890, 0.4846, 0.5475), (0.3652, 0.5100, 0.8350), (0.2773, 0.4391, 0.8440), (0.4052, 0.2474, 0.6183), (0.2890, 0.2423, 0.6675)\}.$$

Theorem 4 (Idempotency) Let $\mathfrak{h}_k \in \mathcal{H}^m$ ($k = 1, 2, \dots, m$). If $\mathfrak{h}_k = \mathfrak{h}$ for $k = 1, 2, \dots, m$, then $\text{HTSDFWGA}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_m) = \mathfrak{h}$.

Proof Straightforward. Therefore, the proof is omitted. $\square$

Hesitant T-spherical Dombi fuzzy ordered weighted arithmetic averaging (HTSDFOWAA) operator

Definition 15 Let $\mathcal{H}^m = \{\mathfrak{h}_k = \{(s_{kj}, i_{kj}, d_{kj}) : 1 \leq j \leq \ell_{\mathfrak{h}_k}, k = 1, 2, \dots, m\}$ be an m dimensional collection of HT-SFEs. An HTSDFOWAA operator is defined by a function HTSDFOWAA: $\mathcal{H}^m \rightarrow \mathcal{H}$ as follows:

$$\begin{aligned}
 & \text{HTSDFOWAA}(\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3, \dots, \mathfrak{h}_m) \\
 &= \bigoplus_{z=1}^m (w_z \mathfrak{h}_{\sigma(z)}) \\
 &= (w_1 \mathfrak{h}_{\sigma(1)}) \oplus (w_2 \mathfrak{h}_{\sigma(2)}) \oplus \dots \oplus (w_m \mathfrak{h}_{\sigma(m)}),
 \end{aligned}$$

where $\mathfrak{h}_{\sigma(z)}$ is the z th largest of $\mathfrak{h}_z$ and w_z is weighted vector of $\mathfrak{h}_z$ ($z = 1, 2, \dots, m$), $0 \leq w_z \leq 1$ and $\sum_{z=1}^m w_z = 1$.

Theorem 5 Let $\mathfrak{h}_k \in \mathcal{H}^m$ ($k = 1, 2, \dots, m$). Then

$$\begin{aligned}
 & \text{HTSDFOWAA}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_m) = \bigoplus_{z=1}^m (w_z \mathfrak{h}_{\sigma(z)}) \\
 &= \bigcup_{\substack{(s_1, i_1, d_1) \in \mathfrak{h}_1 \\ (s_2, i_2, d_2) \in \mathfrak{h}_2 \\ \dots \\ (s_m, i_m, d_m) \in \mathfrak{h}_m}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{s_{\sigma(z)}^n}{1-s_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1-i_{\sigma(z)}^n}{i_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1-d_{\sigma(z)}^n}{d_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right),
 \end{aligned}$$

where $h_{\sigma(z)}$ is the z th largest of h_z and $\omega = (w_1, w_2, \dots, w_m)$ be the m weighted vector of $h_k (k = 1, 2, \dots, m)$, such that $0 < w_k < 1$ and $\sum_{k=1}^m w_k = 1$

Proof The proof is made by similar way to proof of Theorem 1. $\square$

Hesitant T-spherical Dombi fuzzy ordered weighted geometric averaging (HTSDFOWGA) operator

Definition 16 Let $\mathcal{H}^m = \{h_k = \{(s_{kj}, i_{kj}, d_{kj}) : 1 \leq j \leq \ell_{h_k}\}, k = 1, 2, \dots, m\}$ be an m dimensional collection of HT-SFEs. An HTSDFOWGA operator is defined by a function HTSDFOWGA: $\mathcal{H}^m \rightarrow \mathcal{H}$ as follows:

$$\begin{aligned} \text{HTSDFOWGA}(h_1, h_2, h_3, \dots, h_m) \\ &= \bigotimes_{z=1}^m (h_{\sigma(z)}^{w_z}) \\ &= (h_1^{w_1}) \otimes (h_2^{w_2}) \otimes \dots \otimes (h_m^{w_m}), \end{aligned}$$

where w_z is weighted vector of $h_z (z = 1, 2, \dots, m)$, $0 \leq w_z \leq 1$ and $\sum_{z=1}^m w_z = 1$.

Theorem 6 Let $h_k \in \mathcal{H}^m$. Then

$$\begin{aligned} \text{HTSDFOWGA}(h_{\sigma(1)}, h_{\sigma(2)}, \dots, h_m) &= \bigotimes_{z=1}^m (h_{\sigma(z)}^{w_z}) \\ &= \bigcup_{\substack{(s_{\sigma(1)}, i_{\sigma(1)}, d_{\sigma(1)}) \in h_{\sigma(1)} \\ (s_{\sigma(2)}, i_{\sigma(2)}, d_{\sigma(2)}) \in h_{\sigma(2)} \\ \dots \\ (s_{\sigma(m)}, i_{\sigma(m)}, d_{\sigma(m)}) \in h_{\sigma(m)}}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1 - s_{\sigma(z)}^n}{s_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ &\quad \left. \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{1 - i_{\sigma(z)}^n}{i_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ &\quad \left. \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^m w_z \left(\frac{d_{\sigma(z)}^n}{1 - d_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right), \end{aligned}$$

where $h_{\sigma(z)}$ is the z th largest of h_z and $\omega = (w_1, w_2, \dots, w_m)$ be the m weighted vector of $h_k (k = 1, 2, \dots, m)$, such that $0 < w_k < 1$ and $\sum_{k=1}^m w_k = 1$.

Example 8 Let us consider $h_1 = \{(0.6, 0.8, 0.4), (0.4, 0.5, 0.9), (0.6, 0.2, 0.7)\}$, $h_2 = \{(0.3, 0.9, 0.5), (0.2, 0.4, 0.7)\}$ and $h_3 = \{(0.5, 0.4, 0.3)\}$ for $n = 3$. When $\gamma = 1$, with weight vector $\omega = (0.5, 0.3, 0.2)$, we get

$$\text{HTSDFOWGA}(h_{\sigma(1)}, h_{\sigma(2)}, h_3) = \bigotimes_{z=1}^3 (h_{\sigma(z)}^{w_z})$$

$$= \bigcup_{\substack{(s_{\sigma(1)}, i_{\sigma(1)}, d_{\sigma(1)}) \in h_{\sigma(1)} \\ (s_{\sigma(2)}, i_{\sigma(2)}, d_{\sigma(2)}) \in h_{\sigma(2)} \\ (s_{\sigma(3)}, i_{\sigma(3)}, d_{\sigma(3)}) \in h_{\sigma(3)}}} \left(\sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{1 - s_{\sigma(z)}^n}{s_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ \left. \sqrt[n]{\frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{1 - i_{\sigma(z)}^n}{i_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}}, \right. \\ \left. \sqrt[n]{1 - \frac{1}{1 + \left(\sum_{z=1}^3 w_z \left(\frac{d_{\sigma(z)}^n}{1 - d_{\sigma(z)}^n} \right)^\gamma \right)^{\frac{1}{\gamma}}}} \right).$$

Using Eq. (1), SVs of HT-SFEs are obtained as follows:

$$\text{SV}(h_1) = -0.2133,$$

$$\text{SV}(h_2) = -0.2165,$$

$$\text{SV}(h_3) = 0.089.$$

Here, $\text{SV}(h_3) > \text{SV}(h_1) > \text{SV}(h_2)$ and

$$h_{\sigma(1)} = h_3 = \{(0.5, 0.4, 0.3)\}$$

$$h_{\sigma(2)} = h_1 = \{(0.6, 0.8, 0.4), (0.4, 0.5, 0.9), (0.6, 0.2, 0.7)\}$$

$$h_{\sigma(3)} = h_2 = \{(0.3, 0.9, 0.5), (0.2, 0.4, 0.7)\}.$$

Then

$$\begin{aligned} \text{HTSDFOWGA}(h_1, h_2, h_3) &= \{(0.4275, 0.4867, 0.3898), \\ &\quad (0.3961, 0.4569, 0.7716), (0.4275, 0.2799, 0.5496), \\ &\quad (0.3205, 0.3105, 0.4958), (0.3096, 0.3051, 0.7833), \\ &\quad (0.3205, 0.2423, 0.5997)\}. \end{aligned}$$

Theorem 7 (Idempotency property) Let $h_k \in \mathcal{H}^m (k = 1, 2, \dots, m)$. If $h_k = h$ for $k = 1, 2, \dots, m$, then $\text{HTSDFOWGA}(h_1, h_2, \dots, h_m) = h$.

Proof Straightforward. $\square$

Multiple criteria group decision-making (MCGDM) method under HT-SF information

In this section, we develop a multiple criteria group decision-making method. First, we give frequently used notation in Table 2 for convenience.

Let $\kappa = \{\kappa_1, \kappa_2, \dots, \kappa_l\}$ be set of alternatives, $\epsilon = \{\epsilon_1, \epsilon_2, \dots, \epsilon_s\}$ be a set of criteria and $\partial = \{\partial_1, \partial_2, \dots, \partial_t\}$ be a set of decision-makers. Let us consider $w = (w_1, w_2, \dots, w_s)$, such that $w_j \in (0, 1]$ and $\sum_{j=1}^s w_j = 1$ as the weight vector of the criteria which is determined by decision-makers. The steps of the MCGDM method are given as follows:

Step 1: The evaluation of the alternative κ_i according to criteria ϵ_j performed by decision-makers $\partial_y (y =$

Table 2 Frequently used notations in “Multiple criteria group decision-making (MCGDM) method under HT-SF information” and other

$\kappa = \{\kappa_1, \kappa_2, \dots, \kappa_l\}$	Set of alternatives
$\epsilon = \{\epsilon_1, \epsilon_2, \dots, \epsilon_s\}$	Set of criteria
$\partial = \{\partial_1, \partial_2, \dots, \partial_t\}$	Set of decision-makers
D_{κ_i}	For alternative κ_i decision matrix
$HTSF_i$	Collection of i column elements of D_{κ_i} matrix
ζ_{yj}	Element of D_{κ_i} corresponding to y row and j column
$\mathfrak{A}_i$	HT-SFE corresponding to alternative κ_i

$1, 2, \dots, t)$ can be written as $\zeta_{yj} (i = 1, 2, \dots, l; j = 1, 2, \dots, s; y = 1, 2, \dots, t)$. Hence, HT-SF-decision matrix $DM_{\kappa_i} = [\zeta_{yj}]_{t \times s}$ can be constructed as follows:

$$D_{\kappa_i} = [\zeta_{yj}]_{t \times s} = \begin{pmatrix} \zeta_{11} & \zeta_{12} & \cdots & \zeta_{1s} \\ \zeta_{21} & \zeta_{22} & \cdots & \zeta_{2s} \\ \vdots & \vdots & \cdots & \vdots \\ \zeta_{t1} & \zeta_{t2} & \cdots & \zeta_{ts} \end{pmatrix}.$$

Step 2: For all $i = 1, 2, \dots, l$, HT-SFS denoted by $HTSF_i$ is obtained as follows:

$$HTSF_i = \{(\epsilon_j, h_{\epsilon_j}) : j = 1, 2, \dots, s\}.$$

Here $h_{\epsilon_j} = \bigcup_{y=1}^t \{\zeta_{yj}\}$.

Step 3: For $\kappa_i, i = 1, 2, 3, \dots, l$ HT-SF element related to κ_i denoted by $\mathfrak{A}_i$, is defined as follows:

$$\mathfrak{A}_i = \bigoplus_{j=1}^s w_j h_{\epsilon_j}.$$

Step 4: Find score values of $\mathfrak{A}_i$ ($i = 1, 2, 3, \dots, l$).

Step 5: Order score values of $\mathfrak{A}_i$ ($i = 1, 2, 3, \dots, l$).

Step 6: Choose the alternative which has maximum score value.

Flowchart of the algorithm is given in Fig. 1.

Fig. 1 Flowchart of the proposed method

Illustrative example

We consider that a university wants for filling the position of one assistant professorship in a department. After the announcement for this vacant position, seven candidates $\kappa_1, \kappa_2, \dots, \kappa_7$ apply for the position. University rector assigns three experts ∂_1, ∂_2 , and ∂_3 to evaluate alternatives according to criterion ϵ_1 = experience, ϵ_2 = scientific works, and ϵ_3 = quality of the researches. After interview, experts determine the weight vector of the criteria as $(0.35, 0.25, 0.40)^T$.

Step 1: Experts evaluate the alternatives by HT-SFNs corresponding to linguistic variables given in Table 3 for each criteria and $n = 4$.

Table 3 Linguistic variable table for evaluation of the candidates

Grades	HT-SFNs
Very poor (VP)	(0.100, 0.700, 0.900)
Poor (P)	(0.233, 0.634, 0.767)
Medium poor (MP)	(0.367, 0.567, 0.634)
Fairly (F)	(0.500, 0.500, 0.500)
Medium good (MG)	(0.633, 0.436, 0.367)
Good (G)	(0.764, 0.370, 0.234)
Very good (VG)	(0.900, 0.300, 0.100)

		ϵ_1	ϵ_2	ϵ_3
D_{κ_1}	∂_1	(0.633, 0.436, 0.367)	(0.233, 0.634, 0.767)	*
	∂_2	(0.100, 0.700, 0.900)	(0.900, 0.300, 0.100)	(0.500, 0.500, 0.500)
	∂_3	*	(0.764, 0.370, 0.234)	*
D_{κ_2}	∂_1	*	(0.233, 0.634, 0.767)	*
	∂_2	(0.233, 0.634, 0.767)	(0.367, 0.567, 0.634)	(0.500, 0.500, 0.500)
	∂_3	(0.764, 0.370, 0.234)	(0.100, 0.700, 0.900)	(0.633, 0.436, 0.367)
D_{κ_3}	∂_1	(0.367, 0.567, 0.634)	(0.633, 0.436, 0.367)	(0.100, 0.700, 0.900)
	∂_2	(0.900, 0.300, 0.100)	(0.764, 0.370, 0.234)	*
	∂_3	(0.633, 0.436, 0.367)	(0.100, 0.700, 0.900)	(0.500, 0.500, 0.500)
D_{κ_4}	∂_1	(0.900, 0.300, 0.100)	(0.500, 0.500, 0.500)	(0.100, 0.700, 0.900)
	∂_2	(0.900, 0.300, 0.100)	*	(0.500, 0.500, 0.500)
	∂_3	*	(0.100, 0.700, 0.900)	(0.500, 0.500, 0.500)
D_{κ_5}	∂_1	(0.367, 0.567, 0.634)	(0.500, 0.500, 0.500)	(0.100, 0.700, 0.900)
	∂_2	(0.100, 0.700, 0.900)	(0.367, 0.567, 0.634)	(0.633, 0.436, 0.367)
	∂_3	(0.900, 0.300, 0.100)	(0.100, 0.700, 0.900)	(0.500, 0.500, 0.500)
D_{κ_6}	∂_1	(0.500, 0.500, 0.500)	(0.633, 0.436, 0.367)	(0.764, 0.370, 0.234)
	∂_2	*	(0.367, 0.567, 0.634)	(0.633, 0.436, 0.367)
	∂_3	*	(0.233, 0.634, 0.767)	(0.500, 0.500, 0.500)
D_{κ_7}	∂_1	(0.500, 0.500, 0.500)	(0.633, 0.436, 0.367)	(0.500, 0.500, 0.500)
	∂_2	(0.367, 0.567, 0.634)	(0.100, 0.700, 0.900)	(0.233, 0.634, 0.767)
	∂_3	(0.233, 0.634, 0.767)	(0.100, 0.700, 0.900)	*

Step 2: Using HT-SF decision matrices given in Step 1, $HTSF_i$ ($i = 1, 2, \dots, 7$) are obtained as follows:

$$\begin{aligned}
 HTSF_1 &= \left\{ \left(\epsilon_1, \{(0.633, 0.436, 0.367), \right. \right. \\
 &\quad \left. \left(\epsilon_2, \{(0.233, 0.634, 0.767), \right. \right. \\
 &\quad \left. \left. (0.900, 0.300, 0.100), (0.764, 0.370, 0.234)\} \right), \right. \\
 &\quad \left. \left. \left(\epsilon_3, \{(0.500, 0.500, 0.500)\} \right) \right\}, \\
 HTSF_2 &= \left\{ \left(\epsilon_1, \{(0.764, 0.370, 0.234), \right. \right. \\
 &\quad \left. \left. (0.233, 0.634, 0.767)\} \right), \left(\epsilon_2, \{(0.100, 0.700, 0.900), \right. \right. \\
 &\quad \left. \left. (0.233, 0.634, 0.767), (0.367, 0.567, 0.634)\} \right), \right. \\
 &\quad \left. \left. \left(\epsilon_3, \{(0.500, 0.500, 0.500), (0.633, 0.436, 0.367)\} \right) \right\}, \\
 HTSF_3 &= \left\{ \left(\epsilon_1, \{(0.367, 0.567, 0.634), \right. \right. \\
 &\quad \left. \left. (0.900, 0.300, 0.100), (0.633, 0.436, 0.367)\} \right), \right. \\
 &\quad \left. \left(\epsilon_2, \{(0.633, 0.436, 0.367), \right. \right. \\
 &\quad \left. \left. (0.764, 0.370, 0.234), (0.100, 0.700, 0.900)\} \right), \right. \\
 &\quad \left. \left. \left(\epsilon_3, \{(0.100, 0.700, 0.900), (0.500, 0.500, 0.500)\} \right) \right\}, \\
 HTSF_4 &= \left\{ \left(\epsilon_1, \{(0.900, 0.300, 0.100), \right. \right. \\
 &\quad \left. \left. (0.900, 0.300, 0.100)\} \right), \left(\epsilon_2, \{(0.100, 0.700, 0.900), \right. \right. \\
 &\quad \left. \left. (0.500, 0.500, 0.500)\} \right), \left(\epsilon_3, \{(0.100, 0.700, 0.900), \right. \right. \\
 &\quad \left. \left. (0.500, 0.500, 0.500), (0.500, 0.500, 0.500)\} \right) \right\},
 \end{aligned}$$

$$\begin{aligned}
 HTSF_5 &= \left\{ \left(\epsilon_1, \{(0.367, 0.567, 0.634), \right. \right. \\
 &\quad \left. \left. (0.100, 0.700, 0.900), (0.900, 0.300, 0.100)\} \right), \right. \\
 &\quad \left. \left(\epsilon_2, \{(0.500, 0.500, 0.500), \right. \right. \\
 &\quad \left. \left. (0.367, 0.567, 0.634), (0.100, 0.700, 0.900)\} \right), \right. \\
 &\quad \left. \left. \left(\epsilon_3, \{(0.100, 0.700, 0.900), \right. \right. \right. \\
 &\quad \left. \left. \left. (0.633, 0.436, 0.367), (0.500, 0.500, 0.500)\} \right) \right\}, \\
 HTSF_6 &= \left\{ \left(\epsilon_1, \{(0.500, 0.500, 0.500)\} \right), \right. \\
 &\quad \left(\epsilon_2, \{(0.633, 0.436, 0.367), \right. \\
 &\quad \left. \left. (0.367, 0.567, 0.634), (0.233, 0.634, 0.767)\} \right), \right. \\
 &\quad \left(\epsilon_3, \{(0.764, 0.370, 0.234), \right. \\
 &\quad \left. \left. (0.633, 0.436, 0.367), (0.500, 0.500, 0.500)\} \right) \right\}, \\
 HTSF_7 &= \left\{ \left(\epsilon_1, \{(0.500, 0.500, 0.500), \right. \right. \\
 &\quad \left. \left. (0.367, 0.567, 0.634), (0.233, 0.634, 0.767)\} \right), \right. \\
 &\quad \left(\epsilon_2, \{(0.633, 0.436, 0.367), \right. \\
 &\quad \left. \left. (0.100, 0.700, 0.900), (0.100, 0.700, 0.900)\} \right), \right. \\
 &\quad \left. \left. \left(\epsilon_3, \{(0.500, 0.500, 0.500), (0.233, 0.634, 0.767)\} \right) \right\}.
 \end{aligned}$$

Table 4 HTSDFWAA and HTSDFWGA values of $HTSF_i$ ($i = 1, 2, \dots, 7$)

	HTSDFWAA	HTSDFWGA
$\mathfrak{A}_1$	{(0.542, 0.488, 0.441), (0.776, 0.382, 0.141), (0.653, 0.429, 0.309)} {(0.404, 0.571, 0.601), (0.761, 0.401, 0.141), (0.606, 0.466, 0.324)}	{(0.322, 0.488, 0.614), (0.578, 0.382, 0.423), (0.572, 0.429, 0.425), (0.129, 0.571, 0.820), (0.130, 0.401, 0.800), (0.130, 0.466, 0.800)}
$\mathfrak{A}_2$	{(0.408, 0.561, 0.592), (0.519, 0.510, 0.453), (0.421, 0.549, 0.578), (0.525, 0.503, 0.449), (0.405, 0.568, 0.599), (0.518, 0.514, 0.454), (0.644, 0.440, 0.300), (0.673, 0.423, 0.291), (0.647, 0.437, 0.299), (0.675, 0.420, 0.291), (0.644, 0.442, 0.300), (0.673, 0.425, 0.291)}	{(0.263, 0.561, 0.711), (0.264, 0.510, 0.704), (0.291, 0.549, 0.674), (0.293, 0.503, 0.664), (0.140, 0.568, 0.799), (0.140, 0.514, 0.796), (0.323, 0.440, 0.609), (0.326, 0.423, 0.593), (0.466, 0.437, 0.515), (0.490, 0.420, 0.481), (0.141, 0.442, 0.761), (0.141, 0.425, 0.756)}
$\mathfrak{A}_3$	{(0.476, 0.538, 0.496), (0.523, 0.495, 0.460), (0.588, 0.483, 0.328), (0.611, 0.456, 0.323), (0.284, 0.636, 0.750), (0.423, 0.551, 0.578), (0.804, 0.373, 0.130), (0.808, 0.365, 0.130), (0.816, 0.362, 0.129), (0.820, 0.355, 0.129), (0.795, 0.384, 0.130), (0.800, 0.375, 0.130), (0.566, 0.484, 0.415), (0.593, 0.457, 0.399), (0.636, 0.449, 0.314), (0.653, 0.429, 0.309), (0.501, 0.533, 0.471), (0.541, 0.492, 0.443)}	{(0.126, 0.538, 0.821), (0.438, 0.495, 0.548), (0.126, 0.483, 0.821), (0.441, 0.456, 0.543), (0.111, 0.636, 0.868), (0.141, 0.551, 0.776), (0.126, 0.373, 0.812), (0.586, 0.365, 0.417), (0.126, 0.362, 0.811), (0.601, 0.355, 0.404), (0.111, 0.384, 0.863), (0.141, 0.375, 0.761), (0.126, 0.484, 0.813), (0.560, 0.457, 0.437), (0.126, 0.449, 0.812), (0.572, 0.429, 0.425), (0.111, 0.533, 0.863), (0.141, 0.492, 0.762)}
$\mathfrak{A}_4$	{(0.798, 0.378, 0.130), (0.798, 0.378, 0.130), (0.798, 0.378, 0.130), (0.798, 0.378, 0.130), (0.803, 0.370, 0.130), (0.803, 0.370, 0.130), (0.800, 0.375, 0.130), (0.800, 0.375, 0.130), (0.800, 0.375, 0.130), (0.800, 0.375, 0.130), (0.800, 0.375, 0.130), (0.800, 0.375, 0.130)}	{(0.126, 0.378, 0.814), (0.126, 0.378, 0.814), (0.126, 0.378, 0.814), (0.126, 0.378, 0.814), (0.550, 0.370, 0.452), (0.550, 0.370, 0.452), (0.141, 0.375, 0.761), (0.141, 0.375, 0.761), (0.141, 0.375, 0.761), (0.141, 0.375, 0.761), (0.141, 0.375, 0.761), (0.141, 0.375, 0.761)}
$\mathfrak{A}_5$	{(0.388, 0.577, 0.620), (0.549, 0.482, 0.434), (0.467, 0.519, 0.531), (0.324, 0.605, 0.694), (0.533, 0.493, 0.444), (0.437, 0.535, 0.561), (0.284, 0.636, 0.750), (0.526, 0.504, 0.449), (0.423, 0.551, 0.578), (0.358, 0.612, 0.664), (0.540, 0.496, 0.440), (0.452, 0.539, 0.550), (0.261, 0.652, 0.781), (0.523, 0.509, 0.451), (0.417, 0.559, 0.586), (0.100, 0.700, 0.900), (0.516, 0.521, 0.457), (0.402, 0.579, 0.608), (0.798, 0.378, 0.130), (0.811, 0.362, 0.130), (0.803, 0.370, 0.130), (0.796, 0.381, 0.130), (0.809, 0.364, 0.130), (0.801, 0.372, 0.130), (0.795, 0.384, 0.130), (0.808, 0.367, 0.130), (0.800, 0.375, 0.130)}	{(0.126, 0.577, 0.823), (0.444, 0.482, 0.538), (0.428, 0.519, 0.562), (0.125, 0.605, 0.827), (0.409, 0.493, 0.575), (0.399, 0.535, 0.594), (0.111, 0.636, 0.868), (0.141, 0.504, 0.772), (0.141, 0.551, 0.776), (0.107, 0.612, 0.877), (0.130, 0.496, 0.800), (0.130, 0.539, 0.803), (0.107, 0.652, 0.879), (0.130, 0.509, 0.805), (0.130, 0.559, 0.808), (0.100, 0.700, 0.900), (0.114, 0.521, 0.855), (0.114, 0.579, 0.857), (0.126, 0.378, 0.814), (0.614, 0.362, 0.392), (0.550, 0.370, 0.452), (0.126, 0.381, 0.818), (0.494, 0.364, 0.479), (0.469, 0.372, 0.514), (0.111, 0.384, 0.863), (0.141, 0.367, 0.756), (0.141, 0.375, 0.761)}
$\mathfrak{A}_6$	{(0.683, 0.415, 0.284), (0.599, 0.454, 0.394), (0.546, 0.479, 0.444), (0.660, 0.430, 0.290), (0.555, 0.478, 0.430), (0.477, 0.513, 0.521), (0.658, 0.433, 0.291), (0.550, 0.484, 0.433), (0.469, 0.521, 0.529)}	{(0.589, 0.415, 0.410), (0.567, 0.454, 0.430), (0.521, 0.479, 0.477), (0.470, 0.430, 0.510), (0.462, 0.478, 0.520), (0.444, 0.513, 0.547), (0.323, 0.433, 0.607), (0.322, 0.484, 0.612), (0.319, 0.521, 0.627)}
$\mathfrak{A}_7$	{(0.546, 0.479, 0.444), (0.510, 0.508, 0.470), (0.467, 0.526, 0.533), (0.393, 0.575, 0.612), (0.467, 0.526, 0.533), (0.393, 0.575, 0.612), (0.523, 0.495, 0.460), (0.479, 0.529, 0.492), (0.423, 0.551, 0.578), (0.295, 0.617, 0.721), (0.423, 0.551, 0.578), (0.295, 0.617, 0.721), (0.515, 0.504, 0.466), (0.467, 0.542, 0.500), (0.405, 0.568, 0.599), (0.217, 0.648, 0.791), (0.405, 0.568, 0.599), (0.217, 0.648, 0.791)}	{(0.521, 0.479, 0.477), (0.289, 0.508, 0.663), (0.141, 0.526, 0.766), (0.139, 0.575, 0.803), (0.141, 0.526, 0.766), (0.139, 0.575, 0.803), (0.438, 0.495, 0.548), (0.283, 0.529, 0.686), (0.141, 0.551, 0.776), (0.139, 0.617, 0.810), (0.141, 0.551, 0.776), (0.139, 0.617, 0.810), (0.298, 0.504, 0.650), (0.250, 0.542, 0.732), (0.140, 0.568, 0.799), (0.138, 0.648, 0.826), (0.140, 0.568, 0.799), (0.138, 0.648, 0.826)}

Table 5 Score values of $\mathfrak{A}_i$ according to HTSDFWAA and HTSDFWGA values

	HTSDFWAA	HTSDFWGA
SV($\mathfrak{A}_1$)	0.156	−0.208
SV($\mathfrak{A}_2$)	0.076	−0.217
SV($\mathfrak{A}_3$)	0.165	−0.285
SV($\mathfrak{A}_4$)	0.409	−0.305
SV($\mathfrak{A}_5$)	0.063	−0.335
SV($\mathfrak{A}_6$)	0.088	−0.035
SV($\mathfrak{A}_7$)	−0.104	−0.319

Step 3: For $n = 4$ and $\gamma = 1$, HTSDFWAA and HTSDFWGA values of HTSF $_i$, ($i = 1, 2, \dots, 7$) are obtained as in Table 4

Step 4: Score values of $\mathfrak{A}_i$, ($i = 1, 2, \dots, 7$) under score function are obtained as in Table 5.

Step 5: Using Eq. (1), ordering of the candidates is obtained as in Table 6.

Step 6: From the above illustration, although overall ranking values of the alternatives are different through the use of two operators, optimum alternatives are κ_4 and κ_6 for the two operators, respectively.

Analysis of the effect of parameter γ on the results

To show the effect of the γ variable in the formula of HTSDFWAA and HTSDFWGA on MCGDM results, we assign different values to γ from 1 to 10 and order candidates according to score values based on HTSDFWAA and HTSDFWGA. Ranking orders of the candidates according to score

values and their ranking orders based on HTSDFWAA and HTSDFWGA operators are shown in Table 7. It is clear when γ value is changed in the formula HTFD WAS, the optimum candidate is always the same person, and orderings of candidates ($SV(\mathfrak{A}_4) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_6) > SV(\mathfrak{A}_7)$) are same except in condition $\gamma = 1$. By Table 8, it is clear that when the value of γ is changed for HTSDFWGA operator, and the ranking orders of candidates ($SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_4)$) are same except from in case $\gamma = 1$. In addition, best suitable candidate is identical for $1 \leq \gamma \leq 10$.

Graphical representation of Table 7 is given in Fig. 2.

Graphical representation of Table 7 is given in Fig. 3.

Using HTSDFWAA operator and score function, we obtain alternative $\mathfrak{A}_4$ which has maximum score value as an optimum element. Also, using HTSDFWGA operator and score function of HT-SFEs, we obtain alternative $\mathfrak{A}_6$ which has maximum score value. We see that different alternatives which are maximum score values are obtained each of proposed aggregation operators. In Table 5, for $\gamma = 2, 3, 4, \dots, 10$ alternative $\mathfrak{A}_4$ which has minimum score value. In HTSDFWGA operator, since third competent of a TSFE is obtained using Dombi t-conorm and it has a negative effect over score value, alternative $\mathfrak{A}_4$ which has minimum score can be considered optimum element. Furthermore, we can give this relation by $1 - S\mathfrak{A}_i$ for score value obtained using result of HTSDFWGA operator.

Table 6 Score values of $\mathfrak{A}_i$ obtained using HTSDFWAA and HTSDFWGA operators

	Ordering
HTSDFWAA	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_7$)
HTSDFWGA	SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_7$) > SV($\mathfrak{A}_5$)

Table 7 Ranking order for different γ values in the HTSDFWAA operator

γ	SV($\mathfrak{A}_1$)	SV($\mathfrak{A}_2$)	SV($\mathfrak{A}_3$)	SV($\mathfrak{A}_4$)	SV($\mathfrak{A}_5$)	SV($\mathfrak{A}_6$)	SV($\mathfrak{A}_7$)	Ranking order
1	0.273	0.148	0.265	0.573	0.156	0.134	−0.049	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_7$)
2	0.237	0.125	0.235	0.530	0.130	0.118	−0.067	SV4($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)
3	0.273	0.148	0.265	0.573	0.156	0.134	−0.049	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)
4	0.292	0.161	0.281	0.595	0.170	0.143	−0.040	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)
5	0.305	0.169	0.291	0.607	0.178	0.149	−0.033	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)
6	0.313	0.175	0.298	0.615	0.183	0.153	−0.029	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)
7	0.319	0.179	0.303	0.621	0.187	0.157	−0.026	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)
8	0.323	0.182	0.307	0.626	0.190	0.159	−0.023	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)
9	0.327	0.185	0.310	0.629	0.192	0.161	−0.022	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)
10	0.330	0.187	0.312	0.632	0.194	0.162	−0.020	SV($\mathfrak{A}_4$) > SV($\mathfrak{A}_1$) > SV($\mathfrak{A}_3$) > SV($\mathfrak{A}_5$) > SV($\mathfrak{A}_2$) > SV($\mathfrak{A}_6$) > SV($\mathfrak{A}_7$)

Table 8 Ranking order for different γ values in the HTSDFWGA operator

γ	$SV(\mathfrak{A}_1)$	$SV(\mathfrak{A}_2)$	$SV(\mathfrak{A}_3)$	$SV(\mathfrak{A}_4)$	$SV(\mathfrak{A}_5)$	$SV(\mathfrak{A}_6)$	$SV(\mathfrak{A}_7)$	Ranking order
1	-0.316	-0.333	-0.392	-0.465	-0.435	-0.099	-0.434	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_4) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_5)$
2	-0.284	-0.297	-0.363	-0.422	-0.406	-0.076	-0.395	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_4)$
3	-0.316	-0.333	-0.392	-0.465	-0.435	-0.099	-0.434	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_4)$
4	-0.333	-0.352	-0.408	-0.486	-0.449	-0.113	-0.455	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_4)$
5	-0.344	-0.364	-0.417	-0.498	-0.458	-0.123	-0.468	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_4)$
6	-0.350	-0.372	-0.423	-0.507	-0.464	-0.129	-0.477	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_4)$
7	-0.355	-0.378	-0.427	-0.512	-0.468	-0.133	-0.484	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_4)$
8	-0.359	-0.383	-0.431	-0.517	-0.472	-0.137	-0.489	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_4)$
9	-0.362	-0.386	-0.433	-0.520	-0.474	-0.140	-0.492	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_4)$
10	-0.364	-0.389	-0.435	-0.523	-0.476	-0.142	-0.495	$SV(\mathfrak{A}_6) > SV(\mathfrak{A}_1) > SV(\mathfrak{A}_2) > SV(\mathfrak{A}_3) > SV(\mathfrak{A}_5) > SV(\mathfrak{A}_7) > SV(\mathfrak{A}_4)$

Fig. 2 Ranking order for different γ values in the HTSDFWAA operator **Fig. 3** Ranking order for different γ values in the HTSDFWGA operator

Table 9 Comparison table of the HT-SFS with some extensions of fuzzy

	n (degrees of component)			k (length)		Condition
	1	2	> 2	1	> 1	
Fuzzy set [1]	✓	×	×	✓	×	$0 \leq s_k + d_k = 1, i_k = 0$
Intuitionistic fuzzy set [4]	✓	×	×	✓	×	$s_k + i_k + d_k = 1$
Pythagorean fuzzy set [5]	✓	✓	×	✓	×	$0 \leq s_k^2 + d_k^2 \leq 1$
Picture fuzzy set [7]	✓	×	×	✓	×	$0 \leq s_k + i_k + d_k \leq 1$
Spherical fuzzy set [25]	✓	✓	×	✓	×	$0 \leq s_k^2 + i_k^2 + d_k^2 \leq 1$
q-rung orthopair fuzzy set [6]	✓	✓	✓	✓	×	$0 \leq s_k^n + d_k^n \leq 1$
T-spherical fuzzy set [27]	✓	✓	✓	✓	×	$0 \leq s_k^n + i_k^n + d_k^n \leq 1$
Hesitant fuzzy set [42,43]	✓	×	×	✓	✓	$0 \leq s_k + d_k = 1, i_k = 0$
Intuitionistic hesitant fuzzy set [75]	✓	×	×	✓	✓	$0 \leq s_k + d_k = 1$
Hesitant Pythagorean fuzzy set [76]	✓	✓	×	✓	✓	$0 \leq s_k^2 + d_k^2 \leq 1$
q-rung Orthopair hesitant fuzzy set [77]	✓	✓	✓	✓	✓	$0 \leq s_k^n + d_k^n \leq 1$
Picture hesitant fuzzy set [59]	✓	✓	×	✓	✓	$0 \leq s_k + i_k + d_k \leq 1$
Hesitant T-spherical fuzzy set	✓	✓	✓	✓	✓	$0 \leq s_k^n + i_k^n + d_k^n \leq 1$

Comparative analysis and discussion

In this section, we compare HT-SFS with other extensions of the fuzzy sets. Let $\mathbb{T}_H = \{(x, \{(s_k, i_k, d_k) : 1 \leq k \leq l_x\}) : x \in \mathfrak{X}\}$. Then, comparison table can be given as in Table 9.

Here, we see that HT-SFS is an extension of sets specified in the Table 9. Therefore, the set structure defined in this paper has advantages of the other extensions of fuzzy sets specified in Table 9 in the modelling. It also models some problems which cannot be modelled existing set theories. In the following example, relation between these sets is explained.

Conclusion

In this paper, the concept of hesitant T-spherical sets and its set theoretical operations such as union, intersection, and complement have been defined. To be more understandable, some examples are given related to defined operations. Based on Dombi t-norm and Domb t-conorm operations, arithmetic operations between two HT-SFEs and

some aggregation operators such as HTSDFWAA, HTSDFWGA, HTSDFOWAA, and HTSDFOWGA operators have been introduced. Furthermore, an MCGDM method has been developed and presented an application to MCGDM problem involving selecting a person for a vacant position in any department of the university. Obtained results have been compared for different parameters. Also, the proposed set structure has been compared by other extensions of the fuzzy sets and mentioned its advantages. In future, our targets are to study on other aggregation operators, similarity measures,

distance measures, and decision-making method based on TOPSIS, VIKOR, AHP, etc.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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References

1. Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353
2. Gadekallu TR, Gao X-Z (2021) An efficient attribute reduction and fuzzy logic classifier for heart disease and diabetes prediction. Recent Adv Comput Sci Commun 14(1):158–165
3. Sakthidasan K, Gao X-Z, Devabalaji KR, Roopa TM (2021) Energy based random repeat trust computation approach and reliable fuzzy

- and heuristic ant colony mechanism for improving QoS in WSN. *Energy Rep.* <https://doi.org/10.1016/egyr.2021.08.121>
4. Atanassov KT (1986) Intuitionistic fuzzy sets. *Fuzzy Sets Syst* 20(1):87–96
 5. Yager RR (2013) Pythagorean fuzzy subsets (2013) Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS) 57–61
 6. Yager RR (2013) Pythagorean membership grades in multi-criteria decision making. *IEEE Trans Fuzzy Syst* 22(4):958–965
 7. Cuong BC (2013) Picture fuzzy sets-First results, Part 1, In Seminar Neuro-Fuzzy Systems with Applications; Institute of Mathematics. Hanoi, Vietnam, Vietnam Academy of Science and Technology
 8. Cuong BC (2013) Picture fuzzy sets-First results, Part 2, In Seminar Neuro-Fuzzy Systems with Applications; Institute of Mathematics. Hanoi, Vietnam, Vietnam Academy of Science and Technology
 9. Garg H (2017) Some picture fuzzy aggregation operators and their applications to multicriteria decision-making. *Arab J Sci Eng* 42(12):5275–5290
 10. Peng X, Dai J (2017) Algorithm for picture fuzzy multiple attribute decision-making based on new distance measure. *Int J Uncertain Quantif* 7(2):177–187
 11. Wei G (2017) Picture fuzzy aggregation operators and their application to multiple attribute decision making. *J Intell Fuzzy Syst* 33(2):713–724
 12. Wei G (2018) TODIM method for picture fuzzy multiple attribute decision making. *Informatica* 29(3):555–566
 13. Cao G (2020) A multi-criteria picture fuzzy decision-making model for green supplier selection based on fractional programming. *Int J Comput Commun Control* 15(1):1–14
 14. Joshi R (2020) A novel decision-making method using r-norm concept and VIKOR approach under picture fuzzy environment. *Expert Syst Appl* 147:113228
 15. Tian C, Peng J, Zhang W, Zhang S, Wang J (2020) Tourism environmental impact assessment based on improved AHP and picture fuzzy PROMETHEE II methods. *Technol Econ Dev Econ* 26(2):355–378
 16. Wei G (2017) Some cosine similarity measures for picture fuzzy sets and their applications to strategic decision making. *Informatica* 28(3):547–564
 17. Wei G, Gao H (2018) The generalized Dice similarity measures for picture fuzzy sets and their applications. *Informatica* 29(1):107–124
 18. Wei G (2018) Some similarity measures for picture fuzzy sets and their applications. *Iran J Fuzzy Syst* 15(1):77–89
 19. Rafiq M, Ashraf S, Abdullah S, Mahmood T, Muhammad S (2019) The cosine similarity measures of spherical fuzzy sets and their applications in decision making. *J Intell Fuzzy Syst* 36(6):6059–6073
 20. Thao NX (2020) Similarity measures of picture fuzzy sets based on entropy and their application in MCDM. *Pattern Anal Appl* 23(3):1203–1213
 21. Singh P (2015) Correlation coefficients for picture fuzzy sets. *J Intell Fuzzy Syst* 28(2):591–604
 22. Ganie AH, Singh S, Bhatia PK (2020) Some new correlation coefficients of picture fuzzy sets with applications. *Neural Comput Appl* 32:12609–12625
 23. Son LH (2016) Generalized picture distance measure and applications to picture fuzzy clustering. *Appl Soft Comput* 46(C):284–295
 24. Hao ND, Son LH, Thong PH (2016) Some improvements of fuzzy clustering algorithms using picture fuzzy sets and applications for geographic data clustering. *VNU J Sci Comput Sci Commun Eng* 32(3):32–38
 25. Gündoğdu FK, Kahraman C (2019) Spherical fuzzy sets and spherical fuzzy TOPSIS method. *J Intell Fuzzy Syst* 36(1):337–352
 26. Gündoğdu FK, Kahraman C (2020) Spherical fuzzy sets and decision making applications. In: Kahraman C, Cebi S, Cevik Onar S, Oztaysi B, Tolga A, Sari I (eds) *Intelligent and fuzzy techniques in big data analytics and decision making*. INFUS 2019. Advances in intelligent systems and computing. Springer, Cham, p 1029
 27. Mahmood T, Ullah K, Khan Q, Jan N (2019) An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Comput. Appl.* 31:7041–7053
 28. Ullah K, Mahmood T, Jan N (2018) Similarity measures for T-spherical fuzzy sets with applications in pattern recognition. *Symmetry* 10(6):193
 29. Garg H, Munir M, Ullah K, Mahmood T, Jan N (2018) Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators. *Symmetry* 10(12):670
 30. Ullah K, Mahmood T, Jan N, Ali Z (2018) A note on geometric aggregation operators in T-spherical fuzzy environment and their applications in multi-attribute decision making. *J Eng Appl Sci* 37(2):75–86
 31. Ullah K, Hassan N, Mahmood T, Jan N, Hassan M (2019) Evaluation of investment policy based on multi-attribute decision-making using interval valued T-spherical fuzzy aggregation operators. *Symmetry* 11(3):357
 32. Liu P, Khan Q, Mahmood T, Hassan N (2019) T-spherical fuzzy power muirhead mean operator based on novel operational laws and their application in multi-attribute group decision making. *IEEE Access* 7:22613–22632
 33. Wu M, Chen T, Fan J (2020) Divergence measure of T-spherical fuzzy sets and its applications in pattern recognition. *IEEE Access* 8:10208–10221
 34. Zeng S, Garg H, Munir M, Mahmood T, Hussain A (2019) A multi-attribute decision making process with immediate probabilistic interactive averaging aggregation operators of T-spherical fuzzy sets and its application in the selection of solar cells. *Energies* 12(23):4436
 35. Guleria A, Bajaj RK (2019) T-spherical fuzzy soft sets and its aggregation operators with application in decision making. *Sci Iran.* <https://doi.org/10.24200/SCI.2019.53027.3018>
 36. Quek SG, Selvachandran G, Munir M, Mahmood T, Ullah K, Son LH, Thong PH, Kumar R, Priyadarshini I (2019) Multi-attribute multi-perception decision-making based on generalized T-spherical fuzzy weighted aggregation operators on neutrosophic sets. *Mathematics* 7:780
 37. Munir M, Kalsoom H, Ullah K, Mahmood T, Chu YM (2020) T-spherical fuzzy Einstein hybrid aggregation operators and their applications in multi-attribute decision making problems. *Symmetry* 12(3):365
 38. Ullah K, Garg H, Mahmood T, Jan N, Ali Z (2020) Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making. *Soft Comput* 24(3):1647–1659
 39. Ullah K, Mahmood T, Garg H (2020) Evaluation of the performance of search and rescue robots using T-spherical fuzzy hamacher aggregation operators. *Int J Fuzzy Syst* 22(2):570–582
 40. Ali Z, Mahmood T, Yang MS (2020) Complex T-spherical fuzzy aggregation operators with application to multi-attribute decision making. *Symmetry* 12(8):1311
 41. Ozlu S, Karaaslan F (2021) Correlation coefficient of T-spherical type-2 hesitant fuzzy sets and their applications in clustering analysis. *J Ambient Intell Hum Comput.* <https://doi.org/10.1007/s12652-021-02904-8>
 42. Torra V, Narukawa Y On hesitant fuzzy sets and decision. In: 2009 IEEE international conference on fuzzy systems. IEEE, pp 1378–1382
 43. Torra V (2010) Hesitant fuzzy sets. *Int J Intell Syst* 25(6):529–539
 44. Xu Z, Xia M (2011) On distance and correlation measures of hesitant fuzzy information. *Int J Intell Syst* 26(5):410–425
 45. Li D, Zeng W, Li J (2015) New distance and similarity measures on hesitant fuzzy sets and their applications in multiple criteria decision making. *Eng Appl Artif Intell* 40:11–16

46. Zeng S, Xiao Y (2018) A method based on TOPSIS and distance measures for hesitant fuzzy multiple attribute decision making. *Technol Econ Dev Econ* 24(3):969–983
47. Hu J, Yang Y, Zhang X, Chen X (2018) Similarity and entropy measures for hesitant fuzzy sets. *Int Trans Oper Res* 25(3):857–886
48. Xu Z, Xia M (2011) Distance and similarity measures for hesitant fuzzy sets. *Inf Sci* 181(11):2128–2138
49. Zeng W, Li D, Yin Q (2016) Distance and similarity measures between hesitant fuzzy sets and their application in pattern recognition. *Pattern Recognit Lett* 84:267–271
50. Xia M, Xu Z, Chen N (2013) Some hesitant fuzzy aggregation operators with their application in group decision making. *Group Decis Negot* 22:259–279
51. Chen N, Xu Z, Xia M (2013) Interval-valued hesitant preference relations and their applications to group decision making. *Knowl Based Syst* 37:528–540
52. Peng DH, Wang TD, Gao CY, Wang H (2014) Continuous hesitant fuzzy aggregation operators and their application to decision making under interval-valued hesitant fuzzy setting. *Sci World J* 2014:897304. <https://doi.org/10.1155/2014.897304>
53. Mu Z, Zeng S, Baležentis T (2015) A novel aggregation principle for hesitant fuzzy elements. *Knowl Based Syst* 84:134–143
54. Amin F, Fahmi A, Abdullah S, Ali A, Ahmad R, Ghani F (2018) Triangular cubic linguistic hesitant fuzzy aggregation operators and their application in group decision making. *J Intell Fuzzy Syst* 34(4):2401–2416
55. Fahmi A, Abdullah S, Amin F, Ali A, Ahmed R, Shakeel M (2019) Trapezoidal cubic hesitant fuzzy aggregation operators and their application in group decision-making. *J Intell Fuzzy Syst* 36(4):3619–3635
56. Jiang C, Jiang S, Chen J (2019) Interval-valued dual hesitant fuzzy hamacher aggregation operators for multiple attribute decision making. *J Syst Sci Inf* 7(3):227–256
57. Liu HB, Liu Y, Xu L (2020) Dombi interval-valued hesitant fuzzy aggregation operators for information security risk assessment. *Math Probl Eng*. <https://doi.org/10.1155/2020/3198645>
58. Zeng W, Xi Y, Yin Q, Guo P (2018) Weighted dual hesitant fuzzy sets and its application in group decision making. In: 2018 14th international conference on computational intelligence and security (CIS). IEEE, pp 77–82
59. Wang R, Li Y (2018) Picture hesitant fuzzy set and its application to multiple criteria decision-making. *Symmetry* 10(7):295
60. Liang D, Darko AP, Xu Z, Wang M (2019) Aggregation of dual hesitant fuzzy heterogeneous related information with extended Bonferroni mean and its application to MULTIMOORA. *Comput Ind Eng* 135:156–176
61. Liu Y, Rodriguez RM, Alcantud JCR, Qin K, Martinez L (2019) Hesitant linguistic expression soft sets: application to group decision making. *Comput Ind Eng* 136:575–590
62. Qiao J (2019) Hesitant relations: novel properties and applications in three-way decisions. *Inf Sci* 497:165–188
63. Bai W, Ding J, Zhang C (2020) Dual hesitant fuzzy graphs with applications to multi-attribute decision making. *Int J Cogn Comput Eng* 1:18–26
64. Ding Q, Wang YM, Goh M (2020) An extended TODIM approach for group emergency decision making based on bidirectional projection with hesitant triangular fuzzy sets. *Comput Ind Eng* 151:106959
65. Mo X, Zhao H, Xu Z (2020) Feature-based hesitant fuzzy aggregation method for satisfaction with life scale. *Appl Soft Comput* 94:106493
66. Liao H, Jiang L, Fang R, Qin R (2020) A consensus measure for group decision making with hesitant linguistic preference information based on double alpha-cut. *Appl Soft Comput* 98:106890
67. Wu J, Liu F, Rong Y, Liu Y, Liu C (2020) Hesitant fuzzy generalised Bonferroni mean operators based on archimedean copula for multiple-attribute decision-making. *Math Probl Eng* 2020:8712376. <https://doi.org/10.1155/2020/8712376>
68. Liu P, Xu H, Geng Y (2020) Normal wiggly hesitant fuzzy linguistic power Hamy mean aggregation operators and their application to multi-attribute decision-making. *Comput Ind Eng* 140:106224
69. Mahmood T, Ur Rehman U, Ali Z, Chinram R (2020) Jaccard and dice similarity measures based on novel complex dual hesitant fuzzy sets and their applications. *Math Probl Eng* 2020:5920432. <https://doi.org/10.1155/2020/5920432>
70. Wang Z, Nie H, Zhao H (2020) An extended GEDM method with heterogeneous reference points of decision makers and a new hesitant fuzzy distance formula. *Comput Ind Eng* 146:106533
71. Li X, Huang X (2020) A novel three-way investment decisions based on decision-theoretic rough sets with hesitant fuzzy information. *Int J Fuzzy Syst* 22:2708–2719
72. Karamaz F, Karaaslan F (2021) Hesitant fuzzy parameterized soft sets and their applications in decision making. *J Ambient Intell Hum Comput* 12:1869–1878
73. Yager RR (1988) On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Trans Syst Man Cybern* 18(1):183–190
74. Dombi J (1982) A general class of fuzzy operators, the demorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators. *Fuzzy Sets Syst* 8(2):149–163
75. Chen X, Li J, Qian L, Hu X (2016) Distance and similarity measures for intuitionistic hesitant fuzzy sets. *International conference on artificial intelligence: technologies and applications (ICAITA)*
76. Garg G (2018) Hesitant Pythagorean fuzzy sets and their aggregation operators in multiple attribute decision-making. *Int J Uncertain Quantif* 8(3):267–289
77. Yang W, Yongfeng P (2020) New q-Rung orthopair hesitant fuzzy decision making based on linear programming and TOPSIS. *IEEE Access* 8:221299–221311
78. Al-Husseinawi AH (2021) Hesitant T-spherical fuzzy sets and their application in decision-making. M.Sc thesis, Graduate School of Natural and Applied Sciences, Cankiri Karatekin University, Cankiri, Turkey

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