Least-looping stepping-stone-based ASM approach for transportation and triangular intuitionistic fuzzy transportation problems

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Abstract

Transportation problem (TP) is a popular branch of Linear Programming Problem in the field of Transportation engineering. Over the years, attempts have been made in finding improved approaches to solve the TPs. Recently, in Quddoos et al. (Int J Comput Sci Eng (IJCSSE) 4(7): 1271–1274, 2012), an efficient approach, namely ASM, is proposed for solving crisp TPs. However, it is found that ASM fails to provide better optimal solution in some cases. Therefore, a new and efficient ASM approach is proposed in this paper to enhance the inherent mechanism of the existing ASM method to solve both crisp TPs and Triangular Intuitionistic Fuzzy Transportation Problems (TIFTPs). A least-looping stepping-stone method has been employed as one of the key factors to improve the solution quality, which is an improved version of the existing stepping-stone method (Roy and Hossain in, Operation research Titus Publication, 2015). Unlike stepping stone method, least-looping stepping-stone method only deals with few selected non-basic cells under some prescribed conditions and hence minimizes the computational burden. Therefore, the framework of the proposed method (namely LS-ASM) is a combination of ASM (Quddoos et al. 2012) and least-looping stepping-stone approach. To validate the performance of LS-ASM, a set of six case studies and a real-world problem (those include both crisp TPs and TIFTPs) have been solved. The statistical results obtained by LS-ASM have been well compared with the existing popular modified distribution (MODI) method and the original ASM method, as well. The statistical results confirm the superiority of the LS-ASM over other compared algorithms with a less computational effort.

Keywords Transportation problem · Triangular intuitionistic fuzzy number · Stepping-stone method · ASM method · Optimal solution

Introduction

A constrained optimization problem consists of an objective function along with some constraints. If all of them are linear, then it is known as a Linear Programming Problem (LPP). The popular Transportation Problem (TP) is a particular case of LPP. The objective is to minimize the cost

of dispensing products from ‘ m ’ number of suppliers to ‘ n ’ number of different stations. The LPP formulation of the TP is originally developed by Hitchcock [1] in the year 1941, as follows.

Mathematical formulation of TP [1]

Consider a TP with ‘ m ’ sources and ‘ n ’ destinations, where p_{ij} is the unit transportation cost from i th source to j th destination. Let S_i be the supply quantity available at i th source and D_j be the demand quantity of j th destination. The goal of the objective function is to minimize the total transportation cost (Z), subject to the commodity q_{ij} is shipped that met the demand. The mathematical formulation of a balanced TP (where $\sum_{i=1}^m S_i = \sum_{j=1}^n D_j$) is given by

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$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n p_{ij} q_{ij}$$

subject to

$$\sum_{j=1}^n q_{ij} \leq S_i; i = 1, 2, \dots, m$$

$$\sum_{i=1}^m q_{ij} \geq D_j; j = 1, 2, \dots, n$$

$$q_{ij} \geq 0 \forall i, j.$$

The first two set of constraints fortify that the total quantities shipped from the source ‘*i*’ must not exceed its supply and the total quantity shipped to the ‘*j*th’ destination must be greater than or equal to its demand, respectively. The third set of constraints assures that quantity of commodity q_{ij} must be non-negative.

Real-world distribution problems are uncertain in nature because of the changes taking place in the parameters. Zadeh [2] developed the fuzzy set theory to deal with indistinct data computationally in making decisions and succeeded in its applications in different areas. The application of fuzzy set theory landed in the optimization field after the spearheading research carried out by Bellman and Zadeh [3]. Nonetheless, there are uncertainties with hesitation due to numerous factors like need for better communication, inaccuracy in data, comprehension of markets, unawareness of customers, etc. Likewise, the costs are still hanging with uncertainties with hesitation because of different components like diversity in rates of fuels, congested driving conditions, climatic conditions, and so on. In such circumstances, the decision-makers fail to foresee transportation cost precisely. Therefore, to deal with these uncertain data, Atanassov [4] proposed the Intuitionistic Fuzzy Set (IFS) which is validated to be more accurate than the fuzzy set theory proposed by Zadeh [2]. Afterward, quite ample number of successful approaches have been presented in the literature to solve TPs and IFTPs and the range of application has been steadily widened. It is now considered as one of the important tools in business and industry. Several sorts of methods have been proposed for finding the optimal solution. ASM [5] is one of such recent methods which has been applied to solve TPs and IFTPs, as well. The authors of ASM method solved a real-valued TP and then compared its result to that of MODI method. They concluded ASM to be a quick and effective approach. In the year 2012, Mohammad Kamrul Hasan [15] proposed that direct methods (including ASM method) for finding optimal solution of a TP do not reflect optimal solution continuously. Murugesan [17] admitted and perceived Hasan’s claim by verifying the ASM method for several benchmark problems. However, in the mean time, Abdul Quddoes et al. [16] experienced a few problems for which the ASM method does not

happen to provide the optimal solution, but executes well enough to provide a best IBFS, which appears to be close to the optimal solution. To resolve this issue, Abdul Quddoes et al. [16] forwarded a revised approach to the ASM method where MODI method was combined with the existing ASM method. This approach gives the optimal solution directly for most of the problems, and if not, it gives the best IBFS. Later, the claim was established by Murugesan et al. [18–23] in which they tested on 30 balanced and 20 unbalanced benchmark TPs. Over the years, it can be seen that several approaches have been mentioned in the literature for the improvement of ASM method. However, the concept of stepping stone was never used with ASM method in the past research. Therefore, an effort is made in this manuscript to improve the efficiency of the original ASM by a crucial modification in it. The authors developed a new approach, namely, the least-looping stepping-stone approach, and hybridized it with ASM to increase the robustness of the original ASM.

The manuscript is systemized as follows: Section “[Motivation](#)” presents the motivation of the proposed method; Section “[Proposed LS-ASM Method](#)” introduces the proposed algorithm along with six case studies and a real-life problem. Section [Results and Discussion](#) includes the comparative analysis of results. The conclusion part of this paper is produced in Section [Conclusion](#), which comprises some of the future opportunities, as well.

Motivation

To solve TPs, a number of suitable approaches have been presented in the literature. Most of these approaches attempted to calculate the Initial Basic Feasible Solution (IBFS) before employing its different steps. In 2012, a popular method, namely ASM, was proposed [5] to solve the classical TP without finding the IBFS. In fact, it was a direct approach basically named after the first letters of the authors’ names Abdul Quddoes, Shakeel Javaid, and M.M. Khalid. However, later in [8, 11], it is seen that ASM failed to get to the optimal cost for some of the TPs. To validate this fact, two case studies of Crisp TPs are explained below.

Case study-1:

Table 1 considers a balanced TP which is solved by ASM Method in Table 2 with total transportation cost = $7 \times 5 + 5 \times 5 + 9 \times 20 + 3 \times 25 + 3 \times 20 + 2 \times 5 + 3 \times 10 = 415$.

Case study-2:

Table 3 considers a balanced TP which is solved by ASM method in Table 4 with total transportation cost = $13 \times 80 + 15 \times 200 + 22 \times 50 + 11 \times 280 + 19 \times 220 + 11 \times 180 = 14,380$.

Table 1 A balanced minimization TP [8]

	D_1	D_2	D_3	D_4	Supply
S_1	7	5	9	11	30
S_2	4	3	8	6	25
S_3	3	8	10	5	20
S_4	2	6	7	3	15
Demand	30	30	20	10	90

Table 2 Optimal table according to ASM [5]

	D_1	D_2	D_3	D_4	Supply
S_1	7 ₅	5 ₅	9 ₂₀	11	30
S_2	4	3 ₂₅	8	6	25
S_3	3 ₂₀	8	10	5	20
S_4	2 ₅	6	7	3 ₁₀	15
Demand	30	30	20	10	90

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Table 3 A balanced minimization TP [11]

	D_1	D_2	D_3	D_4	Supply
S_1	13	15	16	18	280
S_2	20	22	11	8	330
S_3	19	25	17	11	400
Demand	300	250	280	180	1010

Table 4 Optimal table according to ASM [5]

	D_1	D_2	D_3	D_4	Supply
S_1	13 ₈₀	15 ₂₀₀	16	18	280
S_2	20	22 ₅₀	11 ₂₈₀	8	330
S_3	19 ₂₂₀	25	17	11 ₁₈₀	400
Demand	300	250	280	180	1010

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

It is worth noting that using each of the techniques like Vogel's Approximation Method (VAM)-MODI [9], North-West Corner Method (NWC)-MODI [9], Row Minima method (RMM)-MODI [9], and Column Minima Method (CMM)-MODI [9], the optimal solutions found are **410** and **14,180** for case studies-1 and 2, respectively. It motivated the authors to investigate more key factors of ASM, which needed to be substantially modified to enhance its robustness. Least-looping stepping-stone approach has been employed in the working mechanism of ASM to improve the robustness of ASM. This method is exempted from checking the optimality criteria for all the non-basic cells. Instead, it checks for some selected loops and calculates the net cost

change values. Therefore, it helps to minimize the computational time as compared to stepping-stone approach. This new approach is proven to be more attractive due to easy implementation and minimum calculation, as compared to the classical approaches. To validate the better performance of the proposed method in all sort of TPs, two problems on balanced TPs, two problems on TIFTPs, two problems on unbalanced TPs, and one real-life problem have been considered. The working mechanism of the proposed approach has been explained in details in the following section.

Proposed LS-ASM Method

Least-looping stepping-stone method

The stepping-stone Method [14] emerged from the idea of crossing a pond making use of stepping stones. The entire cost matrix is considered as a pond and the basic cells are the stones required for certain sequential movement within the pond. In other words, it is an iterative approach to reach the optimal solution from an IBFS. Each iteration includes the formation of closed loops* for each of the non-basic cells in the cost matrix. This helps in determining whether the current iteration has all the best routes. If not, then the only penalty imposed is to perform additional iterations.

However in this paper, the stepping-stone Method is suitably modified for its up gradation. Unlike ASM, LS-ASM considers only few selected non-basic cells for forming closed loops in its working mechanism (algorithm) presented below. First two iterations of least-looping stepping-stone are combined with ASM to reach the optimal solution. This new method is named as 'Least-looping stepping-stone based ASM Method' (**LS-ASM Method**). Even after this hybridization, the proposed method proves out to be an easy and time-saving approach. In the later part of this paper, LS-ASM has been compared with other mentioned approaches.

LS-ASM algorithm

Consider the given TP and construct its transportation table.

- Step1: If the transportation table is not balanced, then balance it by adding suitable dummy rows/columns. Else, proceed.
- Step 2: Subtract each row minima from the corresponding row costs and each column minima from the corresponding column costs.
- Step 3: Select a zero appearing in the cost matrix and count the total sum of the number of zeros (precluding the selected zero) in that corresponding row and column. Let us name it as sum-zero. Find the sum-zero for each of the existing zeros in the cost matrix.

Step 4: Choose a cell with the cost zero for which the sum-zero is minimum. Supply maximum possible unit to that zero cell. If tie occurs for some zeros in step 3, then take that zero cell for which total sum of all the costs of respective row and column is maximum and assign maximum possible unit to that cell.

Step 5: Block the row/column where the supply or the demand is met.

Step 6: Now, if the reduced cost matrix does not have at least one zero in each row and each column, then go to step 2; otherwise continue step 3 to step 5 until all the demands and supplies are met.

Step 7: Write the resulting table with both basic and non-basic cells.

Step 8: For each non-basic cell in the resulting table, trace closed loops* back to the original non-basic cell. Choose only those closed loops* where either or both costs of the adjacent two basic cells of a non-basic cell are greater than the costs of the corresponding non-basic cells.

Step 9: For each selected closed loop* by Step 8, assign (+) and (−) sign alternatively on each corner, starting from the original non-basic cell. Choose the minimum allocated value of all the negative positions (−) on the closed loop*. Allocate this value to the selected non-basic cell, so that the non-basic cell becomes basic cell. Add this value to other basic cells with (+) sign and subtract this value from other basic cells with (−) sign.

Step 10: Calculate the sum of the transportation costs of each cell followed in the closed loop*. This sum value is termed as Net Cost Change. Redo this for all other selected closed loops*.

Step 11: If all the net cost change values are non-negative, an optimal value is attained. Then terminate. Else, select the non-basic cell having negative net cost change value and draw a closed loop.

Step 12: Choose the minimum allocated value of all the negative positions (−) on the closed loop. Allocate this value to the selected non-basic cell, so that the non-basic cell becomes basic cell. Add this value to other basic cells with (+) sign and subtract this value from other basic cells with (−) sign.

Step 13: Repeat the steps 8–11 (viz., **steps of Least-looping stepping-stone Method**) once again and the optimal solution will be obtained with all the values of net cost change non-negative for non-basic cells.

Step 14: Evaluate the transportation cost and stop.

* **Closed loop** is a sequence of cells that forms a look in the transportation table with the following conditions.

1. Any two consecutive cells lie either in the similar row/ column.

Fig. 1 One form of closed loop*

Fig. 2 Another form of closed loop*

2. No three successive cells lie either in the similar row/ column.
3. The starting and ending cell of a sequence lies in the similar row/column.
4. No cell shows up more than once in a sequence.
5. Only horizontal and vertical moves are permitted and can change directions at used cells.

To have a quick reference of closed loops* in LS-ASM, two illustrations are presented in Figs. 1 and 2.

To have a quick reference of the work sequence in LS-ASM, the flow diagram also is presented in Fig. 3.

Case studies

In this section, an attempt is made to justify the efficiency of the proposed LS-ASM method in solving TPs. Hence, a set of varieties of TPs are being considered both from crisp and fuzzy environment. Keeping in view the above fact, the TPs of the following categories have been solved by LS-ASM.

Fig. 3 Flowchart for LS-ASM

- Balanced TPs in crisp environment (case studies-1 and 2)
- Balanced TPs in intuitionistic fuzzy environment (case studies-3 and 4)
- Unbalanced TPs in crisp environment (case studies-5 and 6)
- A real-life problem in intuitionistic fuzzy environment (separately discussed in Section “Real-Life IFTP”).

Now, the detailed descriptions of the above TPs are discussed below under individual case studies.

Balanced TPs

Case study-1:

The first case study is already considered in Sect. 2. Assuming the optimal table by ASM, clearly the non-basic cells are S_1D_4 , S_2D_1 , S_2D_3 , S_2D_4 , S_3D_2 , S_3D_3 , S_3D_4 , S_4D_2 , and S_4D_3 .

In Table 5, it is observed from the loop that S_1D_1 and S_4D_4 are adjacent to S_1D_4 . Now, the cost values of S_1D_1 and S_4D_4 (7 and 3, respectively) are less than the cost value of S_1D_4

Table 5 Closed loop for the non-basic cell S_1D_4

	D_1	D_2	D_3	D_4	Supply
S_1	(-)7 5	5 5	9 20	(+)11	30
S_2	4	3 25	8	6	25
S_3	3 20	8	10	5	20
S_4	(+)2 5	6	7	(-)3 10	15
Demand	30	30	20	10	90

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

(i.e., 11). Therefore, this closed loop is not considered to evaluate the net cost change value. Other closed loops can be constructed and checked in the similar way for the other non-basic cells.

Now, apply Step 12 of the LS-ASM algorithm to the non-basic cell S_2D_1 having the only negative net cost change value (Refer to Table 6). It is seen that in the closed loop of Table 7 out of both the negative positions, $5 < 25$. Therefore, 5 is added to the non-basic cell S_2D_1 and basic cell S_1D_2 , whereas it is subtracted from the allocated values of the basic cells S_1D_1 and S_2D_2 . Now, S_2D_1 becomes a basic cell. Since, it can be checked that all the net cost change values of the non-basic cells are positive in the next iteration. Finally, the optimal solution has been reached. The optimal table of the TP is demonstrated in Table 8.

Thus, the total transportation cost according to LS-ASM Method = $5 \times 10 + 9 \times 20 + 4 \times 5 + 3 \times 20 + 3 \times 20 + 2 \times 5 + 3 \times 10 = 410$.

Case study-2:

The case study-2 has already been considered in Sect. 2. Proceeding in the similar way as done for case study-1, the optimal transportation cost by LS-ASM is obtained as **14,180**. Two more case studies with the implementation of LS-ASM for TIFTPs are described following case studies.

Case study-3:

Table 10 uses the ranking function [6] to convert the Triangular Intuitionistic Fuzzy cost values to real values. In Table 11, the total transportation cost according to the LS-ASM Method = $1 \times 4 + 4 \times 1 + 7 \times 5 + 8 \times 1 + 2 \times 7 + 6 \times 3 + 5 \times 7 = 118$.

Case study-4:

Table 6 Selected closed loops for each of the non-basic cells

Non-basic cells	Closed loops	Net cost change
S_2D_1	$(+)S_2D_1(+S_2D_2-(-)S_1D_2-(-)S_1D_1-(+)S_2D_1$	$4-3+5-7=-1$ (negative)
S_2D_3	$(+)S_2D_3(-)S_1D_3-S_1D_2-(-)S_2D_2-(+)S_2D_3$	$8-9+5-3=1$
S_4D_2	$(+)S_4D_2(-)S_4D_1-(-)S_1D_1-(-)S_1D_2-(+)S_4D_2$	$6-2+7-5=6$
S_4D_3	$(+)S_4D_3(-)S_4D_1-(-)S_1D_1-(-)S_1D_3-(+)S_4D_3$	$7-2+7-9=3$

Table 7 Closed loop for the non-basic cell S_2D_1

	D_1	D_2	D_3	D_4	Supply
S_1	(-) 7 ₅	(+) 5 ₅	9 ₂₀	11	30
S_2	(+) 4	(-) 3 ₂₅	8	6	25
S_3	3 ₂₀	8	10	5	20
S_4	2 ₅	6	7	3 ₁₀	15
Demand	30	30	20	10	90

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Table 8 Optimal result table

	D_1	D_2	D_3	D_4	Supply
S_1	7	5 ₁₀	9 ₂₀	11	30
S_2	4 ₅	3 ₂₀	8	6	25
S_3	3 ₂₀	8	10	5	20
S_4	2 ₅	6	7	3 ₁₀	15
Demand	30	30	20	10	90

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Table 13 uses the ranking function [6] to convert the Triangular Intuitionistic Fuzzy cost values to real values. In Table 14, the total transportation cost according to the LS-ASM Method = $(243.33 \times 3500 + 3721.21 \times 1000 + 406.833 \times 1500 + 1050 \times 2000 + 2209.259 \times 1500 + 5621.21 \times 500) = 13,406,940.5$.

Table 9 A balanced minimization TP with TIFNs [10]

	D_1	D_2	D_3	D_4	IF supply
S_1	(14, 16, 18; 12, 16, 20)	(0, 1, 2; -1, 1, 3)	(7, 8, 9; 6, 8, 10)	(11, 13, 15; 10, 13, 16)	(2, 4, 6; 1, 4, 7)
S_2	(8, 11, 14; 7, 11, 15)	(3, 4, 5; 2, 4, 6)	(5, 7, 9; 4, 7, 10)	(8, 10, 12; 6, 10, 14)	(5, 6, 7; 4, 6, 8)
S_3	(6, 8, 10; 5, 8, 11)	(13, 15, 17; 12, 15, 18)	(7, 9, 11; 6, 9, 12)	(1, 2, 3; 0, 2, 4)	(7, 8, 9; 5, 8, 11)
S_4	(5, 6, 7; 4, 6, 8)	(11, 12, 13; 10, 12, 14)	(3, 5, 7; 1, 5, 9)	(12, 14, 16; 11, 14, 17)	(8, 10, 12; 6, 10, 14)
IF Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

Table 10 Transportation of Table 9 after using the ranking function [6]

	D_1	D_2	D_3	D_4	Supply
S_1	16	1	8	13	4
S_2	11	4	7	10	6
S_3	8	15	9	2	8
S_4	6	12	5	14	10
Demand	4	5	12	7	28

Table 11 Optimal solution table using LS-ASM

	D_1	D_2	D_3	D_4	Supply
S_1	16	1 ₄	8	13	4
S_2	11	4 ₁	7 ₅	10	6
S_3	8 ₁	15	9	2 ₇	8
S_4	6 ₃	12	5 ₇	14	10
Demand	4	5	12	7	28

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Unbalanced TPs

In this sub-section, two unbalanced TPs are considered as follows to solve by LS-ASM Method.

Case study-5:

Table 15 considers an unbalanced TP which is solved by LS-ASM in Table 16 with total transportation cost = $25 \times 125 + 15 \times 450 + 0 \times 300 + 10 \times 350 + 20 \times 225 = 17,875$.

Table 12 A balanced minimization TP with TIFNs [7]

	D_1	D_2	D_3	D_4	Supply (in tons)
S_1	(210, 250, 270; 200, 250, 280)	(600, 700, 750; 600, 700, 800)	(950, 1000, 1050; 900, 1000, 1100)	(3500, 3700, 3900; 3400, 3700, 4100)	4500
S_2	(650, 750, 800; 600, 750, 850)	(350, 400, 450; 340, 400, 480)	(1000, 1050, 1100; 950, 1050, 1150)	(3600, 3900, 4000; 3500, 3900, 4600)	3500
S_3	(2600, 2800, 3000; 2500, 2800, 3100)	(2100, 2200, 2300; 2100, 2200, 2350)	(2900, 3100, 3300; 2800, 3100, 3400)	(5400, 5600, 5800; 5300, 5600, 6000)	2000
Demand (in tons)	3500	3000	2000	1500	10,000

Table 13 Transportation of Table 12 after using the ranking function [6]

	D_1	D_2	D_3	D_4	Supply (in tons)
S_1	243.33	692.857	1000	3721.21	4500
S_2	733.33	406.388	1050	4015.87	3500
S_3	2800	2209.259	3100	5621.21	2000
Demand (in tons)	3500	3000	2000	1500	10,000

Table 14 Optimal solution table using LS-ASM

	D_1	D_2	D_3	D_4	Supply (in tons)
S_1	243.33 ₃₅₀₀	692.857	1000	3721.21 ₁₀₀₀	4500
S_2	733.33	406.388 ₁₅₀₀	1050 ₂₀₀₀	4015.87	3500
S_3	2800	2209.259 ₁₅₀₀	3100	5621.21 ₅₀₀	2000
Demand (in tons)	3500	3000	2000	1500	10,000

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Table 15 An unbalanced minimization TP [12]

From/ To Plants	Warehouse(j)			Capacity
	P	Q	R	
A	35	25	15	875
B	10	20	30	575
Demand	350	350	450	1450 1150

Table 16 Optimal solution table using LS-ASM

From/ To Plants	Warehouse(j)				Capacity
	P	Q	R	Dummy	
A	35	25 ₁₂₅	15 ₄₅₀	0 ₃₀₀	875
B	10 ₃₅₀	20 ₂₂₅	30	0	575
Demand	350	350	450	300	1450 1450

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Table 17 An unbalanced minimization TP [13]

$s_i \backslash d_j$	d_1	d_2	d_3	Supply
s_1	5	1	7	10
s_2	6	4	6	80
s_3	3	2	5	15
Demand	75	20	50	105 145

Case study-6:

Table 17 considers an unbalanced TP which is solved by LS-ASM Method in Table 18 with total transportation cost $= 10 \times 1 + 6 \times 60 + 4 \times 10 + 6 \times 10 + 3 \times 15 + 0 \times 40 = 515$.

Real-life IFTP

To realize the effectiveness of LS-ASM in real-life problem, a TIFTP from Kumar et al. [6] has been considered here in this section. The problem comprises of an umbrella company where the company manager wishes to transport umbrellas from three different factories to three different retail stores. The dataset for the transportation table is mentioned in Table 19 (see Table 20).

Table 18 Optimal solution table using LS-ASM

$d_j \backslash s_i$	d_1	d_2	d_3	Supply
s_1	5	1₁₀	7	10
s_2	6₆₀	4₁₀	6₁₀	80
s_3	3₁₅	2	5	15
Dummy	0	0	0₄₀	40
Demand	75	20	50	145 145

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Table 21 gives the optimal solution table for the real-life TIFTP solved by LS-ASM with total transportation cost $t = 9.75 \times 7.125 + 12.375 \times 4.625 + 17 \times 10.25 + 8.75 \times 8.75 + 14.75 \times 1.25 = 349.2656$.

Results and discussion

The techniques LS-ASM, ASM, VAM-MODI are being programmed in C + +. The code is compiled and executed on Intel(R) Core (TM) i5-7500 CPU (3.40 GHz) processor using Dev C + + 5.11 compiler. The number of least-looping stepping-stone iterations required to reach optimality in each of the problems of Table 22 is 1. The optimal transportation costs of all the methods are compared in Table 22. The bold letters represent the improved optimal solution reached by that particular method for the corresponding case study and real-life TIFTP.

The results of REDI [12] and Algorithm of [13] are directly taken from their respective papers. From Table 22, it is observed that the LS-ASM code executes successfully reaching the optimal solution. For the first two case studies, LS-ASM gives better optimal results than ASM [5] method, whereas for case study-3 and case study-4, it executes with same optimal results as other methods. A similar argument has been carried out for the case studies-5 and 6 for

Table 20 Transportation of Table 19 after using the ranking function [6]

	D_1	D_2	D_3	Supply
S_1	4.75	10.75	9.75	7.125
S_2	5.125	12.375	17	14.875
S_3	8.75	14.75	20.125	9.875
Demand	8.75	5.75	17.375	

Table 21 Optimal solution table using LS-ASM

	D_1	D_2	D_3	Supply
S_1	4.75	10.75	9.75_{7.125}	7.125
S_2	5.125	12.375_{4.625}	17_{10.25}	14.875
S_3	8.75_{8.75}	14.75_{1.25}	20.125	9.875
Demand	8.75	5.75	17.375	

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

unbalanced TPs and also for the real-life TIFTP considered in Section 3.4. Though the optimal solutions could not beat its near competitors namely, ASM and VAM-MODI, it has been able to obtain equally good results as that of others. Like balanced TPs, the LS-ASM consumes less time in case of unbalanced TPs too as compared to VAM-MODI over an average of 50 independent runs.

A fair comparison of these methods is done under the fixed platform and each algorithm is allowed to execute for 50 independent runs. The computational times of all the methods are compared in Table 23. Furthermore, the bold letters represent the least time required (in s.) for each individual case study and real-life TIFTP.

It is worth noting that LS-ASM consumes less computational time as compared to VAM-MODI method. Moreover, though LS-ASM takes bit more time than ASM for solving the problems in hand, but in return, LS-ASM provides better optimal solution than ASM as reported in Table 22.

Table 19 A real-life TIFTP

	D_1	D_2	D_3	Supply
S_1	(1, 4, 9; 0, 4, 12)	(3, 13, 14; 2, 13, 15)	(4, 6, 16; 1, 6, 33)	(6, 7, 10; 2, 7, 11)
S_2	(4, 5, 7; 1, 5, 9)	(5, 10, 15; 0, 10, 39)	(7, 16, 24; 0, 16, 41)	(6, 15, 23; 1, 15, 29)
S_3	(1, 3, 6; 0, 3, 10)	(5, 13, 21; 5, 13, 35)	(8, 18, 27; 6, 18, 48)	(2, 10, 16; 0, 10, 21)
Demand	(3, 8, 16; 0, 8, 19)	(1, 6, 7; 0, 6, 14)	(10, 18, 26; 3, 18, 28)

Table 22 Total transportation costs

Transportation problems (TPs)	Transportation cost				
	ASM	VAM-MODI	REDI [12]	Algorithm [13]	LS-ASM
Balanced TP					
Case study-1	415	410	*	*	410
Case study-2	14,380	14,180	*	*	14,180
Case study-3	118	118	*	*	118
Case study-4	13,406,940.5	13,406,940.5	*	*	13,406,940.5
Unbalanced TP					
Case study-5	17,875	17,875	17,875	*	17,875
Case study-6	515	515	*	515	515
Real-life TIFTP	349.2656	349.2656	*	*	Real-life TIFTP

*Values are not available

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Table 23 Average computational run times

Transportation problems (TPs)	Avg. computational run times (in secs)			
	ASM	VAM-MODI	LS-ASM	Number of runs
Balanced TP				
Case study-1	31.41	39.66	36.80	50
Case study-2	35.69	41.38	40.63	50
Case study-3	29.67	40.57	36.94	50
Case study-4	75.87	93.45	78.74	50
Unbalanced TP				
Case study-5	22.57	22.41	19.54	50
Case study-6	23.25	24.55	21.25	50
Real-life TIFTP	26.81	34.57	29.76	50

Bold values in the respective mentioned tables signify equally optimal or better optimal solutions

Conclusion

The manuscript proposes an improved stepping-stone approach (namely least-looping stepping-stone method), which has been synergized with a popular ASM approach. The newly framed approach thus established is named as LS-ASM. Unlike ASM, LS-ASM employs a least number of closed loops on non-basic cells. The proposed algorithm is allowed to solve both TPs (balanced and unbalanced) and a real-life TIFTP. Both the optimal solution obtained and the computational time consumed are being compared separately for LS-ASM with its near competitors like ASM and VAM-MODI. Since LS-ASM utilizes less time than the popular VAM-MODI in solving the TPs and TIFTP, it can be treated as an efficient method. On the other hand, LS-ASM provides equally competent with VAM-MODI. Thus, the efficiency of the LS-ASM over VAM-MODI is well established. Furthermore, though LS-ASM is a hybrid version of ASM, it is quite obvious that LS-ASM takes more

executorial time. However, due to its inherent characteristics of using the closed loops as minimum number of times as possible during simulation, the reachability of finding the optimal solution becomes faster. As a result, the performance of LS-ASM is either equally good or better than its original version, namely ASM. As a whole, it can be concluded that LS-ASM outperforms its near competitors in solving balanced, unbalanced, and TIFTP, as well. As a future scope of research, further improvement of LS-ASM can also be made with an effective modification in its existing algorithmic steps, so that the transportation cost will be further minimized and/or the computational time will be further reduced. Also, it can be efficiently applied on several other unbalanced real-world TPs and IFTPs.

Declarations

Conflict of interest statement On behalf of all authors, the corresponding author states that there is no conflict of interest.

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