Inter-specific competition among trees in pythagorean fuzzy soft environment

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Abstract

A Pythagorean fuzzy set is very effective mathematical framework to represent parameter-wise imprecision which is the property of linguistic communication. A Pythagorean fuzzy soft graph is more potent than the intuitionistic fuzzy soft as well as the fuzzy soft graph as it depicts the interactions among the objects of a system using Pythagorean membership grades with respect to different parameters. This article addresses the content of competition graphs as well as economic competition graphs like k -competition graphs, m -step competition graphs and p -competition graphs in Pythagorean fuzzy soft environment. All these concepts are illustrated with examples and fascinating results. Furthermore, an application which describes the competition among distinct forest trees, that grow together in the mixed conifer forests of California, for plant resources is elaborated graphically. An algorithm is also designed for the construction of Pythagorean fuzzy soft competition graphs. It is worthwhile to express the competing and non-competing interactions in various networks with the help of Pythagorean fuzzy soft competition graphs wherein a variation in competition relative to different attributes is visible.

Keywords Competition graphs · Economic competition graphs · Competition among plants

Introduction

A graph is an effective tool to depict the connectivity and relationship among the objects of a system. For all the straightforward as well as complex networks can be modeled in terms of graph. These networks include the telecommunication web in computer sciences, molecular structures of organic compounds in chemistry, electronic circuits of physics, road networks in transportation engineering, food webs in ecology and cellular dynamics in biology. Thus, all pairwise real-world relationships can be illustrated with the help of graphs. The remarkable work of Swiss mathematician Euler, in 1736, for the Königsberg problem of seven bridges is considered to be the formal commencement of graph theory. Afterwards, these mathematical structures were not only applied in other disciplines but also emerged as an independent subject.

A graph in which a direction is associated with every edge is called directed graph. Cohen [7] considered the model of food web as a directed graph and stated that two species (vertices) are joined by edge in the corresponding competition graph (CG), if these species must have at least one prey in common and vice versa. These CGs were then applied in various systems to depict the competition among the constituents of that system. Various authors also studied different forms of CGs like p -CGs [14], tolerance CGs [4], m -step CGs [6] and competition hypergraphs [33]. Different examples of such systems are competition among sellers for buyers in imperfect markets, among the applicants for a post, among animals for food and shelter, among plants for resources and many more. Since all these systems have uncertainty in their data, therefore it is better to handle them with fuzzy set (FS) theory and its extensions.

The FS [36] was suggested by Zadeh in 1965 as an extension of crisp set to deal with uncertainty. It is a function μ that takes elements of universal set X as an input and maps on the members of unit closed interval, that is, $\mu : X \rightarrow [0, 1]$. The value assigned to the element $x \in X$ in FS is called the membership grade $\mu(x)$ of that element x and illustrates the extent of belongingness of x in this set. So FS represents the evidence for $x \in X$, but it lacks the ability to represent the

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evidence against $x \in X$. To address this issue, Atanassov [3], in 1983, put forward the intuitionistic fuzzy set (IFS) by inserting a new component $\nu : X \rightarrow [0, 1]$ that investigates the non-membership grade such that $\mu + \nu \leq 1$. In this way, IFS provides more flexibility as compared to FS theory due its non-null hesitation part. Yager [35] introduced another non-standard fuzzy subset, the Pythagorean fuzzy set (PFS), by relating the negation operation with Pythagorean theorem. This set assigns the membership and non-membership values so that $\mu^2 + \nu^2 \leq 1$. He also compared the IFS and PFS and declared that the set of intuitionistic membership values is smaller than that of Pythagorean membership grades. All these extensions of crisp sets dealt with many actual-world problems that have fuzziness and uncertainty.

Another form of uncertainty was indicated by Molodtsov [21] in 1999. He gave the idea of soft set (S_fS) which is a parameterized collection of subsets of universe X . It is defined as a set-valued approximate mapping from a set of parameters (or attributes which are selected according to the members of X) to the power set of X . This set was applied to various systems by different authors [5–19]. In order to add the flavor of fuzziness to S_fS , Maji et al. [16] put forward the fuzzy soft set and also presented its properties. This hybrid model contributed to the solution of many decision making problems highlighted by Roy and Maji [28]. Maji et al. [17] introduced the intuitionistic fuzzy soft set and Peng et al. [27] put forth Pythagorean fuzzy soft sets (PFS_fS s). These hybrid models have tremendous applications in various fields. Garg [8–11] considered Pythagorean fuzzy sets with applications in multiple criteria decision-making.

Different graphs were studied in FS theory and its related extensions later on. Kaufmann [13] presented fuzzy graph and its operations were defined by Mordeson and Nair [22]. Paravathi and Karunambigai [26] put forward intuitionistic fuzzy graphs with suitable illustrations to its idea. For the implication of Pythagorean membership grades in graphs, Naz et al. [25] introduced the Pythagorean fuzzy graphs (PFGs). They also determined various properties of these graphs, Pythagorean fuzzy preference relations as well as applications to some decision-making problems modeled in Pythagorean fuzzy environment. Mordeson et al. [23] discussed many fuzzy graph theoretic problems regarding to human trafficking.

Soft graph which is based on the structure of S_fS is, in fact, a parameterized family of graphs and was suggested by Thumbakara and George [34]. Akram and Nawaz [1] combined the concept of fuzzy graphs and soft graph to put forward a new notion of fuzzy soft graphs. For the involvement of Pythagorean membership grades to the idea of soft graph, Akram and Shahzadi [32] introduced the Pythagorean fuzzy soft graph (PFS_fG) and discussed its regularity.

With the passage of time, the concept of CGs was also elaborated in different theories. Samanta and Pal [31] pro-

posed fuzzy CGs and some of its generalizations. The m -step fuzzy CGs [30], intuitionistic fuzzy CGs [29], q -rung orthopair fuzzy CGs [12], q -rung picture fuzzy CGs [2] and fuzzy soft CGs [24] were suggested by different authors. All these research articles include various interesting applications of CGs in social, ecological as well as economic competing network models.

There exist various competing networks that exhibit variation with respect to distinct attributes. To show the competing entities of such systems, it is necessary to include the parametrization factor to Pythagorean fuzzy competition graphs (PFCGs). The motivation of this paper is the following:

1. The PFS_fS is a hybrid model of PFS and S_fS , and has the combined properties of both these ground sets. The PFS_fG s are able to represent all those interactions that have Pythagorean membership grades and depend on different factors.
2. The introduction to Pythagorean fuzzy soft competition graphs (PFS_fCG s) and Pythagorean fuzzy soft economic competition graphs (PFS_fECG s) is essential to depict the competition in the ecological as well as economic networks, respectively, of PFS_fG s so that different measures can be taken in different situations to minimize that competition.

This article contributes to the existing literature as follows:

1. It first defines some important definitions related to PFS_fS s like height, cardinality and support of these sets which are used in further research work.
2. It updates the readers with new concept of PFS_fCG s, PFS_fECG s and various other definitions as well as important results.
3. It provides an example of competing forest trees for the resources in PFS_f environment, where quantity of the resources that are a cause of growth promotion as well as inhibition of trees vary with seasonal changes.
4. It renders an algorithm to evaluate the strength of competing and non-competing interaction of parameter-dependent competing network.

The layout of this article is as follows. The next section gives preliminary concepts. The PFS_fCG s and PFS_fECG s and other related definitions are provided in the next section. All these definitions are also elaborated with the help of examples and explained with the help of theorems. In the following section, we have used the proposed concept of CGs in PFS_fS theory to represent the competition among forest trees of California. The plants require nutrients, water, sunlight, etc. for their growth. When the concentrations of these plant resources are suboptimal or not equally dis-

Table 1 List of acronyms

Acronyms	Description
FS	Fuzzy set
IFS	Intuitionistic Fuzzy set
PFS	Pythagorean fuzzy set
S _f S	Soft set
PFS _f S	Pythagorean fuzzy soft set
PFG	Pythagorean fuzzy graph
PFS _f G	Pythagorean fuzzy soft graph
CG	Competition graph
PFCG	Pythagorean fuzzy competition graph
PFECG	Pythagorean fuzzy economic competition graph
PFS _f CG	Pythagorean fuzzy soft competition graph
PFS _f ECG	Pythagorean fuzzy soft economic competition graph

tributed, they compete with one another for these resources in order to survive. The availability of these natural resources vary from season to season so the discussion of competition in PFS_f environment permits to examine competition with respect to different parameters which prevent the information loss. Moreover, the assignment of Pythagorean membership grades to the constituents of this system provide us information about both the involvement and non-involvement of trees in competition. This competition is also presented graphically. The comparative analysis is given in the following section and the concluding section completes the article. The list of acronyms is provided in Table 1.

Preliminaries

Throughout this section, we will consider X as a non-empty universal set, P as the set of parameters regarding to all objects in the universe and Q as a subset of P :

Definition 1 [35] A PFS A over the universal set X is defined as an object $A = (\mu_A, \nu_A) : X \rightarrow [0, 1] \times [0, 1]$, where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ represent the membership and non-membership functions, respectively, such that $0 \leq \mu_A^2(x) + \nu_A^2(x) \leq 1$ holds for all $x \in X$. The set notation of PFS A is $A = \{(x, (\mu_A(x), \nu_A(x))) \mid x \in X\}$, and $\pi_A(x) = \sqrt{1 - \mu_A^2(x) - \nu_A^2(x)}$ is the Pythagorean fuzzy index of the element x to the set A .

Definition 2 [27] A PFS_fS $(\mathbb{A}, Q)$ over X is defined by the Pythagorean fuzzy approximation function $\mathbb{A} : Q \rightarrow P(X)$, where $P(X)$ denotes the Pythagorean fuzzy power set over X containing all possible PFSs over X . Its set notation is as follows: $\mathbb{A} = \{(q, \mathbb{A}(q)) \mid \forall q \in Q\}$, where $\mathbb{A}(q) = \{(x, (\mu_{\mathbb{A}(q)}(x), \nu_{\mathbb{A}(q)}(x))) \mid \forall x \in X\}$ with $0 \leq$

$\mu_{\mathbb{A}(q)}^2(x) + \nu_{\mathbb{A}(q)}^2(x) \leq 1$ for all $x \in X$, is a PFS relative to parameter q .

Definition 3 [32] A PFS_fG $\mathbb{G}$ on X is an ordered 3-tuple $\mathbb{G} = (\mathbb{A}, \mathbb{B}, Q)$ such that

- (i) Q denotes a non-empty set of attributes,
- (ii) $(\mathbb{A}, Q)$ represents a PFS_fS over X , i.e., $\mathbb{A} : Q \rightarrow P(X)$,
- (iii) $(\mathbb{B}, Q)$ represents a PFS_f relation on X , i.e., $\mathbb{B} : Q \rightarrow P(X \times X)$,
- (iv) $\mathbb{R}(q) = (\mathbb{A}(q), \mathbb{B}(q))$ is a PFG, for all $q \in Q$.

That is, the membership and non-membership values of the Pythagorean fuzzy edge (x, y) in $\mathbb{R}(q)$ is given by

$$\begin{cases} \mu_{\mathbb{B}(q)}(x, y) \leq \min\{\mu_{\mathbb{A}(q)}(x), \mu_{\mathbb{A}(q)}(y)\}, \\ \nu_{\mathbb{B}(q)}(x, y) \leq \max\{\nu_{\mathbb{A}(q)}(x), \nu_{\mathbb{A}(q)}(y)\}, \end{cases}$$

respectively, where $0 \leq \mu_{\mathbb{B}(q)}^2(x, y) + \nu_{\mathbb{B}(q)}^2(x, y) \leq 1$ holds for all $q \in Q$ and $x, y \in X$.

$(\mathbb{A}, Q)$ is called PFS_f vertex set and $(\mathbb{B}, Q)$ is called PFS_f edge set for the PFS_fG $\mathbb{G}$. Moreover, the element x in $(\mathbb{A}, Q)$ is termed as PFS_f vertex and (x, y) in $(\mathbb{B}, Q)$ is known as PFS_f edge.

Definition 4 We define *support* $\text{supp}(\mathbb{A}, Q)$ of a PFS_fS $(\mathbb{A}, Q)$ as $\text{supp}(\mathbb{A}, Q) = \{(q, \text{supp}_\mu(\mathbb{A}(q)) \cup \text{supp}_\nu(\mathbb{A}(q)))\}$, where $\text{supp}_\mu(\mathbb{A}(q)) = \{x \in X \mid \mu_{\mathbb{A}(q)}(x) > 0\}$ and $\text{supp}_\nu(\mathbb{A}(q)) = \{x \in X \mid \nu_{\mathbb{A}(q)}(x) < 1\}$, for each $q \in Q$.

Definition 5 We define *cardinality* $\text{Card}(\mathbb{A}, Q)$ of a PFS_fS $(\mathbb{A}, Q)$ as $\text{Card}(\mathbb{A}, Q) = \sum_{q \in Q} \text{Card}(\mathbb{A}(q))$, where $\text{Card}(\mathbb{A}(q)) = (|\mathbb{A}(q)|_\mu, |\mathbb{A}(q)|_\nu)$ such that $|\mathbb{A}(q)|_\mu = \sum_{x \in X} \mu_{\mathbb{A}(q)}(x)$ and $|\mathbb{A}(q)|_\nu = \sum_{x \in X} \nu_{\mathbb{A}(q)}(x)$, for each $q \in Q$.

Definition 6 We define *height* $h(\mathbb{A}, Q)$ of a PFS_fS $(\mathbb{A}, Q)$ as $h(\mathbb{A}, Q) = (\sup_{q \in Q} h_\mu(\mathbb{A}(q)), \inf_{q \in Q} h_\nu(\mathbb{A}(q)))$, where $h_\mu(\mathbb{A}(q)) = \sup_{x \in X} (\mu_{\mathbb{A}(q)}(x))$ and $h_\nu(\mathbb{A}(q)) = \inf_{x \in X} (\nu_{\mathbb{A}(q)}(x))$, for each $q \in Q$.

Definition 7 The *strength* $\text{Str}(x, y)$ of a PFS_f edge (x, y) in the PFS_fG $\mathbb{G} = (\mathbb{A}, \mathbb{B}, Q)$ is defined as $\text{Str}(x, y) = (\text{Str}(x, y)_\mu, \text{Str}(x, y)_\nu) = (\min_q \{\text{Str}(q; (x, y))_\mu\}, \max_q \{\text{Str}(q; (x, y))_\nu\})$, where

$$\begin{cases} \text{Str}(q; (x, y))_\mu = \frac{\mu_{\mathbb{B}(q)}(x, y)}{\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)}, \\ \text{Str}(q; (x, y))_\nu = \frac{\nu_{\mathbb{B}(q)}(x, y)}{\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)}. \end{cases}$$

Note that $(\text{Str}(q; (x, y))_\mu, \text{Str}(q; (x, y))_\nu)$ denotes the strength of Pythagorean fuzzy edge (x, y) in the PFG $\mathbb{R}(q) = (\mathbb{A}(q), \mathbb{B}(q))$.

Fig. 1 A PFS_fG with strong and weak edges

Definition 8 A PFS_f edge (x, y) is called *independent strong* in $\mathbb{G} = (\mathbb{A}, \mathbb{B}, Q)$ if the Pythagorean fuzzy edge (x, y) is independent strong in $\text{PFG } \mathbb{R}(q) = (\mathbb{A}(q), \mathbb{B}(q))$ for every q , i.e., following inequalities hold for all q :

$$\begin{cases} \mu_{\mathbb{B}(q)}(x, y) \geq \frac{1}{2}(\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)), \\ \nu_{\mathbb{B}(q)}(x, y) \leq \frac{1}{2}(\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)). \end{cases}$$

The edge (x, y) is called *weak* if above-mentioned conditions are not satisfied for some q .

In other words, a PFS_f edge (x, y) is called *independent strong* in $\mathbb{G} = (\mathbb{A}, \mathbb{B}, Q)$ if $\text{Str}(x, y)_\mu \geq 0.5$ and $\text{Str}(x, y)_\nu \leq 0.5$, and *weak* if not.

Example 1 Let us consider a $\text{PFS}_f\text{G } \mathbb{G} = (\mathbb{A}, \mathbb{B}, Q)$ displayed in Fig. 1.

Since $\text{Str}(x, y) = (0.48, 0.92)$, $\text{Str}(x, z) = (0.82, 0.45)$ and $\text{Str}(y, z) = (0.92, 0.47)$, therefore, the PFS_f edges (x, z) and (y, z) are independent strong, whereas (x, y) is weak in the $\text{PFS}_f\text{G } \mathbb{G}$.

Definition 9 A $\text{PFS}_f\text{G } \mathbb{G}' = (\mathbb{A}', \mathbb{B}', Q)$ is called a PFS_f subgraph of $\mathbb{G} = (\mathbb{A}, \mathbb{B}, Q)$ if $\mathbb{A}' \leq \mathbb{A}$ (similarly, $\mathbb{B}' \leq \mathbb{B}$), i.e., $\mu_{\mathbb{A}'(q)}(x) \leq \mu_{\mathbb{A}(q)}(x)$ (similarly, $\mu_{\mathbb{B}'(q)}(x, y) \leq \mu_{\mathbb{B}(q)}(x, y)$) and $\nu_{\mathbb{A}'(q)}(x) \geq \nu_{\mathbb{A}(q)}(x)$ (similarly, $\nu_{\mathbb{B}'(q)}(x, y) \geq \nu_{\mathbb{B}(q)}(x, y)$) for each $x, y \in X$ and $q \in Q$.

Definition 10 A PFS_f digraph $\vec{\mathbb{G}}$ on universal set X is an ordered 3-tuple $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ such that

- (i) Q is a non-void set of attributes,
- (ii) $(\mathbb{A}, Q)$ is a PFS_fS over X , i.e., $\mathbb{A} : Q \rightarrow P(X)$,
- (iii) $(\vec{\mathbb{B}}, Q)$ is a PFS_f relation on X , i.e., $\vec{\mathbb{B}} : Q \rightarrow P(X \times X)$,
- (iv) $\vec{\mathbb{R}}(q) = (\mathbb{A}(q), \vec{\mathbb{B}}(q))$ is a Pythagorean fuzzy digraph, for all $q \in Q$.

That is, the membership and non-membership degrees of a Pythagorean fuzzy arc (x, y) in $\vec{\mathbb{R}}(q)$ is given by

$$\begin{cases} \mu_{\vec{\mathbb{B}}(q)}(x, y) \leq \min\{\mu_{\mathbb{A}(q)}(x), \mu_{\mathbb{A}(q)}(y)\}, \\ \nu_{\vec{\mathbb{B}}(q)}(x, y) \leq \max\{\nu_{\mathbb{A}(q)}(x), \nu_{\mathbb{A}(q)}(y)\}, \end{cases}$$

for each q and $x, y \in X$, such that the condition $0 \leq \mu_{\vec{\mathbb{B}}(q)}^2(x, y) + \nu_{\vec{\mathbb{B}}(q)}^2(x, y) \leq 1$ must holds. Moreover, $(\mathbb{A}, Q)$ is PFS_f vertex set and $(\vec{\mathbb{B}}, Q)$ is PFS_f edge set for the PFS_f digraph $\vec{\mathbb{G}}$.

Pythagorean fuzzy soft competition graphs

Definition 11 The PFS_f out-neighborhood $\mathbb{N}^+(x)$ of a vertex x in PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ is a $\text{PFS}_f\text{S } \mathbb{N}^+(x) = \{\langle q, \mathbb{N}^+(q)(x) \rangle \mid \forall q \in Q\}$, where $\mathbb{N}^+(q)(x) = \{\langle y, (\mu_{\vec{\mathbb{B}}(q)}(x, y), \nu_{\vec{\mathbb{B}}(q)}(x, y)) \rangle \mid \mu_{\vec{\mathbb{B}}(q)}(x, y) > 0 \text{ or } \nu_{\vec{\mathbb{B}}(q)}(x, y) > 0\}$. The PFS_f in-neighborhood $\mathbb{N}^-(x)$ of a vertex x in PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ is a $\text{PFS}_f\text{S } \mathbb{N}^-(x) = \{\langle q, \mathbb{N}^-(q)(x) \rangle \mid \forall q \in Q\}$, where $\mathbb{N}^-(q)(x) = \{\langle y, (\mu_{\vec{\mathbb{B}}(q)}(y, x), \nu_{\vec{\mathbb{B}}(q)}(y, x)) \rangle \mid \mu_{\vec{\mathbb{B}}(q)}(y, x) > 0 \text{ or } \nu_{\vec{\mathbb{B}}(q)}(y, x) > 0\}$.

Example 2 Consider a PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$, displayed in Fig. 2, such that $\vec{\mathbb{G}} = (\vec{\mathbb{R}}(q_1), \vec{\mathbb{R}}(q_2))$, where

$$\begin{aligned} \vec{\mathbb{R}}(q_1) = (\mathbb{A}(q_1), \vec{\mathbb{B}}(q_1)) = (\{ & \langle v, (0.59, 0.65) \rangle, \\ & \langle w, (0.74, 0.42) \rangle, \langle x, (0.82, 0.36) \rangle, \\ & \langle y, (0.68, 0.58) \rangle, \langle z, (0.34, 0.89) \rangle\}, \\ & \{ \langle (v, w), (0.51, 0.62) \rangle, \langle (v, x), (0.54, 0.64) \rangle, \\ & \langle (v, y), (0.55, 0.47) \rangle, \\ & \langle (y, w), (0.66, 0.57) \rangle, \langle (y, x), (0.63, 0.49) \rangle, \\ & \langle (y, z), (0.31, 0.82) \rangle, \langle (z, w), (0.28, 0.78) \rangle \}), \end{aligned}$$

$$\begin{aligned}\vec{\mathbb{R}}(q_2) = (\mathbb{A}(q_2), \vec{\mathbb{B}}(q_2)) = & \{ \langle v, (0.64, 0.54) \rangle, \\ & \langle w, (0.84, 0.32) \rangle, \langle x, (0.75, 0.66) \rangle, \\ & \langle y, (0.39, 0.46) \rangle, \langle z, (0.58, 0.79) \rangle \}, \\ & \{ \langle (w, v), (0.61, 0.49) \rangle, \langle (w, x), (0.72, 0.51) \rangle, \\ & \langle (w, z), (0.57, 0.72) \rangle, \langle (x, v), (0.63, 0.48) \rangle, \\ & \langle (x, y), (0.38, 0.55) \rangle, \\ & \langle (z, v), (0.59, 0.42) \rangle, \langle (z, x), (0.58, 0.73) \rangle, \\ & \langle (z, y), (0.33, 0.69) \rangle \}.\end{aligned}$$

$$\begin{cases} \mu_{\mathbb{D}(q)}(x, y) = (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y))h_{\mu}(\mathbb{N}^+(q)(x) \\ \quad \cap \mathbb{N}^+(q)(y)), \\ \nu_{\mathbb{D}(q)}(x, y) = (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y))h_{\nu}(\mathbb{N}^+(q)(x) \\ \quad \cap \mathbb{N}^+(q)(y)), \end{cases}$$

such that $0 \leq \mu_{\mathbb{D}(q)}^2(x, y) + \nu_{\mathbb{D}(q)}^2(x, y) \leq 1$, for all $x, y \in X$ and $q \in Q$.

Example 3 Consider the PFS_fG $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ of Example 2.

The PFS_f out-neighborhoods of PFS_f vertices of $\vec{\mathbb{G}}$ are

$$\begin{aligned}\mathbb{N}^+(v) &= \left\{ \frac{\{ \langle w, (0.51, 0.62) \rangle, \langle x, (0.54, 0.64) \rangle, \langle y, (0.55, 0.47) \rangle \}}{q_1}, \frac{\{ \}}{q_2} \right\}, \\ \mathbb{N}^+(w) &= \left\{ \frac{\{ \}}{q_1}, \frac{\{ \langle v, (0.61, 0.49) \rangle, \langle x, (0.72, 0.51) \rangle, \langle z, (0.57, 0.72) \rangle \}}{q_2} \right\}, \\ \mathbb{N}^+(x) &= \left\{ \frac{\{ \}}{q_1}, \frac{\{ \langle v, (0.63, 0.48) \rangle, \langle y, (0.38, 0.55) \rangle \}}{q_2} \right\}, \\ \mathbb{N}^+(y) &= \left\{ \frac{\{ \langle w, (0.66, 0.57) \rangle, \langle x, (0.63, 0.49) \rangle, \langle z, (0.31, 0.82) \rangle \}}{q_1}, \frac{\{ \}}{q_2} \right\}, \\ \mathbb{N}^+(z) &= \left\{ \frac{\{ \langle w, (0.28, 0.78) \rangle \}}{q_1}, \frac{\{ \langle v, (0.59, 0.42) \rangle, \langle x, (0.58, 0.73) \rangle, \langle y, (0.33, 0.69) \rangle \}}{q_2} \right\}.\end{aligned}$$

The PFS_f in-neighborhoods of PFS_f vertices of $\vec{\mathbb{G}}$ are

$$\begin{aligned}\mathbb{N}^-(v) &= \left\{ \frac{\{ \}}{q_1}, \frac{\{ \langle w, (0.61, 0.49) \rangle, \langle x, (0.63, 0.48) \rangle, \langle z, (0.59, 0.42) \rangle \}}{q_2} \right\}, \\ \mathbb{N}^-(w) &= \left\{ \frac{\{ \langle v, (0.51, 0.62) \rangle, \langle y, (0.66, 0.57) \rangle, \langle z, (0.28, 0.78) \rangle \}}{q_1}, \frac{\{ \}}{q_2} \right\}, \\ \mathbb{N}^-(x) &= \left\{ \frac{\{ \langle v, (0.54, 0.64) \rangle, \langle y, (0.63, 0.49) \rangle \}}{q_1}, \frac{\{ \langle w, (0.72, 0.51) \rangle, \langle z, (0.58, 0.73) \rangle \}}{q_2} \right\}, \\ \mathbb{N}^-(y) &= \left\{ \frac{\{ \langle v, (0.55, 0.47) \rangle \}}{q_1}, \frac{\{ \langle x, (0.38, 0.55) \rangle, \langle z, (0.33, 0.69) \rangle \}}{q_2} \right\}, \\ \mathbb{N}^-(z) &= \left\{ \frac{\{ \langle y, (0.31, 0.82) \rangle \}}{q_1}, \frac{\{ \langle w, (0.57, 0.72) \rangle \}}{q_2} \right\}.\end{aligned}$$

Definition 12 The PFS_fCG $\mathbb{C}(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$ of a PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ is an undirected PFS_fG that has same PFS_f vertex set as in $\vec{\mathbb{G}}$ and for all $q \in Q$, it has a Pythagorean fuzzy edge between points $x, y \in X$ in $\mathbb{S}(q) = (\mathbb{A}(q), \mathbb{D}(q))$ if and only if $\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)$ is non-empty set. The membership as well as non-membership grade of edges in $\mathbb{S}(q)$ are computed as follows:

We can calculate the edge membership and non-membership values for the corresponding PFS_fCG $\mathbb{C}(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$ as

$$\begin{aligned}\mathbb{N}^+(q_1)(v) \cap \mathbb{N}^+(q_1)(y) \\ &= \{ \langle w, (0.51, 0.62) \rangle, \langle x, (0.54, 0.64) \rangle \} \\ &\Rightarrow h(\mathbb{N}^+(q_1)(v) \cap \mathbb{N}^+(q_1)(y)) = (0.54, 0.62)\end{aligned}$$

Fig. 2 A PFS_f digraph

$$\begin{cases} \mu_{\mathbb{D}(q_1)}(v, y) = (\mu_{\mathbb{A}(q_1)}(v) \wedge \mu_{\mathbb{A}(q_1)}(y))h_{\mu}(\mathbb{N}^+(q_1)(v) \\ \quad \times \cap \mathbb{N}^+(q_1)(y)) = (0.59 \wedge 0.68)(0.54) \\ \quad = 0.32, \\ \nu_{\mathbb{D}(q_1)}(v, y) = (\nu_{\mathbb{A}(q_1)}(v) \vee \nu_{\mathbb{A}(q_1)}(y))h_{\nu}(\mathbb{N}^+(q_1)(v) \\ \quad \times \cap \mathbb{N}^+(q_1)(y)) = (0.65 \vee 0.58)(0.62) \\ \quad = 0.40, \end{cases}$$

where $\mu_{\mathbb{D}(q_1)}^2(v, w) + \nu_{\mathbb{D}(q_1)}^2(v, w) \leq 1$. Hence, we have $\mathbb{C}(\vec{\mathbb{G}}) = (\mathbb{S}(q_1), \mathbb{S}(q_2))$ displayed in Fig. 3, where

$$\begin{aligned} \mathbb{S}(q_1) &= (\mathbb{A}(q_1), \mathbb{D}(q_1)) = (\{\langle v, (0.59, 0.65) \rangle, \\ &\quad \langle w, (0.74, 0.42) \rangle, \langle x, (0.82, 0.36) \rangle, \\ &\quad \langle y, (0.68, 0.58) \rangle, \langle z, (0.34, 0.89) \rangle\}, \\ &\quad \{\langle (v, y), (0.32, 0.40) \rangle, \langle (v, z), (0.09, 0.69) \rangle, \\ &\quad \langle (y, z), (0.09, 0.69) \rangle\}), \\ \mathbb{S}(q_2) &= (\mathbb{A}(q_2), \mathbb{D}(q_2)) = (\{\langle v, (0.64, 0.54) \rangle, \\ &\quad \langle w, (0.84, 0.32) \rangle, \langle x, (0.75, 0.66) \rangle, \\ &\quad \langle y, (0.39, 0.46) \rangle, \langle z, (0.58, 0.79) \rangle\}, \\ &\quad \{\langle (w, z), (0.34, 0.39) \rangle, \langle (x, z), (0.19, 0.54) \rangle\}). \end{aligned}$$

Definition 13 The PFS_f ECG $\mathbb{E}(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{F}, \mathbb{Q})$ of a PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, \mathbb{Q})$ is an undirected PFS_fG that has same PFS_f vertex set as in $\vec{\mathbb{G}}$ and for all $q \in \mathbb{Q}$, it has a Pythagorean fuzzy edge between points $x, y \in X$ in $\mathbb{T}(q) = (\mathbb{A}(q), \mathbb{F}(q))$ if and only if $\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)$ is non-empty set. The membership as well as non-membership grade of edges in $\mathbb{T}(q)$ are computed as follows:

$$\begin{cases} \mu_{\mathbb{F}(q)}(x, y) = (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y))h_{\mu}(\mathbb{N}^-(q)(x) \\ \quad \times \cap \mathbb{N}^-(q)(y)), \\ \nu_{\mathbb{F}(q)}(x, y) = (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y))h_{\nu}(\mathbb{N}^-(q)(x) \\ \quad \times \cap \mathbb{N}^-(q)(y)), \end{cases}$$

such that $0 \leq \mu_{\mathbb{F}(q)}^2(x, y) + \nu_{\mathbb{F}(q)}^2(x, y) \leq 1$, for all $x, y \in X$ and $q \in \mathbb{Q}$.

Example 4 The PFS_f ECG of $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, \mathbb{Q})$ given in Example 2 is $\mathbb{E}(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{F}, \mathbb{Q})$ shown in Fig. 4.

The edge membership and non-membership values of $\mathbb{E}(\vec{\mathbb{G}})$ are calculated as

$$\begin{aligned} &\mathbb{N}^-(q_1)(w) \cap \mathbb{N}^-(q_1)(x) \\ &= \{\langle v, (0.51, 0.64) \rangle, \langle y, (0.63, 0.57) \rangle\} \\ &\Rightarrow h(\mathbb{N}^-(q_1)(w) \cap \mathbb{N}^-(q_1)(x)) = (0.63, 0.57) \\ &\begin{cases} \mu_{\mathbb{F}(q_1)}(w, x) = (\mu_{\mathbb{A}(q_1)}(w) \wedge \mu_{\mathbb{A}(q_1)}(x))h_{\mu}(\mathbb{N}^-(q_1)(w) \\ \quad \times \cap \mathbb{N}^-(q_1)(x)) = (0.74 \wedge 0.82)(0.63) \\ \quad = 0.47, \\ \nu_{\mathbb{F}(q_1)}(w, x) = (\nu_{\mathbb{A}(q_1)}(w) \vee \nu_{\mathbb{A}(q_1)}(x))h_{\nu}(\mathbb{N}^-(q_1)(w) \\ \quad \times \cap \mathbb{N}^-(q_1)(x)) = (0.42 \vee 0.36)(0.57) \\ \quad = 0.24, \end{cases} \end{aligned}$$

where $\mu_{\mathbb{F}(q_1)}^2(w, x) + \nu_{\mathbb{F}(q_1)}^2(w, x) \leq 1$. Thus, we have $\mathbb{E}(\vec{\mathbb{G}}) = (\mathbb{T}(q_1), \mathbb{T}(q_2))$, where

$$\mathbb{T}(q_1) = (\mathbb{A}(q_1), \mathbb{F}(q_1)) = (\{\langle v, (0.59, 0.65) \rangle,$$

Fig. 3 A PFS_f CG

Fig. 4 A PFS_f ECG

$$\begin{aligned} & \langle w, (0.74, 0.42) \rangle, \langle x, (0.82, 0.36) \rangle, \\ & \langle y, (0.68, 0.58) \rangle, \langle z, (0.34, 0.89) \rangle \}, \\ & \{ \langle (w, x), (0.47, 0.24) \rangle, \langle (w, y), (0.35, 0.36) \rangle, \\ & \langle (w, z), (0.10, 0.73) \rangle, \langle (x, y), (0.37, 0.37) \rangle, \\ & \langle (x, z), (0.10, 0.73) \rangle \}, \\ & \mathbb{T}(q_2) = (\mathbb{A}(q_2), \mathbb{F}(q_2)) = (\{ \langle v, (0.64, 0.54) \rangle, \\ & \langle w, (0.84, 0.32) \rangle, \langle x, (0.75, 0.66) \rangle, \\ & \langle y, (0.39, 0.46) \rangle, \langle z, (0.58, 0.79) \rangle \}, \\ & \{ \langle (v, x), (0.39, 0.34) \rangle, \langle (v, y), (0.15, 0.30) \rangle, \\ & \langle (v, z), (0.33, 0.57) \rangle, \langle (x, y), (0.13, 0.48) \rangle, \\ & \langle (x, z), (0.33, 0.57) \rangle \}). \end{aligned}$$

Theorem 1 Let $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph on a non-void set X .

1. If $\text{supp}(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y))$ is a singleton subset of X , then the Pythagorean fuzzy edge (x, y) of $\mathbb{S}(q) = (\mathbb{A}(q), \mathbb{D}(q))$ in $\mathbb{C}(\vec{\mathbb{G}})$ is independent strong if and only if $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\mu > 0.5$ and $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\nu < 0.5$.
2. If $\text{supp}(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y))$ is a singleton subset of X , then the Pythagorean fuzzy edge (x, y) of $\mathbb{T}(q) = (\mathbb{A}(q), \mathbb{F}(q))$ in $\mathbb{E}(\vec{\mathbb{G}})$ is independent strong if and only

if $|\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_\mu > 0.5$ and $|\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_\nu < 0.5$.

In general if, in both cases, the sufficient condition holds relative to all attributes q , then the PFS_f edge is said to be strong and vice versa.

Proof Let $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph. Assume that z is the only element with membership degree $\mu_{(q)}(z)$ and non-membership degree $\nu_{(q)}(z)$ such that $\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y) = \{z, (\mu_{(q)}(z), \nu_{(q)}(z))\}$. Clearly $\text{supp}(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)) = \{z\}$. Observe that $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\mu = \mu_{(q)}(z) = h_\mu(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y))$ and $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\nu = \nu_{(q)}(z) = h_\nu(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y))$. Let $\mathbb{C}(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$ be a PFS_f CG of $\vec{\mathbb{G}}$. Then $\mu_{\mathbb{D}(q)}(x, y) = (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)) \times \mu_{(q)}(z)$ and $\nu_{\mathbb{D}(q)}(x, y) = (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)) \times \nu_{(q)}(z)$ give the membership and non-membership values of edge (x, y) in $\mathbb{S}(q) = (\mathbb{A}(q), \mathbb{D}(q))$. As a consequence, we have $\frac{\mu_{\mathbb{D}(q)}(x, y)}{\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)} = \mu_{(q)}(z)$ and $\frac{\nu_{\mathbb{D}(q)}(x, y)}{\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)} = \nu_{(q)}(z)$.

Assume the Pythagorean fuzzy edge (x, y) of $\mathbb{S}(q)$ in $\mathbb{C}(\vec{\mathbb{G}})$ to be independent strong so that $\mu_{\mathbb{D}(q)}(x, y) > \frac{1}{2}(\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y))$ and $\nu_{\mathbb{D}(q)}(x, y) < \frac{1}{2}(\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y))$, i.e., $\frac{\mu_{\mathbb{D}(q)}(x, y)}{\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)} > \frac{1}{2}$ and $\frac{\nu_{\mathbb{D}(q)}(x, y)}{\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)} < \frac{1}{2}$. Both results combine to give $\mu_{(q)}(z) > \frac{1}{2}$ as well as $\nu_{(q)}(z) < \frac{1}{2}$. Consequently, $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\mu > \frac{1}{2}$ and $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\nu < \frac{1}{2}$. $\square$

Conversely, let $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\mu > \frac{1}{2}$ and $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\nu < \frac{1}{2}$ which implies $\mu_{(q)}(z) > \frac{1}{2}$ and $\nu_{(q)}(z) < \frac{1}{2}$, respectively. Both these relations combine to give $\frac{\mu_{\mathbb{D}(q)}(x, y)}{\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)} > \frac{1}{2}$ and $\frac{\nu_{\mathbb{D}(q)}(x, y)}{\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)} < \frac{1}{2}$. It follows that the edge (x, y) of $\mathbb{S}(q) = (\mathbb{A}(q), \mathbb{D}(q))$ in $\mathbb{C}(\vec{\mathbb{G}})$ is independent strong.

The strategy of the proof of (2) is similar to the above.

Theorem 2 Let $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph on X .

1. If all PFS_f directed edges of $\vec{\mathbb{G}}$ are independent strong, then $\min_q \left(\frac{\mu_{\mathbb{D}(q)}(x, y)}{[\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)]^2} \right) > \frac{1}{2}$ as well as $\max_q \left(\frac{\nu_{\mathbb{D}(q)}(x, y)}{[\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)]^2} \right) < \frac{1}{2}$ for all PFS_f edges (x, y) in $\mathbb{C}(\vec{\mathbb{G}})$.
2. If all PFS_f directed edges of $\vec{\mathbb{G}}$ are independent strong, then $\min_q \left(\frac{\mu_{\mathbb{F}(q)}(x, y)}{[\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)]^2} \right) > \frac{1}{2}$ as well as $\max_q \left(\frac{\nu_{\mathbb{F}(q)}(x, y)}{[\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)]^2} \right) < \frac{1}{2}$ for all PFS_f edges (x, y) in $\mathbb{E}(\vec{\mathbb{G}})$.

Theorem 3 The PFS_f CG and PFS_f ECG are equal for a complete PFS_f digraph.

Proof Let us consider a complete PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ on X and assume that $\mathbb{C}(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$ and $\mathbb{E}(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{F}, Q)$ are the corresponding PFS_f CG as well as PFS_f ECG, respectively. Since in a complete PFS_f G, $\mathbb{N}_m^+(x) = \mathbb{N}_m^-(x)$ which certainly means that $\mathbb{N}_m^+(q)(x) = \mathbb{N}_m^-(q)(x)$ as well as $\mathbb{N}_m^+(q)(x) \cap \mathbb{N}_m^+(q)(y) = \mathbb{N}_m^-(q)(x) \cap \mathbb{N}_m^-(q)(y)$ also holds for all $x, y \in X, q \in Q$. Consequently,

$$\begin{cases} (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y))h_\mu(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)) \\ = (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y))h_\mu(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)), \\ (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y))h_\nu(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)) \\ = (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y))h_\nu(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)). \end{cases}$$

This implies $\mu_{\mathbb{D}(q)}(x, y) = \mu_{\mathbb{F}(q)}(x, y)$ and $\nu_{\mathbb{D}(q)}(x, y) = \nu_{\mathbb{F}(q)}(x, y)$, where $\mu_{\mathbb{D}(q)}^2(x, y) + \nu_{\mathbb{D}(q)}^2(x, y) \leq 1$ for all $x, y \in X$ and $q \in Q$. Thus, $\mathbb{C}(\vec{\mathbb{G}}) = \mathbb{E}(\vec{\mathbb{G}})$. $\square$

Definition 14 For a non-negative real number k , the Pythagorean fuzzy soft k -competition graph (PFS_f k -CG) of a PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ is an undirected PFS_f G $\mathbb{C}_k(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$ which has the same PFS_f vertex set as in $\vec{\mathbb{G}}$ and for all $q \in Q$, it has a Pythagorean fuzzy edge between points $x, y \in X$ in $\mathbb{S}(q) = (\mathbb{A}(q), \mathbb{D}(q))$ if and only if $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\mu > k$ and $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\nu > k$. The membership as well as non-membership grade of edges in $\mathbb{S}(q)$ are computed as follows:

$$\begin{cases} \mu_{\mathbb{D}(q)}(x, y) = \frac{k_1 - k}{k_1}(\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y))h_\mu(\mathbb{N}^+(q)(x) \\ \cap \mathbb{N}^+(q)(y)), \\ \nu_{\mathbb{D}(q)}(x, y) = \frac{k_2 - k}{k_2}(\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y))h_\nu(\mathbb{N}^+(q)(x) \\ \cap \mathbb{N}^+(q)(y)), \end{cases}$$

such that $0 \leq \mu_{\mathbb{D}(q)}^2(x, y) + \nu_{\mathbb{D}(q)}^2(x, y) \leq 1$ for all $x, y \in X, q \in Q$, where $k_1 = |\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\mu$, $k_2 = |\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_\nu$.

Example 5 Consider the PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ of Example 2.

We can construct the corresponding PFS_f 0.3-CG $\mathbb{C}_{0.3}(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$, whose edge membership and non-membership degrees are computed as follows:

$$k_1 = |\mathbb{N}^+(q_1)(v) \cap \mathbb{N}^+(q_1)(y)|_\mu = 1.05,$$

$$k_2 = |\mathbb{N}^+(q_1)(v) \cap \mathbb{N}^+(q_1)(y)|_\nu = 1.26,$$

$$\left\{ \begin{array}{l} \mu_{\mathbb{D}(q_1)}(v, y) \\ = \frac{k_1 - k}{k_1} (\mu_{\mathbb{A}(q_1)}(v) \wedge \mu_{\mathbb{A}(q_1)}(y)) h_{\mu}(\mathbb{N}^+(q_1)(v)) \\ \cap \mathbb{N}^+(q_1)(y)) = (0.71)(0.59 \wedge 0.68)(0.54) = 0.23, \\ \nu_{\mathbb{D}(q_1)}(v, y) \\ = \frac{k_2 - k}{k_2} (\nu_{\mathbb{A}(q_1)}(v) \vee \nu_{\mathbb{A}(q_1)}(y)) h_{\nu}(\mathbb{N}^+(q_1)(v)) \\ \cap \mathbb{N}^+(q_1)(y)) = (0.76)(0.65 \vee 0.58)(0.62) = 0.31, \end{array} \right.$$

where $\mu_{\mathbb{D}(q_1)}^2(v, w) + \nu_{\mathbb{D}(q_1)}^2(v, w) \leq 1$. The $\text{PFS}_f G$ $C_{0.3}(\vec{G}) = (\mathbb{A}, \mathbb{D}, Q)$ is given in Fig. 5.

Definition 15 For a non-negative real number k , the *Pythagorean fuzzy soft k -economic competition graph* (PFS_f k -economic CG) of a PFS_f digraph $\vec{G} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ is an undirected $\text{PFS}_f G$ $\mathbb{E}_k(\vec{G}) = (\mathbb{A}, \mathbb{F}, Q)$ which has the same PFS_f vertex set as in $\vec{G}$ and for all $q \in Q$, the Pythagorean fuzzy points $x, y \in X$ are joined in $\mathbb{T}(q) = (\mathbb{A}(q), \mathbb{F}(q))$ if and only if $|\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_{\mu} > k$ and $|\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_{\nu} > k$. The membership as well as non-membership grade of edges in $\mathbb{T}(q)$ are computed as follows:

$$\left\{ \begin{array}{l} \mu_{\mathbb{F}(q)}(x, y) = \frac{k_1 - k}{k_1} (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)) h_{\mu}(\mathbb{N}^-(q)(x) \\ \cap \mathbb{N}^-(q)(y)), \\ \nu_{\mathbb{F}(q)}(x, y) = \frac{k_2 - k}{k_2} (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)) h_{\nu}(\mathbb{N}^-(q)(x) \\ \cap \mathbb{N}^-(q)(y)), \end{array} \right.$$

such that $0 \leq \mu_{\mathbb{F}(q)}^2(x, y) + \nu_{\mathbb{F}(q)}^2(x, y) \leq 1$ for all $x, y \in X$, $q \in Q$, where $k_1 = |\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_{\mu}$, $k_2 = |\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_{\nu}$.

Example 6 Consider the PFS_f digraph $\vec{G} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ of Example 2.

We can construct the corresponding PFS_f 0.5-economic CG $\mathbb{E}_{0.5}(\vec{G}) = (\mathbb{A}, \mathbb{F}, Q)$, whose edge membership and non-membership degrees are computed as

$$k_1 = |\mathbb{N}^-(q_1)(w) \cap \mathbb{N}^-(q_1)(x)|_{\mu} = 1.14,$$

$$k_2 = |\mathbb{N}^-(q_1)(w) \cap \mathbb{N}^-(q_1)(x)|_{\nu} = 1.21,$$

$$\left\{ \begin{array}{l} \mu_{\mathbb{F}(q_1)}(w, x) \\ = \frac{k_1 - k}{k_1} (\mu_{\mathbb{A}(q_1)}(w) \wedge \mu_{\mathbb{A}(q_1)}(x)) h_{\mu}(\mathbb{N}^-(q_1)(w) \\ \cap \mathbb{N}^-(q_1)(x)) = (0.56)(0.74 \wedge 0.82)(0.63) = 0.26, \\ \nu_{\mathbb{F}(q_1)}(w, x) \\ = \frac{k_2 - k}{k_2} (\nu_{\mathbb{A}(q_1)}(w) \vee \nu_{\mathbb{A}(q_1)}(x)) h_{\nu}(\mathbb{N}^-(q_1)(w) \\ \cap \mathbb{N}^-(q_1)(x)) = (0.59)(0.42 \vee 0.36)(0.57) = 0.14, \end{array} \right.$$

where $\mu_{\mathbb{F}(q_1)}^2(w, x) + \nu_{\mathbb{F}(q_1)}^2(w, x) \leq 1$. The $\text{PFS}_f G$ $\mathbb{E}_{0.5}(\vec{G}) = (\mathbb{A}, \mathbb{F}, Q)$ is given in Fig. 6.

Theorem 4 Let $\vec{G} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph on X .

1. If $h_{\mu}(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)) = 1 = h_{\nu}(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y))$, $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_{\mu} > 2k$ and $|\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y)|_{\nu} < 2k$, then the Pythagorean fuzzy edge (x, y) of $\mathbb{S}(q) = (\mathbb{A}(q), \mathbb{D}(q))$ in $\mathbb{C}_k(\vec{G})$ is independent strong.
2. If $h_{\mu}(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)) = 1 = h_{\nu}(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y))$, $|\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_{\mu} > 2k$ and $|\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_{\nu} < 2k$, then the Pythagorean fuzzy edge (x, y) of $\mathbb{T}(q) = (\mathbb{A}(q), \mathbb{F}(q))$ in $\mathbb{E}_k(\vec{G})$ is independent strong.

In general if, in both cases, the statement holds for all parameters q , then (x, y) is said to be independent strong PFS_f edge.

Proof Let $\vec{G} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph and let $\mathbb{E}_k(\vec{G}) = (\mathbb{A}, \mathbb{F}, Q)$ be the corresponding PFS_f k -economic CG. Given that for $x, y \in X$ and $q \in Q$, $h_{\mu}(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)) = 1$ and $k_1 = |\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_{\mu} > 2k$. Combining both these relations to the definition of $\mu_{\mathbb{F}(q)}(x, y)$, we have $\mu_{\mathbb{F}(q)}(x, y) = \frac{k_1 - k}{k_1} (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y))$. This yields $\frac{\mu_{\mathbb{F}(q)}(x, y)}{\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)} = \frac{k_1 - k}{k_1} > \frac{k_1 - \frac{k_1}{2}}{k_1} = \frac{1}{2}$. Now let $h_{\nu}(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)) = 1$ and $k_2 = |\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y)|_{\nu} < 2k$, for $x, y \in X$ and $q \in Q$. The definition of $\nu_{\mathbb{F}(q)}(x, y)$ becomes $\nu_{\mathbb{F}(q)}(x, y) = \frac{k_2 - k}{k_2} (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y))$ which implies $\frac{\nu_{\mathbb{F}(q)}(x, y)}{\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)} = \frac{k_2 - k}{k_2} < \frac{k_2 - \frac{k_2}{2}}{k_2} = \frac{1}{2}$ which shows that the Pythagorean fuzzy edge (x, y) is independent strong in the Pythagorean fuzzy subgraph $\mathbb{T}(q)$ of $\mathbb{E}_k(\vec{G})$.

The proof to statement (1) is similar. $\square$

Definition 16 Let $\vec{G} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph on universal set X . The m -step PFS_f digraph of $\vec{G}$ is a PFS_f digraph $\vec{G}_m = (\mathbb{A}, \vec{\mathbb{H}}, Q)$ which has the same PFS_f vertex set as in $\vec{G}$ and for all $q \in Q$, the Pythagorean fuzzy points $x, y \in X$ are joined in $\vec{\mathbb{M}}(q) = (\mathbb{A}(q), \vec{\mathbb{H}}(q))$ if there exist a fuzzy directed path $\vec{P}^m_{(q);(x,y)}$ in $\vec{\mathbb{R}}(q)$. The membership and non-membership degrees of directed edges in $\vec{\mathbb{M}}(q)$ are evaluated as

$$\left\{ \begin{array}{l} \mu_{\vec{\mathbb{H}}(q)}(x, y) \\ = \min\{\mu_{\vec{\mathbb{B}}(q)}(u, v) \mid (u, v) \text{ is an arc of } \vec{P}^m_{(q);(x,y)}\}; \\ \nu_{\vec{\mathbb{H}}(q)}(x, y) \\ = \max\{\nu_{\vec{\mathbb{B}}(q)}(u, v) \mid (u, v) \text{ is an arc of } \vec{P}^m_{(q);(x,y)}\}, \end{array} \right.$$

where $0 \leq \mu_{\vec{\mathbb{H}}(q)}^2(x, y) + \nu_{\vec{\mathbb{H}}(q)}^2(x, y) \leq 1$, for all $x, y \in X$ and $q \in Q$.

Fig. 5 A PFS_f 0.3-CG Fig. 6 A PFS_f 0.5-economic CG

Example 7 Consider the $PFS_f G \vec{G} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ of Example 2. We can construct its 2-step PFS_f digraph $\vec{G}_2 = (\mathbb{A}, \vec{\mathbb{H}}, Q)$ as shown in Fig. 7 with

$$\begin{aligned} \vec{M}(q_1) = (\mathbb{A}(q_1), \vec{\mathbb{H}}(q_1)) = & \{ \langle v, (0.59, 0.65) \rangle, \\ & \langle w, (0.74, 0.42) \rangle, \langle x, (0.82, 0.36) \rangle, \\ & \langle y, (0.68, 0.58) \rangle, \langle z, (0.34, 0.89) \rangle \}, \\ & \{ \langle (w, v), (0.55, 0.57) \rangle, \langle (v, x), (0.55, 0.49) \rangle, \\ & \langle (v, z), (0.31, 0.82) \rangle, \\ & \langle (y, w), (0.28, 0.82) \rangle \}, \end{aligned}$$

$$\vec{M}(q_2) = (\mathbb{A}(q_2), \vec{\mathbb{H}}(q_2)) = \{ \langle v, (0.64, 0.54) \rangle,$$

$$\begin{aligned} & \langle w, (0.84, 0.32) \rangle, \langle x, (0.75, 0.66) \rangle, \\ & \langle y, (0.39, 0.46) \rangle, \langle z, (0.58, 0.79) \rangle \}, \\ & \{ \langle (w, v), (0.57, 0.72) \rangle, \langle (w, x), (0.57, 0.73) \rangle, \\ & \langle (w, y), (0.33, 0.72) \rangle, \langle (z, v), (0.58, 0.73) \rangle, \\ & \langle (z, y), (0.38, 0.73) \rangle \}. \end{aligned}$$

Definition 17 An m -step PFS_f out-neighborhood $N_m^+(x)$ of a vertex x of a PFS_f digraph $\vec{G}$ is a $PFS_f S$ $N_m^+(x) = \{ \langle q, N_m^+(q)(x) \rangle \mid \forall q \in Q \}$ such that $N_m^+(q)(x) = \{ \langle y, (\mu_{\vec{\mathbb{B}}(q)}(y), v_{\vec{\mathbb{B}}(q)}(y)) \rangle \mid \vec{P}^m_{(q;(x,y))} \text{ exists} \}$ where $\mu_{\vec{\mathbb{B}}(q)}(y) = \min \{ \mu_{\vec{\mathbb{B}}(q)}(u, v) \mid (u, v) \text{ is an arc of } \vec{P}^m_{(q;(x,y))} \}$ and $v_{\vec{\mathbb{B}}(q)}(y) = \max \{ v_{\vec{\mathbb{B}}(q)}(u, v) \mid (u, v) \text{ is an arc of } \vec{P}^m_{(q;(x,y))} \}$.

Fig. 7 A 2-step PFS_f digraph

an arc of $\vec{P}^m_{(q;(y,x))}$. An m -step PFS_f in-neighborhood $N_m^-(x)$ of a vertex x of a PFS_f digraph $\vec{G}$ is a PFS_fS $N_m^-(x) = \{\langle q, N_m^-(q)(x) \rangle \mid \forall q \in Q\}$ such that $N_m^-(q)(x) = \{\langle y, (\mu_{\vec{B}(q)}(y), \nu_{\vec{B}(q)}(y)) \rangle \mid \vec{P}^m_{(q;(y,x))} \text{ exists} \}$ where $\mu_{\vec{B}(q)}(y) = \min\{\mu_{\vec{B}(q)}(u, v) \mid (u, v) \text{ is an arc of } \vec{P}^m_{(q;(y,x))}\}$ and $\nu_{\vec{B}(q)}(y) = \max\{\nu_{\vec{B}(q)}(u, v) \mid (u, v) \text{ is an arc of } \vec{P}^m_{(q;(y,x))}\}$.

Example 8 Consider again the PFS_f digraph $\vec{G} = (\mathbb{A}, \vec{B}, Q)$, displayed in Fig. 2.

The PFS_f 2-step out-neighborhoods of vertices of $\vec{G}$ are

$$\begin{aligned} N_2^+(v) &= \left\{ \frac{\{\langle w, (0.55, 0.57) \rangle, \langle x, (0.55, 0.49) \rangle, \langle z, (0.31, 0.82) \rangle\}}{q_1}, \frac{\{\}}{q_2} \right\}, \\ N_2^+(w) &= \left\{ \frac{\{\}}{q_1}, \frac{\{\langle v, (0.57, 0.72) \rangle, \langle x, (0.57, 0.73) \rangle, \langle y, (0.33, 0.72) \rangle\}}{q_2} \right\}, \\ N_2^+(x) &= \left\{ \frac{\{\}}{q_1}, \frac{\{\}}{q_2} \right\}, \\ N_2^+(y) &= \left\{ \frac{\{\langle w, (0.28, 0.82) \rangle\}}{q_1}, \frac{\{\}}{q_2} \right\}, \\ N_2^+(z) &= \left\{ \frac{\{\}}{q_1}, \frac{\{\langle v, (0.58, 0.73) \rangle, \langle y, (0.38, 0.73) \rangle\}}{q_2} \right\}. \end{aligned}$$

The PFS_f 2-step in-neighborhoods of vertices of $\vec{G}$ are

$$\begin{aligned} N_2^-(v) &= \left\{ \frac{\{\}}{q_1}, \frac{\{\langle w, (0.57, 0.72) \rangle, \langle z, (0.58, 0.73) \rangle\}}{q_2} \right\}, \\ N_2^-(w) &= \left\{ \frac{\{\langle v, (0.55, 0.57) \rangle, \langle y, (0.28, 0.82) \rangle\}}{q_1}, \frac{\{\}}{q_2} \right\}, \end{aligned}$$

$$\begin{aligned} N_2^-(x) &= \left\{ \frac{\{\langle v, (0.55, 0.49) \rangle\}}{q_1}, \frac{\{\langle w, (0.57, 0.73) \rangle\}}{q_2} \right\}, \\ N_2^-(y) &= \left\{ \frac{\{\}}{q_1}, \frac{\{\langle w, (0.33, 0.72) \rangle, \langle z, (0.38, 0.73) \rangle\}}{q_2} \right\}, \\ N_2^-(z) &= \left\{ \frac{\{\langle v, (0.31, 0.82) \rangle\}}{q_1}, \frac{\{\}}{q_2} \right\}. \end{aligned}$$

Definition 18 An m -step Pythagorean fuzzy soft competition graph (m -step PFS_f CG) $\mathbb{C}_m(\vec{G}) = (\mathbb{A}, \mathbb{D}, Q)$ of a PFS_f digraph $\vec{G} = (\mathbb{A}, \vec{B}, Q)$ is an undirected PFS_fG which has the same PFS_f vertex set as in $\vec{G}$ and for all $q \in Q$, the Pythagorean fuzzy points $x, y \in X$ are joined in $\mathbb{S}(q) =$

$(\mathbb{A}(q), \mathbb{D}(q))$ if and only if $N_m^+(q)(x) \cap N_m^+(q)(y)$ is non-empty set. The membership as well as non-membership grade of edges in $\mathbb{S}(q)$ are evaluated as follows:

$$\begin{cases} \mu_{\mathbb{D}(q)}(x, y) = (\mu_{\mathbb{A}(q)} \wedge \nu_{\mathbb{A}(q)})h_{\mu}(N_m^+(q)(x) \cap N_m^+(q)(y)), \\ \nu_{\mathbb{D}(q)}(x, y) = (\nu_{\mathbb{A}(q)} \vee \mu_{\mathbb{A}(q)})h_{\nu}(N_m^+(q)(x) \cap N_m^+(q)(y)), \end{cases}$$

such that $0 \leq \mu_{\mathbb{D}(q)}^2(x, y) + \nu_{\mathbb{D}(q)}^2(x, y) \leq 1$, for all $x, y \in X$ and $q \in Q$.

Example 9 The 2-step $\text{PFS}_f\text{CG } \mathbb{C}_2(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$, for the $\text{PFS}_f\text{G } \vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ of Example 2, is shown in Fig. 8.

The edge membership values can be calculated as

$$\begin{aligned} & \mathbb{N}_2^+(\mathbf{q}_1)(v) \cap \mathbb{N}_2^+(\mathbf{q}_1)(y) \\ &= \{\langle w, (0.28, 0.82) \rangle\} \\ &\Rightarrow h(\mathbb{N}_2^+(\mathbf{q}_1)(v) \cap \mathbb{N}_2^+(\mathbf{q}_1)(y)) = (0.28, 0.82), \\ &\left\{ \begin{array}{l} \mu_{\mathbb{D}(\mathbf{q}_1)}(v, y) = (\mu_{\mathbb{A}(\mathbf{q}_1)}(v) \wedge \mu_{\mathbb{A}(\mathbf{q}_1)}(y))h_{\mu}(\mathbb{N}_2^+(\mathbf{q}_1)(v) \\ \quad \cap \mathbb{N}_2^+(\mathbf{q}_1)(y)) = (0.59 \wedge 0.68)(0.25) \\ \quad = 0.16, \\ \nu_{\mathbb{D}(\mathbf{q}_1)}(v, y) = (\nu_{\mathbb{A}(\mathbf{q}_1)}(v) \vee \nu_{\mathbb{A}(\mathbf{q}_1)}(y))h_{\nu}(\mathbb{N}_2^+(\mathbf{q}_1)(v) \\ \quad \cap \mathbb{N}_2^+(\mathbf{q}_1)(y)) = (0.65 \vee 0.58)(0.82) \\ \quad = 0.53, \end{array} \right. \end{aligned}$$

such that $\mu_{\mathbb{D}(\mathbf{q}_1)}^2(v, w) + \nu_{\mathbb{D}(\mathbf{q}_1)}^2(v, w) \leq 1$. Thus, we have $\mathbb{C}_2(\vec{\mathbb{G}}) = (\mathbb{S}(\mathbf{q}_1), \mathbb{S}(\mathbf{q}_2))$, where

$$\begin{aligned} \mathbb{S}(\mathbf{q}_1) &= (\mathbb{A}(\mathbf{q}_1), \mathbb{D}(\mathbf{q}_1)) = (\{\langle v, (0.59, 0.65) \rangle, \\ &\quad \langle w, (0.74, 0.42) \rangle, \langle x, (0.82, 0.36) \rangle, \\ &\quad \langle y, (0.68, 0.58) \rangle, \langle z, (0.34, 0.89) \rangle\}, \\ &\quad \{\langle (v, y), (0.16, 0.53) \rangle\}), \\ \mathbb{S}(\mathbf{q}_2) &= (\mathbb{A}(\mathbf{q}_2), \mathbb{D}(\mathbf{q}_2)) = (\{\langle v, (0.64, 0.54) \rangle, \\ &\quad \langle w, (0.84, 0.32) \rangle, \langle x, (0.75, 0.66) \rangle, \\ &\quad \langle y, (0.39, 0.46) \rangle, \langle z, (0.58, 0.79) \rangle\}, \\ &\quad \{\langle (w, z), (0.33, 0.58) \rangle\}). \end{aligned}$$

Definition 19 An m -step Pythagorean fuzzy soft economic competition graph (m -step PFS_fECG) $\mathbb{E}_m(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{F}, Q)$ of a PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ is an undirected PFS_fG which has the same PFS_f vertex set as in $\vec{\mathbb{G}}$ and for all $q \in Q$, the Pythagorean fuzzy points $x, y \in X$ are joined in $\mathbb{T}(q) = (\mathbb{A}(q), \mathbb{F}(q))$ if and only if $\mathbb{N}_m^-(q)(x) \cap \mathbb{N}_m^-(q)(y)$ is non-empty set. The membership as well as non-membership grade of edges in $\mathbb{T}(q)$ are evaluated as follows:

$$\left\{ \begin{array}{l} \mu_{\mathbb{F}(q)}(x, y) = (\mu_{\mathbb{A}(q)} \wedge \mu_{\mathbb{A}(q)})h_{\mu}(\mathbb{N}_m^-(q)(x) \cap \mathbb{N}_m^-(q)(y)), \\ \nu_{\mathbb{F}(q)}(x, y) = (\nu_{\mathbb{A}(q)} \vee \nu_{\mathbb{A}(q)})h_{\nu}(\mathbb{N}_m^-(q)(x) \cap \mathbb{N}_m^-(q)(y)), \end{array} \right.$$

such that $0 \leq \mu_{\mathbb{F}(q)}^2(x, y) + \nu_{\mathbb{F}(q)}^2(x, y) \leq 1$, for all $x, y \in X$ and $q \in Q$.

Example 10 The 2-step $\text{PFS}_f\text{ECG } \mathbb{E}_2(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{F}, Q)$, for the $\text{PFS}_f\text{G } \vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ of Example 2, is presented in Fig. 9.

The edge membership values can be calculated as

$$\begin{aligned} & \mathbb{N}_2^-(\mathbf{q}_1)(w) \cap \mathbb{N}_2^-(\mathbf{q}_1)(x) \\ &= \{\langle v, (0.55, 0.57) \rangle\} \\ &\Rightarrow h(\mathbb{N}_2^-(\mathbf{q}_1)(w) \cap \mathbb{N}_2^-(\mathbf{q}_1)(x)) = (0.55, 0.57), \\ &\left\{ \begin{array}{l} \mu_{\mathbb{F}(\mathbf{q}_1)}(w, x) = (\mu_{\mathbb{A}(\mathbf{q}_1)}(w) \wedge \mu_{\mathbb{A}(\mathbf{q}_1)}(x))h_{\mu}(\mathbb{N}_2^-(\mathbf{q}_1)(w) \\ \quad \cap \mathbb{N}_2^-(\mathbf{q}_1)(x)) = (0.74 \wedge 0.82)(0.55) \\ \quad = 0.41, \\ \nu_{\mathbb{F}(\mathbf{q}_1)}(w, x) = (\nu_{\mathbb{A}(\mathbf{q}_1)}(w) \vee \nu_{\mathbb{A}(\mathbf{q}_1)}(x))h_{\nu}(\mathbb{N}_2^-(\mathbf{q}_1)(w) \\ \quad \cap \mathbb{N}_2^-(\mathbf{q}_1)(x)) = (0.42 \vee 0.36)(0.57) \\ \quad = 0.24, \end{array} \right. \end{aligned}$$

such that $\mu_{\mathbb{F}(\mathbf{q}_1)}^2(w, x) + \nu_{\mathbb{F}(\mathbf{q}_1)}^2(w, x) \leq 1$. Thus, we have $\mathbb{E}_2(\vec{\mathbb{G}}) = (\mathbb{T}(\mathbf{q}_1), \mathbb{T}(\mathbf{q}_2))$, where

$$\begin{aligned} \mathbb{T}(\mathbf{q}_1) &= (\mathbb{A}(\mathbf{q}_1), \mathbb{F}(\mathbf{q}_1)) = (\{\langle v, (0.59, 0.65) \rangle, \\ &\quad \langle w, (0.74, 0.42) \rangle, \langle x, (0.82, 0.36) \rangle, \\ &\quad \langle y, (0.68, 0.58) \rangle, \langle z, (0.34, 0.89) \rangle\}, \\ &\quad \{\langle (w, x), (0.41, 0.24) \rangle, \langle (w, z), (0.10, 0.73) \rangle, \\ &\quad \langle (x, z), (0.10, 0.73) \rangle\}), \\ \mathbb{T}(\mathbf{q}_2) &= (\mathbb{A}(\mathbf{q}_2), \mathbb{F}(\mathbf{q}_2)) = (\{\langle v, (0.64, 0.54) \rangle, \\ &\quad \langle w, (0.84, 0.32) \rangle, \langle x, (0.75, 0.66) \rangle, \\ &\quad \langle y, (0.39, 0.46) \rangle, \langle z, (0.58, 0.79) \rangle\}, \\ &\quad \{\langle (v, x), (0.36, 0.48) \rangle, \langle (v, y), (0.15, 0.39) \rangle, \\ &\quad \langle (x, y), (0.13, 0.48) \rangle\}). \end{aligned}$$

Theorem 5 Let $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph on X . If $m > |X|$, then $\mathbb{C}_m(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$ as well as $\mathbb{E}_m(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{F}, Q)$ has no edge.

Proof Let $\vec{\mathbb{R}}(q) = (\mathbb{A}(q), \vec{\mathbb{B}}(q))$ be a fuzzy sub-digraph in $\vec{\mathbb{G}}$ and $\mathbb{S}(q) = (\mathbb{A}(q), \mathbb{D}(q))$ be the corresponding PFCG in $\mathbb{C}_m(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$. If $m > |X|$, then for all q , it is obvious that there exist no directed path of length m in $\vec{\mathbb{R}}(q)$, i.e., $\mathbb{N}_m^+(q)(x) = \{\}$ for all $x \in X$. Consequently, $\mathbb{N}_m^+(q)(x) \cap \mathbb{N}_m^+(q)(y) = \{\}$ and hence $\mu_{\mathbb{D}(q)}(x, y) = 0 = \nu_{\mathbb{D}(q)}(x, y)$, $\forall x, y \in X$. As $\mathbb{S}(q)$ has no edge and q is arbitrary, it is clear that $\mathbb{C}_m(\vec{\mathbb{G}})$ is also void of edges.

The proof for the result of $\mathbb{E}_m(\vec{\mathbb{G}})$ is similar to the above.

Theorem 6 If $\vec{\mathbb{G}}$ is a PFS_f digraph and $\vec{\mathbb{G}}_m$ is the m -step PFS_f digraph of $\vec{\mathbb{G}}$, then

1. $\mathbb{C}_m(\vec{\mathbb{G}}) = \mathbb{C}(\vec{\mathbb{G}}_m)$,
2. $\mathbb{E}_m(\vec{\mathbb{G}}) = \mathbb{E}(\vec{\mathbb{G}}_m)$.

Fig. 8 A 2-step PFS_f CG

Fig. 9 A 2-step PFS_f ECG

Theorem 7 If $\vec{G}'$ is a PFS_f sub-digraph of $\vec{G}$, then

1. $C_m(\vec{G}') \subset C_m(\vec{G})$,
2. $E_m(\vec{G}') \subset E_m(\vec{G})$.

Proof Let $\vec{G}' = (A', \vec{B}', Q)$ be a PFS_f sub-digraph of $\vec{G} = (A, \vec{B}, Q)$. Then $\mu_{A'(q)}(x) \leq \mu_{A(q)}(x)$ as well as $\nu_{A'(q)}(x) \geq \nu_{A(q)}(x)$ and $\mu_{B'(q)}(x, y) \leq \mu_{B(q)}(x, y)$ as well as $\nu_{B'(q)}(x, y) \geq \nu_{B(q)}(x, y)$ for each $x, y \in X$ and $q \in Q$.

To prove the statement (2), consider the m -step PFS_f ECG $E_m(\vec{G}') = (A', D', Q)$ and $E_m(\vec{G}) = (A, D, Q)$ of $\vec{G}'$ and $\vec{G}$, respectively. As the vertex set of a digraph and

its corresponding CG is same so are the PFS_f vertex sets of $\vec{G}$ and $E_m(\vec{G})$ as well as $\vec{G}'$ and $E_m(\vec{G}')$. This shows that m -step PFS_f in-neighborhood of any vertex x in A' is contained in the m -step PFS_f in-neighborhood of the same vertex in A . So for any $x, y \in X$, $N_m^-(x) \cap N_m^-(y)$ in $\vec{G}'$ is a PFS_f subset of that in $\vec{G}$. In other words, for each $q \in Q$, $N_m^-(q)(x) \cap N_m^-(q)(y)$ in $\vec{G}'$ is a Pythagorean fuzzy subset of that in $\vec{G}$, i.e., $\mu_{D'(q)}(x, y) \leq \mu_{D(q)}(x, y)$ and $\nu_{D'(q)}(x, y) \geq \nu_{D(q)}(x, y)$. Consequently, $E_m(\vec{G}') \subset E_m(\vec{G})$. $\square$

Definition 20 Let $\vec{G} = (A, \vec{B}, Q)$ be a PFS_f digraph on X . Let s be a common Pythagorean fuzzy prey of

m -step Pythagorean fuzzy out-neighborhoods of vertices $x_1, x_2, \dots, x_t$ in Pythagorean fuzzy sub-digraph $\vec{\mathbb{R}}(q) = (\mathbb{A}(q), \vec{\mathbb{B}}(q))$ of $\vec{\mathbb{G}}$ such that $\mu_{\vec{\mathbb{B}}(q)}(u_1, v_1), \mu_{\vec{\mathbb{B}}(q)}(u_2, v_2), \dots, \mu_{\vec{\mathbb{B}}(q)}(u_t, v_t)$ be the smallest membership degree and $\nu_{\vec{\mathbb{B}}(q)}(u_1, v_1), \nu_{\vec{\mathbb{B}}(q)}(u_2, v_2), \dots, \nu_{\vec{\mathbb{B}}(q)}(u_t, v_t)$ be the largest non-

membership degree of edges in the Pythagorean fuzzy directed paths $\vec{P}_{(q;(x_1,s))}^m, \vec{P}_{(q;(x_2,s))}^m, \dots, \vec{P}_{(q;(x_t,s))}^m$, respectively. We call the m -step Pythagorean fuzzy prey $s \in X$, a strong Pythagorean fuzzy prey in $\vec{\mathbb{R}}(q)$ if for all $1 \leq i \leq t$, $\mu_{\vec{\mathbb{B}}(q)}(u_i, v_i) > 0.5$ and $\nu_{\vec{\mathbb{B}}(q)}(u_i, v_i) < 0.5$. If these condition are satisfied in $\vec{\mathbb{R}}(q)$ for all q , then s is termed as a strong PFS_f prey.

The strength of a PFS_f prey s is denoted by $\text{Str}(s) = (\text{Str}(s)_\mu, \text{Str}(s)_\nu) = (\min_q \{\text{Str}(q; s)_\mu\}, \max_q \{\text{Str}(q; s)_\nu\})$, where the mappings $\text{Str}(q; s)_\mu : X \rightarrow [0, 1]$ and $\text{Str}(q; s)_\nu : X \rightarrow [0, 1]$ are defined as

$$\begin{cases} \text{Str}(q; s)_\mu = \frac{\sum_{i=1}^t \mu_{\vec{\mathbb{B}}(q)}(u_i, v_i)}{t}, \\ \text{Str}(q; s)_\nu = \frac{\sum_{i=1}^t \nu_{\vec{\mathbb{B}}(q)}(u_i, v_i)}{t}. \end{cases}$$

Definition 21 Let $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph on X . Let b be a common Pythagorean fuzzy buyer of m -step fuzzy in-neighborhoods of vertices $x_1, x_2, \dots, x_t$ in Pythagorean fuzzy sub-digraph $\vec{\mathbb{R}}(q) = (\mathbb{A}(q), \vec{\mathbb{B}}(q))$ of $\vec{\mathbb{G}}$ such that $\mu_{\vec{\mathbb{B}}(q)}(u_1, v_1), \mu_{\vec{\mathbb{B}}(q)}(u_2, v_2), \dots, \mu_{\vec{\mathbb{B}}(q)}(u_t, v_t)$ be the smallest membership degree and $\nu_{\vec{\mathbb{B}}(q)}(u_1, v_1), \nu_{\vec{\mathbb{B}}(q)}(u_2, v_2), \dots, \nu_{\vec{\mathbb{B}}(q)}(u_t, v_t)$ be the largest non-membership degree of edges in the Pythagorean fuzzy directed paths $\vec{P}_{(q;(b,x_1))}^m, \vec{P}_{(q;(b,x_2))}^m, \dots, \vec{P}_{(q;(b,x_t))}^m$, respectively. We call the m -step Pythagorean fuzzy buyer $b \in X$, a strong Pythagorean fuzzy buyer in $\vec{\mathbb{R}}(q)$ if for all $1 \leq i \leq t$, $\mu_{\vec{\mathbb{B}}(q)}(u_i, v_i) > 0.5$ and $\nu_{\vec{\mathbb{B}}(q)}(u_i, v_i) < 0.5$. If these condition are satisfied in $\vec{\mathbb{R}}(q)$ for all q , then b is termed as a strong PFS_f buyer.

The strength of a PFS_f buyer b is denoted by $\text{Str}(b) = (\text{Str}(b)_\mu, \text{Str}(b)_\nu) = (\min_q \{\text{Str}(q; b)_\mu\}, \max_q \{\text{Str}(q; b)_\nu\})$, where the mappings $\text{Str}(q; b)_\mu : X \rightarrow [0, 1]$ and $\text{Str}(q; b)_\nu : X \rightarrow [0, 1]$ are defined as

$$\begin{cases} \text{Str}(q; b)_\mu = \frac{\sum_{i=1}^t \mu_{\vec{\mathbb{B}}(q)}(u_i, v_i)}{t}, \\ \text{Str}(q; b)_\nu = \frac{\sum_{i=1}^t \nu_{\vec{\mathbb{B}}(q)}(u_i, v_i)}{t}. \end{cases}$$

Example 11 Consider the PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ displayed in Fig. 2.

Note that y is a common Pythagorean fuzzy prey of 2-step Pythagorean fuzzy out-neighborhoods of vertices v and z in $\vec{\mathbb{R}}(q_1)$ as well as $\vec{\mathbb{R}}(q_2)$, and its strength is $\text{Str}(y) = (0.35, 0.71)$. Similarly, w is a common Pythagorean fuzzy buyer of 2-step Pythagorean fuzzy in-neighborhoods of vertices x and z in $\vec{\mathbb{R}}(q_1)$ as well as $\vec{\mathbb{R}}(q_2)$, and its strength is $\text{Str}(w) = (0.47, 0.69)$.

Theorem 8 Let $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ be a PFS_f digraph on a non-void set X .

1. If a PFS_f prey s of $\vec{\mathbb{G}}$ is independent strong then the strength of s satisfies $\text{Str}(s)_\mu > 0.5$ and $\text{Str}(s)_\nu < 0.5$.
2. If a PFS_f buyer b of $\vec{\mathbb{G}}$ is independent strong then the strength of b satisfies $\text{Str}(b)_\mu > 0.5$ and $\text{Str}(b)_\nu < 0.5$.

Proof Let s be a common PFS_f vertex of m -step PFS_f out-neighborhoods of PFS_f vertices $x_1, x_2, \dots, x_t$, i.e., s is a common Pythagorean fuzzy vertex of m -step Pythagorean fuzzy out-neighborhoods of Pythagorean fuzzy vertices $x_1, x_2, \dots, x_t$ in Pythagorean fuzzy sub-digraph $\vec{\mathbb{R}}(q) = (\mathbb{A}(q), \vec{\mathbb{B}}(q))$ of $\vec{\mathbb{G}}$, for all q . This means that there exist the Pythagorean fuzzy directed paths $\vec{P}_{(q;(x_1,s))}^m, \vec{P}_{(q;(x_2,s))}^m, \dots, \vec{P}_{(q;(x_t,s))}^m$ of length m in $\vec{\mathbb{R}}(q)$, $\forall q$. Suppose $\mu_{\vec{\mathbb{B}}(q)}(u_1, v_1), \mu_{\vec{\mathbb{B}}(q)}(u_2, v_2), \dots, \mu_{\vec{\mathbb{B}}(q)}(u_t, v_t)$ be the smallest membership degree and $\nu_{\vec{\mathbb{B}}(q)}(u_1, v_1), \nu_{\vec{\mathbb{B}}(q)}(u_2, v_2), \dots, \nu_{\vec{\mathbb{B}}(q)}(u_t, v_t)$ be the largest non-membership degree of edges in paths $\vec{P}_{(q;(x_1,s))}^m, \vec{P}_{(q;(x_2,s))}^m, \dots, \vec{P}_{(q;(x_t,s))}^m$, respectively. Assume that prey s is independent strong in $\vec{\mathbb{R}}(q)$ then the Pythagorean fuzzy directed edges fulfill the conditions $\mu_{\vec{\mathbb{B}}(q)}(u_i, v_i) > 0.5$ and $\nu_{\vec{\mathbb{B}}(q)}(u_i, v_i) < 0.5$, for all $1 \leq i \leq t$. This implies

$$\text{Str}(q; s)_\mu > \frac{0.5 + 0.5 + \dots + 0.5(t \text{ times})}{t} = 0.5,$$

$$\text{Str}(q; s)_\nu < \frac{0.5 + 0.5 + \dots + 0.5(t \text{ times})}{t} = 0.5.$$

Since q is arbitrary, $0.5 < \min_q \text{Str}(q; s)_\mu = \text{Str}(s)_\mu$ and $0.5 > \max_q \text{Str}(q; s)_\nu = \text{Str}(s)_\nu$.

The proof to statement (2) is similar. $\square$

Definition 22 For a positive integer p , the p -competition Pythagorean fuzzy soft graph (p -competition PFS_fG) of a PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ is an undirected PFSG $\mathbb{C}^p(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{D}, Q)$ which has the same PFS_f vertex set as in $\vec{\mathbb{G}}$ and for all $q \in Q$, the Pythagorean fuzzy points $x, y \in X$ are joined in $\mathbb{S}(q) = (\mathbb{A}(q), \mathbb{D}(q))$ if and only if for each $q \in Q$ $|supp(N^+(q)(x) \cap N^+(q)(y))| \geq p$. The membership as well as non-membership grade of edges in

Fig. 10 A PFS_f digraph

$\mathbb{S}(q)$ are computed as follows:

$$\begin{cases} \mu_{\mathbb{D}(q)}(x, y) \\ = \frac{(n-p)+1}{n} (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)) h_{\mu}(\mathbb{N}^+(q)(x) \\ \cap \mathbb{N}^+(q)(y)), \\ \nu_{\mathbb{D}(q)}(x, y) \\ = \frac{(n-p)+1}{n} (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)) h_{\nu}(\mathbb{N}^+(q)(x) \\ \cap \mathbb{N}^+(q)(y)), \end{cases}$$

such that $0 \leq \mu_{\mathbb{D}(q)}^2(x, y) + \nu_{\mathbb{D}(q)}^2(x, y) \leq 1$, for all $x, y \in X$ and $q \in Q$, where $n = |\text{supp}(\mathbb{N}^+(q)(x) \cap \mathbb{N}^+(q)(y))|$.

Example 12 Let us take into consideration a PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ presented in the Fig. 10 such that

$$\begin{aligned} \vec{\mathbb{R}}(q_1) = (\mathbb{A}(q_1), \vec{\mathbb{B}}(q_1)) = & \{ \langle u, (0.59, 0.71) \rangle, \\ & \langle v, (0.92, 0.21) \rangle, \langle w, (0.83, 0.45) \rangle, \\ & \langle x, (0.63, 0.55) \rangle, \langle y, (0.48, 0.69) \rangle, \\ & \langle z, (0.79, 0.46) \rangle \}, \{ \langle (u, x), (0.58, 0.47) \rangle, \\ & \langle (v, u), (0.52, 0.61) \rangle, \langle (v, w), (0.81, 0.39) \rangle, \\ & \langle (v, x), (0.63, 0.50) \rangle, \\ & \langle (w, u), (0.46, 0.65) \rangle, \langle (w, x), (0.60, 0.54) \rangle, \\ & \langle (y, u), (0.42, 0.70) \rangle, \\ & \langle (y, v), (0.48, 0.68) \rangle, \langle (y, w), (0.46, 0.53) \rangle, \\ & \langle (y, x), (0.41, 0.67) \rangle, \langle (z, u), (0.59, 0.70) \rangle, \\ & \langle (z, v), (0.76, 0.41) \rangle, \langle (z, w), (0.74, 0.45) \rangle, \\ & \langle (z, x), (0.57, 0.48) \rangle, \\ & \langle (z, y), (0.43, 0.66) \rangle \}. \end{aligned}$$

Fig. 11 A 2-competition PFS_f G

In order to have a 2-competition PFS_f G, we can compute the edge membership and non-membership degrees as

$$\begin{aligned} & \mathbb{N}^+(q_1)(v) \cap \mathbb{N}^+(q_1)(w) \\ & = \{ \langle u, (0.46, 0.65) \rangle, \langle x, (0.60, 0.54) \rangle \} \\ & \Rightarrow n = |\text{supp}(\mathbb{N}^+(q_1)(v) \cap \mathbb{N}^+(q_1)(w))| = 2, \\ & \begin{cases} \mu_{\mathbb{D}(q_1)}(v, w) \\ = \frac{(n-2)+1}{n} (\mu_{\mathbb{A}(q_1)}(v) \wedge \mu_{\mathbb{A}(q_1)}(w)) h_{\mu}(\mathbb{N}^+(q_1)(v) \\ \cap \mathbb{N}^+(q_1)(w)) = \frac{1}{2} (0.92 \wedge 0.83) (0.60) = 0.25, \\ \nu_{\mathbb{D}(q_1)}(v, w) \\ = \frac{(n-2)+1}{n} (\nu_{\mathbb{A}(q_1)}(v) \vee \nu_{\mathbb{A}(q_1)}(w)) h_{\nu}(\mathbb{N}^+(q_1)(v) \\ \cap \mathbb{N}^+(q_1)(w)) = \frac{1}{2} (0.21 \vee 0.45) (0.54) = 0.12, \end{cases} \end{aligned}$$

where $\mu_{\mathbb{D}(q_1)}^2(u, v) + \nu_{\mathbb{D}(q_1)}^2(u, v) \leq 1$. The PFS_f G $\mathbb{C}^2(\vec{\mathbb{G}})$ obtained is given in Fig. 11.

Definition 23 For a positive integer p , the p -economic competition Pythagorean fuzzy soft graph (p -economic competition PFS_f G) of a PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ is an undirected PFSG $\mathbb{E}^p(\vec{\mathbb{G}}) = (\mathbb{A}, \mathbb{F}, Q)$ which has the same PFS_f vertex set as in $\vec{\mathbb{G}}$ and for all $q \in Q$, the Pythagorean fuzzy points $x, y \in X$ are joined in $\mathbb{T}(q) = (\mathbb{A}(q), \mathbb{F}(q))$ if and only if for each $q \in Q$ $|\text{supp}(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y))| \geq p$. The membership as well as non-membership grade of

Fig. 12 A 2-economic competition PFS_f G

edges in $\mathbb{T}(q)$ are computed as follows:

$$\begin{cases} \mu_{\mathbb{F}(q)}(x, y) \\ = \frac{(n-p)+1}{n} (\mu_{\mathbb{A}(q)}(x) \wedge \mu_{\mathbb{A}(q)}(y)) h_{\mu}(\mathbb{N}^-(q)(x) \\ \cap \mathbb{N}^-(q)(y)), \\ \nu_{\mathbb{F}(q)}(x, y) \\ = \frac{(n-p)+1}{n} (\nu_{\mathbb{A}(q)}(x) \vee \nu_{\mathbb{A}(q)}(y)) h_{\nu}(\mathbb{N}^-(q)(x) \\ \cap \mathbb{N}^-(q)(y)), \end{cases}$$

such that $0 \leq \mu_{\mathbb{F}(q)}^2(x, y) + \nu_{\mathbb{F}(q)}^2(x, y) \leq 1$, for all $x, y \in X$ and $q \in Q$, where $n = |\text{supp}(\mathbb{N}^-(q)(x) \cap \mathbb{N}^-(q)(y))|$.

Example 13 Consider again the PFS_f digraph $\vec{\mathbb{G}} = (\mathbb{A}, \vec{\mathbb{B}}, Q)$ of Example 12. In order to have a 2-economic competition PFS_f G, we can compute the edge membership and non-membership degrees as

$$\begin{aligned} \mathbb{N}^-(q_1)(u) \cap \mathbb{N}^-(q_1)(v) &= \{\langle y, (0.42, 0.70) \rangle, \langle z, (0.59, 0.70) \rangle\} \Rightarrow n \\ &= |\text{supp}(\mathbb{N}^-(q_1)(u) \cap \mathbb{N}^-(q_1)(v))| = 2, \\ \begin{cases} \mu_{\mathbb{F}(q_1)}(u, v) \\ = \frac{(n-2)+1}{n} (\mu_{\mathbb{A}(q_1)}(u) \wedge \mu_{\mathbb{A}(q_1)}(v)) h_{\mu}(\mathbb{N}^-(q_1)(u) \\ \cap \mathbb{N}^-(q_1)(v)) = \frac{1}{2} (0.59 \wedge 0.92)(0.59) = 0.17, \\ \nu_{\mathbb{F}(q_1)}(u, v) \\ = \frac{(n-2)+1}{n} (\nu_{\mathbb{A}(q_1)}(u) \vee \nu_{\mathbb{A}(q_1)}(v)) h_{\nu}(\mathbb{N}^-(q_1)(u) \\ \cap \mathbb{N}^-(q_1)(v)) = \frac{1}{2} (0.71 \wedge 0.21)(0.70) = 0.25, \end{cases} \end{aligned}$$

where $\mu_{\mathbb{F}(q_1)}^2(u, v) + \nu_{\mathbb{F}(q_1)}^2(u, v) \leq 1$. The PFS_f G $\mathbb{E}^2(\vec{\mathbb{G}})$ obtained is given in Fig. 12.

Competition among forest trees

Competition among organisms is a universal phenomenon that varies with respect to the factors involved in their development and growth. Its importance lies in the fact that severe competition among the species of a community can alter its structure. Competition inspires human beings to enhance their performance by putting more effort that brings satisfaction of doing things better. They usually compete for food, riches, prestige and mates. Animals and plants also struggle and compete with their own species members as well as with the individuals of other species for their survival. Competition among the individuals of a species is termed as intra-specific competition, whereas that within different species is called inter-specific competition.

Plants compete slowly which often went on for years as compared to animals. The resources that are essential for the survival and development of a plant are not necessarily available in sufficient quantity in the environment where it has to grow. There are various resources whose abundance or shortage influence the plants to compete for their survival. These are: (i) light (when one plant is shaded by nearby plants), (ii) water (when the dampness in soil is suboptimal), (iii) heat (when radiant energy is not available in cold areas), (iv) space (when the seeds sown are dense), (v) nutrients (when nutrients in soil are not ideally distributed), (vi) carbon dioxide (during vigorous photosynthesis in dense communities), and (vii) oxygen (as required by roots) [20]. Plants require nutrients throughout the year which include the macronutrients like nitrogen, potassium, phosphorus, calcium, sulfur and magnesium as well as micronutrients such as manganese, copper, iron, zinc etc., but their amounts needed are different for distinct plants. A plant can acquire nutrients from atmosphere, native soil nutrients, weathering of rocks and dropped litter of plants. Especially, nitrogen-fixing bacteria provide lots of nitrogen in soil. The amount of these resources in the environment of plants varies from season to season.

Consider the example of forest trees found in the mixed conifer forests of California. The climate of California changes to a great extent from burning hot to freezing cold. Some mixed conifer forests of California are found on the mountains of Sierra Nevada. In these mountains, the climate at the beginning of year is freezing cold which gradually changes to a pleasant weather during the early months of summer, hot during the summer season and then again turns to snowy winter at the end. During winter season in mixed conifer forests, the plants vigorously require water, sunlight and heat. Winter in these forests is so white that leaves and limbs of some trees cannot bear the snow load and thus fall while the other trees that have flexible branches shed heavy snow and thus survive. However, they require more space and light in summer because of regrowth of new limbs and leaves which steal the light as well as space of the relatively

small or the new growing trees. Moreover, the plants that are shade-intolerant like Ponderosa pine which die in the constant absence of light require sunlight in all seasons. Similarly, white fir which is drought-intolerant need sufficient water in each season for its survival and development [15].

Consider a set {Ponderosa pine, Sugar pine, White fir, Incense cedar, Douglas-fir, Red fir, Lodgepole pine, Jaffrey pine} of forest trees and a set {Water, Nutrients, Sunlight, Heat, Space} of resources that are required for the development of the considered trees. Also, consider a set $\{q_1, q_2\}$ of attributes where q_1 symbolizes winter season and q_2 symbolizes the summer season. A PFS_f digraph that presents the forest trees and their interaction with resources is given in Fig. 13. With regards to the considered parameter (season), the membership as well as non-membership grade of (a) trees represent their growth and growth inhibition, (b) resources denote the resource availability and shortage, and (c) directed edges narrates the accessibility and inaccessibility of resource to tree, in that season. For example, in $\vec{\mathbb{R}}(q_1)$, the directed edge from the Ponderosa pine to space has degree (0.82, 0.21) which shows that the space available to plant is 67.24%, not available is 4.41%, and the hesitation is 28.35%. Table 2 represents the PFS_f out-neighborhoods of forest trees for growth resources.

The competition among trees for resources relative to winter (by $\mathbb{S}(q_1)$) and summer (by $\mathbb{S}(q_2)$) seasons is shown in Fig. 14 as a PFS_f CG. The edge joining Ponderosa pine and Lodgepole pine has degree (0.4290, 0.0630) in $\mathbb{S}(q_1)$ which narrates that they are competing (for sunlight, heat as well as space) with strength 18.4% and are not involved in competition 0.4%. Table 3 gives the membership and non-membership values of PFS_f edges of PFS_f CG. Note that the Pythagorean fuzzy subgraph $\mathbb{S}(q_1)$ is more dense than that of $\mathbb{S}(q_2)$. It is because of the fact that the resources are more depleted in winter as compared to summer, so those plants which are not tolerant to deficiency of a resource will not be able to survive and may pass by death.

The Algorithm 41 presents the procedure of the construction of PFS_f CG for the PFS_f digraph. It takes the PFS_f digraph and for each of its parameter, computes the PFS_f out-neighborhoods corresponding to each tree. The heights of intersecting vertex neighborhoods and the vertex degrees give rise to the strength of competing and non-competing interactions among the forest trees.

Comparative analysis

The PFS_f S proposed by Peng et al. [27] is very assistive in order to deal with the parameterized family of PFSs. The competition in problems that have parameterized Pythagorean uncertainty should be dealt with PFS_f CGs; oth-

Algorithm 4.1

Input: A PFS_f digraph $\vec{\mathbb{G}} = (\vec{\mathbb{R}}(q_l) \mid 1 \leq l \leq m, q_l \in Q)$, where $\vec{\mathbb{R}}(q_l) = (\mathbb{A}(q_l), \mathbb{B}(q_l))$ and

$\mathbb{A}(q_l) = \{x_i, (\mu_{\mathbb{A}(q_l)}(x_i), \nu_{\mathbb{A}(q_l)}(x_i)) \mid 1 \leq i \leq n (= r + s)\}$, x_i 's include resources c_j , $1 \leq j \leq s$ and forest trees f_k , $1 \leq k \leq r$.

Output: Competing and non-competing strength among forest trees f_{k_u} , $1 \leq k_u \leq r - 1$ and f_{k_v} , $k_u + 1 \leq k_v \leq r$.

procedure Strength of competing and non-competing interaction

$\{(\mu_{\mathbb{D}(q_l)}(f_{k_u}, f_{k_v}), \nu_{\mathbb{D}(q_l)}(f_{k_u}, f_{k_v})) \mid 1 \leq l \leq m\}$

for $l := 1$ to m **do**

for $k := 1$ to r **do**

for $j := 1$ to s **do**

if $\mu_{\mathbb{B}(q_l)}(f_k, c_j) > 0$ or $\nu_{\mathbb{B}(q_l)}(f_k, c_j) > 0$ **then**

$(c_j, (\mu_{\mathbb{B}(q_l)}(f_k, c_j), \nu_{\mathbb{B}(q_l)}(f_k, c_j))) \in$

$\mathbb{N}^+(q_l)(f_k);$

else

$(c_j, (\mu_{\mathbb{B}(q_l)}(f_k, c_j), \nu_{\mathbb{B}(q_l)}(f_k, c_j))) \notin$

$\mathbb{N}^+(q_l)(f_k);$

end if

end for

end for

for $k_u := 1$ to $r - 1$ **do**

for $k_v := k_u + 1$ to r **do**

if $\mathbb{N}^+(q_l)(f_{k_u}) \cap \mathbb{N}^+(q_l)(f_{k_v}) \neq \{\}$ **then**

$\mu_{\mathbb{D}(q_l)}(f_{k_u}, f_{k_v}) := (\mu_{\mathbb{A}(q_l)}(f_{k_u}) \wedge \mu_{\mathbb{A}(q_l)}(f_{k_v})) \times$

$h_{\mu}(\mathbb{N}^+(q_l)(f_{k_u}) \cap \mathbb{N}^+(q_l)(f_{k_v}));$

$\nu_{\mathbb{D}(q_l)}(f_{k_u}, f_{k_v}) := (\nu_{\mathbb{A}(q_l)}(f_{k_u}) \vee \nu_{\mathbb{A}(q_l)}(f_{k_v})) \times$

$h_{\nu}(\mathbb{N}^+(q_l)(f_{k_u}) \cap \mathbb{N}^+(q_l)(f_{k_v}));$

else

$(\mu_{\mathbb{D}(q_l)}(f_{k_u}, f_{k_v}), \nu_{\mathbb{D}(q_l)}(f_{k_u}, f_{k_v})) = (0, 0);$

end if

end for

end for

end for

$((\mu_{\mathbb{D}(q_l)}(f_{k_u}, f_{k_v}), \nu_{\mathbb{D}(q_l)}(f_{k_u}, f_{k_v})) \mid 1 \leq l \leq m);$

end procedure

erwise, the competing and non-competing interactions could be inappropriate. We consider the PFG to present a comparison with the above discussed example of plant competition. Consider a PFS $\{(Water, (0.63, 0.41)), (Nutrients, (0.58, 0.37)), (Sunlight, (0.73, 0.25)), (Heat, (0.32, 0.86)), (Space, (0.87, 0.24))\}$ of resources available for the growth of forest trees given by the PFS $\{(Ponderosapine, (0.45, 0.59)), (Whitefir, (0.77, 0.38)), (Douglas - fir, (0.82, 0.29)), (Lodgepolepine, (0.59, 0.53)), (Jaffrey pine, (0.74, 0.34)), (Redfir, (0.64, 0.27)), (Incense cedar, (0.55, 0.63)), (Sugarpine, (0.62, 0.38))\}$.

The interaction of trees with the available resources is given by a set of Pythagorean fuzzy directed edges $\{((Douglas - fir, Water), (0.58, 0.40)), ((Whitefir, Nutrients), (0.57, 0.36)), ((Ponderosapine, Sunlight), (0.44, 0.52)), ((Sugarpine, Water), (0.61, 0.26)), ((Sugarpine, Heat), (0.21, 0.34)), ((Jaffrey pine, Space), (0.69, 0.31)), ((Redfir, Sunlight), (0.63, 0.26)), ((Incense cedar, Nutrients), (0.53, 0.52)), ((Douglas - fir, Space), (0.79, 0.23)), ((Lodgepolepine, Heat), (0.28, 0.77)), ((Jaffrey pine, Sunlight), (0.73, 0.29)), ((Ponderosapine, Water), (0.36, 0.47))\}$.

Table 2 PFS_f out-neighborhoods of forest trees

Forest trees	$\mathbb{N}^+$ (Forest trees)
Ponderosa pine	$\left\{ \frac{\{ \langle \text{sunlight}, (0.18, 0.44) \rangle, \langle \text{heat}, (0.07, 0.71) \rangle, \langle \text{space}, (0.82, 0.21) \rangle \}}{q_1}, \frac{\{ \langle \text{sunlight}, (0.28, 0.13) \rangle, \langle \text{heat}, (0.41, 0.23) \rangle \}}{q_2} \right\}$
Sugar pine	$\left\{ \frac{\{ \langle \text{water}, (0.45, 0.53) \rangle, \langle \text{nutrients}, (0.43, 0.47) \rangle \}}{q_1}, \frac{\{ \langle \text{nutrients}, (0.71, 0.31) \rangle, \langle \text{sunlight}, (0.82, 0.09) \rangle \}}{q_2} \right\}$
White fir	$\left\{ \frac{\{ \langle \text{water}, (0.33, 0.62) \rangle, \langle \text{nutrients}, (0.36, 0.22) \rangle \}}{q_1}, \frac{\{ \langle \text{water}, (0.34, 0.36) \rangle, \langle \text{nutrients}, (0.41, 0.43) \rangle \}}{q_2} \right\}$
Incense cedar	$\left\{ \frac{\{ \langle \text{water}, (0.46, 0.53) \rangle, \langle \text{heat}, (0.23, 0.75) \rangle, \langle \text{space}, (0.70, 0.36) \rangle \}}{q_1}, \frac{\{ \langle \text{space}, (0.30, 0.42) \rangle \}}{q_2} \right\}$
Douglas-fir	$\left\{ \frac{\{ \langle \text{water}, (0.42, 0.60) \rangle, \langle \text{nutrients}, (0.50, 0.41) \rangle \}}{q_1}, \frac{\{ \langle \text{water}, (0.63, 0.43) \rangle, \langle \text{space}, (0.27, 0.41) \rangle \}}{q_2} \right\}$
Red fir	$\left\{ \frac{\{ \langle \text{water}, (0.40, 0.52) \rangle, \langle \text{heat}, (0.21, 0.74) \rangle \}}{q_1}, \frac{\{ \langle \text{sunlight}, (0.46, 0.33) \rangle, \langle \text{heat}, (0.50, 0.11) \rangle \}}{q_2} \right\}$
Lodgepole pine	$\left\{ \frac{\{ \langle \text{sunlight}, (0.58, 0.40) \rangle, \langle \text{heat}, (0.22, 0.65) \rangle, \langle \text{space}, (0.65, 0.21) \rangle \}}{q_1}, \frac{\{ \langle \text{nutrients}, (0.71, 0.32) \rangle, \langle \text{space}, (0.24, 0.33) \rangle \}}{q_2} \right\}$
Jaffrey pine	$\left\{ \frac{\{ \langle \text{nutrients}, (0.55, 0.36) \rangle, \langle \text{sunlight}, (0.23, 0.44) \rangle, \langle \text{space}, (0.77, 0.18) \rangle \}}{q_1}, \frac{\{ \langle \text{space}, (0.17, 0.43) \rangle \}}{q_2} \right\}$

Table 3 The competing and non-competing strength of forest trees for resources

Forest trees	Strength of competition
(Ponderosa pine, Sugar pine)	((0, 0), (0.1316, 0.0338))
(Ponderosa pine, Incense cedar)	((0.4970, 0.1332), (0, 0))
(Ponderosa pine, Red fir)	((0.0308, 0.4292), (0.1927, 0.0989))
(Ponderosa pine, Lodgepole pine)	((0.4290, 0.0630), (0, 0))
(Ponderosa pine, Jaffrey pine)	((0.6391, 0.0462), (0, 0))
(Sugar pine, White fir)	((0.1332, 0.3854), (0.1722, 0.2279))
(Sugar pine, Incense cedar)	((0.2025, 0.2067), (0, 0))
(Sugar pine, Douglas-fir)	((0.1935, 0.2162), (0, 0))
(Sugar pine, Red fir)	((0.1640, 0.3074), (0.2346, 0.1419))
(Sugar pine, Lodgepole pine)	((0, 0), (0.5822, 0.1088))
(Sugar pine, Jaffrey pine)	((0.1935, 0.1833), (0, 0))
(White fir, Incense cedar)	((0.1221, 0.5084), (0, 0))
(White fir, Douglas-fir)	((0.1332, 0.3362), (0.1428, 0.2279))
(White fir, Red fir)	((0.1221, 0.5084), (0, 0))
(White fir, Lodgepole pine)	((0, 0), (0.1722, 0.2279))
(White fir, Jaffrey pine)	((0.1332, 0.2952), (0, 0))
(Incense cedar, Douglas-fir)	((0.2142, 0.2760), (0.1728, 0.1680))
(Incense cedar, Red fir)	((0.1640, 0.3074), (0, 0))
(Incense cedar, Lodgepole pine)	((0.4290, 0.1332), (0.1608, 0.1680))
(Incense cedar, Jaffrey pine)	((0.4970, 0.1332), (0.1139, 0.1720))
(Douglas-fir, Red fir)	((0.1640, 0.3480), (0, 0))
(Douglas-fir, Lodgepole pine)	((0, 0), (0.1536, 0.1505))
(Douglas-fir, Jaffrey pine)	((0.2550, 0.1886), (0.1088, 0.1505))
(Red fir, Lodgepole pine)	((0.0924, 0.4292), (0, 0))
(Lodgepole pine, Jaffrey pine)	((0.4290, 0.0630), (0.1326, 0.1462))

Fig. 13 A PFS_f digraph presenting the interaction among forest trees and their resources

If we consider this system as to depict the annual interaction of forest trees and resources, the resulting PFCG would be incorrect. Its reason is that the resources that are responsible for the growth promotion and inhibition of plants do not remain same throughout the year in the surroundings of trees. Due to this reason, the competition among plants is not constant and varies from season to season. So, this system should be studied in the PFS_f environment so that the forest management can take measures according to each season

for the growth of plants. The analysis through PFS_f CGs will also be helpful to understand that the resource for which most of the trees are competing is deficient and should be provided in sufficient amounts.

Fig. 14 A PFS_f CG

Conclusion

We often experience different types of competitions in our daily life that can be modeled as networks or graphs. The CGs of directed graphs suggested by Cohen [7] are effective to view the competing entities of system. The presence of uncertainty in actual-world problems intended the researchers to illustrate CGs in different theories. The PFS_f theory is a good blend of PFS and S_fS , which permits the assignment

of Pythagorean membership values with respect to different parameters. The purpose of this article is to present competition as well as economic CGs in the PFS_f environment. Additionally, we have discussed PFS_fk -CGs which are, in fact, the PFS_f CGs at level k , where k is a non-negative real number. The m -step PFS_f CGs describe the competition among organisms for their m -step common preys. The edges in p -competition PFS_f Gs highlight all those pairwise competitors that have at least p -common preys. We have

explained all these variations of PFS_f CGs through examples and also provided some interesting results. All these generalizations of PFS_f CGs have significance at their own as they can represent different forms of competition in a parameter-dependent system. An example of competing forest trees for growth resources have also been discussed using PFS_f CGs. The considered set of forest trees is common in the mixed conifer forests of California. The seasonal variations are responsible for changing concentrations of resources in the environment, so it is better to study such competitions in PFS_f theory. An algorithm which calculates the strength of competing and non-competing interactions of the competing constituents of a system have also been given. The future research directions might address various topics. It is desirable to discuss (1) Pythagorean fuzzy soft competition hypergraphs, (2) Pythagorean fuzzy soft graph structure and (3) Interval-valued Pythagorean fuzzy competition graphs.

Declarations

Conflict of interest The authors declare that they have no conflict of interest regarding the publication of the research article.

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