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Decision-making methods based on fuzzy soft competition hypergraphs

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Abstract
Fuzzy soft set theory is an effective framework that is utilized to determine the uncertainty and plays a major role to identify vague objects in a parametric manner. The existing methods to discuss the competitive relations among objects have some limitations due to the existence of different types of uncertainties in a single mathematical structure. In this research article, we define a novel framework of fuzzy soft hypergraphs that export the qualities of fuzzy soft sets to hypergraphs. The effectiveness of competition methods is enhanced with the novel notion of fuzzy soft competition hypergraphs. We study certain types of fuzzy soft competition hypergraphs to illustrate different relations in a directed fuzzy soft network using the concepts of height, depth, union, and intersection simultaneously. We introduce the notions of fuzzy soft k-competition hypergraphs and fuzzy soft neighborhood hypergraphs. We design certain algorithms to compute the strength of competition in fuzzy soft directed graphs that reduce the calculation complexity of existing fuzzy-based non-parameterized models. We analyze the significance of our proposed theory with a decision-making problem. Finally, we present graphical, numerical, as well as theoretical comparison analysis with existing methods that endorse the applicability and advantages of our proposed approach.

Keywords Fuzzy soft competition hypergraphs · Fuzzy soft common enemy hypergraphs · Fuzzy soft neighborhood hypergraphs · Fuzzy soft k-competition hypergraphs · Decision-making

Introduction

Fuzzy set theory, initiated by Zadeh [1] in 1965, is a powerful approach to study partial existence of objects between absolute true and absolute false. This technique of fuzziness has numerous applications in wireless communication for selecting appropriate network, information technology, hydrocarbon industry for food safety and piping risk assessment, artificial neural networks, biotechnology management, social networking, decision analysis for risk assessment caused due to accidental chemical release, economy management, and also applicable in plenty of other research domains. Fuzzy sets are applicable in various domains where it is difficult to study approximate and uncertain relations using crisp set theory. Conclusively, the notion of a fuzzy set as a generalization of a crisp set estimates the occurrence of uncertainty factor in real-world problems from mathematical perspective. Kaufmann [2] initiated the most implemented technique of a fuzzy graph based on fuzzy relations. Rosenfeld [3] introduced a new definition of fuzzy graph considering fuzzy relations based on fuzzy sets. It is important to consider that the uncertainty arising from various sources has different nature and components which cannot be discussed using single mathematical structure of fuzzy set theory. In recent years, a number of theories have been proposed for dealing with such systems in an effective way, for instance, intuitionistic fuzzy sets [4,5], vague sets [6], theory of interval mathematics [5,7], rough set theory [8], etc. All these the-

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ories, however, are associated with an inherent limitation, which is the inadequacy of the parametrization tool.

In 1999, Molodtsov [9] introduced the idea of a soft set in which each element is connected with a parameter. Maji et al. [10] presented a hybrid technique by integrating soft sets with fuzzy sets, and studied the properties and applications of fuzzy soft sets. Kharal and Ahmad [11] presented the notion of mappings on classes of fuzzy soft sets and soft sets. Certain hybrid models including rough soft sets, soft rough sets, and soft rough fuzzy sets by combining the theories of fuzzy sets, rough sets, and soft sets were obtained by Feng et al. [12]. Graph theory is an active domain of research due to its applications in engineering, communication networks, computer science, and biomedical sciences. Graphs are used to paradigm any physical situation engaging the correlation among discrete objects. Digraphs are powerful mathematical structures to depict point-to-point relationships among objects connected in a directed network. Some useful results on fuzzy graphs were explored by Bhattacharya [14]. Mordeson and Nair [15] studied certain operations on fuzzy graphs. Many researchers studied and analyzed the idea of fuzzy graphs in recent decades [16,17]. Certain operations and properties of soft graphs and fuzzy soft graphs were studied by Akram and Nawaz [18].

In 1968, Cohen [19] developed a new mathematical structure of competition graphs to discuss competition among species in ecological networks. The main advantage of this technique is to identify the explicit behavior of objects, especially predator–prey relations. Many researchers studied competition graphs and introduced double competition graphs of digraphs [20], p-competition graphs [21], tolerance competition graphs [22], and m-step competition graphs [23]. However, in all this work, the theory of competition graphs is inadequate to handle competition or relations among three or more objects. The idea of competition hypergraphs was initiated by Sonntag and Teichert [24] in 2004. These are crisp hypergraphs in which nodes and edges are explicitly defined. However, to handle uncertainty and to describe all real-world competitions including predator–prey relations, powerful communities in a social network, rivalries in the business market, and signal influence of wireless tools, the idea of fuzzy sets is widely applied in competition graphs and competition hypergraphs. The idea of fuzzy competition graphs was initiated by Samanta and Pal [25] with some generalizations including fuzzy k-competition graphs and p-competition fuzzy graphs. The researchers have extended the notion of fuzzy competition graphs to intuitionistic fuzzy competition graphs [26], m-step fuzzy competition graphs [27], q-rung orthopair fuzzy competition graphs [28], and complex fuzzy competition graphs [29]. Sarwar et al. [30] proposed the idea of fuzzy competition hypergraphs and discussed its certain invariants with decision-making problems. Nawaz and Akram [31] put forward a new approach

to evaluate competition in several directions using fuzzy soft competition graphs.

The historical progress of various researchers toward the analysis of competition graphs, fuzzy competition graphs, and related extensions is given in Table 1.

The limitations of the existing approaches are to find the solution in a group-wise competitive network in the presence of parameters. Considering the advantages of extensions of FSs and the benefits of competition hypergraphs, we propose a novel technique of FS competition hypergraphs. The researchers are working on various parameterized models including: decision-making based on FS sets [34], fuzzy partition based on fuzzy hypergraphs [35], Hebbian structures based on fuzzy hypergraphs [36], decision-making based on intuitionistic FS sets [37], extensions of fuzzy hypergraphs [38], and bipolar fuzzy competition graphs [39]. For more terminologies and concepts, we refer the reader to [40–43].

Motivation and contribution

To increase the characterization of vagueness and to overcome the limitations entailed in existing fuzzy competition graphs, FS competition graphs, competition hypergraphs, and fuzzy competition hypergraphs, we integrate the notion of fuzzy competition hypergraphs with soft sets. This hybrid model helps to study strength of competition in parameterized directed networks having relations among two or more objects. The proposed model is known as FS competition hypergraph. The main motivation of this article can be summarized by the following points:

1. Fuzzy hypergraph theory is one of the most emerging research areas that have frequent usage in different domains. Due to the inadequacy of parametrization tool, it is difficult to handle complex decision-making problems. To deal with this difficulty, soft set theory is combined with fuzzy competition hypergraphs.

2. In fuzzy competition hypergraphs, we discuss group-wise conflicts, relations, and influences among objects that arise in real-world situations. However, this existing graphical model is insufficient to discuss fuzziness in several directions. To handle this loss of data, there is a need to interpret the existing graphical model in terms of FS competition hypergraphs.

3. The proposed method offers a more rational and effective framework for evaluating the strength of competition in various directions and makes an efficient use of the given information in the presence of parameters. The most important feature of the proposed approach is that it generalizes all the existing techniques.

The main contribution of this article is as follows:

Table 1 Literature review of different techniques of graphs and hypergraphs

Authors	Existing approaches of hypergraphs
Cohen [19]	Introduced the notion of competition graphs to study the competition among species in ecological networks
Sonntag and Teichert [24]	Presented the idea of competition hypergraphs to discuss competition among two or more species
Samanta and Pal [25]	Competition graphs and competition hypergraphs cannot deal with uncertainty among objects. To describe all real-world competitions, fuzzy competition graphs were introduced by integrating powerful techniques of fuzzy sets and competition graphs
Sarwar et al. [30]	Suggested a powerful novel framework of fuzzy competition hypergraphs to sort out the gaps of above-mentioned techniques
Shahzadi et al. [32]	To handle positive as well as negative degrees, the idea of BF competition hypergraphs, as a generalized form of fuzzy competition hypergraphs was introduced
Akram and Nawaz [18]	A new mathematical approach FS graphs based on parameters was presented to define uncertainty in several directions
Sarwar et al. [33]	Due to the loss of important information, e.g., given objects satisfying identical characterization, there is a need to depict the data as a hypergraph under bipolar FS information.
Nawaz and Akram [31]	Introduced fuzzy soft competition graphs to study uncertain competitions in a parametric manner

1. The concept of FS hypergraphs is initiated by integrating the notion of fuzzy hypergraphs with soft set theory.
2. Two FS hypergraphs, named as FS competition hypergraphs and FS common enemy hypergraphs, are defined to discuss competition in decision-making problems.
3. FS neighborhoods graphs are proposed to evaluate the interrelations between FS k-competition hypergraphs and FS neighborhood graphs.
4. The significance of our research work is studied with an application in business marketing. An algorithm is designed to explain the step-by-step procedure of the proposed model.

Framework of the paper

The paper is organized as follows:

1. Section Motivation and contribution gives some important preliminaries related to this study.
2. In Sect. Framework of the paper, the notion of FS hypergraphs and strong hyperedges is discussed with suitable examples.
3. In Sect. Preliminaries, the notions of FS competition hypergraphs, FS common enemy hypergraphs, and generalizations of these two types of fuzzy hypergraphs are discussed.
4. Section Fuzzy soft hypergraphs establishes the notions of FS open neighborhood hypergraphs, FS closed neigh-

borhood hypergraphs, FS k-neighborhood hypergraphs of open and closed types, and FS underlying graphs.
5. In Sect. Fuzzy soft competition hypergraphs, a decision-making method is proposed to study the importance and significance of proposed research study.
6. Section Fuzzy soft neighborhood hypergraphs concerns its effectiveness and provides the comparison of proposed method with some existing techniques.

Preliminaries

In this section, we review some basic terminologies relating to FS sets and FS graphs. Throughout the paper, we will consider $\mathcal{R}$ as a non-empty crisp set, $\mathcal{P}$ as the set of all parameters referring to the objects in $\mathcal{R}$ and $\mathcal{W} \subseteq \mathcal{P}$. A crisp graph $\mathcal{G}^*$ on $\mathcal{R}$ is a pair $(\mathcal{R}, \mathcal{E})$, where $\mathcal{R}$ is called a vertex set and $\mathcal{E} \subseteq \mathcal{R} \times \mathcal{R}$ is the set of all edges of $\mathcal{G}^*$ called edge set.

Definition 1 [40] A fuzzy digraph on $\mathcal{R}$ is a pair $\overrightarrow{\mathcal{J}} = (\mathcal{A}, \overrightarrow{\mathcal{B}})$, where $\mathcal{A}$ is a fuzzy set on $\mathcal{R}$ and $\overrightarrow{\mathcal{B}}$ is a fuzzy relation on $\mathcal{R}$ with the property that

$$\zeta_{\overrightarrow{\mathcal{B}}}(r_1 r_2) \le \zeta_{\mathcal{A}}(r_1) \wedge \zeta_{\mathcal{A}}(r_2), \text{ for all } r_1, r_2 \in \mathcal{R}.$$

Definition 2 [10] Let $P(\mathcal{R})$ represents the power sets of $\mathcal{R}$. A pair $(\phi, \mathcal{W})$ is called a soft set on $\mathcal{R}$, where $\phi : \mathcal{W} \longrightarrow P(\mathcal{R})$ is a mapping called approximate function of the soft

set $(\phi, \mathcal{W})$. In a set of ordered pairs, it is represented as $(\phi, \mathcal{W}) = \{(q, \phi(w)) \mid w \in \mathcal{W}, \phi(w) \subseteq \mathcal{R}\}$.

Definition 3 [10] Let $P_f(\mathcal{R})$ denotes the collection of all fuzzy subsets of $\mathcal{R}$. A pair $(\mathcal{X}, \mathcal{W})$ is called an FS set on $\mathcal{R}$, where $\mathcal{X} : \mathcal{W} \longrightarrow P_f(\mathcal{R})$ is a mapping called fuzzy approximate function of the FS set $(\mathcal{X}, \mathcal{W})$. In a set form, it is represented as

$$(\mathcal{X}, \mathcal{W}) = \{\mathcal{X}(w) \in P_f(\mathcal{R}) \mid w \in \mathcal{W}, \ \mathcal{X}(w) = \phi \ if \ w \notin \mathcal{W}\}.$$

It is also represented as a set of ordered pairs $(\mathcal{X}, \mathcal{W}) = \{(w, \mathcal{X}(w)) \mid w \in \mathcal{W}\}$, where $\mathcal{X}(w) = \{r, \zeta_{\mathcal{X}(w)}(r) \mid r \in \mathcal{R}\}$ is a fuzzy set corresponding to parameter w.

Definition 4 [18] An FS graph on $\mathcal{R}$ is a 3-tuple $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$, where

  (i) $(\mathcal{X}, \mathcal{W})$ is an FS set on $\mathcal{R}$.
 (ii) $(\mathcal{K}, \mathcal{W})$ is an FS relation on $\mathcal{R}$, i.e., $\mathcal{K} : \mathcal{W} \longrightarrow P_f(\mathcal{R} \times \mathcal{R})$, where $P_f(\mathcal{R} \times \mathcal{R})$ is a fuzzy power set on $\mathcal{R} \times \mathcal{R}$.
(iii) For each $w \in \mathcal{W}$, $(\mathcal{X}(w), \mathcal{K}(w))$ is a fuzzy graph. That is

$$\zeta_{\mathcal{K}(w)}(r_1 r_2) \leq \zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2), \text{ for all } r_1, r_2 \in \mathcal{R}.$$

It is denoted as $\mathcal{G}(w) = (\mathcal{X}(w), \mathcal{K}(w))$, where $w \in \mathcal{W}$. Hence, the set of all fuzzy graphs corresponding to parameters $w \in \mathcal{W}$ is called an FS graph $\mathcal{G} = \{\mathcal{G}(w) : w \in \mathcal{W}\}$.

Definition 5 [41] An FS digraph on $\mathcal{R}$ is a 3-tuple $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$, where

  (i) $(\mathcal{X}, \mathcal{W})$ is an FS set on $\mathcal{R}$.
 (ii) $(\overrightarrow{\mathcal{K}}, \mathcal{W})$ is a FS relation on $\mathcal{R}$, i.e., $\overrightarrow{\mathcal{K}} : \mathcal{W} \longrightarrow P_f(\mathcal{R} \times \mathcal{R})$, where $P_f(\mathcal{R} \times \mathcal{R})$ is a fuzzy power set on $\mathcal{R} \times \mathcal{R}$.
(iii) For each $w \in \mathcal{W}$, $(\mathcal{X}(w), \overrightarrow{\mathcal{K}}(w))$ is a fuzzy digraph. That is

$$\zeta_{\overrightarrow{\mathcal{K}}(w)}(r_1 r_2) \leq \zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2), \text{ for all } r_1, r_2 \in \mathcal{R}.$$

It is denoted as $\overrightarrow{\mathcal{G}}(w) = (\mathcal{X}(w), \overrightarrow{\mathcal{K}}(w))$, where $w \in \mathcal{W}$. Hence, the set of all fuzzy digraphs corresponding to parameters $w \in \mathcal{W}$ is called an FS digraph $\overrightarrow{\mathcal{G}} = \{\overrightarrow{\mathcal{G}}(w) : w \in \mathcal{W}\}$.

Definition 6 [31] An FS out neighborhood of a vertex r of an FS digraph $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ is an FS set

$$(\mathcal{N}^+(r), \mathcal{W}) = \{(w, \mathcal{N}^+(r)(w)) \mid w \in \mathcal{W}\},$$

where

$$\mathcal{N}^+(r)(w) = \{r', \zeta_{\overrightarrow{\mathcal{K}}(w)}(rr') \mid \zeta_{\overrightarrow{\mathcal{K}}(w)}(rr') > 0, w \in \mathcal{W}\}.$$

Definition 7 [31] An FS in neighborhood of a vertex r of an FS digraph $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ is an FS set

$$(\mathcal{N}^-(r), \mathcal{W}) = \{(w, \mathcal{N}^-(r)(w)) \mid w \in \mathcal{W}\},$$

where

$$\mathcal{N}^-(r)(w) = \{r', \zeta_{\overrightarrow{\mathcal{K}}(w)}(r'r) \mid \zeta_{\overrightarrow{\mathcal{K}}(w)}(r'r) > 0, w \in \mathcal{W}\}.$$

Definition 8 [42] The cardinality of an FS set $(\mathcal{X}, \mathcal{W})$ is defined as follows:

$$|(\mathcal{X}, \mathcal{W})| = \sum_{w \in \mathcal{W}} |\mathcal{X}(w)|,$$

where $|\mathcal{X}(w)| = \sum_{r \in \mathcal{R}} \zeta_{\mathcal{X}(w)}(r)$ for each $w \in \mathcal{W}$.

Definition 9 [31] The support of a FS set $(\mathcal{X}, \mathcal{W})$ is defined as follows:

$$supp(\mathcal{X}, \mathcal{W}) = \{(q, supp(\mathcal{X}(w))) \mid w \in \mathcal{W}\},$$

where $supp(\mathcal{X}(w)) = \{r \in \mathcal{R} \mid \zeta_{\mathcal{X}(w)}(r) > 0\}$ for each $w \in \mathcal{W}$.

Definition 10 [31] The height of an FS set $(\mathcal{X}, \mathcal{W})$ is defined as follows:

$$h(\mathcal{X}, \mathcal{W}) = \max_{w \in \mathcal{W}} h(\mathcal{X}(w)),$$

where $h(\mathcal{X}(w)) = \max_{r \in \mathcal{R}}(\zeta_{\mathcal{X}(w)}(r))$ for each $w \in \mathcal{W}$.

Fuzzy soft hypergraphs

In this section, we define an FS hypergraph technique for FS competition and common enemy hypergraphs. Strong hyperedges are evaluated with numerical examples which are also discussed. These terms are defined as follows:

Definition 11 Let $(\mathcal{X}, \mathcal{W})$ be an FS set on $\mathcal{R}$ and $(\mathcal{K}, \mathcal{W})$ is an FS relation on $\mathcal{R}$. We say $(\mathcal{K}, \mathcal{W})$ is an FS relation on $(\mathcal{X}, \mathcal{W})$ if it satisfies the following condition:

$$\zeta_{\mathcal{K}(w)}(r_1 r_2) \leq \zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2),$$

for all $r_1, r_2 \in \mathcal{R}$ and $w \in \mathcal{W}$.

Definition 12 An FS hypergraph on $\mathcal{R}$ is a 3-tuple $\mathcal{D} = (\mathcal{U}, \mathcal{V}, \mathcal{W})$, where $\mathcal{U} = \{(\mathcal{U}_1, \mathcal{W}), (\mathcal{U}_2, \mathcal{W}) \cdots, (\mathcal{U}_s, \mathcal{W})\}$ is a collection of FS subsets on $\mathcal{R}$, such that $\bigcup_{w \in \mathcal{W}} \bigcup_{1 \leq i \leq s} supp$

Table 2 FS subsets corresponding to w_1

$r \in \mathcal{R}$	$\mathcal{U}_1(w_1)$	$\mathcal{U}_2(w_1)$	$\mathcal{U}_3(w_1)$	$\mathcal{U}_4(w_1)$	$\mathcal{U}_5(w_1)$
r_1	0.2	0	0	0	0
r_2	0.3	0	0	0	0
r_3	0.3	0.3	0	0	0
r_4	0	0.1	0	0	0
r_5	0	0.3	0.3	0	0
r_6	0.9	0	0.9	0.9	0
r_7	0	0	0	0.2	0.2

Table 3 FS subsets corresponding to w_2

$r \in \mathcal{R}$	$\mathcal{U}_1(w_2)$	$\mathcal{U}_2(w_2)$	$\mathcal{U}_3(w_2)$	$\mathcal{U}_4(w_2)$	$\mathcal{U}_5(w_2)$
r_1	0.2	0	0	0	0
r_2	0.3	0	0	0	0
r_3	0.3	0.3	0	0	0
r_4	0	0.2	0	0	0
r_5	0	0.3	0.3	0.3	0
r_6	0.6	0	0.6	0	0
r_7	0	0	0	0.2	0.2

Fig. 1 FS hypergraph $\mathcal{D} = \{\mathcal{D}(w_1), \mathcal{D}(w_2)\}$

$(\mathcal{U}_i(w)) = \mathcal{R}$. $(\mathcal{V}, \mathcal{W})$ is a, FS relation on FS subsets $(\mathcal{U}_i, \mathcal{W})$, such that

$$\zeta_{\mathcal{V}(w)}(E_i) = \zeta_{\mathcal{V}(w)}(r_1, r_2, \cdots, r_p)$$
$$\leq \min\{\zeta_{\mathcal{U}_i(w)}(r_1), \zeta_{\mathcal{U}_i(w)}(r_2), \cdots, \zeta_{\mathcal{U}_i(w)}(r_p)\},$$

for all $r_1, r_2, \cdots, r_p \in \mathcal{R}$ and $w \in \mathcal{W}$.

Example 1 Let $\mathcal{R} = \{r_1, r_2, r_3, r_4, r_5, r_6, r_7\}$ be a crisp set and $\mathcal{W} = \{w_1, w_2\}$ is a set of parameter. Suppose $\{(\mathcal{U}_1, \mathcal{W}), (\mathcal{U}_2, \mathcal{W}), (\mathcal{U}_3, \mathcal{W}), (\mathcal{U}_4, \mathcal{W}), (\mathcal{U}_5, \mathcal{W})\}$ be the collection of FS sets on $\mathcal{R}$ as specified in Tables 2 and 3.

Let $(\mathcal{V}, \mathcal{W})$ be an FS relation on $(\mathcal{U}_i, \mathcal{W})$, $1 \leq i \leq 5$, given as $\zeta_{\mathcal{V}(w_1)}(\{r_1, r_2, r_3, r_6\}) = 0.2$, $\zeta_{\mathcal{V}(w_1)}(\{r_3, r_4, r_5\}) = 0.1$, $\zeta_{\mathcal{V}(w_1)}(\{r_5, r_6\}) = 0.3$, $\zeta_{\mathcal{V}(w_1)}(\{r_6, r_7\}) = 0.2$, $\zeta_{\mathcal{V}(w_1)}(\{r_7\}) = 0.2$, $\zeta_{\mathcal{V}(w_2)}(\{r_1, r_2, r_3, r_6\}) = 0.2$, $\zeta_{\mathcal{V}(w_2)}(\{r_3, r_4, r_5\}) = 0.2$, $\zeta_{\mathcal{V}(w_2)}(\{r_5, r_6\}) = 0.2$, $\zeta_{\mathcal{V}(w_2)}(\{r_5, r_7\}) = 0.2$, and $\zeta_{\mathcal{V}(w_2)}(\{r_7\}) = 0.2$. The FS hypergraph $\mathcal{D} = \{\mathcal{D}(w_1), \mathcal{D}(w_2)\}$ is shown in Fig. 1. Here, $\mathcal{D}(w_1)$ is a fuzzy hypergraph corresponding to parameter w_1 and $\mathcal{D}(w_2)$ is a fuzzy hypergraph corresponding to parameter w_2.

Definition 13 Let $\mathcal{D} = (\mathcal{U}, \mathcal{V}, \mathcal{W})$ be an FS hypergraph on $\mathcal{R}$. A hyperedge $E_i = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ of an FS hypergraph $\mathcal{D}$ is called strong if $\zeta_{\mathcal{V}(w)}(E_i) \geq \frac{1}{2} \bigwedge_{k=1}^{p} \zeta_{\mathcal{U}_i(w)}(r_k)$ for all $w \in \mathcal{W}$.
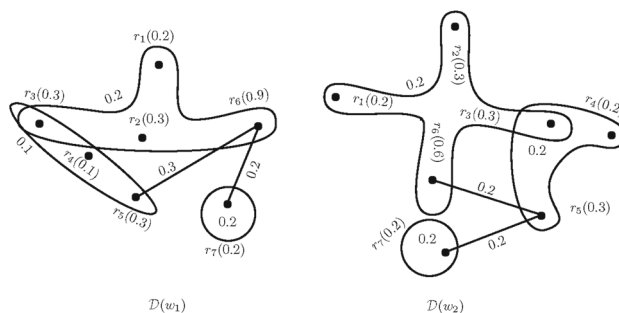
Example 2 Consider an FS hypergraph $\mathcal{D}$ as specified in Fig. 1. Since $E_1 = \{r_1, r_2, r_3, r_6\}$ is a hyperedge of a FS hypergraph $\mathcal{D}$

$$\zeta_{\mathcal{V}(w_1)}(E_1) = \zeta_{\mathcal{V}(w_1)}(\{r_1, r_2, r_3, r_6\}) = 0.2,$$
$$\zeta_{\mathcal{U}_1(w_1)}(r_1) \wedge \zeta_{\mathcal{U}_1(w_1)}(r_2) \wedge \zeta_{\mathcal{U}_1(w_1)}(r_3) \wedge \zeta_{\mathcal{U}_1(w_1)}(r_6)$$
$$= 0.2, 0.2 > 0.1.$$
$$\zeta_{\mathcal{V}(w_2)}(E_1) = \zeta_{\mathcal{V}(w_2)}(\{r_1, r_2, r_3, r_6\}) = 0.2,$$
$$\zeta_{\mathcal{U}_1(w_2)}(r_1) \wedge \zeta_{\mathcal{U}_1(w_2)}(r_2) \wedge \zeta_{\mathcal{U}_1(w_2)}(r_3) \wedge \zeta_{\mathcal{U}_1(w_2)}(r_6) = 0.2,$$
$$0.2 > 0.1.$$

Therefore, the hyperedge $E_1 = \{r_1, r_2, r_3, r_6\}$ is strong in an FS hypergraph $\mathcal{D}$. Similarly, $\{r_3, r_4, r_5\}$, $\{r_5, r_6\}$, and $\{r_7\}$ are strong hyperedges in an FS hypergraph $\mathcal{D}$.

Fuzzy soft competition hypergraphs

To resolve competition difficulties in several directions, FS competition hypergraphs and FS common enemy hypergraphs are discussed in this section. Different consequences of strong hyperedges are evaluated by utilizing the definition of FS strong hyperedges. Two generalizations of FS hypergraphs are discussed to investigate the strength of competition using crisp values.

Definition 14 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$. The FS competition hypergraph $\mathcal{CH}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{J}, \mathcal{W})$ corresponding to $\overrightarrow{\mathcal{G}}$ containing the FS vertex set same as in FS digraph $\overrightarrow{\mathcal{G}}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is a hyperedge of $\mathcal{CH}(\overrightarrow{\mathcal{G}}(w)) = (\mathcal{X}(w), \mathcal{J}(w))$ if $\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w) \neq \phi$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ for each parameter w is defined as

$$\zeta_{\mathcal{J}(w)}(E) = [\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)).$$

The method for the formation of FS competition hypergraph $\mathcal{CH}(\overrightarrow{\mathcal{G}})$ of an FS digraph $\overrightarrow{\mathcal{G}}$ is explained in Algorithm 4.

Method for construction of FS competition hypergraph

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1. Input the FS set $(\mathcal{X}, \mathcal{W})$ on set of vertices $\mathcal{R} = \{r_1, r_2, \cdots, r_n\}$.
2. For each $w \in \mathcal{W}$, the adjacency matrix $A(w) = [r_{ij}]_{n \times n}$ of $\overrightarrow{\mathcal{G}}(w) = (\mathcal{X}(w), \overrightarrow{\mathcal{K}}(w))$ such that $\zeta_{\overrightarrow{\mathcal{K}}(w)}(r_i r_j) = r_{ij}$ as shown in Table 4.
3. For each $w \in \mathcal{W}$, define a relation such that $f(w) : \mathcal{R} \longrightarrow \mathcal{R}$ by $f(w)(r_i) = r_j$ if $r_{ij} > 0$.
4. If $r_{ij} > 0$ then (r_j, r_{ij}) belongs to the FS out neighborhood $\mathcal{N}^+(r_i)(w)$ corresponding to parameter w.
5. For each $w \in \mathcal{W}$, determine the family of sets such that $\mathcal{Z} = \{E_i = f^{-1}(w)(r_i) : |f^{-1}(w)(r_i)| \geq 2, r_i \in \mathcal{R}\}$, where $E_i = \{r_{i_1}, r_{i_2}, \cdots, r_{i_r}\}$ is a hyperedge of $\mathcal{CH}(\overrightarrow{\mathcal{G}}(w))$.
6. Compute the grade of membership of each hyperedge corresponding to parameter w by using Definition 14.

Example 3 Let $\mathcal{R} = \{r_1, r_2, r_3, r_4, r_5\}$ is a crisp set and $\mathcal{W} = \{w_1, w_2\}$ is a set of parameter. Suppose $(\mathcal{X}, \mathcal{W})$ is an FS set on $\mathcal{R}$ and $(\overrightarrow{\mathcal{K}}, \mathcal{W})$ is an FS relation on $\mathcal{R}$ as specified in Tables 5 and 6, respectively.

The FS digraph $\overrightarrow{\mathcal{G}} = \{\overrightarrow{\mathcal{G}}(w_1), \overrightarrow{\mathcal{G}}(w_2)\}$ is given in Fig. 2. Here, $\overrightarrow{\mathcal{G}}(w_1) = (\mathcal{X}(w_1), \overrightarrow{\mathcal{K}}(w_1))$ and $\overrightarrow{\mathcal{G}}(w_2) = (\mathcal{X}(w_2), \overrightarrow{\mathcal{K}}(w_2))$ are fuzzy digraphs corresponding to parameters w_1 and w_2, respectively.

The FS out neighborhood of all vertices of $\overrightarrow{\mathcal{G}}$ are specified in Table 7.

Table 4 Adjacency matrix

$A(w)$	r_1	r_2	...	r_n
r_1	r_{11}	r_{12}	...	r_{1n}
r_2	r_{21}	r_{22}	...	r_{2n}
.	.	.	...	.
.	.	.	...	.
.	.	.	...	.
r_n	r_{n1}	r_{n2}	...	r_{nn}

Table 5 FS set on $\mathcal{R}$

r	$\mathcal{X}(w_1)$	r	$\mathcal{X}(w_2)$
r_1	0.8	r_1	0.4
r_2	0.4	r_2	0.6
r_3	0.6	r_3	0.9
r_4	0.3	r_4	0.2
r_5	1.0	r_5	0.8

Table 6 FS relation on $\mathcal{R}$

r	$\overrightarrow{\mathcal{K}}(w_1)$	r	$\overrightarrow{\mathcal{K}}(w_2)$
$r_1 r_3$	0.6	$r_1 r_3$	0.2
$r_2 r_1$	0.4	$r_2 r_3$	0.5
$r_2 r_5$	0.3	$r_3 r_4$	0.1
$r_3 r_4$	0.2	$r_5 r_1$	0.3
$r_4 r_5$	0.1	$r_5 r_2$	0.6
$r_5 r_1$	0.7	$r_5 r_3$	0.6
$r_5 r_3$	0.5		
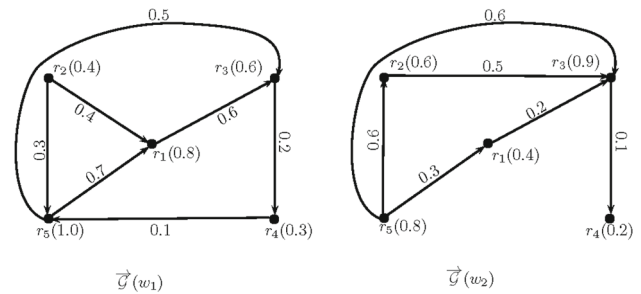
Fig. 2 FS digraph $\overrightarrow{\mathcal{G}} = \{\overrightarrow{\mathcal{G}_1}(w_1), \overrightarrow{\mathcal{G}_1}(w_2)\}$

The adjacency matrices $A(w_1)$ and $A(w_2)$ corresponding to fuzzy digraphs $\overrightarrow{\mathcal{G}}(w_1)$ and $\overrightarrow{\mathcal{G}}(w_2)$ are specified in Table 8 and 9, respectively. Hence, $A = \{A(w_1), A(w_2)\}$ is a adjacency matrix of FS digraph $\overrightarrow{\mathcal{G}}$.

Using Algorithm 4, the relations $f(w_1) : \mathcal{R} \longrightarrow \mathcal{R}$ and $f(w_2) : \mathcal{R} \longrightarrow \mathcal{R}$ are given in Fig. 3.

Using fuzzy relation in $\overrightarrow{\mathcal{G}}(w_1)$, there are three hyperedges $f^{-1}(w_1)(r_1) = E_1 = \{r_2, r_5\}$, $f^{-1}(w_1)(r_3) = E_3 = \{r_1, r_5\}$, and $f^{-1}(w_1)(r_5) = E_5 = \{r_2, r_4\}$ of $\mathcal{CH}(\overrightarrow{\mathcal{G}}(w_1)) = (\mathcal{X}(w_1), \mathcal{J}(w_1))$. Now, we calculate the membership grade of each hyperedge corresponding to parameter w_1

$$\zeta_{\mathcal{J}(w_1)}(E_1) = \zeta_{\mathcal{X}(w_1)}(r_2) \wedge \zeta_{\mathcal{X}(w_1)}(r_5)$$
$$\times h(\mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_5)(w_1)) = 0.4 \times 0.4 = 0.16.$$

$$\zeta_{\mathcal{J}(w_1)}(E_3) = \zeta_{\mathcal{X}(w_1)}(r_1) \wedge \zeta_{\mathcal{X}(w_1)}(r_5)$$
$$\times h(\mathcal{N}^+(r_1)(w_1) \cap \mathcal{N}^+(r_5)(w_1)) = 0.8 \times 0.5 = 0.4.$$

$$\zeta_{\mathcal{J}(w_1)}(E_5) = \zeta_{\mathcal{X}(w_1)}(r_2) \wedge \zeta_{\mathcal{X}(w_1)}(r_4)$$
$$\times h(\mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_4)(w_1)) = 0.3 \times 0.1 = 0.03.$$

Using fuzzy relation in $\overrightarrow{\mathcal{G}}(w_2)$, there is only one hyperedge $f^{-1}(w_2)(r_3) = E_3 = \{r_1, r_2, r_5\}$ of $\mathcal{CH}(\overrightarrow{\mathcal{G}}(w_2)) = (\mathcal{X}(w_2), \mathcal{J}(w_2))$. Now, we calculate the membership grade of hyperedge E_3 corresponding to parameter w_2

$$\zeta_{\mathcal{J}(w_2)}(E_3) = \zeta_{\mathcal{X}(w_2)}(r_1) \wedge \zeta_{\mathcal{X}(w_2)}(r_2) \wedge \zeta_{\mathcal{X}(w_2)}(r_5)$$
$$\times h(\mathcal{N}^+(r_1)(w_2)$$

Table 7 FS out neighborhood of vertices in $\overrightarrow{\mathcal{G}}$

$r \in \mathcal{R}$	$\mathcal{N}^+(r)(w_1)$	$\mathcal{N}^+(r)(w_2)$	$(\mathcal{N}^+(r), \mathcal{W})$
r_1	$\{(r_3, 0.6)\}$	$\{(r_3, 0.2)\}$	$\{\frac{(r_3, 0.6)}{w_1}, \frac{(r_3, 0.2)}{w_2}\}$
r_2	$\{(r_1, 0.4), (r_5, 0.3)\}$	$\{(r_3, 0.5)\}$	$\{\frac{(r_1, 0.4), (r_5, 0.3)}{w_1}, \frac{(r_3, 0.5)}{w_2}\}$
r_3	$\{(r_4, 0.2)\}$	$\{(r_4, 0.1)\}$	$\{\frac{(r_4, 0.2)}{w_1}, \frac{(r_4, 0.1)}{w_2}\}$
r_4	$\{(r_5, 0.1)\}$	$\{\}$	$\{\frac{(r_5, 0.1)}{w_1}, \frac{\{\}}{w_2}\}$
r_5	$\{(r_1, 0.7), (r_3, 0.5)\}$	$\{(r_1, 0.3), (r_2, 0.6), (r_3, 0.6)\}$	$\{\frac{(r_1, 0.7), (r_3, 0.5)}{w_1}, \frac{(r_1, 0.3), (r_2, 0.6), (r_3, 0.6)}{w_2}\}$

Table 8 Adjacency matrix of $\overrightarrow{\mathcal{G}}(w_1)$

$A(w_1)$	r_1	r_2	r_3	r_4	r_5
r_1	0	0	0.6	0	0
r_2	0.4	0	0	0	0.3
r_3	0	0	0	0.2	0
r_4	0	0	0	0	0.1
r_5	0.7	0	0.5	0	0

Table 9 Adjacency matrix of $\overrightarrow{\mathcal{G}}(w_2)$

$A(w_2)$	r_1	r_2	r_3	r_4	r_5
r_1	0	0	0.2	0	0
r_2	0	0	0.5	0	0
r_3	0	0	0	0.1	0
r_4	0	0	0	0	0
r_5	0.3	0.6	0.6	0	0

$$\cap \mathcal{N}^+(r_2)(w_2) \cap \mathcal{N}^+(r_5)(w_2))$$
$$= 0.4 \times 0.2 = 0.08.$$

The FS competition hypergraph $\mathcal{CH}(\overrightarrow{\mathcal{G}}) = (\mathcal{CH}(\overrightarrow{\mathcal{G}}(w_1)), \mathcal{CH}(\overrightarrow{\mathcal{G}}(w_2)))$ is given in Fig. 4.

Definition 15 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$. The FS common enemy hypergraph $\mathcal{CEH}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{A}, \mathcal{W})$ corresponding to $\overrightarrow{\mathcal{G}}$ containing the FS vertex set same as in FS digraph $\overrightarrow{\mathcal{G}}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is a hyperedge of $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w)) = (\mathcal{X}(w), \mathcal{A}(w))$ if $\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w) \neq \phi$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ corresponding to parameter w is defined as

$$\zeta_{\mathcal{A}(w)}(E) = [\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)).$$

The method for the construction of FS common enemy hypergraph $\mathcal{CEH}(\overrightarrow{\mathcal{G}})$ of a FS digraph $\overrightarrow{\mathcal{G}}$ is given in Algorithm 4.

Method for construction of FS common enemy hypergraph

1. Follow first two steps of Algorithm 4.
2. For each $w \in \mathcal{W}$, define a relation such that $f(w)$: $\mathcal{R} \longrightarrow \mathcal{R}$ by $f(w)(r_i) = r_j$ if $r_{ij} > 0$.
3. If $r_{ij} > 0$ then (r_i, r_{ij}) belongs to the FS in neighborhood $\mathcal{N}^-(r_j)(w)$ corresponding to parameter w.
4. For each $w \in \mathcal{W}$, determine the family of sets such that $\mathcal{Z} = \{E_i = f(w)(r_i) : |f(w)(r_i)| \geq 2, r_i \in \mathcal{R}\}$, where $E_i = \{r_{i_1}, r_{i_2}, \cdots, r_{i_r}\}$ is a hyperedge of $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w))$.
5. Compute the grade of membership of each hyperedge corresponding to parameter w using Definition 15.

Example 4 Consider an FS digraph $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ as given in Fig. 2. The FS in neighborhood of all vertices of $\overrightarrow{\mathcal{G}}$ are specified in Table 10.

Using Algorithm 4, we compute the hyperedges of $\mathcal{CEH}(\overrightarrow{\mathcal{G}})$. Using fuzzy relation in $\overrightarrow{\mathcal{G}}(w_1)$, there are two hyperedges $f(w_1)(r_2) = E_2 = \{r_1, r_5\}$ and $f(w_1)(r_5) = E_5 = \{r_1, r_3\}$ of $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_1))$. Now, we compute the membership grades of these two hyperedges corresponding to parameter w_1

$$\zeta_{\mathcal{A}(w_1)}(E_2) = \zeta_{\mathcal{X}(w_1)}(r_1) \wedge \zeta_{\mathcal{X}(w_1)}(r_5)$$
$$\times h(\mathcal{N}^-(r_1)(w_1) \cap \mathcal{N}^-(r_5)(w_1)) = 0.8 \times 0.3 = 0.24.$$
$$\zeta_{\mathcal{A}(w_1)}(E_5) = \zeta_{\mathcal{X}(w_1)}(r_1) \wedge \zeta_{\mathcal{X}(w_1)}(r_3)$$
$$\times h(\mathcal{N}^-(r_1)(w_1) \cap \mathcal{N}^-(r_3)(w_1)) = 0.6 \times 0.5 = 0.3.$$

Using fuzzy relation in $\overrightarrow{\mathcal{G}}(w_2)$, there is only one hyperedge $f(w_2)(r_5) = E_5 = \{r_1, r_2, r_3\}$ of $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_2))$. Now, we compute the membership grade of this hyperedge corresponding to parameter w_2

$$\zeta_{\mathcal{A}(w_2)}(E_5) = \zeta_{\mathcal{X}(w_2)}(r_1) \wedge \zeta_{\mathcal{X}(w_2)}(r_2) \wedge \zeta_{\mathcal{X}(w_2)}(r_3)$$
$$\times h(\mathcal{N}^-(r_1)(w_2)$$
$$\cap \mathcal{N}^-(r_2)(w_2) \cap \mathcal{N}^-(r_3)(w_2))$$
$$= 0.4 \times 0.3 = 0.12.$$

The FS common enemy hypergraph $\mathcal{CEH}(\overrightarrow{\mathcal{G}}) = (\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_1)), \mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_2)))$ is given in Fig. 5.

Fig. 3 Representation of FS relation in $\overrightarrow{\mathcal{G}}$
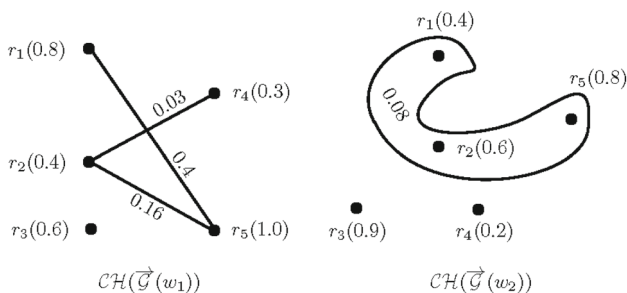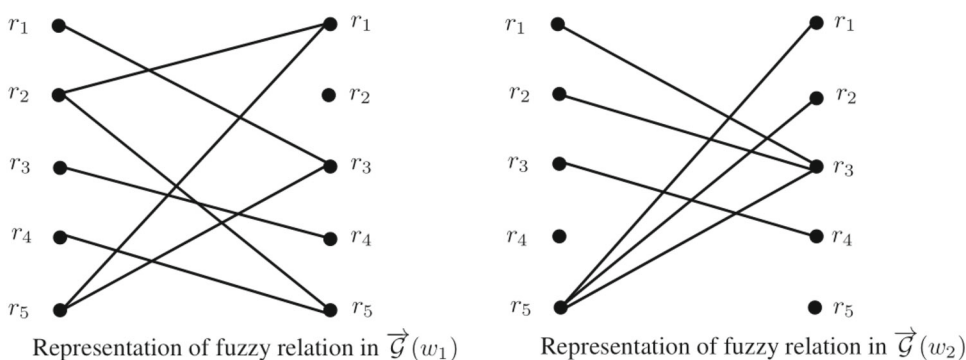

Representation of fuzzy relation in $\overrightarrow{\mathcal{G}}(w_1)$

Representation of fuzzy relation in $\overrightarrow{\mathcal{G}}(w_2)$

Fig. 4 FS competition hypergraph $\mathcal{CH}(\overrightarrow{\mathcal{G}})$
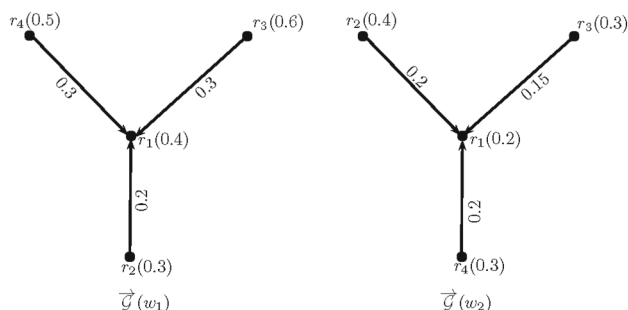
Fig. 6 FS digraph $\overrightarrow{\mathcal{G}} = \{\overrightarrow{\mathcal{G}}(w_1), \overrightarrow{\mathcal{G}}(w_2)\}$
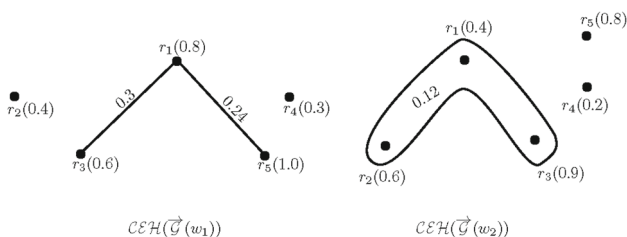
Fig. 5 FS common enemy hypergraph $\mathcal{CEH}(\overrightarrow{\mathcal{G}})$

Theorem 1 *Let* $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ *be an FS digraph on* $\mathcal{R}$.

(1) If for each $w \in \mathcal{W}$, $\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)$ *contains only one vertex. Then, the hyperedge* $E = \{r_1, r_2, \cdots, r_p\}$ *of an FS competition hypergraph* $\mathcal{CH}(\overrightarrow{\mathcal{G}})$ *is strong iff* $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)^+(w)| > \frac{1}{2}$ *for all* $w \in \mathcal{W}$.

(2) If for each $w \in \mathcal{W}$, $\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)$ *contains only one vertex. Then, the hyperedge* $E = \{r_1, r_2, \cdots, r_p\}$ *of an FS common enemy hypergraph* $\mathcal{CEH}(\overrightarrow{\mathcal{G}})$ *is strong iff* $|\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)^-(w)| > \frac{1}{2}$ *for all* $w \in \mathcal{W}$.

Proof (1) Let $\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w) = \{r, \zeta_{\mathcal{X}(w)}(r)\}$ for all $w \in \mathcal{W}$. For each $w \in \mathcal{W}$, $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)| = \{\zeta_{\mathcal{X}(w)}(r)\}$. Since $h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap$

$\mathcal{N}^+(r_p)(w)) = \zeta_{\mathcal{X}(w)}(r)$ for all $w \in \mathcal{W}$. Therefore

$$\zeta_{\mathcal{J}(w)}(E) = \zeta_{\mathcal{X}(w)}(r) \times [\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)],$$

for all $w \in \mathcal{W}$. Hence, the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ in $\mathcal{CH}(\overrightarrow{\mathcal{G}})$ is strong iff $\zeta_{\mathcal{X}(w)}(r) > 0.5$ for all $w \in \mathcal{W}$.

(2) Let $\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w) = \{r, \zeta_{\mathcal{X}(w)}(r)\}$ for all $w \in \mathcal{W}$. For each $w \in \mathcal{W}$, $|\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)| = \{\zeta_{\mathcal{X}(w)}(r)\}$. Since $h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)) = \zeta_{\mathcal{X}(w)}(r)$ for all $w \in \mathcal{W}$. Therefore

$$\zeta_{\mathcal{A}(w)}(E) = \zeta_{\mathcal{X}(w)}(r) \times [\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)],$$

for all $w \in \mathcal{W}$. Hence, the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ in $\mathcal{CEH}(\overrightarrow{\mathcal{G}})$ is strong iff $\zeta_{\mathcal{X}(w)}(r) > 0.5$ for all $w \in \mathcal{W}$. $\square$

We now discuss an example that if all the edges of an FS digraph are strong, then it is not necessary the hyperedges of an FS competition hypergraph are strong.

Example 5 Consider $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph as given in Fig. 6.

Clearly, all edges of FS digraph are strong. The FS competition hypergraph corresponding to $\overrightarrow{\mathcal{G}}$ is depicted in Fig. 5.

FS competition hypergraph $\mathcal{CH}(\overrightarrow{\mathcal{G}})$

The hyperedge $\{r_2, r_3, r_4\}$ of FS competition hypergraph is not strong, because $0.06 \not\geq 0.15$ and $0.045 \not\geq 0.15$.

Theorem 2 *If $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be a complete FS digraph, then the FS competition hypergraph and FS common enemy hypergraph corresponding to $\overrightarrow{\mathcal{G}}$ are same.*

Proof Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be a complete FS digraph on $\mathcal{R}$. Now, we want to proof that FS competition hypergraph $\mathcal{CH}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{J}, \mathcal{W})$ and FS common enemy hypergraph $\mathcal{CEH}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{A}, \mathcal{W})$ are same. Since $\mathcal{N}^+(r)(w) = \mathcal{N}^-(r)(w)$ for all $r \in \mathcal{R}$ and $w \in \mathcal{W}$. Then, for each $w \in \mathcal{W}$ and $r_1, r_2, \cdots, r_p \in \mathcal{R}$, $\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w) = \mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)$. Therefore

$$
\begin{aligned}
&[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)] \\
&\quad \times h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)) \\
&= [\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)] \\
&\quad \times h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)).
\end{aligned}
$$

This shows that $\zeta_{\mathcal{J}(w)}(E) = \zeta_{\mathcal{A}(w)}(E)$ for all $r_1, r_2, \cdots, r_p \in \mathcal{R}$ and $w \in \mathcal{W}$. Hence, it follows that $\mathcal{CH}(\overrightarrow{\mathcal{G}}) = \mathcal{CEH}(\overrightarrow{\mathcal{G}})$.

$\square$

The extensions of FS competition hypergraphs and FS common enemy hypergraphs named as FS k-competition hypergraphs and FS k-common enemy hypergraphs are given in Definition 16 and 17.

Definition 16 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$ and suppose k be a non-negative number. The FS k-competition hypergraphs $\mathcal{C}_k(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{M}, \mathcal{W})$ of $\overrightarrow{\mathcal{G}}$ containing the FS vertex set same as in FS digraph $\overrightarrow{\mathcal{G}}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is the hyperedge of $\mathcal{C}_k(\overrightarrow{\mathcal{G}}(w)) = (\mathcal{X}(w), \mathcal{M}(w))$ if $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap$

$\cdots \cap \mathcal{N}^+(r_p)(w)| > k$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ for each parameter w is defined as

$$
\begin{aligned}
\zeta_{\mathcal{M}(w)}(E) =& \frac{k_1 - k}{k_1}[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)] \\
&\times h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)),
\end{aligned}
$$

where $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)| = k_1$.

Example 6 Consider $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph as given in Fig. 2. Since we know that $E_1 = \{r_2, r_5\}$, $E_3 = \{r_1, r_5\}$, and $E_5 = \{r_2, r_4\}$ (see Example 3). Now, we check whether these are hyperedges of $\mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}}(w_1)) = (\mathcal{X}(w_1), \mathcal{M}(w_1))$ or not. Here

$$
\begin{aligned}
&\mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_5)(w_1) = \{(r_1, 0.4)\}, \\
&|\mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_5)(w_1)| = k_1 = 0.4.
\end{aligned}
$$

Therefore, $|\mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_5)(w_1)| = 0.4 > 0.1$, and by Definition 16, E_1 is the hyperedge of $\mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}}(w_1))$. Now, we calculate the grade of membership of this hyperedge corresponding to parameter w_1

$$
\begin{aligned}
\zeta_{\mathcal{M}(w_1)}(E_1) =& \frac{0.4 - 0.1}{0.4}(\zeta_{\mathcal{X}(w_1)}(r_2) \wedge \zeta_{\mathcal{X}(w_1)}(r_5)) \\
&\times h(\mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_5)(w_1)) = 0.12.
\end{aligned}
$$

Similarly, $E_3 = \{r_1, r_5\}$ is the hyperedge of $\mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}}(w_1))$ and membership grade of E_3 is $\zeta_{\mathcal{M}(w_1)}(E_3) = 0.32$. E_5 is not the hyperedge of $\mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}}(w_1))$, because $|\mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_4)(w_1)| = 0.1 \not> 0.1$.

Since $E_3 = \{r_1, r_2, r_5\}$ (see Example 3). Here

$$
\begin{aligned}
&\mathcal{N}^+(r_1)(w_2) \cap \mathcal{N}^+(r_2)(w_2) \cap \mathcal{N}^+(r_5)(w_2) = \{(r_3, 0.2)\}, \\
&|\mathcal{N}^+(r_1)(w_2) \cap \mathcal{N}^+(r_2)(w_2) \cap \mathcal{N}^+(r_5)(w_2)| = k_1 = 0.2.
\end{aligned}
$$

Therefore, $|\mathcal{N}^+(r_1)(w_2) \cap \mathcal{N}^+(r_2)(w_2) \cap \mathcal{N}^+(r_5)(w_2)| = 0.2 > 0.1$, and by Definition 16, E_3 is the hyperedge of $\mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}}(w_2)) = (\mathcal{X}(w_2), \mathcal{M}(w_2))$. Now, we calculate the

Table 10 FS in neighborhood of vertices in $\overrightarrow{\mathcal{G}}$

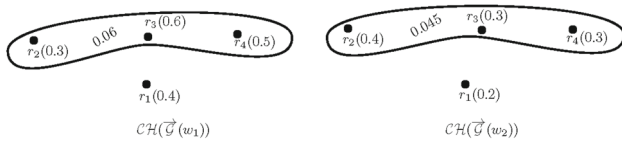| $r \in \mathcal{R}$	$\mathcal{N}^-(r)(w_1)$	$\mathcal{N}^-(r)(w_2)$	$(\mathcal{N}^-(r), \mathcal{W})$
r_1	$\{(r_2, 0.4), (r_5, 0.7)\}$	$\{(r_5, 0.3)\}$	$\{\frac{(r_2,0.4),(r_5,0.7)}{w_1}, \frac{(r_5,0.3)}{w_2}\}$
r_2	$\{\}$	$\{(r_5, 0.6)\}$	$\{\frac{\{\}}{w_1}, \frac{(r_5,0.6)}{w_2}\}$
r_3	$\{(r_1, 0.6), (r_5, 0.5)\}$	$\{(r_1, 0.2), (r_2, 0.5), (r_5, 0.6)\}$	$\{\frac{(r_1,0.6),(r_5,0.5)}{w_1}, \frac{(r_1,0.2),(r_2,0.5),(r_5,0.6)}{w_2}\}$
r_4	$\{(r_3, 0.2)\}$	$\{(r_3, 0.1)\}$	$\{\frac{(r_3,0.2)}{w_1}, \frac{\{(r_3,0.1)\}}{w_2}\}$
r_5	$\{(r_2, 0.3), (r_4, 0.1)\}$	$\{\}$	$\{\frac{(r_2,0.3),(r_4,0.1)}{w_1}, \frac{\{\}}{w_2}\}$

Fig. 7 FS competition hypergraph $\mathcal{CH}(\overrightarrow{\mathcal{G}})$

grade of membership of this hyperedge corresponding to parameter w_2

$$\zeta_{\mathcal{M}(w_2)}(E_3) = \frac{0.2 - 0.1}{0.2}(\zeta_{\mathcal{X}(w_2)}(r_1) \wedge \zeta_{\mathcal{X}(w_2)}(r_2) \wedge \zeta_{\mathcal{X}(w_2)}(r_5))$$
$$\times h(\mathcal{N}^+(r_1)(w_2) \cap \mathcal{N}^+(r_2)(w_2) \cap \mathcal{N}^+(r_5)(w_2)) = 0.04.$$

The FS 0.1-competition hypergraph $\mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}}) = (\mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}}(w_1)), \mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}}(w_2)))$ of $\overrightarrow{\mathcal{G}}$ is given in Fig. 7.

Definition 17 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$ and suppose k be a non-negative number. The FS k-common enemy hypergraph $\mathcal{C}_k\mathcal{E}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{N}, \mathcal{W})$ of $\overrightarrow{\mathcal{G}}$ containing the FS vertex set same as in FS digraph $\overrightarrow{\mathcal{G}}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is the hyperedge of $\mathcal{C}_k\mathcal{E}(\overrightarrow{\mathcal{G}}(w)) = (\mathcal{X}(w), \mathcal{N}(w))$ if $|\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)| > k$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ for each parameter w is defined as

$$\zeta_{\mathcal{N}(w)}(E) = \frac{k_1 - k}{k_1}[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)),$$

where $|\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)| = k_1$.

Remark 1 At $k = 0$, the FS k-competition hypergraphs $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$ and FS k-common enemy hypergraphs $\mathcal{C}_k\mathcal{E}(\overrightarrow{\mathcal{G}})$ are FS competition hypergraphs $\mathcal{CH}(\overrightarrow{\mathcal{G}})$ and FS common enemy hypergraphs $\mathcal{CEH}(\overrightarrow{\mathcal{G}})$, respectively.

Theorem 3 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$.

(1) If for each $w \in \mathcal{W}$, $h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \mathcal{N}^+(r_p)(w)) = 1$ and $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \mathcal{N}^+(y_p)(w)| > 2k$ for some $r_1, r_2, \cdots, r_p \in \mathcal{R}$, then the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ is strong in $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$.

(2) If for each $w \in \mathcal{W}$, $h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \mathcal{N}^-(r_p)(w)) = 1$ and $|\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \mathcal{N}^-(y_p)(w)| > 2k$ for some $r_1, r_2, \cdots, r_p \in \mathcal{R}$, then the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ is strong in $\mathcal{C}_k\mathcal{E}(\overrightarrow{\mathcal{G}})$.

Proof (1) Let $\mathcal{C}_k(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{M}, \mathcal{W})$ be an FS k-competition hypergraph corresponding to the $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$. Suppose for each $w \in \mathcal{W}$, $h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \mathcal{N}^+(y_p)(w)) = 1$ and $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \mathcal{N}^+(y_p)(w)| > 2k$ for some $r_1, r_2, \cdots, r_p$. Also, let $E = \{r_1, r_2, \cdots, r_p\}$ is any hyperedge of $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$. Now

$$\zeta_{\mathcal{M}(w)}(E) = \frac{k_1}{k_1}[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)),$$

for all $w \in \mathcal{W}$. Here, $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \mathcal{N}^+(y_p)(w)| = k_1$ and $h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \mathcal{N}^+(r_p)(w)) = 1$ for all $w \in \mathcal{W}$

$$\zeta_{\mathcal{M}(w)}(E) = \frac{k_1 - k}{k_1}(\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)),$$
$$\zeta_{\mathcal{M}(w)}(E) > \frac{1}{2}(\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)),$$

for all $w \in \mathcal{W}$. Hence, the hyperedge E of $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$ is strong.

(2) Let $\mathcal{C}_k\mathcal{E}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{N}, \mathcal{W})$ be a FS k-common enemy hypergraph corresponding to the $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$. Suppose for each $w \in \mathcal{W}$, $h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \mathcal{N}^-(r_p)(w)) = 1$ and $|\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \mathcal{N}^-(r_p)(w)| > 2k$ for some $r_1, r_2, \cdots, r_p$. Also, let $E = \{r_1, r_2, \cdots, r_p\}$ is any hyperedge of $\mathcal{C}_k\mathcal{E}(\overrightarrow{\mathcal{G}})$. Now

$$\zeta_{\mathcal{N}(w)}(E) = \frac{k_1 - k}{k_1}[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)),$$

for all $w \in \mathcal{W}$. Here, $|\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \mathcal{N}^-(y_p)(w)| = k_1$ and $h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \mathcal{N}^-(r_p)(w)) = 1$ for all $w \in \mathcal{W}$

$$\zeta_{\mathcal{N}(w)}(E) = \frac{k_1 - k}{k_1}(\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)),$$
$$\zeta_{\mathcal{N}(w)}(E) > \frac{1}{2}(\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)),$$

for all $w \in \mathcal{W}$. Hence, the hyperedge E of $\mathcal{C}_k\mathcal{E}(\overrightarrow{\mathcal{G}})$ is strong.

$\square$

Now, we define the notions of p-competition FS hypergraphs and p-common enemy FS hypergraphs, where p be any positive integer.

Definition 18 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$ and suppose p be a positive integer. The p-competition FS hypergraph $\mathcal{C}^p(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{D}, \mathcal{W})$ of $\overrightarrow{\mathcal{G}}$ containing the FS vertex set same as in FS digraph $\overrightarrow{\mathcal{G}}$ and for each $w \in \mathcal{W}$,

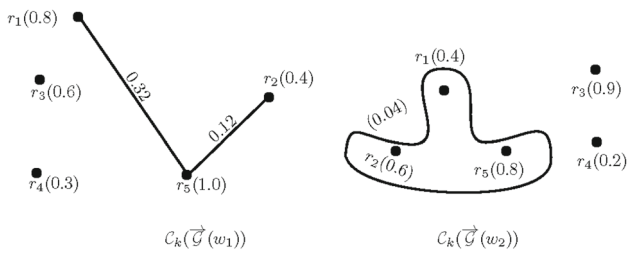**Fig. 8** FS 0.1-competition hypergraph $\mathcal{C}_{0.1}(\overrightarrow{\mathcal{G}})$

Fig. 9 FS digraph $\overrightarrow{\mathcal{G}} = \{\overrightarrow{\mathcal{G}}(w_1), \overrightarrow{\mathcal{G}}(w_2)\}$

$E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is the hyperedge of $\mathcal{C}^p(\overrightarrow{\mathcal{G}}(w))$ $= (\mathcal{X}(w), \mathcal{D}(w))$ if $|supp(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w))| \geq p$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ for each parameter w is defined as

$$\zeta_{\mathcal{D}(w)}(E) = \frac{(n-p)+1}{n}[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots$$
$$\wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)),$$

where $|supp(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w))| = n$.

Example 7 Consider an FS digraph $\overrightarrow{\mathcal{G}}$, as shown in Fig. 8. Using Algorithm 4, $E_3 = \{r_1, r_2, r_4\}$. Here, $\mathcal{N}^+(r_1)(w_1) = \{(r_2, 0.1), (r_3, 0.1), (r_5, 0.2)\}$, $\mathcal{N}^+(r_2)(w_1) = \{(r_3, 0.3), (r_5, 0.3)\}$, and $\mathcal{N}^+(r_4)(w_1) = \{(r_3, 0.2), (r_5, 0.4)\}$. Hence, $\mathcal{N}^+(r_1)(w_1) \cap \mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_4)(w_1) = \{(r_3, 0.1), (r_5, 0.2)\}$. Now, $n = |supp(\mathcal{N}^+(r_1)(w_1) \cap \mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_4)(w_1))| = 2$. Therefore, by Definition 18, $E_3 = \{r_1, r_2, r_4\}$ is the hyperedge of $\mathcal{C}^2(\overrightarrow{\mathcal{G}}(w_1)) = (\mathcal{X}(w_1), \mathcal{D}(w_1))$. Now, we calculate the membership grade of this hyperedge corresponding to parameter w_1

$$\zeta_{\mathcal{D}(w_1)}(E) = \frac{(2-2)+1}{2}[\zeta_{\mathcal{X}(w_1)}(r_1) \wedge \zeta_{\mathcal{X}(w_1)}(r_2) \wedge \zeta_{\mathcal{X}(w_1)}(r_4)]$$
$$\times h(\mathcal{N}^+(r_1)(w_1) \cap \mathcal{N}^+(r_2)(w_1) \cap \mathcal{N}^+(r_4)(w_1)) = 0.02.$$

Using Algorithm 4, $E_3 = \{r_1, r_2\}$. Here, $\mathcal{N}^+(r_1)(w_2) = \{(r_3, 0.7), (r_4, 0.7)\}$ and $\mathcal{N}^+(r_2)(w_2) = \{(r_3, 0.6), (r_4, 0.8), (r_5, 0.8)\}$. Hence, $\mathcal{N}^+(r_1)(w_2) \cap \mathcal{N}^+(r_2)(w_2) = \{(r_3, 0.6), (r_4, 0.7)\}$. Since we know that $n = |supp(\mathcal{N}^+(r_1)(w_2) \cap \mathcal{N}^+(r_2)(w_2))|$, so $n = 2$. By Definition 18, $E_3 = \{r_1, r_2\}$ is the hyperedge of $\mathcal{C}^2(\overrightarrow{\mathcal{G}}(w_2)) = (\mathcal{X}(w_2), \mathcal{D}(w_2))$. Now, we calculate the membership grade of this hyperedge corresponding to parameter w_2

$$\zeta_{\mathcal{D}(w_2)}(E) = \frac{(2-2)+1}{2}[\zeta_{\mathcal{X}(w_2)}(r_1) \wedge \zeta_{\mathcal{X}(w_2)}(r_2)]$$
$$\times h(\mathcal{N}^+(r_1)(w_2) \cap \mathcal{N}^+(r_2)(w_2)) = 0.245.$$

The 2-competition FS hypergraph $\mathcal{C}^2(\overrightarrow{\mathcal{G}}) = (\mathcal{C}^2(\overrightarrow{\mathcal{G}}(w_1)), \mathcal{C}^2(\overrightarrow{\mathcal{G}}(w_2)))$ is given in Fig. 9.
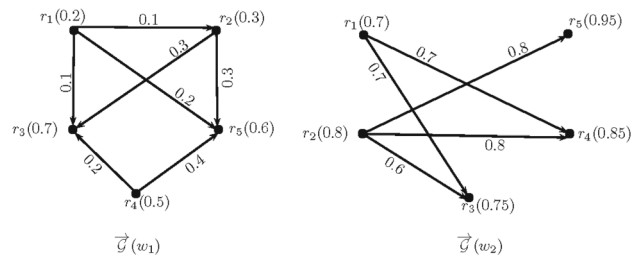
Definition 19 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$ and suppose p be a positive integer. The p-common enemy FS hypergraph $\mathcal{C}^p\mathcal{E}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{T}, \mathcal{W})$ of $\overrightarrow{\mathcal{G}}$ containing the FS vertex set same as in FS digraph $\overrightarrow{\mathcal{G}}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is the hyperedge of $\mathcal{C}^p\mathcal{E}(\overrightarrow{\mathcal{G}}(w))$ $= (\mathcal{X}(w), \mathcal{T}(w))$ if $|supp(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w))| \geq p$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ corresponding to the parameter w is defined as

$$\zeta_{\mathcal{T}(w)}(E) = \frac{(n-p)+1}{n}[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots$$
$$\wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)),$$

where $|supp(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w))| = n$.

Remark 2 At $p = 1$, the p-competition FS hypergraphs $\mathcal{C}^p(\overrightarrow{\mathcal{G}})$ and p-common enemy FS hypergraphs $\mathcal{C}^p\mathcal{E}(\overrightarrow{\mathcal{G}})$ are FS competition hypergraphs $\mathcal{CH}(\overrightarrow{\mathcal{G}})$ and FS common enemy hypergraphs $\mathcal{CEH}(\overrightarrow{\mathcal{G}})$, respectively.

Theorem 4 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$.

(1) If for each $w \in \mathcal{W}$, $h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)) = 1$ in $\mathcal{C}^{[\frac{n}{2}]}(\overrightarrow{\mathcal{G}}(w))$, where $n = |supp(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w))|$, then the hyperedge $\{r_1, r_2, \cdots, r_p\}$ is strong in $\mathcal{C}^{[\frac{n}{2}]}(\overrightarrow{\mathcal{G}})$. (Note that r be any real number, $[r]$ = greatest integer not greater than r.)

(2) If for each $w \in \mathcal{W}$, $h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)) = 1$ in $\mathcal{C}^{[\frac{n}{2}]}\mathcal{E}(\overrightarrow{\mathcal{G}}(w))$, where $n = |supp(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w))|$, then the hyperedge $\{r_1, r_2, \cdots, r_p\}$ is strong in $\mathcal{C}^{[\frac{n}{2}]}\mathcal{E}(\overrightarrow{\mathcal{G}})$.

Proof (1) Here, $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$. Let for each $w \in \mathcal{W}$, $h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)) = 1$ in $\mathcal{C}^{[\frac{n}{2}]}(\overrightarrow{\mathcal{G}}(w))$ corresponding to parameter w, where $n = |supp(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w))|$. Now, for each $w \in \mathcal{W}$

$$\zeta_{\mathcal{D}(w)}(\{r_1, r_2, \cdots, r_p\})$$
$$= \frac{n - [\frac{n}{2}] + 1}{n} \zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots$$
$$\wedge \zeta_{\mathcal{X}(w)}(r_p), \frac{\zeta_{\mathcal{D}(w)}(\{r_1, r_2, \cdots, r_p\})}{\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)}$$
$$= \frac{n - [\frac{n}{2}] + 1}{n} > 0.5.$$

Hence, the hyperedge $\{r_1, r_2, \cdots, r_p\}$ is strong in $\mathcal{C}^{[\frac{n}{2}]}(\overrightarrow{\mathcal{G}})$.

(2) Here $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph. Let for each $w \in \mathcal{W}$, $h(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w)) = 1$ in $\mathcal{C}^{[\frac{n}{2}]}\mathcal{E}(\overrightarrow{\mathcal{G}}(w))$ corresponding to parameter w, where $n = |supp(\mathcal{N}^-(r_1)(w) \cap \mathcal{N}^-(r_2)(w) \cap \cdots \cap \mathcal{N}^-(r_p)(w))|$. Now, for each $w \in \mathcal{W}$

$$\zeta_{\mathcal{T}(w)}(\{r_1, r_2, \cdots, r_p\}) = \frac{n - [\frac{n}{2}] + 1}{n} \zeta_{\mathcal{X}(w)}(r_1)$$
$$\wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p),$$
$$\frac{\zeta_{\mathcal{T}(w)}(\{r_1, r_2, \cdots, r_p\})}{\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)}$$
$$= \frac{n - [\frac{n}{2}] + 1}{n} > 0.5.$$

Hence, the hyperedge $\{r_1, r_2, \cdots, r_p\}$ is strong in $\mathcal{C}^{[\frac{n}{2}]}\mathcal{E}(\overrightarrow{\mathcal{G}})$.

$\square$
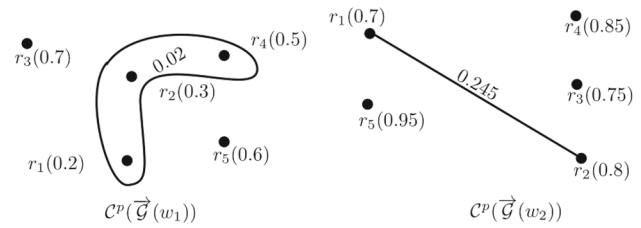
Fig. 10 2-Competition FS hypergraph $\mathcal{C}^2(\overrightarrow{\mathcal{G}})$

Table 11 FS set $(\mathcal{X}, \mathcal{W})$

$\mathcal{R}$	$\mathcal{X}(w_1)$	$\mathcal{R}$	$\mathcal{X}(w_2)$
r_1	0.4	r_1	0.4
r_2	0.5	r_2	0.6
r_3	0.6	r_3	0.7
r_4	0.3	r_4	0.2

Fuzzy soft neighborhood hypergraphs

The notions of FS open neighborhood and FS closed neighborhood of a vertex in the FS graph are illustrated in this section. The concepts of FS open neighborhood hypergraphs and FS closed neighborhood hypergraphs are given.

Definition 20 An FS open neighborhood of a vertex r in an FS graph $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$ is an FS set

$$(\mathcal{N}(r), \mathcal{W}) = \{(w, \mathcal{N}(r)(w)) \mid w \in \mathcal{W}\},$$

where $\mathcal{N}(r)(w)$ is a fuzzy set defined as

$$\mathcal{N}(r)(w) = \{r', \zeta_{\mathcal{K}(w)}(rr') \mid \zeta_{\mathcal{K}(w)}(rr') > 0, w \in \mathcal{W}\}, \text{ for each } w \in \mathcal{W}.$$

The Venn diagram Fig.10 shows the similarities and differences between FS k-competition hypergraphs $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$ and p-competition FS hypergraph $\mathcal{C}^p(\overrightarrow{\mathcal{G}})$.

Both FS k-competition hypergraphs and p-competition FS hypergraphs are generalizations of FS competition hypergraphs. The positive real number k in FS k-competition hypergraphs and the positive integer p in p-competition FS hypergraphs measure the strength of competitions of corresponding FS competition hypergraphs. A real positive number k is related to the cardinality of an FS set and p is related to the cardinality of a soft set. FS k-competition hypergraph is an intersection hypergraph of FS out neighborhood of vertices in an FS digraph. The other is p-competition FS hypergraph and is also an intersection hypergraph of supports of FS out neighborhood of vertices in an FS digraph.

Definition 21 An FS closed neighborhood of a vertex r in an FS graph $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$ is an FS set

$$(\mathcal{N}[r], \mathcal{W}) = \{(w, \mathcal{N}[r](w)) \mid w \in \mathcal{W}\},$$

where $\mathcal{N}[r](w)$ is defined as

$$\mathcal{N}[r](w) = \mathcal{N}(r)(w) \cup \{r, \zeta_{\mathcal{X}(w)}(r)\},$$
$$\text{for each } w \in \mathcal{W}.$$

Example 8 Let $(\mathcal{X}, \mathcal{W})$ be an FS set on $\mathcal{R} = \{r_1, r_2, r_3, r_4\}$ and $(\mathcal{K}, \mathcal{W})$ is an FS relation on $\mathcal{R}$, as shown in Table 11 and 12, respectively. Here, $\mathcal{W} = \{w_1, w_2\}$ is a parameter set. The FS graph $\mathcal{G} = \{\mathcal{G}(w_1), \mathcal{G}(w_2)\}$ is given in Fig. 11.

The FS open neighborhood of all vertices in FS graph $\mathcal{G}$ is given in Table 13.

Table 12 FS relation $(\mathcal{K}, \mathcal{W})$

$\mathcal{R}$	$\mathcal{K}(w_1)$	$\mathcal{R}$	$\mathcal{K}(w_2)$
$r_1 r_2$	0.3	$r_1 r_2$	0.3
$r_1 r_3$	0.2	$r_2 r_3$	0.5
$r_2 r_3$	0.5	$r_2 r_4$	0.1
$r_3 r_4$	0.1	$r_3 r_4$	0.2
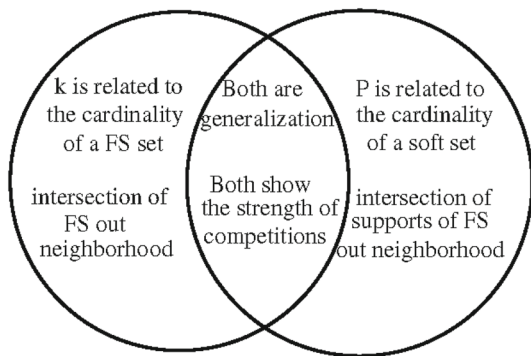
Fig. 11 $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$ vs $\mathcal{C}^p(\overrightarrow{\mathcal{G}})$

The FS closed neighborhood of all vertices in $\mathcal{G}(w_1)$ and $\mathcal{G}(w_2)$ is given in Table 14.
The FS closed neighborhood of all vertices in $\mathcal{G}$ is the following:

$(\mathcal{N}[r_1], \mathcal{W})$
$$= \left\{ \frac{(r_1, 0.4), (r_2, 0.3), (r_3, 0.2)}{w_1}, \frac{(r_1, 0.4), (r_2, 0.3)}{w_2} \right\},$$
$(\mathcal{N}[r_2], \mathcal{W})$
$$= \left\{ \frac{(r_1, 0.3), (r_2, 0.5), (r_3, 0.5)}{w_1}, \frac{(r_1, 0.3), (r_2, 0.6), (r_3, 0.5), (r_4, 0.1)}{w_2} \right\},$$
$(\mathcal{N}[r_3], \mathcal{W})$
$$= \left\{ \frac{(r_1, 0.2), (r_2, 0.5), (r_3, 0.6), (r_4, 0.1)}{w_1}, \frac{(r_2, 0.5), (r_3, 0.7), (r_4, 0.2)}{w_2} \right\},$$

$(\mathcal{N}[r_4], \mathcal{W})$
$$= \left\{ \frac{(r_3, 0.1), (r_4, 0.3)}{w_1}, \frac{(r_2, 0.1), (r_3, 0.2), (r_4, 0.2)}{w_2} \right\}.$$

Definition 22 Let $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$ be an FS graph on $\mathcal{R}$. The FS open neighborhood hypergraph $\mathcal{N}(\mathcal{G}) = (\mathcal{X}, \mathcal{E}, \mathcal{W})$ of $\mathcal{G}$ containing the FS vertex set same as in FS graph $\mathcal{G}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is the hyperedge of $\mathcal{N}(\mathcal{G}(w)) = (\mathcal{X}(w), \mathcal{E}(w))$ if $\mathcal{N}(r_1)(w) \cap \mathcal{N}(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)(w) \neq \emptyset$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ corresponding to parameter w is defined as

$$\zeta_{\mathcal{E}(w)}(E) = [\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)] \times h(\mathcal{N}(r_1)(w) \cap \mathcal{N}(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)(w)).$$

The technique is given in Algorithm 5 for proceeding FS open neighborhood hypergraph of an FS graph $\mathcal{G}$.

Method for construction of FS open neighborhood hypergraph

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1. Input the FS set on set of vertices $\mathcal{R} = \{r_1, r_2, \cdots, r_n\}$.
2. Compute the FS open neighborhood of each vertex in FS graph $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$.
3. For each $w \in \mathcal{W}$, define a relation in such a way that $f(w) : \mathcal{R} \longrightarrow \mathcal{R}$ by $f(w)(r_i)=r_j$ if $r_j \in supp(\mathcal{N}(r_i)(w))$.
4. Determine the category of sets for each $w \in \mathcal{W}$, $Z = \{E_i = f^{-1}(w)(r_i) : |f^{-1}(w)(r_i)| \geq 2, r_i \in \mathcal{R}\}$, where $E_i = \{r_{i_1}, r_{i_2}, \cdots, r_{i_r}\}$ is a hyperedge of $\mathcal{N}(\mathcal{G}(w))$.
5. Compute the grade of membership of each hyperedge corresponding to parameter w by using Definition 5.4.
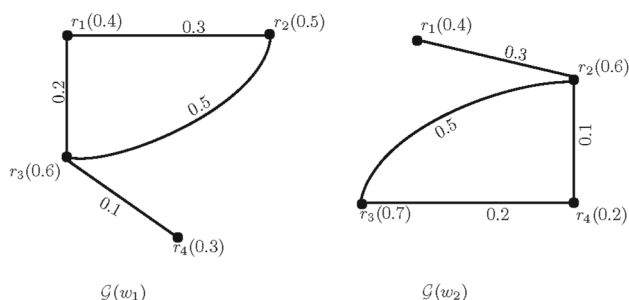

Table 13 FS open neighborhood of vertices in $\mathcal{G}$

$r \in \mathcal{R}$	$\mathcal{N}(r)(w_1)$	$\mathcal{N}(r)(w_2)$	$(\mathcal{N}(r), \mathcal{W})$
r_1	$\{(r_2, 0.3), (r_3, 0.2)\}$	$\{(r_2, 0.3)\}$	$\left\{ \frac{(r_2, 0.3), (r_3, 0.2)}{w_1}, \frac{(r_2, 0.3)}{w_2} \right\}$
r_2	$\{(r_1, 0.3), (r_3, 0.5)\}$	$\{(r_1, 0.3), (r_3, 0.5), (r_4, 0.1)\}$	$\left\{ \frac{(r_1, 0.3), (r_3, 0.5)}{w_1}, \frac{(r_1, 0.3), (r_3, 0.5), (r_4, 0.1)}{w_2} \right\}$
r_3	$\{(r_1, 0.2), (r_2, 0.5), (r_4, 0.1)\}$	$\{(r_2, 0.5), (r_4, 0.2)\}$	$\left\{ \frac{(r_1, 0.2), (r_2, 0.5), (r_4, 0.1)}{w_1}, \frac{(r_2, 0.5), (r_4, 0.2)}{w_2} \right\}$
r_4	$\{(r_3, 0.1)\}$	$\{(r_2, 0.1), (r_3, 0.2)\}$	$\left\{ \frac{(r_3, 0.1)}{w_1}, \frac{\{(r_2, 0.1), (r_3, 0.2)\}}{w_2} \right\}$

Table 14 FS closed neighborhood of vertices in $\mathcal{G}(w_1)$ and $\mathcal{G}(w_2)$

$r \in \mathcal{R}$	$\mathcal{N}[r](w_1)$	$\mathcal{N}[r](w_2)$
r_1	$\{(r_1, 0.4), (r_2, 0.3), (r_3, 0.2)\}$	$\{(r_1, 0.4), (r_2, 0.3)\}$
r_2	$\{(r_1, 0.3), (r_2, 0.5), (r_3, 0.5)\}$	$\{(r_1, 0.3), (r_2, 0.6), (r_3, 0.5), (r_4, 0.1)\}$
r_3	$\{(r_1, 0.2), (r_2, 0.5), (r_3, 0.6), (r_4, 0.1)\}$	$\{(r_2, 0.5), (r_3, 0.7), (r_4, 0.2)\}$
r_4	$\{(r_3, 0.1), (r_4, 0.3)\}$	$\{(r_2, 0.1), (r_3, 0.2), (r_4, 0.2)\}$

Fig. 12 FS graph $\mathcal{G} = (\{\mathcal{G}(w_1), \mathcal{G}(w_2)\}$

Example 9 Consider an FS graph $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$, as shown in Fig. 11. The FS open neighborhood of all vertices in $\mathcal{G}$ is given in Table 13. The support of $(\mathcal{N}(r), \mathcal{W})$ is given in Table 15.

The relations $f(w_1) : \mathcal{R} \longrightarrow \mathcal{R}$ and $f(w_2) : \mathcal{R} \longrightarrow \mathcal{R}$ are given in Fig. 12.

Using fuzzy relation in $\mathcal{G}(w_1)$, there are three hyperedges $f^{-1}(r_1)(w_1) = E_1 = \{r_2, r_3\}$, $f^{-1}(r_2)(w_1) = E_2 = \{r_1, r_3\}$, and $f^{-1}(r_3)(w_1) = E_3 = \{r_1, r_2, r_4\}$ of $\mathcal{N}(\mathcal{G}(w_1)) = (\mathcal{X}(w_1), \mathcal{E}(w_1))$. Membership grades of these hyperedges are computed as follows:

$$\zeta_{\mathcal{E}(w_1)}(E_1) = \zeta_{\mathcal{X}(w_1)}(r_2) \wedge \zeta_{\mathcal{X}(w_1)}(r_3)$$
$$\times h(\mathcal{N}(r_2)(w_1) \cap \mathcal{N}(r_3)(w_1))$$
$$= 0.5 \times 0.2 = 0.1.$$

$$\zeta_{\mathcal{E}(w_1)}(E_2) = \zeta_{\mathcal{X}(w_1)}(r_1) \wedge \zeta_{\mathcal{X}(w_1)}(r_3)$$
$$\times h(\mathcal{N}(r_1)(w_1) \cap \mathcal{N}(r_3)(w_1))$$
$$= 0.4 \times 0.3 = 0.12.$$

$$\zeta_{\mathcal{E}(w_1)}(E_3) = \zeta_{\mathcal{X}(w_1)}(r_1) \wedge \zeta_{\mathcal{X}(w_1)}(r_2) \wedge \zeta_{\mathcal{X}(w_1)}(r_4)$$
$$\times h(\mathcal{N}(r_1)(w_1) \cap \mathcal{N}(r_2)(w_1) \cap \mathcal{N}(r_4)(w_1))$$
$$= 0.3 \times 0.1 = 0.03.$$

Using fuzzy relation in $\mathcal{G}(w_2)$, there are three hyperedges $f^{-1}(r_2)(w_2) = E_2 = \{r_1, r_3, r_4\}$, $f^{-1}(r_3)(w_2) = E_3 = \{r_2, r_4\}$, and $f^{-1}(r_4)(w_2) = E_4 = \{r_2, r_3\}$ of $\mathcal{N}(\mathcal{G}(w_2)) = (\mathcal{X}(w_2), \mathcal{E}(w_2))$. Membership grades of these hyperedges are the following:

$$\zeta_{\mathcal{E}(w_2)}(E_2) = \zeta_{\mathcal{X}(w_2)}(r_1) \wedge \zeta_{\mathcal{X}(w_2)}(r_3) \wedge \zeta_{\mathcal{X}(w_2)}(r_4)$$
$$\times h(\mathcal{N}(r_1)(w_2) \cap \mathcal{N}(r_3)(w_2) \cap \mathcal{N}(r_4)(w_2))$$

$$= 0.2 \times 0.1 = 0.02.$$

$$\zeta_{\mathcal{E}(w_2)}(E_3) = \zeta_{\mathcal{X}(w_2)}(r_2) \wedge \zeta_{\mathcal{X}(w_2)}(r_4)$$
$$\times h(\mathcal{N}(r_2)(w_2) \cap \mathcal{N}(r_4)(w_2)) = 0.2 \times 0.2 = 0.04.$$

$$\zeta_{\mathcal{E}(w_2)}(E_4) = \zeta_{\mathcal{X}(w_2)}(r_2) \wedge \zeta_{\mathcal{X}(w_2)}(r_3)$$
$$\times h(\mathcal{N}(r_2)(w_2) \cap \mathcal{N}(r_3)(w_2)) = 0.6 \times 0.1 = 0.06.$$

The FS open neighborhood hypergraph $\mathcal{N}(\mathcal{G}) = (\mathcal{N}(\mathcal{G}(w_1)), \mathcal{N}(\mathcal{G}(w_2)))$ is given in Fig. 13.

Definition 23 Let $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$ be an FS graph on $\mathcal{R}$. The FS closed neighborhood hypergraph $\mathcal{N}[\mathcal{G}] = (\mathcal{X}, \mathcal{O}, \mathcal{W})$ of $\mathcal{G}$ containing the FS vertex set same as in FS graph $\mathcal{G}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is the hyperedge of $\mathcal{N}[\mathcal{G}(w)] = (\mathcal{X}(w), \mathcal{O}(w))$ if $\mathcal{N}[r_1](w) \cap \mathcal{N}[r_2](w) \cap \cdots \cap \mathcal{N}[r_p](w) \neq \emptyset$. The grade of membership of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ for each parameter w is defined as

$$\zeta_{\mathcal{O}(w)}(E) = [\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}[r_1](w) \cap \mathcal{N}[r_2](w) \cap \cdots \cap \mathcal{N}[r_p](w)).$$
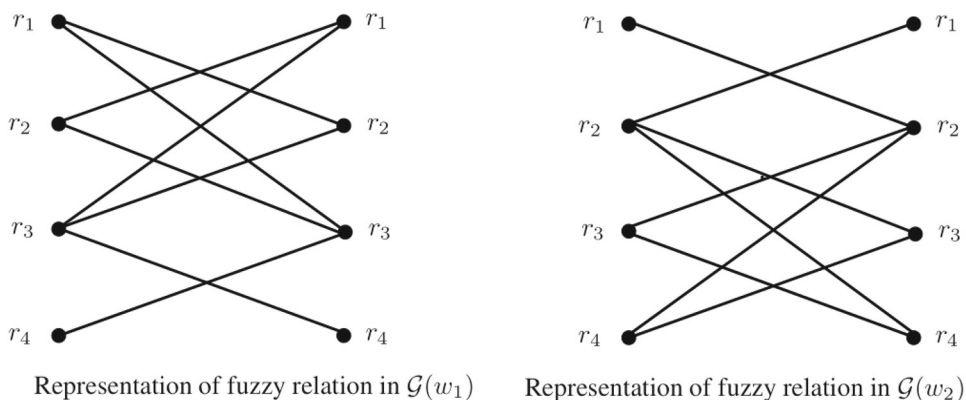
Definition 24 Let $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$ be an FS graph on $\mathcal{R}$ and k be a non-negative real number. The FS (k)-competition hypergraph $\mathcal{N}_k(\mathcal{G}) = (\mathcal{X}, \mathcal{S}, \mathcal{W})$ of $\mathcal{G}$ containing the FS vertex set same as in FS graph $\mathcal{G}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is the hyperedge of $\mathcal{N}_k(\mathcal{G}(w)) = (\mathcal{X}(w), \mathcal{S}(w))$ if $|\mathcal{N}(r_1)(w) \cap \mathcal{N}(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)(w)| > k$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ for each parameter w is defined as

$$\zeta_{\mathcal{S}(w)}(E) = \frac{k_2 - k}{k_2}[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2)$$
$$\wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)]$$
$$\times h(\mathcal{N}(r_1)(w) \cap \mathcal{N}(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)(w)),$$

where $|\mathcal{N}(r_1)(w) \cap \mathcal{N}(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)(w)| = k_2$.

Definition 25 Let $\mathcal{G} = (\mathcal{X}, \mathcal{K}, \mathcal{W})$ be an FS graph on $\mathcal{R}$ and k be a non-negative real number. The FS $[k]$-competition hypergraph $\mathcal{N}_k[\mathcal{G}] = (\mathcal{X}, \mathcal{B}, \mathcal{W})$ of $\mathcal{G}$ containing the FS vertex set same as in FS graph $\mathcal{G}$ and for each $w \in \mathcal{W}$, $E = \{r_1, r_2, \cdots, r_p\} \subseteq \mathcal{R}$ is the hyperedge of $\mathcal{N}_k[\mathcal{G}(w)] = (\mathcal{X}(w), \mathcal{B}(w))$ if $|\mathcal{N}[r_1](w) \cap \mathcal{N}[r_2](w) \cap \cdots \cap \mathcal{N}[r_p](w)| > k$. The membership grade of the hyperedge $E = \{r_1, r_2, \cdots, r_p\}$ for each parameter w is defined as

$$\zeta_{\mathcal{B}(w)}(E) = \frac{k_2 - k}{k_2}[\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_p)]h(\mathcal{N}[r_1](w) \cap \mathcal{N}[r_2](w) \cap \cdots \cap \mathcal{N}[r_p](w)),$$

Table 15 Support of $(\mathcal{N}(r), \mathcal{W})$

$r \in \mathcal{R}$	$supp(\mathcal{N}(r)(w_1))$	$supp(\mathcal{N}(r)(w_2))$	$supp(\mathcal{N}(r), \mathcal{W})$
r_1	$\{r_2, r_3\}$	$\{r_2\}$	$\{\frac{\{r_2,r_3\}}{w_1}, \frac{\{r_2\}}{w_2}\}$
r_2	$\{r_1, r_3\}$	$\{r_1, r_3, r_4\}$	$\{\frac{\{r_1,r_3\}}{w_1}, \frac{\{r_1,r_3,r_4\}}{w_2}\}$
r_3	$\{r_1, r_2, r_4\}$	$\{r_2, r_4\}$	$\{\frac{\{r_1,r_2,r_4\}}{w_1}, \frac{\{r_2,r_4\}}{w_2}\}$
r_4	$\{r_3\}$	$\{r_2, r_3\}$	$\{\frac{\{r_3\}}{w_1}, \frac{\{r_2,r_3\}}{w_2}\}$

Fig. 13 Representation of FS relation in $\mathcal{G}$

Representation of fuzzy relation in $\mathcal{G}(w_1)$

Representation of fuzzy relation in $\mathcal{G}(w_2)$

where $|\mathcal{N}[r_1](w) \cap \mathcal{N}[r_2](w) \cap \cdots \cap \mathcal{N}[r_p](w)| = k_2$.

Definition 26 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be an FS digraph on $\mathcal{R}$. The underlying FS graph of $\overrightarrow{\mathcal{G}}$ is an FS graph $\mathcal{U}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{K}, \mathcal{W})$, such that for each $w \in \mathcal{W}$

$$\zeta_{\mathcal{K}(w)}(r_1 r_2) = \begin{cases} \zeta_{\overrightarrow{\mathcal{K}}(w)}(r_1 r_2), & \text{if } \overrightarrow{r_2 r_1} \notin \overrightarrow{E} \\ \zeta_{\overrightarrow{\mathcal{K}}(w)}(r_2 r_1), & \text{if } \overrightarrow{r_1 r_2} \notin \overrightarrow{E} \\ \zeta_{\overrightarrow{\mathcal{K}}}(r_1 r_2) \wedge \zeta_{\overrightarrow{\mathcal{K}}}(r_2 r_1), & \text{if } \overrightarrow{r_1 r_2}, \overrightarrow{r_2 r_1} \in \overrightarrow{E}, \end{cases}$$

where $\overrightarrow{E} = supp(\overrightarrow{\mathcal{K}}(w))$.

Example 10 Consider an FS digraph on $\mathcal{R}$, as shown in Fig. 2. The support of $(\overrightarrow{\mathcal{K}}, \mathcal{W})$ is the following:

$supp(\overrightarrow{\mathcal{K}}(w_1)) = \{r_1 r_3, r_2 r_1, r_2 r_5, r_3 r_4, r_4 r_5, r_5 r_1, r_5 r_3\},$
$supp(\overrightarrow{\mathcal{K}}(w_2)) = \{r_1 r_3, r_2 r_3, r_3 r_4, r_5 r_1, r_5 r_2, r_5 r_3\},$
$supp(\overrightarrow{\mathcal{K}}, \mathcal{W}) = \left\{ \frac{\{r_1 r_3, r_2 r_1, r_2 r_5, r_3 r_4, r_4 r_5, r_5 r_1, r_5 r_3\}}{w_1}, \right.$
$\left. \frac{\{r_1 r_3, r_2 r_3, r_3 r_4, r_5 r_1, r_5 r_2, r_5 r_3\}}{w_2} \right\}.$

The underlying FS graph $\mathcal{U}(\overrightarrow{\mathcal{G}})$ of FS digraph $\overrightarrow{\mathcal{G}}$ 2 is given in Fig. 14.

We now study the relationship between FS open neighborhood hypergraphs $\mathcal{N}_k(\mathcal{G})$ and FS k-competition hypergraphs $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$.

Theorem 5 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be a symmetric FS digraph having no loops, then $\mathcal{C}_k(\overrightarrow{\mathcal{G}}) = \mathcal{N}_k(\mathcal{U}(\overrightarrow{\mathcal{G}}))$.
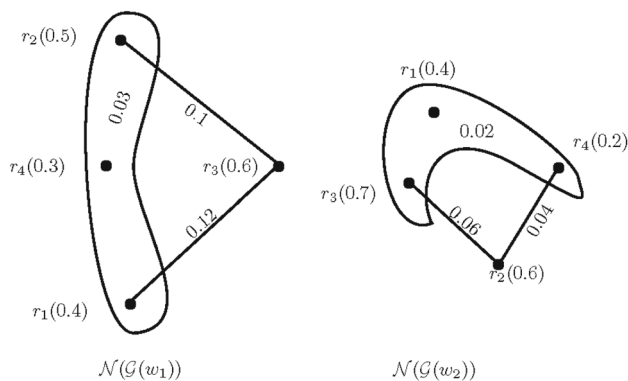

Fig. 14 FS open neighborhood hypergraph $\mathcal{N}(\mathcal{G})$

Proof Let $\mathcal{U}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{K}, \mathcal{W})$ be an underlying FS graph of an FS digraph $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$. Also, let $\mathcal{N}_k(\mathcal{U}(\overrightarrow{\mathcal{G}})) = (\mathcal{X}, \mathcal{S}, \mathcal{W})$ and $\mathcal{C}_k(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{M}, \mathcal{W})$. The FS k-competition hypergraph $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$ and the underlying FS graph $\mathcal{U}(\overrightarrow{\mathcal{G}})$ have the same vertex set as in $\overrightarrow{\mathcal{G}}$. Hence, $\mathcal{N}_k(\mathcal{U}(\overrightarrow{\mathcal{G}}))$ has the same vertex set as in $\overrightarrow{\mathcal{G}}$. Now, we want to show that $\zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\}) = \zeta_{\mathcal{S}(w)}(\{r_1, r_2, \cdots, r_p\})$ for all $w \in \mathcal{W}$ and $r_1, r_2, \cdots, r_p \in \mathcal{R}$.

Case 1: If for each $w \in \mathcal{W}$ and $r_1, r_2, \cdots, r_p \in \mathcal{R}$, $\zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\}) = 0$ in a FS k-competition hypergraph $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$. Then, for each $w \in \mathcal{W}$, $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)| \not> k$. Since $\overrightarrow{\mathcal{G}}$ is symmetric, then $|\mathcal{N}(r_1)(w) \cap \mathcal{N}(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)(w)| \not> k$ for all $w \in \mathcal{W}$ in $\mathcal{U}(\overrightarrow{\mathcal{G}})$. Consequently, $\zeta_{\mathcal{S}(w)}(\{r_1, r_2, \cdots, r_p\}) = 0$ for all $w \in \mathcal{W}$ in $\mathcal{N}_k(\mathcal{U}(\overrightarrow{\mathcal{G}}))$.

Case 2: If for some $r_1, r_2, \cdots, r_p \in \mathcal{R}, \zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\}) \neq 0$ for all $w \in \mathcal{W}$ in an FS k-competition hypergraph $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$, then $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)| > k$ for all $w \in \mathcal{W}$. Since $\overrightarrow{\mathcal{G}}$ is symmetric FS digraph, then $h(\mathcal{N}(r_1)(w) \cap \mathcal{N}(r_2)(w) \cap \cdots \cap \mathcal{N}(r_p)(w)) = h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w))$ for all $w \in \mathcal{W}$. Therefore

$$\zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\})$$
$$= \frac{k_1 - k}{k_1}(\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_P))$$
$$\times h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w))$$
$$= \zeta_{\mathcal{S}(w)}(\{r_1, r_2, \cdots, r_p\}),$$

for all $w \in \mathcal{W}$. Hence, $\zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\}) = \zeta_{\mathcal{S}(w)}(\{r_1, r_2, \cdots, r_p\})$ for all $w \in \mathcal{W}$ and $r_1, r_2, \cdots, r_p$. □

Similarly, a relationship between FS closed neighborhood hypergraphs $\mathcal{N}_k[\mathcal{G}]$ and FS k-competition hypergraphs $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$ is given as follows.

Theorem 6 Let $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$ be a symmetric FS digraph with loops at every vertex, and then, $\mathcal{C}_k(\overrightarrow{\mathcal{G}}) = \mathcal{N}_k[\mathcal{U}(\overrightarrow{\mathcal{G}})]$.

Proof Let $\mathcal{U}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{K}, \mathcal{W})$ be an underlying FS graph of an FS digraph $\overrightarrow{\mathcal{G}} = (\mathcal{X}, \overrightarrow{\mathcal{K}}, \mathcal{W})$. Also, let $\mathcal{N}_k[\mathcal{U}(\overrightarrow{\mathcal{G}})] = (\mathcal{X}, \mathcal{B}, \mathcal{W})$ and $\mathcal{C}_k(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{M}, \mathcal{W})$. The FS k-competition hypergraph $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$ and the underlying FS graph $\mathcal{U}(\overrightarrow{\mathcal{G}})$ have the same vertex set as in $\overrightarrow{\mathcal{G}}$. Hence, $\mathcal{N}_k[\mathcal{U}(\overrightarrow{\mathcal{G}})]$ has the same vertex set as in $\overrightarrow{\mathcal{G}}$. Now, we want to show that $\zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\}) = \zeta_{\mathcal{B}(w)}(\{r_1, r_2, \cdots, r_p\})$ for all $w \in \mathcal{W}$ and $r_1, r_2, \cdots, r_p \in \mathcal{R}$. As the FS digraph has a loop at every vertex, the FS out neighborhood contains the vertex itself. There are two cases.

Case 1: If for each $w \in \mathcal{W}$ and $r_1, r_2, \cdots, r_p \in \mathcal{R}$, $\zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\}) = 0$ in an FS k-competition hypergraph $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$. Then, for each $w \in \mathcal{W}$, $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)| \not> k$. Since $\overrightarrow{\mathcal{G}}$ is symmetric, then $|\mathcal{N}[r_1](w) \cap \mathcal{N}[r_2](w) \cap \cdots \cap \mathcal{N}[r_p](w)| \not> k$ for all $w \in \mathcal{W}$ in $\mathcal{U}(\overrightarrow{\mathcal{G}})$. Consequently, $\zeta_{\mathcal{B}(w)}(\{r_1, r_2, \cdots, r_p\}) = 0$ for all $w \in \mathcal{W}$ in $\mathcal{N}_k[\mathcal{U}(\overrightarrow{\mathcal{G}})]$.

Case 2: If for some $r_1, r_2, \cdots, r_p \in \mathcal{R}, \zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\}) \neq 0$ for all $w \in \mathcal{W}$ in an FS k-competition hypergraph $\mathcal{C}_k(\overrightarrow{\mathcal{G}})$ then $|\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w)| > k$ for all $w \in \mathcal{W}$. Since $\overrightarrow{\mathcal{G}}$ is symmetric FS digraph, then $h(\mathcal{N}[r_1](w) \cap \mathcal{N}[r_2](w) \cap \cdots \cap \mathcal{N}[r_p](w)) = h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w))$ for all $w \in \mathcal{W}$. Therefore

$$\zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\})$$

$$= \frac{k_1 - k}{k_1}(\zeta_{\mathcal{X}(w)}(r_1) \wedge \zeta_{\mathcal{X}(w)}(r_2) \wedge \cdots \wedge \zeta_{\mathcal{X}(w)}(r_P))$$
$$\times h(\mathcal{N}^+(r_1)(w) \cap \mathcal{N}^+(r_2)(w) \cap \cdots \cap \mathcal{N}^+(r_p)(w))$$
$$= \zeta_{\mathcal{B}(w)}(\{r_1, r_2, \cdots, r_p\}),$$

for all $w \in \mathcal{W}$. Hence, $\zeta_{\mathcal{M}(w)}(\{r_1, r_2, \cdots, r_p\}) = \zeta_{\mathcal{B}(w)}(\{r_1, r_2, \cdots, r_p\})$ for all $w \in \mathcal{W}$ and $r_1, r_2, \cdots, r_p$. □

Application to decision-making

Artificial intelligence (AI) is the core of active research that has embraced new developments in information technology. Although AI has roots which trace back to several years ago, it has endowed automatic intelligent machines with learning, reasoning, and adaptable capabilities. Explainable AI (XAI) refers to techniques and models to produce explainable and accurate approaches to show how a machine learning algorithm reaches a decision which is understandable by humans. XAI helps to understand transparency, fairness, model accuracy, and outcomes occurred in AI decision-making. With the advancement of AI models, it has become a challenge for humans to comprehend how an algorithm approaches an accurate result. XAI helps decision-makers and developers to ensure that a system is working properly to meet the the basic standards. News channels play a vital role in any field as they make us aware of latest trends, changes, and the circumstances of the world. News channels can be beneficial for every class person in their own way. For instance, Students get the latest knowledge concerning current affairs of the nation and the world, Businessmen watch the news to check the current status of market all over the world, and so on. Each news channel has a competitive interrelationship and entanglement among themselves on the basis of their superior content including news headlines and multifarious informative discussion programs. Following we discuss an application of FS common enemy hypergraph (which is a type of FS competition hypergraph) in news channels and study how to apply the notion of FS common enemy hypergraphs in the competitive domain. The strength of power of each news channels in different cities in an FS common enemy hypergraph can be computed by Algorithm 6.

Method to evaluate the strength of power of each news channel in different cities

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1. Input the most suitable set of parameters $\mathcal{W}$ on n news channels $r_1, r_2, \cdots, r_n$.
2. Define the FS set $(\mathcal{X}, \mathcal{W})$ on n news channels and cities and FS relation $(\mathcal{K}, \mathcal{W})$ between n news channels and cities.
3. Calculate the FS in neighborhood of each news channel.

4. Construct the FS common enemy hypergraph $\mathcal{CEH}(\overrightarrow{\mathcal{G}}) = (\mathcal{X}, \mathcal{A}, \mathcal{W})$ by using Algorithm 4, where $\overrightarrow{\mathcal{G}}$ is a FS digraph.

5. The strength of each news channel can be computed by following formula:

$$(S(r_i), \mathcal{W}) = \sum_{w \in \mathcal{W}} (S(r_i)(w)),$$

where $S(r_i)(w) = \sum_{r_i \in E} \zeta_{\mathcal{A}(w)}(E)$, E is a hyperedge of fuzzy common enemy hypergraph corresponding to parameter w.

6. The decision is $S(r_i)$ if $S(r_i)$ has highest value among all other news channels, $1 \leq i \leq n$.

Consider the top 5 most popular news channels of the world BBC World News, Fox News, Cable News Network (CNN), Sky News, and MSNBC. In a set form, these news channels can be represented as

$$\mathcal{R} = \{(r_1 = \text{BBC World News}), (r_2 = \text{Fox News}),$$
$$(r_3 = \text{CNN}), (r_4 = \text{Sky News}), (r_5 = \text{MSNBC})\}.$$

Every news channel competes with another for the parameters $\{w_1, w_2, w_3, w_4\}$, where w_1 symbolizes current affairs talk shows' every news channel want to make their talk shows best to aware the audience for all the government exams. w_2 symbolizes 'news programs' the news keeps you updated and provides every type of important information, so that every news channel makes an effort to give accurate news to the audience. w_3 symbolizes 'interview programs' these types of shows are an excellent format for discussing different topics, so every news channel wants to invite whose guests which give valid information to viewers. w_4 symbolizes 'political news & discussion programs' every news channel try to give exact report of politicians and make authentic discussion programs for public. The membership degree of every news channel corresponding to parameters w_1, w_2, w_3, and w_4 is given in Table 16 which indicates the degree of trp of different programs including current affairs talk shows, news programs, interview programs, and political news & discussion programs that airing every day or once in a weak on these considered news channels.

The maximum membership value of Sky News corresponding to parameter w_1 indicates that current affairs talk show of Sky News has the highest trp as compared to other news channels. Also, consider a set

{London, New York, Paris, Moscow, Tokyo,

Singapore, Barcelona, Los Angeles},

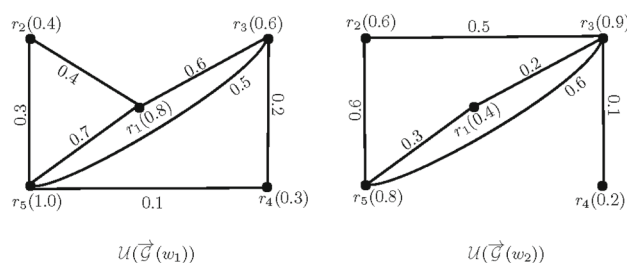**Fig. 15** Underlying FS graph $\mathcal{U}(\overrightarrow{\mathcal{G}}) = \{\mathcal{U}(\overrightarrow{\mathcal{G}}(w_1)), \mathcal{U}(\overrightarrow{\mathcal{G}}(w_2))\}$

of cities that are availing from these considered news channels. The membership grades of all cities are given in Table 17 which tell us about the interest or requirement of audiences for the above-mentioned parameters of the news channels. The membership value of London corresponding to parameter w_1 is 0.95 means that 95% people who are living in London want to watch current affairs talk shows on news channels. Furthermore, let

{(A1 = London), (A2 = New York City), (A3 = Paris),

(A4 = Mascow), (A5 = Tokyo), (A6 = Singapore),

(A7 = Barcelona), (A8 = Los Angeles)}.

The relationship between cities and news channels relative to above given four parameters is presented graphically in Fig. 15. This is the FS digraph and the directed edges between cities and news channels show the influences of audiences toward news channels. For example, the membership grade of London and Fox News is 0.75 in $\overrightarrow{\mathcal{G}}(w_1)$ means the people who are living in London only 75% like current affairs talk show of Fox News.

The FS in neighborhood of FS digraph 15 is given in Table 18. Using Algorithm 4, compute the hyperedges of FS common enemy hypergraph of FS digraph 15. By direct calculation, it is easy to check that

1. {BBC World News, CNN}, {BBC World News, Fox News}, and {Fox News, Sky News, MSNBC} are hyperedges of $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_1))$.

2. {BBC World News, MSNBC}, {BBC World News, Fox News}, and {BBC World News, CNN, Sky News} are hyperedges of $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_2))$.

3. {Fox News, MSNBC}, {BBC World News, Fox News, CNN}, {CNN, Sky News}, and {BBC World News, Sky News} are hyperedges of $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_3))$.

4. {BBC World News, MSNBC}, {BBC World News, Fox News, CNN}, and {CNN, Sky News} are hyperedges of $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_4))$.

Calculate the membership grades of these hyperedges using Definition 4.3. The FS common enemy hypergraph

Table 16 Membership grades of news channels corresponding to parameters

News Channels	Degrees of w_1	Degrees of w_2	Degrees of w_3	Degrees of w_4
BBC World News	0.68	0.86	0.51	0.68
Fox News	0.79	0.70	0.48	0.79
CNN	0.74	0.64	0.57	0.74
Sky News	0.83	0.93	0.66	0.83
MSNBC	0.82	0.72	0.21	0.82

Table 17 Membership values of cities corresponding to parameters

Cities	Degrees of w_1	Degrees of w_2	Degrees of w_3	Degrees of w_4
London	0.95	0.56	0.60	0.65
New York	0.76	0.56	0.72	0.99
Paris	0.81	0.71	0.43	0.71
Mascow	0.78	0.80	0.54	0.81
Tokyo	0.37	0.77	0.31	0.57
Singapore	0.87	0.77	0.85	0.77
Barcelona	0.54	0.50	0.79	0.32
Los Angeles	0.42	0.62	0.28	0.47

$\mathcal{CEH}(\mathcal{G})$ is given in Fig. 16. The hyperedge {BBC World News, CNN} of $\mathcal{CEH}\overrightarrow{\mathcal{G}}(w_1)$ show that there are common cities A2 and A8 between BBC World News and CNN, and both these channels compete for cities A2 and A8. The membership grade of each hyperedge between news channels indicates the influences of audiences of common cities toward news channels.

Now, we evaluate the strength of power of each news channel using FS common enemy hypergraph. The strength of power of each news channel is computed in Table 19 which indicates the dominant worth of each news channel in different cities. However, Table 15 indicates that BBC World News is the most powerful/dominant news channel than others. In other words, it is the most watchable news channel in different cities.

Discussion and comparison analysis

In this section, we discuss the comparison of proposed FS competition hypergraphs with FS competition graphs and fuzzy competition hypergraphs.

Comparison with fuzzy soft common enemy graphs

FS common enemy graphs [31] deal with real-world situations in the presence of parameters and it is successfully manipulated in different research domains. The decision-making problem that we presented in Sect. Fuzzy soft hypergraphs can also be discussed using FS common enemy graphs. Now, we study the above news channels problem by

applying the framework of FS common enemy graph. The FS common enemy graph of FS digraph 15 is specified in Fig. 17.

The edge {Fox News, Sky News} of $\mathcal{CE}(\overrightarrow{\mathcal{G}}(w_1))$ states that there is a common city A6 between Fox News and Sky News, and both these channels compete for A6. Moreover, Fox News and MSNBC, and Sky News and MSNBC also compete for common city A6. However, if we see in FS common enemy hypergraph 16, there is a hyperedge {Fox News, Sky News, MSNBC} in $\mathcal{CEH}(\overrightarrow{\mathcal{G}}(w_1))$ which indicates that these are the only channels which compete for A6. Therefore, we concluded two consequences from this example which are the following:

▶ FS common enemy graphs produce pair-wise relations, conflicts, and influences among objects. This model fails to tell whether there is a correspondence or rivalry among more than two objects.
▶ FS common enemy hypergraphs give information not merely as a couple but also provide group-wise relationships between objects. Therefore, our proposed model generalizes the existing model.

Now, we evaluate the strength of power of each news channel by utilizing the above-stated technique 17. Using FS common enemy graph 17, the strength of power of each news channel is given in Table 20. Meanwhile, Table 20 specifies that BBC World News is the most powerful/dominant news channel among other channels. In short, it is the most watchable news channel in different cities.

Table 18 FS in neighborhood of news channels

News channels	N^-(News channels)
BBC World News	$\{\frac{(A2,0.26),(A5,0.19),(A8,0.31)}{w_1}, \frac{(A1,0.46),(A2,0.35),(A5,0.19),(A8,0.31)}{w_2}, \frac{(A1,0.47),(A2,0.49),(A5,0.20),(A7,0.45)}{w_3}, \frac{(A2,0.65),(A4,0.56),(A5,0.39)}{w_4}\}$
Fox News	$\{\frac{(A1,0.75),(A5,0.33),(A6,0.59)}{w_1}, \frac{(A5,0.63),(A7,0.48)}{w_2}, \frac{(A3,0.26),(A5,0.18)}{w_3}, \frac{(A1,0.49),(A2,0.38),(A4,0.52)}{w_4}\}$
CNN	$\{\frac{(A2,0.29),(A8,0.36)}{w_1}, \frac{(A2,0.40),(A4,0.55),(A8,0.60)}{w_2}, \frac{(A4,0.41),(A5,0.28),(A6,0.55)}{w_3}, \frac{(A2,0.69),(A4,0.71),(A6,0.60)}{w_4}\}$
Sky News	$\{\frac{(A3,0.79),(A4,0.74),(A6,0.77)}{w_1}, \frac{(A2,0.50),(A8,0.60)}{w_2}, \frac{(A6,0.65),(A7,0.61)}{w_3}, \frac{(A3,0.38),(A6,0.75),(A8,0.34)}{w_4}\}$
MSNBC	$\{\frac{(A6,0.62),(A7,0.43)}{w_1}, \frac{(A1,0.40),(A3,0.70),(A6,0.62)}{w_2}, \frac{(A3,0.14),(A8,0.07)}{w_3}, \frac{(A5,0.32),(A7,0.21)}{w_4}\}$
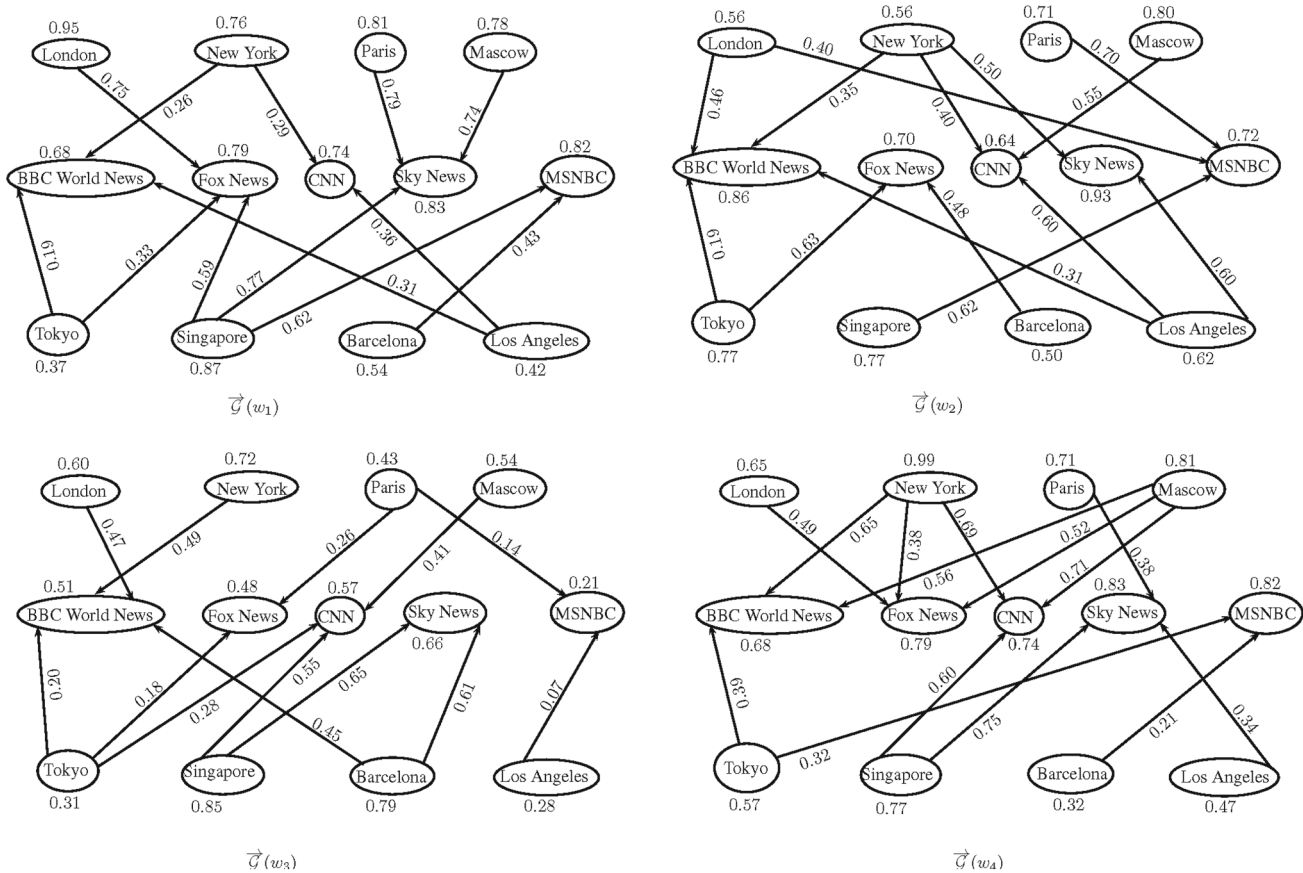
Fig. 16 FS digraph representing the interaction among cities and news channels

Table 19 Strength of power of each news channel

News channels	$S(r_i)(w_1)$	$S(r_i)(w_2)$	$S(r_i)(w_3)$	$S(r_i)(w_4)$	$(S(r_i), \mathcal{W})$
BBC World News	0.34	0.645	0.3159	0.5712	1.8721
Fox News	0.5953	0.133	0.1158	0.3536	1.1977
CNN	0.2108	0.224	0.3999	0.7976	1.6323
Sky News	0.4661	0.224	0.543	0.444	1.6771
MSNBC	0.4661	0.288	0.0294	0.2176	1.0011
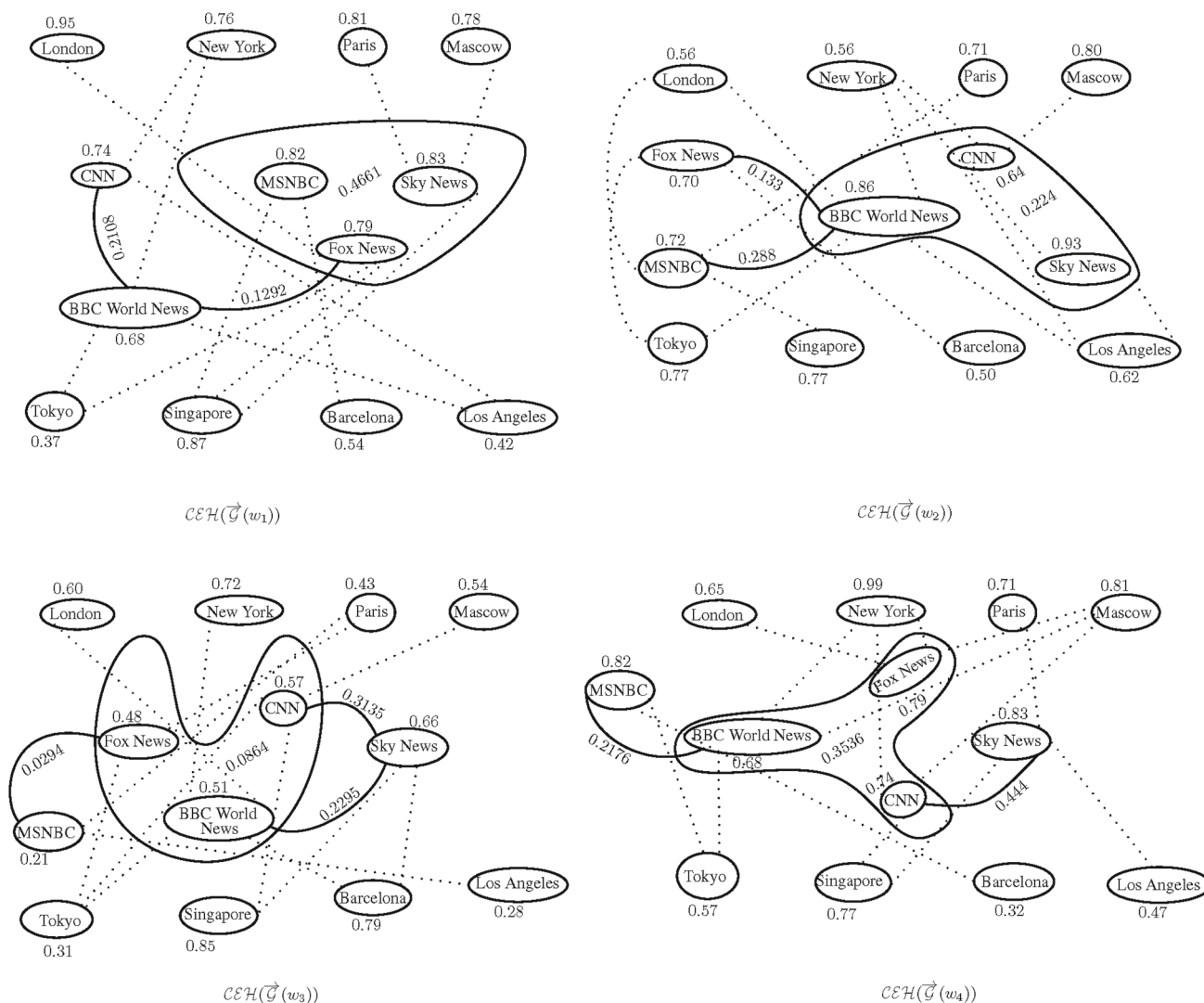
Fig. 17 FS common enemy hypergraph $\mathcal{CEH}(\vec{\mathcal{G}})$

Table 20 Strength of power of each news channel

News channels	$S(r_i)(w_1)$	$S(r_i)(w_2)$	$S(r_i)(w_3)$	$S(r_i)(w_4)$	$(S(r_i), \mathcal{W})$
BBC World News	0.34	0.946	0.4179	1.0132	2.7171
Fox News	1.0614	0.133	0.2022	0.7384	2.135
CNN	0.2108	0.608	0.5019	1.2708	2.5915
Sky News	0.9745	0.685	0.543	0.444	2.6465
MSNBC	0.9745	0.288	0.0294	0.2176	1.5095

These calculations and conclusion lead us to the following two questions.

If the consequence of existing technique and proposed model are analogous then

(i) Why we choose and support the proposed technique in decision making queries rather than this existing approach?

(ii) Which type of piece of information we acquired from proposed model?

Table 22 Strength of news channel

News channels	$S(r_i)$
BBC World News	0.34
Fox News	0.5953
CNN	0.2108
Sky News	0.4661
MSNBC	0.4661

Table 23 Strength of news channel

News Channels	$S(r_i)$
BBC World News	0.34
Fox News	1.0614
CNN	0.2108
Sky News	0.9745
MSNBC	0.9745

The following illustration will provide the answers. The strength of power of each news channel which is obtained from proposed and existing technique is given in Table 21. It is observed from Table 21 that the consequences of FS common enemy graph and FS common enemy hypergraph are similar. However, the divergence in significant values gives distinct and unique information in both cases. For instance, in fuzzy common enemy graph, the significant values of each news channel tell us the strength of power of news channels just in pair-wise and fail to tell the strength of power of each news channel in group-wise conflicts. However, our proposed model is a key approach to handle this loss of information and give more accurate, precise, and flexibility to the system as compared to the existing methods discussed in literature. Additionally, the final evaluation of our proposed model is that it generalizes the existing model and provides a better illustration of real-world phenomenon regarding fuzziness. This discussion manifests the effectiveness of our proposed model and yields a reason to prefer the proposed model in such decision-making problems.

Comparison with fuzzy common enemy hypergraphs

Fuzzy common enemy hypergraphs [30] play a key role in different domains of technology, social networking, and biological sciences for demonstrating real-world problems using fuzzy models. However, in all these problems, the relations among objects were considered only in one direction. For instance, in Sect. 4, the decision-making problem tackles four parameters at a time, but in fuzzy common enemy hypergraphs, we can deal with only one parameter in a context. Either, we can examine the connection between news channels and different cities for 'current affairs talk shows' or 'news programs' or 'interview programs' or 'political news & discussion programs'. Now, we check the relationship among news channels and different cities for 'current affairs talk shows' and evaluate the conclusions. The fuzzy common enemy hypergraph corresponding to fuzzy 'current affairs talk shows' digraph is shown in Fig. 19. The hyperedge {Fox News, Sky News, MSNBC} of fuzzy common enemy hypergraph $\mathcal{CEH}(\overrightarrow{\mathcal{G}})$ express that Fox News, Sky News, and MSNBC compete for common city

A6. The membership grade of each hyperedge points out the influences of audiences of common cities toward news channels. For example, the membership grade of hyperedge {Fox News, Sky News, MSNBC} is 0.4661 asserts that almost 46% residential people of city A6 which like the current affairs talk shows of given news channels. However, FS common enemy hypergraph 16 shows the multiple interest of people including 'current affairs talk shows', 'news programs', 'interview programs', and 'political news & discussion programs' in one frame. This technique is incapable to deal with the conflicts, correlations, or communications among objects in the proximity of parameters.

Using fuzzy common enemy hypergraph 18, the strength of power of each news channel is given in Table 22. Moreover, Table 22 indicates that Fox News is the most powerful/dominant news channel among other channels. In other terms, it is the most watchable news channel in different cities.

Comparison with fuzzy common enemy graphs

The existing fuzzy common enemy graph technique is beneficial to evaluate the strength of competition interrelationships among objects. This technique also resolved various problems emerging in crisp methods of evaluation. In this method, the fuzziness was studied only in one direction. Now, we discuss the above-stated application with this technique by taking one parameter 'current affairs talk shows' and evaluate the strength of power and observe the divergence in conclusions. The fuzzy common enemy graph corresponding to fuzzy 'current affairs talk shows' digraph is shown in Fig. 20.

Now, we calculate the strength of power of each news channel by utilizing this technique. Table 23, depicts that the most powerful/dominant news channel among other channels is Fox News.

The main disadvantages of this technique are:

► The relative attributes of above-stated problem cannot be taken under consideration in this technique.
► The consequences that we attain from this technique only describe competition or conflicts between two objects or

Table 21 Strength of power of news channels using different techniques

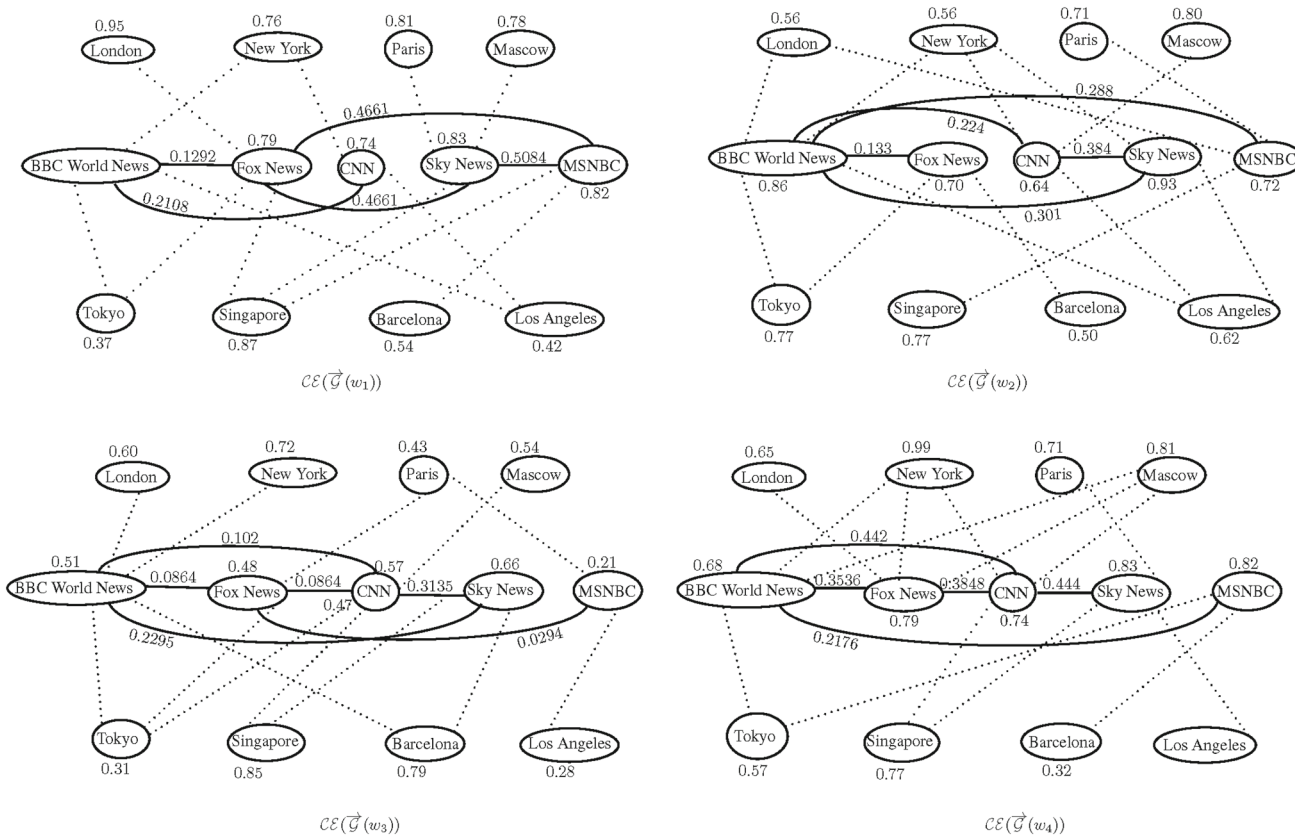| News channels	Proposed model	FS common enemy graph
BBC World News	1.8721	2.7171
Fox News	1.1977	2.135
CNN	1.6323	2.5915
Sky News	1.6771	2.6465
MSNBC	1.0011	1.5095

Fig. 18 FS common enemy graph $\mathcal{CE}(\overrightarrow{\mathcal{G}})$

entities. These shortcoming flaws can be handled through FS common enemy hypergraph, i.e., our proposed model.

Comparison of strength of powers by applying the proposed technique and existing methods, i.e., FS common enemy hypergraph, fuzzy common enemy hypergraph, and fuzzy common enemy graph, is given in Fig. 21.

The conclusions that we attain through fuzzy common enemy hypergraph and fuzzy common enemy graph exhibit the strength of power relative to 'current affairs talk shows' and ignoring the other parameters, because in these existing techniques, we tackle with only one parameter at a time. From above discussion and analysis, we observed that present theories concerning competition lack a lot of crucial facts and hides many errors and flaws. However, our proposed technique has overcome these limitations, permits to handling

this diversity, and assists to approach such decision-making problems not only pair-wise but also in group-wise rivalries and relations. These discussions show the validity of our proposed technique.

Conclusion and future directions

In most of the real-world problems, data have uncertain behavior and changes its nature with respect to different parameters which motivates the necessity of fuzzy soft modeling techniques. Fuzzy soft models play a vital in various domains of science and technology and give more accuracy, precision, and flexibility to the system as compared to crisp, fuzzy, and soft models. In this research article, we have introduced a novel framework as extension of compe-
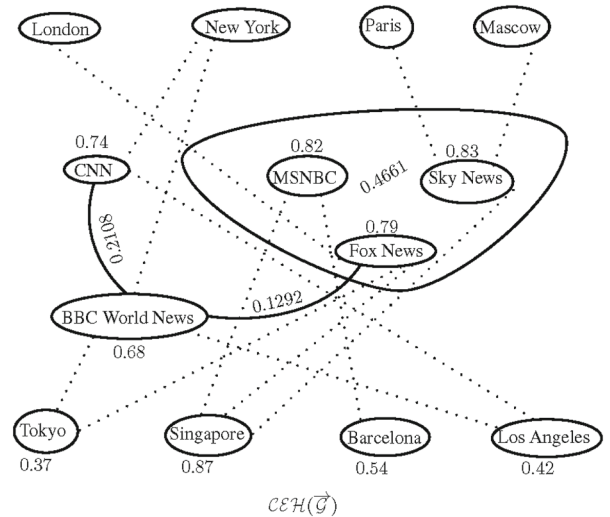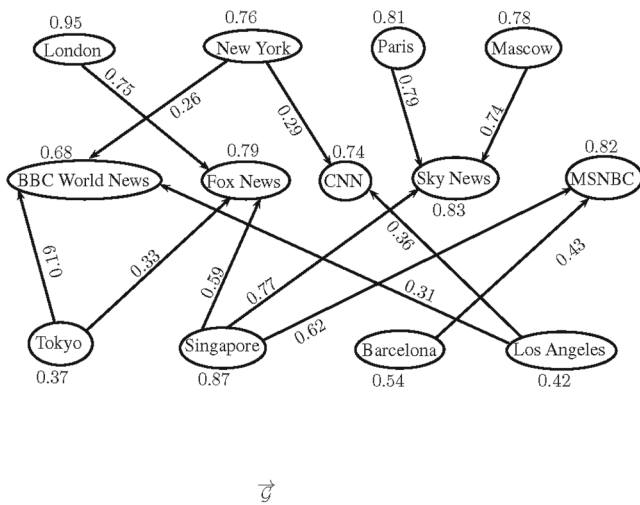
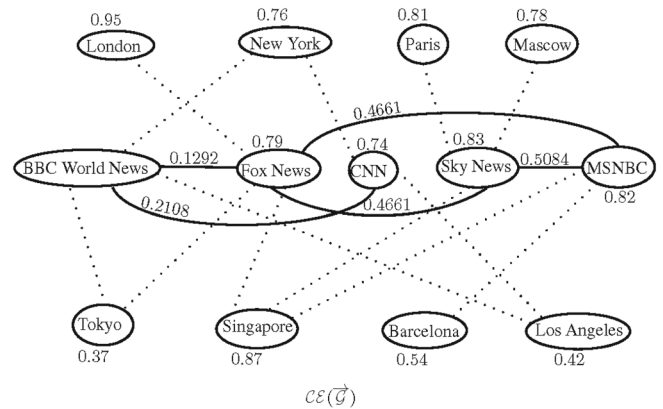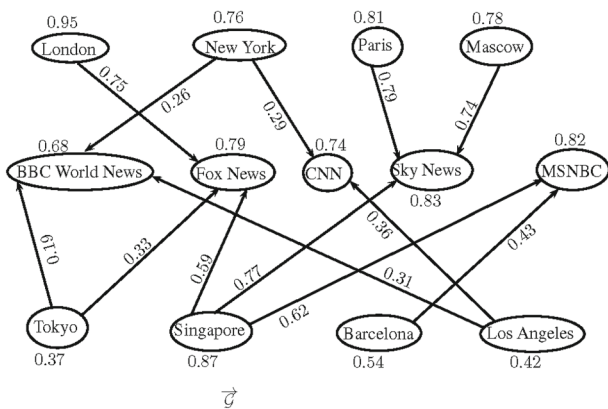**Fig. 19** Example of fuzzy common enemy hypergraph

Fig. 20 Example of fuzzy common enemy graph

tition hypergraphs and fuzzy soft competition hypergraphs by integrating fuzzy soft sets with competition hypergraphs. We have discussed certain types of fuzzy competition hypergraphs to illustrate different competitions in directed fuzzy soft networks. We have also introduced the notions of fuzzy soft k-competition hypergraphs and fuzzy soft neighborhood hypergraphs. We have designed certain algorithms to reduce the calculation procedure while computing these mathematical structures. We have studied the importance of our proposed theory with an application. The proposed model is more effective as it discussed two types of uncertainties in competition networks. On the other hand, it also has some limitations as it increases the calculation complexity with the increase of parameters. This research work can be further extended to 1) neutrosophic soft competition hypergraphs and 2) bipolar fuzzy soft competition hypergraphs.
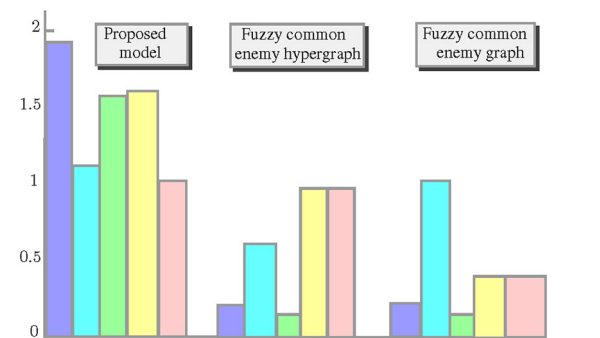
Fig. 21 Graphical representation of proposed model and existing techniques

Declarations

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