Critic-observer-based decentralized force/position approximate optimal control for modular and reconfigurable manipulators with uncertain environmental constraints

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Abstract

A critic-observer decentralized force/position approximate optimal control method is presented to address the joint trajectory and contacted force tracking problem of modular and reconfigurable manipulators (MRMs) with uncertain environmental constraints. The dynamic model of the MRM systems is formulated as an integration of joint subsystems via extensive state observer (ESO) associated with the effect of interconnected dynamic coupling (IDC). A radial basis function neural network (RBF-NN) is developed to deal with the IDC effects among the independent joint subsystems. Based on adaptive dynamic programming (ADP) approach and policy iteration (PI) algorithm, the Hamilton–Jacobi–Bellman (HJB) equation is approximately solved by establishing critic NN structure and then the approximated optimal control policy can be derived. The closed-loop manipulator system is proved to be asymptotic stable by using the Lyapunov theory. Finally, simulation results are provided to demonstrate the effectiveness and advantages of the proposed control method.

Keywords Modular robot manipulators · Adaptive dynamic programming · Force/position optimal control · Neural network

Introduction

Modular and reconfigurable manipulators (MRMs) have attracted attentions in manipulator community. Until now, MRMs possess widely applications, e.g. space explorations, medical assistance, etc. The appropriate control schemes are required to guarantee robustness as well as efficiency of MRMs facing complex and uncertain environmental constraints.

Force/position control, as a useful way to tackle contacting environment tasks of manipulators, have attracted extensive attentions in practices. Recently, force/position control

approaches have been divided into parallel force/position control [1], hybrid force/position control [2], impedance [3] etc. Hu et al. [4] addressed force/position control problem of the manipulator via sliding mode control. A novel estimator based on sparse Gaussian process regression is presented for approximating interaction force [5]. However, in the complex and extreme environments, it is an urgent problem to prove robustness of manipulator systems, and consider comprehensive optimization of tracking performance and energy consumption.

Optimal control method developed 6 decades ago by Pontryagin [6], Bellman [7], and many papers have been reported [8–11]. Recently, adaptive dynamic programming (ADP) have been used in discrete-time systems [12–14], continuous-time systems [15–17], data driven systems [18–20], input/output constraints [21,22], uncertain external disturbance [23–25] as well as actuator failures [26], etc. Some investigations reported ADP-based manipulator systems. An ADP-based NN method was presented for manipulator based on learning [27]. An RL optimal control method was proposed to solve dynamic compensation in manipulator tasks [28]. The methods are centralized control which we have mentioned above. An significant property of MRMs is that the modules can adjust without control parameters variation

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of other modules. Furthermore, local controllers need possess large storage capacity for calculating information under the centralized control policy. To solve above difficulties, decentralized control only utilizes the dynamic information and local states of each corresponding subsystem, which is considered an efficient method for MRMs. A network control method based on local information is presented by Jia [29]. An decentralized feedback control is presented for dead zone systems [30]. A decentralized method is developed for large-scale systems by Zhao [31]. A robust control method for reconfigurable manipulator is presented by Li [32]. Some researchers are proposed decentralized ADP-based method. Bian [33] developed a decentralized controller for unmatched uncertainties system. Zhao [34] developed a policy iteration (PI) for nonlinear systems. To the authors' point of view, there are few researches concentrated on dealing with optimal decentralized force/position control of manipulator systems, especially for MRM with uncertain environmental constraints.

Motivated by the researches above, a critic-observer decentralized force/position approximate optimal control method is developed to solve joint trajectory and contacted force tracking problem of MRMs with uncertain environmental constraints. The dynamic model of the MRM systems is formulated via extensive state observer (ESO) associated with interconnected dynamic coupling (IDC). A radial basis function neural network (RBF-NN) is developed to tackle IDC effects among the independent joint subsystems. Based on ADP and PI algorithms, the Hamilton-Jacobi-Bellman (HJB) equation is approximately addressed using critic NN and then approximated optimal control policy can be derived. The closed-loop manipulator system is proved to be asymptotic stable. Simulation results are provided to show advantage of proposed control method.

The major contributions can be summarized as:

1. To the best of author's knowledge, it is first time to extend ADP to solve decentralized force/position control problem of MRMs with uncertain environmental constraints.
2. Because of different properties of the MRM dynamics, we present two compensator structure to deal with the robot system. ESO is proposed to solve unknown dynamic model because of different configurations and RBF-NN identifies the IDC effect among the modules. Through different compensate scheme, the tracking performance of MRM system can be improved by simulation verification.

The other part is organized as follows: Section "Problem statement and dynamic model" is problem statement and dynamic model formulation. The decentralized approximate optimal force/position control scheme is shown in section "Decentralized approximate optimal force/position control".

Simulations are verified the effectiveness of developed control method in section "Simulation". Section "Conclusion" presents conclusion.

Problem statement and dynamic model

Considering an MRM system is:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + D = u + \tau_c, \quad (1)$$

where $q \in R^n$ means joint movements, $M(q)$ denotes inertia matrix, $C(q, \dot{q})$ indicates centripetal Coriolis matrix, $G(q)$ means gravity term, D denotes uncertain environment, $u \in R^n$ is control torque, $\tau_c \in R^n$ means constraint torque caused by end-effector contact with the environment, and there has $\tau_c = J_\Phi^T(q) \cdot f_{for}$, where $J_\Phi(q) \in R^{n \times n}$ means Jacobian matrix, $\Phi(q)$ is constraint equation, f_{for} means constrained space force. The contact force is vertical to the motion method, which means the force will not change the directions.

Then, according to the reference [35], by separating terms depend on $(q_i, \dot{q}_i, \ddot{q}_i)$, each subsystem dynamic model is given as:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) + Z_i(q, \dot{q}, \ddot{q}) + D_i = u_i + \tau_{ci}, \quad (2)$$

where

$$\begin{aligned} Z_i = & \left(\sum_{j=1, j \neq i}^n (M_{ii}(q) - M_i(q_i)) \ddot{q}_i + M_{ij}(q) \ddot{q}_j \right) \\ & + \left(\sum_{j=1, j \neq i}^n (-C_i(q_i, \dot{q}_i) + C_{ii}(q, \dot{q})) \dot{q}_i + C_{ij}(q, \dot{q}) \dot{q}_j \right) \\ & + (\bar{G}_i(q) - G_i(q_i)), \end{aligned} \quad (3)$$

in which $q_i, \dot{q}_i, \ddot{q}_i, u_i, \tau_{ci}, D_i$ are i th element of the vectors $q, \dot{q}, \ddot{q}, u, \tau_c, D$; $M_i(q_i), C_i(q_i, \dot{q}_i), G_i(q_i)$ are i th separate local element of $M(q), C(q, \dot{q}), G(q)$; Z_i is IDC term; $\bar{G}_i$ is the i th element of G ; M_{ij}, C_{ij} are ij th element of M and C .

Remark 1 D_i is considered as an uncertain environmental constraints term, that caused by the unmeasured environmental contact/collisions, rough surface of constraint, etc. The up-bound can be given as $|D_i| \leq T_{dM}$, where T_{dM} means positive constant. In the two-link robot control case, the phenomenon of constraint is mainly resulting from the external environmental contact on the end-effector.

Define the system state as $x_i = [x_{i1}, x_{i2}]^T = [q_i, \dot{q}_i]^T$, then, (2) can change as following

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = f_i(x_{i1}, x_{i2}) + g_i(x_{i1})(u_i + \tau_{ci}) + h_i(x) + d_i, \\ y_i = x_{i1} \end{cases} \quad (4)$$

in which the nonlinear terms $f_i(x_{i1}, x_{i2})$, $g_i(x_{i1})$, $h_i(x)$ and d_i are formulated as:

$$f_i(x_{i1}, x_{i2}) = M_i^{-1}(q_i)(-C_i(q_i, \dot{q}_i)\dot{q}_i - G_i(q_i))$$

$$g_i(x_{i1}) = M_i^{-1}(q_i)$$

$$h_i(x) = -M_i^{-1}(q_i)Z_i(q, \dot{q}, \ddot{q})$$

$$d_i = -M_i^{-1}(q_i)D_i,$$

where x is global state of the whole MRM and $f_i(x_{i1}, x_{i2})$, $g_i(x_{i1})$ and $h_i(x)$ are all Lipschitz.

Next, a decentralized approximate optimal force/position control method is developed for MRM with uncertain environment to ensure trajectory tracking of MRM systems are asymptotically stable as well as the desired contact force can be satisfied.

Decentralized approximate optimal force/position control

Derivation of the force/position optimal control

For constrained MRM system (2), we can define the error function which is related to the trajectory tracking error as well as force tracking error:

$$e_i(t) = k_{qwei}\dot{e}_{qi} + k_{wei}e_{qi} + k_{\tau i} \int_0^t e_{\tau i} dt, \quad (5)$$

where $e_{qi} = x_{i1} - x_{i1d}$ is position tracking error, x_{i1d} means desired joint position, $e_{\tau i} = \tau_{ci} - \tau_{di}$ is constrained torque deviation of end-effector, τ_{di} means desired constraint torque, $\dot{e}_{qi} = \dot{x}_{i1} - \dot{x}_{i1d}$ represents the velocity tracking error, k_{qwei} , k_{wei} , $k_{\tau i}$ are the function coefficients. The time derivative of e_i is represented:

$$\dot{e}_i(t) = k_{qwei}(f_i(x_{i1}, x_{i2}) + g_i(x_{i1})(u_i + \tau_{ci}) + h_i(x) + d_i) + v_i, \quad (6)$$

where $v_i = -k_{qwei}\ddot{x}_{i1d} + k_{wei}\dot{e}_{qi} + k_{\tau i}e_{\tau i}$.

Define performance index function as follows which is related to the error performance and control strategy:

$$J_i(e_i) = \int_0^\infty (\beta_i T_{dM}^2 + N_i(e_i, u_i) + \gamma_i (\nabla J_i^*(e_i))^2) dt, \quad (7)$$

where T_{dM} is the known up-bound function for d_i , $N_i(e_i, u_i) = e_i^T Q_i e_i + u_i^T R_i u_i$ means utility function composed of force/position deviation fusion, $N_i(0, 0) = 0$ as well as $N_i(e_i, u_i) \geq 0$ for all e_i, u_i . Q_i, R_i mean positive definite matrices and γ_i means determined constant. $\Psi(\Omega)$ means admissible control policies.

Definition 1 Consider system (2), u_i is admissible for performance index (7) on Ω , and $J_i(e_i)$ is finite, $\forall e_i \in \Omega$. Give admissible control $u_i \in Z(\Omega)$, where $Z(\Omega)$ means admissible control set, and infinite form of (8) is Lyapunov equation,

$$\begin{aligned} 0 &= N_i(e_i, u_i) + \gamma_i (\nabla J_i^*(e_i))^2 + \beta_i T_{dM}^2 + (\nabla J_i(e_i))^T \cdot \dot{e}_i \\ &= N_i(e_i, u_i) + \gamma_i (\nabla J_i^*(e_i))^2 + \beta_i T_{dM}^2 + (\nabla J_i(e_i))^T \cdot \\ &\quad (k_{qwei}(f_i(x_{i1}, x_{i2}) + g_i(x_{i1})(u_i + \tau_{ci}) + h_i(x) + d_i) + v_i), \end{aligned} \quad (8)$$

with $J_i^*(e_i)$ means optimal performance index function and $\nabla J_i(e_i)$ is partial derivative of $J_i(e_i)$ which has $\nabla J(e_i) = \frac{\partial J(e_i)}{\partial e_i}$.

The HJB function of the force/position control under uncertain environmental constraints is:

$$\begin{aligned} H_i(e_i, u_i, \nabla J_i(e_i)) &= \beta_i T_{dM}^2 + \gamma_i (\nabla J_i^*(e_i))^2 + N_i(e_i, u_i) \\ &\quad + (\nabla J_i(e_i))^T \cdot \dot{e}_i = \beta_i T_{dM}^2 + e_i^T Q_i e_i + u_i^T R_i u_i \\ &\quad + \gamma_i (\nabla J_i^*(e_i))^2 + (\nabla J_i(e_i))^T \cdot (k_{qwei}(f_i(x_{i1}, x_{i2}) \\ &\quad + g_i(x_{i1})(u_i + \tau_{ci}) + h_i(x) + d_i) + v_i), \end{aligned} \quad (9)$$

and the optimal performance index function is:

$$J_i^*(e_i) = \min_{u_i} \int_0^\infty (\beta_i T_{dM}^2 + N_i(e_i, u_i) + \gamma_i (\nabla J_i^*(e_i))^2) dt. \quad (10)$$

$J_i^*(e_i)$ satisfies HJB equation.

$$0 = \min_{u_i} H_i(e_i, u_i, \nabla J_i^*(e_i)), \quad (11)$$

where $\nabla J^*(e_i) = \frac{\partial J^*(e_i)}{\partial e_i}$, if $J_i^*(e_i)$ is continuously differentiable, the optimal force/position control is

$$u_i^* = -\frac{1}{2} k_{qwei} R_i^{-1} g_i^T \nabla J_i^*(e_i). \quad (12)$$

Dealing with dynamic identification of MRM, IDC term and uncertain environmental constraints, respectively, rewrite

optimal control law u_i^* as $u_i^* = u_{i1} + u_{i2} + u_{i3}^*$, so that HJB equation can be modified as:

$$0 = \beta_i T_{dM}^2 + e_i^T Q_i e_i + u_i^T R_i u_i + \gamma_i (\nabla J_i^*(e_i))^2 + (\nabla J_i(e_i))^T \cdot (k_{qwei} (f_i(x_{i1}, x_{i2}) + g_i(x_{i1})(u_{i1} + u_{i2} + u_{i3}^* + \tau_{ci}) + h_i(x) + d_i) + v_i). \quad (13)$$

ESO-based dynamic identification of MRM

As we all know that the dynamic model of MRM is hard to directly obtain, an ESO is proposed to approximate nonlinear term $f_i(x_{i1}, x_{i2})$, $g_i(x_{i1})$.

According to the state space (4), we consider the MRM system without the IDC effects and uncertainty environmental constraints. Based on the idea of ESO, (4) can be augmented as

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = x_{i3} + g_i(x_{i1})(u_{i1} + \tau_{ci}) \\ \dot{x}_{i3} = F_i(x_{i1}, x_{i2}) \\ y_i = x_{i1} \end{cases}, \quad (14)$$

where $F_i(x_{i1}, x_{i2}) = \dot{f}_i(x_{i1}, x_{i2})$. The estimation error is defined as $e_{io} = [e_{io1} \ e_{io2} \ e_{io3}]^T = [\hat{x}_{i1} \ \hat{x}_{i2} \ \hat{x}_{i3}]^T - [x_{i1} \ x_{i2} \ x_{i3}]^T$, then the ESO is constructed as

$$\begin{cases} \dot{\hat{x}}_{i1} = \hat{x}_{i2} - \beta_{i01}\varphi(\hat{x}_{i1} - x_{i1}) \\ \dot{\hat{x}}_{i2} = \hat{x}_{i3} - \beta_{i02}\varphi(\hat{x}_{i1} - x_{i1}) + \hat{g}_i(\hat{x}_{i1})(u_{i1} + \tau_{ci}) \\ \dot{\hat{x}}_{i3} = -\beta_{i03}\varphi(\hat{x}_{i1} - x_{i1}) \end{cases}, \quad (15)$$

where φ means a function about the estimation error, β_{i0j} ($j = 1, 2, 3$) is adjustable observer gain. The estimation error dynamic can be expressed as

$$\begin{cases} \dot{e}_{io1} = e_{io2} - \beta_{i01}\varphi(e_{io1}) \\ \dot{e}_{io2} = e_{io3} - \beta_{i02}\varphi(e_{io1}) \\ \dot{e}_{io3} = -\beta_{i03}\varphi(e_{io1}) - F_i(\hat{x}_{i1}, \hat{x}_{i2}) \end{cases}. \quad (16)$$

Motivated by finite-time stable system, $\varphi(e_{io1})$ can be defined as follows:

$$\varphi(e_{io1}) = -k_i \operatorname{sgn}(e_{io1}) |e_{io1}|^{\kappa_i}, \kappa_i \in (0, 1). \quad (17)$$

A finite-time convergent ESO can be constructed as

$$\begin{cases} \dot{e}_{io1} = e_{io2} - \beta_{i01} |e_{io1}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{io1}) \\ \dot{e}_{io2} = e_{io3} - \beta_{i02} |e_{io1}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{io1}) \\ \dot{e}_{io3} = -\beta_{i03} |e_{io1}|^{\kappa_i} \operatorname{sgn}(e_{io1}) - F_i(\hat{x}_{i1}, \hat{x}_{i2}) \end{cases}. \quad (18)$$

Theorem 1 Consider the dynamic system with $f_i(x_{i1}, x_{i2})$, $g_i(x_{i1})$ in (4) and the dynamic system is formulated in (14) as well as the up-bound of $F_i(x_{i1}, x_{i2})$ is F_{iM} . The ESO presents in (15) and there exist constants $\beta_{i0j} > 0$ ($j = 1, 2, 3$) and $(\beta_{i01}\beta_{i02})^{\frac{\kappa_i+1}{2}} - \beta_{i03} > 0$ such that can ensure $\lim_{t \rightarrow \infty} e_{io}(t) = 0$.

Proof For the ESO in Eq. (18), define

$$\zeta_i = \begin{bmatrix} |e_{io1}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{io1}) & e_{io2} & e_{io3} \end{bmatrix}^T$$

$$P_i = \begin{bmatrix} \frac{2\beta_{i01}}{\alpha_i+1} + \beta_{i02}^2 + \beta_{i03}^2 & -\beta_{i02} & -\beta_{i03} \\ -\beta_{i02} & 2 & 0 \\ -\beta_{i03} & 0 & 2 \end{bmatrix}.$$

Then, the Lyapunov candidate function is chosen as

$$V_i(\beta_{i0j}, \zeta_i) = \zeta_i^T P_i \zeta_i. \quad (19)$$

When $\beta_{i0j} > 0$, P_i is a symmetric positive definite matrix. Let λ_i represents eigenvalue sequence,

$$\lambda_{\min}(P_i) \|\zeta_i\|_2^2 \leq V_i \leq \lambda_{\max}(P_i) \|\zeta_i\|_2^2, \quad (20)$$

where $\|\zeta_i\|_2^2 = |e_{io1}|^{\kappa_i+1} + e_{io2}^2 + e_{io3}^2$, and

$$|e_{io1}|^{\frac{\kappa_i+1}{2}} \leq \|\zeta_i\|_2 \leq \sqrt{\frac{V_i}{\lambda_i \min(P_i)}}. \quad (21)$$

To apply Lyapunov theorem, we derivative V_i respect to time, thus we can obtain that

$$\begin{aligned} \dot{V}_i &= \begin{bmatrix} |e_{io1}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{io1}) & e_{io2} & e_{io3} \end{bmatrix}^T \\ &\quad Y_i \begin{bmatrix} |e_{io1}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{io1}) & e_{io2} & e_{io3} \end{bmatrix}^T \\ &\quad + 2F_i(\beta_{i03}|e_{io1}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{io1}) + 2e_{io3}) \\ &= -\zeta_i^T Y_i \zeta_i + 2F_i(\beta_{i03}|e_{io1}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{io1}) + 2e_{io3}) \\ &= V_{i1} + V_{i2} \end{aligned} \quad (22)$$

where Y_i can be defined as follows

$$Y_i = \begin{bmatrix} -\beta_{i01}(\kappa_i + 1) \left(\frac{2\beta_{i01}}{\kappa_i + 1} + \beta_{i02}^2 + \beta_{i03}^2 \right) |e_{i01}|^{\frac{\kappa_i-1}{2}} + 2\beta_{i03}^2 |e_{i01}|^{\frac{\kappa_i-1}{2}} + 2\beta_{i02}^2, \\ \frac{\kappa_i + 1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} \left(\frac{2\beta_{i01}}{\kappa_i + 1} + \beta_{i02}^2 + \beta_{i03}^2 + \beta_{i01}\beta_{i02} \right) - 2\beta_{i02}, \beta_{i01}\beta_{i03} \frac{\kappa_i + 1}{2} \\ - 2\beta_{i03} |e_{i01}|^{\frac{\kappa_i-1}{2}} - \beta_{i02}; \frac{\kappa_i + 1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} \left(\frac{2\beta_{i01}}{\kappa_i + 1} + \beta_{i02}^2 + \beta_{i03}^2 + \beta_{i01}\beta_{i02} \right) \\ - 2\beta_{i02}, -\beta_{i02}(\kappa_i + 1) |e_{i01}|^{\frac{\kappa_i-1}{2}}, -\beta_{i03} \frac{\kappa_i + 1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} + 2, \beta_{i01}\beta_{i03} \\ \frac{\kappa_i + 1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}}; -2\beta_{i03} |e_{i01}|^{\frac{\kappa_i-1}{2}} - \beta_{i02}, -\beta_{i03} \frac{\kappa_i + 1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} + 2, 0; \end{bmatrix},$$

and can be equivalent to

$$A_i^T P_i + P_i A_i = -Y_i, \quad (23)$$

where

$$A_i = \begin{bmatrix} -\beta_{i01} \frac{\kappa_i+1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} & \frac{\kappa_i+1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} & 0 \\ -\beta_{i02} & 0 & 1 \\ -\beta_{i03} |e_{i01}|^{\frac{\kappa_i-1}{2}} & 0 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{\kappa_i+1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -|e_{i01}|^{\frac{\kappa_i-1}{2}} \end{bmatrix} \begin{bmatrix} \beta_{i01} & -1 & 0 \\ \beta_{i02} & 0 & -1 \\ \beta_{i03} & 0 & 0 \end{bmatrix}. \quad (24)$$

According to Eq. (24), we can obtain

$$\begin{aligned} |sI - A_i| &= \begin{vmatrix} s + \beta_{i01} \frac{\kappa_i+1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} & -\frac{\kappa_i+1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} & 0 \\ \beta_{i02} & s & -1 \\ \beta_{i03} |e_{i01}|^{\frac{\kappa_i-1}{2}} & 0 & s \end{vmatrix} \\ &= s^3 + \beta_{i01} \frac{\kappa_i + 1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} s^2 + \beta_{i02} \frac{\kappa_i + 1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} s \\ &\quad + \beta_{i03} \frac{\kappa_i + 1}{2} |e_{i01}|^{\kappa_i-1}. \end{aligned} \quad (25)$$

From Eq. (25), the polynomial coefficients are all positive. If $\beta_{i01}, \beta_{i02}, \beta_{i03}, \kappa_i$ are chosen properly, and satisfy

$$\begin{cases} \beta_{i01} \frac{\kappa_i+1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} > 0 \\ \beta_{i02} \frac{\kappa_i+1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} > 0 \\ \beta_{i03} \frac{\kappa_i+1}{2} |e_{i01}|^{\kappa_i-1} > 0 \\ \beta_{i01}\beta_{i02} \frac{\kappa_i+1}{2} - \beta_{i03} > 0, \end{cases} \quad (26)$$

then A_i is Hurwitz. $\square$

Remark 2 According to formulation of A_i in (24), only guarantee A_i is Hurwitz in (26), the observer can be effectiveness. First of all, we choose $\beta_{i01}, \beta_{i02}, \beta_{i03}$ positive as well as, the first of three equations in (26) can be satisfied. That means if $\beta_{i01}\beta_{i02} \frac{\kappa_i+1}{2} > \beta_{i03}$ satisfies, the parameters can choose properly.

According to the Lyapunov theory, there exists solution P_i in Eq. (23).

$$V_{i1} = -\zeta_i^T Y_i \zeta_i \leq -\lambda_{\min}(Y_i) \|\zeta_i\|_2^2. \quad (27)$$

V_{i2} can be re-expressed as follows:

$$\begin{aligned} V_{i2} &= 2F_i(\beta_{i03} |e_{i01}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{i01}) + 2e_{i03}) \\ &= [2F_i \ 0 \ 4F_i] \begin{bmatrix} |e_{i01}|^{\frac{\kappa_i+1}{2}} \operatorname{sgn}(e_{i01}) \\ e_{i02} \\ e_{i03} \end{bmatrix} \\ &= [2F_i \ 0 \ 4F_i] \zeta_i \leq 2\sqrt{5}F_{iM} \|\zeta_i\|_2. \end{aligned} \quad (28)$$

Combing Eqs. (27), (28), we can know that

$$\begin{aligned} \dot{V}_i &= V_{i1} + V_{i2} \leq -\lambda_{\min}(Y_i) \|\zeta_i\|_2^2 + 2\sqrt{5}F_{iM} \|\zeta_i\|_2 \\ &= -(\lambda_{\min}(Y_i) \|\zeta_i\|_2 - 2\sqrt{5}F_{iM}) \|\zeta_i\|_2. \end{aligned} \quad (29)$$

Y_i is positive definite and nonsingular, then

$$\begin{aligned} \lambda_{\min}(Y_i) &= \sigma_{\min}(Y_i) = \sigma_{\min}(-A_i^T P_i + P_i A_i) \\ &= \sigma_{\min}\left((-A_i)^T P_i + ((-A_i)^T P_i)^T\right) \\ &= 2\sigma_{\min}\left((-A_i)^T P_i\right) \\ &\geq 2\sigma_{\min}(-A_i)^T \sigma_{\min}(P_i) \\ &= 2 \min\left(\frac{\kappa_i + 1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}}, 1\right) \sigma_{\min}(\Xi_i) \lambda_{\min}(P_i), \end{aligned} \quad (30)$$

where

$$(-A_i)^T = \begin{bmatrix} \frac{\kappa_i+1}{2} |e_{i01}|^{\frac{\kappa_i-1}{2}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & |e_{i01}|^{\frac{\kappa_i-1}{2}} \end{bmatrix} \begin{bmatrix} \beta_{i01} & \beta_{i02} & \beta_{i03} \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \Theta_i \Xi_i.$$

When $|e_{i01}| > 1, \|\zeta_i\|_2 \geq |e_{i01}|^{\frac{\kappa_i+1}{2}}, \lambda_{\min}(Y_i) \geq (\kappa_i + 1)|e_{i01}|^{\frac{\kappa_i-1}{2}} \sigma_{\min}(\Xi_i) \lambda_{\min}(P_i)$, thus

$$\lambda_{\min}(Y_i) \|\zeta_i\|_2 - 2\sqrt{5}F_{iM} \geq (\kappa_i + 1)\sigma_{\min}(\mathcal{E}_i)\lambda_{\min}(P_i) - 2\sqrt{5}F_{iM}. \quad (31)$$

$$\begin{aligned} \dot{V} &\leq -\left(\lambda_{\min}(Y_i) \|\zeta_i\|_2 - 2\sqrt{5}F_{iM}\right) \|\zeta_i\|_2 \\ &\leq -\left((\kappa_i + 1)\sigma_{\min}(\mathcal{E}_i)\lambda_{\min}(P_i) - 2\sqrt{5}F_{iM}\right) \|\zeta_i\|_2 \\ &\leq -\left((\kappa_i + 1)\sigma_{\min}(\mathcal{E}_i)\lambda_{\min}(P_i) - 2\sqrt{5}F_{iM}\right) \|\zeta_i\|_2 \\ &\quad \sqrt{\frac{V_i}{\lambda_{\max}(P_i)}} \\ &= -\frac{\left((\kappa_i + 1)\sigma_{\min}(\mathcal{E}_i)\lambda_{\min}(P_i) - 2\sqrt{5}F_{iM}\right)}{\sqrt{\lambda_{\max}(P_i)}} V_i^{\frac{1}{2}} \leq 0. \end{aligned} \quad (32)$$

This completes the proof of the Theorem 1.

Remark 3 Different from the traditional ESO, the proposed ESO solves the problem of complicated parameter setting of the controller as well as reduce computation burden of MRM.

Therefore, u_{i1} can be defined as:

$$u_{i1} = k_{ip}(x_{id} - \hat{x}_{i1}) - k_{id}\hat{x}_{i2} - \hat{x}_{i3}, \quad (33)$$

where k_{ip}, k_{id} are controller gains.

Identification of the IDC

Because of the IDC effects among independent joint subsystems, it will reduce the tracking performance via force/position optimal control law. Furthermore, the magnitudes of IDC is totally different with system dynamics $f_i(x_{i1}, x_{i2}), g_i(x_{i1})$. In this part, an RBF-NN is proposed to approximate the term $h_i(x)$.

Using RBF-NN, one can obtain that

$$h_i = \omega_i^T \varphi_i(x_i) + \varepsilon_i, \quad (34)$$

where $\varphi_i = \exp(-\frac{\|x_i - c_{ij}\|^2}{2b_{ij}^2})$ denotes Gaussian basis function and c_{ij}, b_{ij} are the radial basis function neural network centers and widths. ω_i is ideal neural network weights, ε_i is neural network approximation errors. Define $\hat{\omega}_i$ as estimations of ω_i , $\hat{h}_i$ is estimation value of $h_i(x)$, which can be expressed as:

$$\hat{h}_i = \hat{\omega}_i^T \varphi_i(x_i). \quad (35)$$

Define estimation errors as $\tilde{\omega}_i = \hat{\omega}_i - \omega_i$. Therefore, we have

$$h_i - \hat{h}_i = \omega_i^T \varphi_i + \varepsilon_i - \hat{\omega}_i^T \varphi_i = -\tilde{\omega}_i^T \varphi_i + \varepsilon_i. \quad (36)$$

The updated law can be defined as follows:

$$\dot{\tilde{\omega}}_i = \gamma_i e_i \varphi_i(x_i). \quad (37)$$

The RBF-NN-based control law is given as:

$$u_{i2} = -\lambda_i \dot{e}_i - \hat{h}_i + \ddot{x}_{id} - \eta_i \operatorname{sgn}(e_i). \quad (38)$$

Theorem 2 Considering the IDC $h_i(x)$ in (4) and the dynamic system regarding of the IDC is formulated. The RBF-NN presents in (35), and the control law denotes in (38), such that can ensure the IDC term to be asymptotically identified, $\lim_{t \rightarrow \infty} \tilde{h}_i(t) = 0$.

Proof Choose the Lyapunov function as

$$v_i = \frac{1}{2} e_i^2 + \frac{1}{2\gamma_i} \tilde{\omega}_i^T \tilde{\omega}_i, \quad (39)$$

where γ_i denotes a positive constant.

To apply Lyapunov theorem, the time derivative of v_i is obtained as

$$\begin{aligned} \dot{v}_i &= e_i(h_i - \hat{h}_i - \eta_i \operatorname{sgn}(e_i)) + \frac{1}{\gamma_i} \tilde{\omega}_i^T \dot{\tilde{\omega}}_i \\ &= e_i(-\tilde{\omega}_i^T \varphi_i(x) + \varepsilon_i - \eta_i \operatorname{sgn}(e_i)) + \frac{1}{\gamma_i} \tilde{\omega}_i^T \dot{\tilde{\omega}}_i \\ &= e_i \varepsilon_i - \eta_i |e_i| + \tilde{\omega}_i^T \left(\frac{1}{\gamma_i} \dot{\tilde{\omega}}_i - e_i \varphi_i \right). \end{aligned} \quad (40)$$

Then, we can get

$$\dot{v}_i = \varepsilon_i e_i - \eta_i |e_i| \leq 0, \quad (41)$$

where $\eta_i > |\varepsilon_i|_{\max}$. This completes the Theorem 2. $\square$

Critic NN implementation

For finding the optimal control of MRM system (2), one need to solve (13) for the optimal performance index function ∇J_i^* and then obtain optimal control u_i^* . However, the HJB equation (13) is a nonlinear partial differential equation and hard to solve. Therefore, we can apply critic NN to approximate J_i^* .

Using critic NN, we have

$$J_i(e_i) = w_{iF}^T \sigma_{iF}(e_i) + \varepsilon_{iF}, \quad (42)$$

where $w_{iF} \in R^{N_i}$ denotes desired weight, N_i means amount of neurons in hidden-layer, $\sigma_{iF}(e_i)$ is activation function, ε_{iF} means critic NN approximation error. The partial derivative of $J_i(e_i)$ is:

$$\nabla J_i(e_i) = (\nabla \sigma_{iF}(e_i))^T w_{iF} + \nabla \varepsilon_{iF}, \quad (43)$$

where $\nabla \sigma_{iF}(e_i) = \frac{\partial \sigma_{iF}(e_i)}{\partial e_i} \in R^{N_i \times n}$ is partial derivative of activation function, $\nabla \varepsilon_{iF}$ is approximate error.

Combining (4), (8), (43), one obtain

$$\begin{aligned} 0 &= \beta_i T_{dM}^2 + N_i(e_i, u_i) + \gamma_i \\ &\quad \left((\nabla \sigma_{iF}(e_i))^T w_{iF} + \nabla \varepsilon_{iF} \right)^T \left((\nabla \sigma_{iF}(e_i))^T w_{iF} + \nabla \varepsilon_{iF} \right) \\ &\quad + \left((\nabla \sigma_{iF}(e_i))^T w_{iF} + \nabla \varepsilon_{iF} \right)^T \dot{e}_i. \end{aligned} \quad (44)$$

Therefore, the Hamiltonian function is:

$$\begin{aligned} H_i(e_i, u_i, w_{iF}) &= \beta_i T_{dM}^2 + N_i(e_i, u_i) \\ &\quad + \gamma_i \times \left((\nabla \sigma_{iF}(e_i))^T w_{iF} + \nabla \varepsilon_{iF} \right)^T \\ &\quad \times \left((\nabla \sigma_{iF}(e_i))^T w_{iF} \right. \\ &\quad \left. + \nabla \varepsilon_{iF} \right) + \left((\nabla \sigma_{iF}(e_i))^T w_{iF} + \nabla \varepsilon_{iF} \right)^T \dot{e}_i = e_{iJh}, \end{aligned} \quad (45)$$

where e_{iJh} means residual error that is caused of NN approximation. Because w_{iF} is unknown, $J_i(e_i)$ can be approximated by critic NN:

$$\hat{J}_i(e_i) = \hat{w}_{iF}^T \sigma_{iF}(e_i), \quad (46)$$

where $\hat{J}_i(e_i)$ is estimation of $J_i(e_i)$, $\hat{w}_{iF}$ denotes estimation of w_{iF} . Then, partial derivative of $\hat{J}_i(e_i)$ is:

$$\nabla \hat{J}_i(e_i) = (\nabla \sigma_{iF}(e_i))^T \hat{w}_{iF}. \quad (47)$$

Therefore, HJB function can be written as follows:

$$\begin{aligned} H_i(e_i, u_i, \hat{w}_{iF}) &= \beta_i T_{dM}^2 + \gamma_i \left((\nabla \sigma_{iF}(e_i))^T \hat{w}_{iF} \right)^T \\ &\quad \left((\nabla \sigma_{iF}(e_i))^T \hat{w}_{iF} \right) + \left((\nabla \sigma_{iF}(e_i))^T \hat{w}_{iF} \right)^T \dot{e}_i \\ &\quad + N_i(e_i, u_i) = e_{iJ}. \end{aligned} \quad (48)$$

Define objective function $E_{iJ} = \frac{1}{2} e_{iJ}^T e_{iJ}$ which is minimized by gradient decent algorithm for adjusting $\hat{w}_{iF}$, which is updated by

$$\dot{\hat{w}}_{iF} = -\alpha_{iF} e_{iJ} \nabla \sigma_{iF}(e_i) \dot{e}_i, \quad (49)$$

where α_{iF} means updated rate of critic NN. Denote $v_i = \nabla \sigma_{iF}(e_i) \dot{e}_i$, and consider there exists a positive constant v_{iL} that $\|v_i\| \leq v_{iL}$.

Define weight approximation error as

$$\tilde{w}_{iF} = w_{iF} - \hat{w}_{iF}. \quad (50)$$

By (45), (48) and (49), we have

$$e_{iJ} = e_{iJh} - \tilde{w}_{iF}^T v_i. \quad (51)$$

The weight approximation error is

$$\dot{\tilde{w}}_{iF} = -\dot{\hat{w}}_{iF} = \alpha_{iF} e_{iJ} v_i = \alpha_{iF} (e_{iJh} - \tilde{w}_{iF}^T v_i) v_i. \quad (52)$$

Combining (12) and (43), the desired control policy is as follows:

$$u_{i3}^* = -\frac{1}{2} k_{qwei} R_i^{-1} g_i^T \left((\nabla \sigma_{iF}(e_i))^T w_{iF} + \nabla \varepsilon_{iF} \right). \quad (53)$$

Therefore, the approximated force/position optimal control policy $\hat{u}_{i3}^*$ is

$$\hat{u}_{i3}^* = -\frac{1}{2} k_{qwei} R_i^{-1} \hat{g}_i^T (\nabla \sigma_{iF}(e_i))^T \hat{w}_{iF}. \quad (54)$$

By combining (33), (38) with (54), the proposed decentralized force/position approximate optimal control law u_i^* is

$$\begin{aligned} \hat{u}_i^* &= k_{ip}(x_{id} - \hat{x}_{i1}) - k_{id}\hat{x}_{i2} - \hat{x}_{i3} - \lambda_i \dot{e}_i - \hat{h}_i + \ddot{x}_{id} \\ &\quad - \eta_i \operatorname{sgn}(e_i) - \frac{1}{2} k_{qwei} R_i^{-1} \hat{g}_i^T (\nabla \sigma_{iF}(e_i))^T \hat{w}_{iF}. \end{aligned} \quad (55)$$

Policy iteration algorithm

To address HJB equation, PI algorithm is shown as follows.

Step 1: Let $k = 0$, start with an initial admissible control $u_i^{(0)}$, and select a small positive constant ε_i .

Step 2: Let $k > 0$, based on $u_i^{(k)}$, solve $\hat{J}_i^{(k)}(e_i)$ via $0 = \beta_i T_{dM}^2 + N_i(e_i, u_i^{(k)}) + \gamma_i (\nabla \hat{J}_i^{(k+1)}(e_i))^2 + \nabla \hat{J}_i^{(k+1)}(e_i)^T \cdot (k_{qwei}(f_i(\hat{x}_{i1}, \hat{x}_{i2}) + \hat{g}_i(\hat{x}_{i1}))(u_{i1} + u_{i2} + u_{i3}^* + \tau_{ci}) + \hat{h}_i(x) + d_i) + v_i$, with $\hat{J}^{(k+1)}(0) = 0$.

Step 3: Update $u_i^{(k)}$ through $u_i^{(k+1)} = -\frac{1}{2} k_{qwei} R_i^{-1} \hat{g}_i^T(\hat{x}_{i1}) \nabla \hat{J}_i^{(k+1)}(e_i)$.

Step 4: While $\|\hat{J}_i^{(k+1)}(e_i) - \hat{J}_i^{(k)}(e_i)\| \leq \varepsilon_i$, stop and get optimal control; else, let $k = k + 1$ and back to Step 2.

The algorithm will converge to optimal performance index function and control policy, $\hat{J}_i^{(k)}(e_i) \rightarrow \hat{J}_i^*(e_i)$, $u_i^{(k)} \rightarrow u_i^*$ while $t \rightarrow \infty$.

Stability analysis

We investigate stability of closed-loop MRM system under the force/position optimal control law (55), which is:

Theorem 3 Considering an n -DOF MRM system, with the dynamic model formulation (2) and the IDC existed in (4).

The approximated force/position optimal tracking control policy (55) with ESO (15) and RBF-NN (35) can ensure the asymptotic stability of the closed-loop manipulator system with approximate (46) that is updated by (49).

Proof Choose the Lyapunov candidate function as:

$$V(t) = \sum_{i=1}^n V_i(t) = \sum_{i=1}^n \left(\frac{1}{2} e_i^T e_i + J_i^*(e_i) + \frac{1}{2} \int_t^\infty (\tau_{ci}^2 + T_{dM}^2) dt + \frac{1}{2\alpha_{iF}} \tilde{w}_{iF}^T \tilde{w}_{iF} \right). \quad (56)$$

The time derivative of $V(t)$ is:

$$\dot{V}(t) = \sum_{i=1}^n \left(e_i^T \dot{e}_i + (\nabla J_i^*(e_i))^T \dot{e}_i - \frac{1}{2} \|\tau_{ci}\|^2 - \frac{1}{2} \|T_{dM}\|^2 + \frac{1}{\alpha_{iF}} \tilde{w}_{iF}^T \dot{\tilde{w}}_{iF} \right). \quad (57)$$

According to (8) and (13), we have

$$\dot{V}(t) = \sum_{i=1}^n \left(e_i^T \cdot \left(k_{qwei} \begin{pmatrix} f_i(\hat{x}_{i1}, \hat{x}_{i2}) + \hat{g}_i(\hat{x}_{i1})(u_{i1} + u_{i2} + u_{i3}^* + \tau_{ci}) + \hat{h}_i(x) + d_i \end{pmatrix} + v_i \right) - \beta_i T_{dM}^2 - e_i^T Q_i e_i - u_i^T R_i u_i - \gamma_i (\nabla J_i^*(e_i))^2 - \frac{1}{2} \|\tau_{ci}\|^2 - \frac{1}{2} \|T_{dM}\|^2 + \tilde{w}_{iF}^T e_{iJh} v_i - \|\tilde{w}_{iF} v_i\|^2 \right). \quad (58)$$

Because of $f_i(\hat{x}_{i1}, \hat{x}_{i2})$ is locally Lipchitz, assuming $L_{if} > 0$ is a positive constant and $\|f_i(\hat{x}_{i1}, \hat{x}_{i2})\| \leq L_{if} \|e_i\|$, $\|\hat{g}(\hat{x}_{i1})\| \leq \omega_{ig}$ with $\omega_{ig} > 0$. Assume $\|\ddot{q}_{id}\| \leq \delta_i$, $\|\dot{e}_{qi}\| \leq \xi_i$, $\|e_{\tau i}\| \leq \eta_i$, by Young's inequality, we have

$$\dot{V}(t) \leq \sum_{i=1}^n \left(\|e_i\| (k_{qwei} \omega_{ig} (u_i + \tau_{ci} + T_{dM}) + L_{if} \|e_i\|) + v_i \right) - \beta_i T_{dM}^2 - e_i^T Q_i e_i - u_i^T R_i u_i - \gamma_i (\nabla J_i^*(e_i))^2 - \frac{1}{2} \|\tau_{ci}\|^2 - \frac{1}{2} \|T_{dM}\|^2 + \tilde{w}_{iF}^T e_{iJh} v_i - \|\tilde{w}_{iF} v_i\|^2$$

Fig. 1 MRMs with different configurations under uncertain environmental constraints

$$\begin{aligned} &= \sum_{i=1}^n \left(v_i \|e_i\| + k_{qwei} L_{if} \|e_i\|^2 + k_{qwei} \omega_{ig} \|e_i\| \|u_i\| + k_{qwei} \omega_{ig} \|e_i\| \|\tau_{ci}\| + k_{qwei} \omega_{ig} \|e_i\| \|T_{dM}\| - \beta_i T_{dM}^2 - e_i^T Q_i e_i - u_i^T R_i u_i - \gamma_i (\nabla J_i^*(e_i))^2 - \frac{1}{2} \|\tau_{ci}\|^2 - \frac{1}{2} \|T_{dM}\|^2 + \tilde{w}_{iF}^T e_{iJh} v_i - \|\tilde{w}_{iF} v_i\|^2 \right) \\ &\leq \sum_{i=1}^n \left(v_i \|e_i\| + k_{qwei} L_{if} \|e_i\|^2 + \frac{3}{2} k_{qwei}^2 \omega_{ig}^2 \|e_i\|^2 + \frac{1}{2} \|u_i\|^2 + \frac{1}{2} \|\tau_{ci}\|^2 + \frac{1}{2} \|T_{dM}\|^2 - \lambda_{\min}(Q_i) \|e_i\|^2 - \lambda_{\min}(R_i) \|u_i\|^2 - \gamma_i (\nabla J_i^*(e_i))^2 - \frac{1}{2} \|\tau_{ci}\|^2 - \frac{1}{2} \|T_{dM}\|^2 - \frac{1}{2} \left(\|\tilde{w}_{iF} v_i\|^2 - \|e_{iJh}\|^2 \right) - \beta_i T_{dM}^2 \right) \\ &\leq \sum_{i=1}^n \left(\|e_i\| \left(v_i - \lambda_{\min}(Q_i) \|e_i\| + k_{qwei} L_{if} \|e_i\| + \frac{3}{2} k_{qwei}^2 \omega_{ig}^2 \|e_i\| \right) - \gamma_i (\nabla J_i^*(e_i))^2 - \left(\lambda_{\min}(R_i) - \frac{1}{2} \right) \|u_i\|^2 - \frac{1}{2} \left(\|\tilde{w}_{iF} v_i\|^2 - \|e_{iJh}\|^2 \right) - \beta_i T_{dM}^2 \right) \\ &\leq \sum_{i=1}^n \left(-\|e_i\| \left(\|e_i\| (\lambda_{\min}(Q_i) - \frac{3}{2} k_{qwei}^2 \omega_{ig}^2 - k_{qwei} L_{if}) - v_i \right) - \gamma_i (\nabla J_i^*(e_i))^2 - \left(\lambda_{\min}(R_i) - \frac{1}{2} \right) \|u_i\|^2 - \frac{1}{2} \left(\|\tilde{w}_{iF} v_i\|^2 - \|e_{iJh}\|^2 \right) - \beta_i T_{dM}^2 \right). \quad (59) \end{aligned}$$

Define $\Psi_{i1} = \lambda_{\min}(Q_i) - \frac{3}{2} k_{qwei}^2 \omega_{ig}^2 - k_{qwei} L_{if}$, $\Psi_{i2} = \lambda_{\min}(R_i) - \frac{1}{2}$. Then

$$\dot{V}(t) \leq \sum_{i=1}^n \left(-\|e_i\| (\Psi_{i1} \|e_i\| + k_{qwei} \delta_i - k_{wei} \xi_i - k_{\tau i} \eta_i) - \Psi_{i2} \|u_i\|^2 - \gamma_i (\nabla J_i^*(e_i))^2 - \frac{1}{2} \left(\|\tilde{w}_{iF} v_i\|^2 - \|e_{iJh}\|^2 \right) - \beta_i T_{dM}^2 \right). \quad (60)$$

So that, we have $\dot{V}(t) \leq 0$ while e_i lies outside the compact set $\Omega_{i1} = \left\{ e_i : \|e_i\| \leq \frac{-k_{qwei} \delta_i + k_{wei} \xi_i + k_{\tau i} \eta_i}{\Psi_{i1}} \right\}$, and $\Omega_{i2} = \left\{ \tilde{w}_{iF} \| \tilde{w}_{iF} \| \leq \left\| \frac{e_{iJh}}{v_{iL}} \right\| \right\}$, if following

Table 1 Control parameter setting

Name	Value	Name	Value	Name	Value	Name	Value	Name	Value	Name	Value
k_{ip}	1.5	k_{id}	0.3	λ_i	0.5	η_i	0.8	α_{iF}	0.9	k_{qwei}	1
β_i	1	β_{i01}	2.33	β_{i02}	1.56	β_{i03}	0.3	T_{dM}	2	γ_i	1

conditions hold

$$\begin{cases} \lambda_{\min}(Q_i) \geq k_{qwei} L_{if} + \frac{3}{2} k_{qwei}^2 \omega_{ig}^2 \\ \lambda_{\min}(R_i) \geq \frac{1}{2} \end{cases} \quad (61)$$

Moreover, (60) means that $\dot{V}(t) \leq 0$ for $e_i \neq 0$ while only (61) is satisfied. Hence, decentralized force/position approximate optimal controllers asymptotically stable.

Simulation

Simulation setup

By referring Newton-Euler iteration method and reference [36], two MRMs (see Fig. 1) are given for simulation. The end-effector is operating along with the constrained plane (the constrained equation). According to the constrained equation, we can obtain the desired trajectory and the MRM system is under uncertain environment such as contact or collision. The dynamic model for configuration A is:

$$M(q) = \begin{bmatrix} 0.6066 + 0.36 \cos(q_2) & 0.1233 + 0.18 \cos(q_2) \\ 0.1233 + 0.18 \cos(q_2) & 0.1233 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} -0.36 \sin(q_2) \dot{q}_2 & -0.18 \dot{q}_2 \sin(q_2) \\ 0.18 \sin(q_2) (\dot{q}_1 - \dot{q}_2) & 0.18 \dot{q}_1 \sin(q_2) \end{bmatrix}$$

$$G(q) = \begin{bmatrix} -5.88 \sin(q_1 + q_2) - 17.64 \sin(q_1) \\ -5.88 \sin(q_1 + q_2) \end{bmatrix},$$

and configuration B is:

$$M(q) = \begin{bmatrix} 0.17 - 0.1166 \cos^2(q_2) & -0.06 \cos(q_2) \\ -0.06 \cos(q_2) & 0.1233 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} -0.1166 \dot{q}_2 \sin(2q_2) & -0.06 \sin(q_2) \dot{q}_2 \\ (0.06 \sin(q_2) \dot{q}_2 - 0.0583 \sin(2q_2) \dot{q}_1) & -0.06 \dot{q}_1 \sin(q_2) \end{bmatrix}$$

and for configuration B can be described as

$$G(q) = \begin{bmatrix} 0 \\ -5.88 \sin(q_1 + q_2) \end{bmatrix}.$$

For configuration A, the constrained equation for MRM is: $\Phi(q) = -1.5 + l_2 \cos q_2 + l_1 = 0$ and for configuration B is $\Phi(q) = -1 + l_2 \cos q_2 + l_1 \cos q_1 = 0$, where l_1, l_2 are the lengths of two manipulator links. The desired trajectory as well as desired contact force for configuration A are:

$$q_{1d} = \sin(2t) + 0.2 \cos(t), \quad q_{2d} = \frac{\pi}{3}, \quad f_d = 10,$$

$$q_{1d} = 0.5 \sin(2t) + \cos(t),$$

$$q_{2d} = \arccos\left(\frac{1 - l_1 \cos(\cos(t) + 0.5 \sin(2t))}{l_2}\right), \quad f_d = 10.$$

The parameter definition is: $Q_i = R_i = I$. The critic NN is selected as 2-3-1, and $\hat{w}_{iF} = [\hat{w}_{iF1}, \hat{w}_{iF2}, \hat{w}_{iF3}]^T$ with the initial values $\hat{w}_{iF} = [10 \ 15 \ 20]^T$. The active function

Fig. 2 Trajectory tracking for configuration A with proposed methods. **a** Joint one **b** Joint two

Fig. 3 Position error for configuration A with existing methods. **a** Joint one **b** Joint Two

is selected as $\sigma_{iF} = [e_i^2, e_i^2, e_i^2]$, and the control parameters are in Table 1.

Simulation results

(1) Trajectory tracking performance

Figure 2 is joint position curve of configuration A under proposed method. The proposed method is effectiveness because

Fig. 4 Position error for configuration A with proposed methods. **a** Joint one **b** Joint two

Fig. 6 Velocity error for configuration A with existing methods. **a** Joint one **b** Joint two

Fig. 5 Velocity curves for configuration A with proposed methods. **a** Joint one **b** Joint Two

Fig. 7 Velocity error for configuration A with proposed methods. **a** Joint one **b** Joint two

the actual position can follow the desired trajectory very fast. Figures 3 and 4 illustrate joint position tracking error curves under existing and proposed method. In Fig. 3, we can observe position errors are obvious at initial times and amplitude of steady-state error is 0.04 rad. From Fig. 4, position errors can follow very fast. Figure 5 is the velocity tracking curves under the proposed method of configuration A. Figures 6 and 7 are velocity error curves under existing method and proposed method. Because of the existing methods have not considered the influence of IDC effects, so that the tracking error is larger than the proposed method.

(2) Control torque

Figure 8 is control torque curve with existing method of configuration A. For joint one, because of adjustment of the weight of critic NN, the initial control torque is large. For joint two, the control torque has a sudden variety in some time. That is because the method without dynamic compensation. Figure 9 is torque curve of configuration A with the proposed method. The output torques have been optimized with an appropriate behavior because of the developed scheme.

Fig. 8 Control torque for configuration A with existing methods. **a** Joint one **b** Joint two

Fig. 9 Control torque for configuration A with proposed methods. **a** Joint one **b** Joint two

(3) Force performance

Figures 10 and 11 represent the contact force and force error curves under the existence and proposed methods. One can observe that the proposed method can follow the contact force quickly and force error can be a very small value in a very short time which is attributed to the decentralized optimal force/position control method.

(4) Critic NN adjustment

Figure 12 is position error performance with different learning rate for configuration A. It is obvious that different learning rate can achieve different solution. If the parameter is too big, it will lead overtraining for iteration process and

Fig. 10 Force curve for configuration A with existing methods. **a** Contact force. **b** Force error

Fig. 11 Force curve for configuration A with proposed methods. **a** Contact force. **b** Force error

Fig. 12 Position error performance with different learning rate for configuration A. **a** Joint one **b** Joint two

Fig. 13 Trajectory tracking for configuration B with proposed methods. **a** Joint one **b** Joint two

Fig. 14 Position error for configuration B with existing methods. **a** Joint one **b** Joint two

the performance index will not improve. Besides the properties of accuracy in robotic system, fast response is also an important index. If α_{iF} select very small, the accuracy of tracking index will be discounted that means undertraining. Based on different simulation analysis, we select α_{iF} as 0.9.

Figures 13, 14, 15, 16, 17, 18, 19, 20, 21 and 22 are position tracking curves, position error, velocity curves, velocity error, control torque and contact force, force error curves for configuration B. The results are similar with configuration A. It improves that proposed method without the requirements of adjusting parameters. For the simulations, one can get the decentralized optimal force/position method can guarantee stability and accuracy.

Fig. 15 Position error for configuration B with proposed methods. **a** Joint one **b** Joint two

Fig. 16 Velocity curves for configuration B with proposed methods. **a** Joint one **b** Joint two

Fig. 17 Velocity error for configuration B with existing methods. **a** Joint one. **b** Joint two

Fig. 18 Velocity error for configuration B with proposed methods. **a** Joint one. **b** Joint two

Fig. 19 Control torque for configuration B with existing methods. **a** Joint one. **b** Joint two

Fig. 20 Control torque for configuration B with proposed methods. **a** Joint one. **b** Joint two

Fig. 21 Force curve for configuration B with existing methods. **a** Contact force. **b** Force error

Fig. 22 Force curve for configuration B with proposed methods. **a** Contact force. **b** Force error

Conclusion

A critic-observer decentralized force/position approximate optimal control method is presented to address the problems of optimal trajectory and force tracking problem for MRMs with uncertain environmental constraints. The dynamic model of the MRM systems is formulated as an integration of joint subsystems and the unknown manipulator dynamics are identified by the developed ESO. Then, an RBF-NN is developed to deal with IDC effects among the independent joint subsystems. Based on ADP and PI algorithms, HJB equation is approximately solved using critic NN and then the approximated optimal control policy can be derived. The closed-loop manipulator system is proved to be asymptotic stable by using the Lyapunov theory. Simulation results are provided to demonstrate the effectiveness and advantage of the proposed control method finally.

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