Complex & Intelligent Systems (2021) 7:2895–2905
https://doi.org/10.1007/s40747-021-00438-2

ORIGINAL ARTICLE

Complement of max product of intuitionistic fuzzy graphs

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Received: 27 April 2019 / Accepted: 9 June 2021 / Published online: 27 July 2021

Abstract

In this paper, the complement of max product of two intuitionistic fuzzy graphs is defined. The degree of a vertex in the complement of max product of intuitionistic fuzzy graph is studied. Some results on complement of max product of two regular intuitionistic fuzzy graphs are stated and proved. Finally, we provide an application of intuitionistic fuzzy graphs in school determination using normalized Hamming distance.

Keywords Max product · Intuitionistic fuzzy graph · Complement

Introduction

Graph theory has been considered to play an important role when it comes to its application in dealing with real-life situations. The fuzzy graph theory has its own significance as application of fuzzy set theory has no limits. In 1975, Rosenfeld [9] introduced the concept of fuzzy graphs. Yeh and Bang [22] also introduced fuzzy graphs independently. Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graphs. It has numerous applications to problems in computer science, electrical engineering, system analysis, operations research, economics, networking routing, transportation, etc. After Rosenfeld [9], the fuzzy graph theory increases with its various types of branches, such as fuzzy tolerance graph [13], fuzzy threshold graph [12], bipolar fuzzy graphs [7, 8], balanced interval-valued fuzzy graphs [4, 6], fuzzy planar graphs [11], etc. Also, several works have been done on fuzzy graphs by Samanta and Pal [14].

Atanassov [1, 2] developed the concept of intuitionistic fuzzy set (IFS) as an extension of fuzzy set that [23] deals with uncertain situations in a better way as its structure is not limited to membership grades only. The concept of intuitionistic fuzzy sets is a better tool to use due to its diverse structure describing membership as well as non-membership grades of an element. The theory of intuitionistic fuzzy sets has been remarkably used in some areas so far. Shannon and Atanassov [15] introduced the concept of intuitionistic fuzzy graphs in 1994. Parvathi and Karunambigai [5] gave a definition for intuitionistic fuzzy graph as a special case of intuitionistic fuzzy graphs defined by Shannon and Atanassov [16]. Sankar Sahoo and Madhumangal Pal [10] defined three types of products, namely direct product, semi-strong product, and strong product. Yaqoob et al. [21] discussed the four basic operations, namely Cartesian product, composition, union, and join of complex intuitionistic fuzzy graphs. Yahya Mohamed and Mohamed Ali [19, 20] defined modular and max product on intuitionistic fuzzy graph. In this paper, the complement of max product of two intuitionistic fuzzy graphs and the degree of a vertex in the complement of this product are studied under regularity conditions. The max product of intuitionistic fuzzy graphs is applied to solve the decision-making problem in school determination. The research paper is organized as follows: "Introduction" presents the literature review of fuzzy graphs and intuitionistic fuzzy graphs. In "Preliminaries", we have provided some basic concepts of intuitionistic fuzzy graphs. The definition of complement of max product on two intuitionistic fuzzy graphs and its degree are given in "Complement of max product of intuitionistic fuzzy graphs". In "Applications of max product of intuitionistic fuzzy graphs in school determination", we studied an application of intuitionistic fuzzy

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graphs in school determination using normalized hamming distance. This distance function was used to measure the distance between each student and each school. The schools in which each of the students has been enrolled were determined using normalized hamming distance function based on examination that is performed for transition to high school education. The result is determined by calculating the smallest distance between each student and each school. In "Conclusion", we conclude present studies and recommendations for future studies.

Preliminaries

Throughout this paper, assume that $G^* = (V, E)$ is a crisp graph and G is an intuitionistic fuzzy graph, where V is a non-empty vertex set and E is an edge set.

Definition 2.1 [5] An *intuitionistic fuzzy graph* is of the form $G = \left(\left(\sigma_1, \sigma_2\right), \left(\mu_1, \mu_2\right)\right)$ on $G^* = (V, E)$ and

1. $V = \left\{x_1, x_2, \ldots, x_n\right\}$, such that $\sigma_1 : V \to [0,1]$ and $\sigma_2 : V \to [0,1]$ denote the degree of membership and non-membership of the element $x_i \in V$ respectively, such that $0 \leq \sigma_1\left(x_i\right) + \sigma_2\left(x_i\right) \leq 1$ for all $x_i \in V$ $(i = 1, 2, 3, \ldots, n)$.
2. $\mu_1 : V \times V \to [0,1]$ and $\mu_2 : V \times V \to [0,1]$, where $\mu_1\left(x_i, x_j\right)$ and $\mu_2\left(x_i, x_j\right)$ denote the degree of membership and degree of non-membership values of the edge $\left(x_i, x_j\right)$, respectively, such that $\mu_1\left(x_i, x_j\right) \leq \sigma_1\left(x_i\right) \wedge \sigma_1\left(x_j\right)$ and $\mu_2\left(x_i, x_j\right) \leq \sigma_2\left(x_i\right) \vee \sigma_2\left(x_j\right)$; $0 \leq \mu_1\left(x_i x_j\right) + \mu_2\left(x_i x_j\right) \leq 1$, for every edge $\left(x_i, x_j\right)$.

For notational convenience, instead of representing an edge as (x, y), we denote this simply by xy.

Definition 2.2 [5] An intuitionistic fuzzy graph $G = \left(\left(\sigma_1, \sigma_2\right), \left(\mu_1, \mu_2\right)\right)$ is called *strong* intuitionistic fuzzy graph if $\mu_1\left(x_i x_j\right) = \sigma_1\left(x_i\right) \wedge \sigma_1\left(x_j\right)$ and $\mu_2\left(x_i x_j\right) = \sigma_2\left(x_i\right) \vee \sigma_2\left(x_j\right)$, $\forall\, x_i x_j \in E$, $i \neq j$.

Definition 2.3 [5] An intuitionistic fuzzy graph $G = \left(\left(\sigma_1, \sigma_2\right), \left(\mu_1, \mu_2\right)\right)$ is called *complete* intuitionistic fuzzy graph if $\mu_1\left(x_i x_j\right) = \sigma_1\left(x_i\right) \wedge \sigma_1\left(x_j\right)$ and $\mu_2\left(x_i x_j\right) = \sigma_2\left(x_i\right) \vee \sigma_2\left(x_j\right)$, $\forall\, x_i, x_j \in V, i \neq j$.

Definition 2.4 [3] Let $G = \left(\left(\sigma_1, \sigma_2\right), \left(\mu_1, \mu_2\right)\right)$ be an intuitionistic fuzzy graph, and then, the order of G is defined to be $O(G) = \left(O_{\sigma_1}(G), O_{\sigma_2}(G)\right)$ where $O_{\sigma_1}(G) = \sum_{x \in V} \sigma_1(x)$ and $O_{\sigma_2}(G) = \sum_{x \in V} \sigma_2(x)$.

Definition 2.5 [3] Let $G = \left(\left(\sigma_1, \sigma_2\right), \left(\mu_1, \mu_2\right)\right)$ be an intuitionistic fuzzy graph, then the size of G is defined to be $S(G) = \left(S_{\mu_1}(G), S_{\mu_2}(G)\right)$ where $S_{\mu_1}(G) = \sum_{xy \in E} \mu_1(xy)$ and $S_{\mu_2}(G) = \sum_{xy \in E} \mu_2(xy)$.

Definition 2.6 [5] The complement of an intuitionistic fuzzy graph $G = (V, E)$ is an intuitionistic fuzzy graph $\bar{G} = \left(\overline{\left(\sigma_1, \sigma_2\right)}, \overline{\left(\mu_1, \mu_2\right)}\right)$, where $\overline{\left(\sigma_1, \sigma_2\right)} = \left(\sigma_1, \sigma_2\right)$ and $\overline{\left(\mu_1, \mu_2\right)} = \left(\bar{\mu}_1, \bar{\mu}_2\right)$, where $\bar{\mu}_1(xy) = \sigma_1(x) \wedge \sigma_1(y) - \mu_1(xy)$ and $\bar{\mu}_2(xy) = \sigma_2(x) \vee \sigma_2(y) - \mu_2(xy)$, $\forall xy \in E$.

Definition 2.7 [5] Let $G = \left(\left(\sigma_1, \sigma_2\right), \left(\mu_1, \mu_2\right)\right)$ be an intuitionistic fuzzy graph. The degree of a vertex x in G is denoted by $d_G(x) = \left(d_1^G(x), d_2^G(x)\right)$ and defined by $d_1^G(x) = \sum_{x \neq y} \mu_1^G(xy) = \sum_{(x,y) \in E} \mu_1^G(xy)$ and $d_2^G(x) = \sum_{x \neq y} \mu_2^G(xy) = \sum_{(x,y) \in E} \mu_2^G(xy)$, where $d_1^G(x)$ is the sum of membership grades of the edges incident to the vertex x and $d_2^G(x)$ is the sum of non-membership grades of the edges incident to the vertex x.

Definition 2.8 [19] An intuitionistic fuzzy graph $G = \left(\left(\sigma_1, \sigma_2\right), \left(\mu_1, \mu_2\right)\right)$ is called *regular* intuitionistic fuzzy graph if $d_G(x) = \left(d_1^G(x), d_2^G(x)\right) = \left(k_1, k_2\right)$ for all $x \in V$, where k_1 and k_2 are constants.

Definition 2.9 [20] Let $G_1 : \left(\left(\sigma_1^{G_1}, \sigma_2^{G_1}\right), \left(\mu_1^{G_1}, \mu_2^{G_1}\right)\right)$ and $G_2 : \left(\left(\sigma_1^{G_2}, \sigma_2^{G_2}\right), \left(\mu_1^{G_2}, \mu_2^{G_2}\right)\right)$ be two intuitionistic fuzzy graphs. The max product of two intuitionistic fuzzy graph G_1 and G_2 is denoted by $G_1 \times_m G_2 = \left(V_1 \times_m V_2, E_1 \times_m E_2\right)$, $E_1 \times_m E_2 = \{\left(x_1, y_1\right)\left(x_1, y_2\right) / x_1 = x_2, y_1 y_2 \in E_2$ or $y_1 = y_2, x_1 x_2 \in E_1\}$ by $\sigma_1^{G_1 \times_m G_2}\left(x_1, y_1\right) = \sigma_1^{G_1}\left(x_1\right) \vee \sigma_1^{G_2}\left(y_1\right)$, $\sigma_2^{G_1 \times_m G_2}\left(x_1, y_1\right) = \sigma_2^{G_1}\left(x_1\right) \wedge \sigma_2^{G_2}\left(y_1\right)$, for all $\left(u_1, v_1\right) \in V_1 \times V_2$ and

$$\mu_1^{G_1 \times_m G_2}\left(\left(x_1, y_1\right)\left(x_2, y_2\right)\right) = \left\{ \begin{array}{ll} \sigma_1^{G_1}\left(x_1\right) \vee \mu_1^{G_2}\left(y_1 y_2\right) & \text{if } x_1 = x_2,\ y_1 y_2 \in E_2 \\ \mu_1^{G_1}\left(x_1 x_2\right) \vee \sigma_1^{G_2}\left(y_1\right) & \text{if } y_1 = y_2,\ x_1 x_2 \in E_1 \end{array} \right\} \tag{1}$$

$$\mu_2^{G_1 \times_m G_2}\left(\left(x_1, y_1\right)\left(x_2, y_2\right)\right) = \left\{ \begin{array}{ll} \sigma_2^{G_1}\left(x_1\right) \wedge \mu_2^{G_2}\left(y_1 y_2\right) & \text{if } x_1 = x_2,\ y_1 y_2 \in E \\ \mu_2^{G_1}\left(x_1 x_2\right) \wedge \sigma_2^{G_2}\left(y_1\right) & \text{if } y_1 = y_2,\ x_1 x_2 \in E_1 \end{array} \right\} \tag{2}$$

Example 2.1 Let $G_1^* = \left(V_1, E_1\right)$ and $G_2^* = \left(V_2, E_2\right)$ be two crisp graphs, such that $V_1 = \left\{u_1, u_2, u_3\right\}$, $V_2 = \left\{v_1, v_2\right\}$, $E_1 = \left\{u_1 u_3, u_2 u_3\right\}$, $E_2 = \left\{v_1 v_2\right\}$. Consider two intuitionistic fuzzy graphs $G_1 = \left(\left(\sigma_1^{G_1}, \sigma_2^{G_1}\right), \left(\mu_1^{G_1}, \mu_2^{G_1}\right)\right)$ and $G_2 = \left(\left(\sigma_1^{G_2}, \sigma_2^{G_2}\right), \left(\mu_1^{G_2}, \mu_2^{G_2}\right)\right)$ and $G_1 \times_m G_2$ as follows (Tables 1, 2):

Theorem 2.1 [20] *If* $G_1 : \left(\left(\sigma_1^{G_1}, \sigma_2^{G_1}\right), \left(\mu_1^{G_1}, \mu_2^{G_1}\right)\right)$ *and* $G_2 : \left(\left(\sigma_1^{G_2}, \sigma_2^{G_2}\right), \left(\mu_1^{G_2}, \mu_2^{G_2}\right)\right)$ *are two intuitionistic fuzzy graphs. Then,* $G_1 \times_m G_2$ *is also an intuitionistic fuzzy graph.*

Theorem 2.2 [20] *If* $G_1 : \left(\left(\sigma_1^{G_1}, \sigma_2^{G_1}\right), \left(\mu_1^{G_1}, \mu_2^{G_1}\right)\right)$ *and* $G_2 : \left(\left(\sigma_1^{G_2}, \sigma_2^{G_2}\right), \left(\mu_1^{G_2}, \mu_2^{G_2}\right)\right)$ *are two strong intuitionistic fuzzy graphs. Then,* $G_1 \times_m G_2$ *is also a strong intuitionistic fuzzy graph.*

Theorem 2.3 [20] *If* G_1 *and* G_2 *are two complete intuitionistic fuzzy graphs, then* $G_1 \times_m G_2$ *is not a complete intuitionistic fuzzy graph.*

Theorem 2.4 [20] *If* G_1 *and* G_2 *are two connected intuitionistic fuzzy graph, then* $G_1 \times_m G_2$ *is also a connected intuitionistic fuzzy graph.*

Complement of max product of intuitionistic fuzzy graphs

Definition 3.1 The complement of max product of two intuitionistic fuzzy graphs $G_1 = \left(\left(\sigma_1^{G_1}, \sigma_2^{G_1}\right), \left(\mu_1^{G_1}, \mu_2^{G_1}\right)\right)$ and $G_2 = \left(\left(\sigma_1^{G_2}, \sigma_2^{G_2}\right), \left(\mu_1^{G_2}, \mu_2^{G_2}\right)\right)$ is an intuitionistic fuzzy graphs $\overline{G_1 \times_m G_2} = \left(\left(\overline{\sigma_1^{G_1} \times_m \sigma_1^{G_2}}\right)\left(\overline{\sigma_2^{G_1} \times_m \sigma_2^{G_2}}\right), \left(\overline{\mu_1^{G_1} \times_m \mu_1^{G_2}}\right)\left(\overline{\mu_2^{G_1} \times_m \mu_2^{G_2}}\right)\right)$ on $G^* = (V, E)$ and $\overline{V_1 \times_m V_2} = V_1 \times_m V_2$ and $\overline{E_1 \times_m E_2} = \left\{ (x_1, y_1)(x_2, y_2) \mid \begin{array}{c} x_1 = x_2,\ y_1y_2 \in E_2\ (or) y_1 = y_2,\ x_1x_2 \in E_1\ (or) \\ x_1x_2 \in E_1,\ y_1y_2 \notin E_2\ (or)\ x_1x_2 \notin E_1,\ y_1y_2 \in E_2\ (or) \\ x_1x_2 \in E_1,\ y_1y_2 \in E_2\ (or)\ x_1x_2 \notin E_1,\ y_1y_2 \notin E_2 \end{array} \right\}$

$\left(\overline{\sigma_1^{G_1} \times_m \sigma_1^{G_2}}\right)(x_1, y_1) = \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) = \sigma_1^{G_1}(x_1) \vee \sigma_1^{G_2}(y_1)$,

$\left(\overline{\sigma_2^{G_1} \times_m \sigma_2^{G_2}}\right)(x_1, y_1) = \left(\sigma_2^{G_1} \times_m \sigma_2^{G_2}\right)(x_1, y_1) = \sigma_2^{G_1}(x_1) \wedge \sigma_2^{G_2}(y_1)$,

where $x_1 \in V_1$ and $y_1 \in V_2$.

$$\left(\overline{\mu_1^{G_1} \times_m \mu_1^{G_2}}\right)((x_1, y_1), (x_2, y_2)) = \begin{cases} \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) - \left(\mu_1^{G_1} \times_m \mu_1^{G_2}\right)((x_1, y_1), (x_2, y_2)) & \text{if } x_1 = x_2,\ y_1y_2 \in E_2 \\ \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) - \left(\mu_1^{G_1} \times_m \mu_1^{G_2}\right)((x_1, y_1), (x_2, y_2)) & \text{if } y_1 = y_2,\ x_1x_2 \in E_1 \\ \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) & \text{otherwise} \end{cases} \tag{3}$$

$$\left(\overline{\mu_2^{G_1} \times_m \mu_2^{G_2}}\right)((x_1, y_1), (x_2, y_2)) = \begin{cases} \left(\sigma_2^{G_1} \times_m \sigma_2^{G_2}\right)(x_1, y_1) \vee \left(\sigma_2^{G_1} \times_m \sigma_2^{G_2}\right)(x_2, y_2) - \left(\mu_2^{G_1} \times_m \mu_2^{G_2}\right)((x_1, y_1), (x_2, y_2)) & \text{if } x_1 = x_2,\ y_1y_2 \in E_2 \\ \left(\sigma_2^{G_1} \times_m \sigma_2^{G_2}\right)(x_1, y_1) \vee \left(\sigma_2^{G_1} \times_m \sigma_2^{G_2}\right)(x_2, y_2) - \left(\mu_2^{G_1} \times_m \mu_2^{G_2}\right)((x_1, y_1), (x_2, y_2)) & \text{if } y_1 = y_2,\ x_1x_2 \in E_1\ . \\ \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \vee \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) & \text{otherwise} \end{cases} \tag{4}$$

Table 1 Vertex set of $G_1 \times_m G_2$

$V_1 \times_m V_2$	u_1v_1	u_2v_1	u_3v_1	u_1v_2	u_2v_2	u_3v_2
$\sigma_1^{G_1} \times_m \sigma_1^{G_2}$	0.6	0.5	0.6	0.6	0.5	0.6
$\sigma_2^{G_1} \times_m \sigma_2^{G_2}$	0.3	0.5	0.3	0.2	0.2	0.2

Table 2 Edge set of $G_1 \times_m G_2$

$E_1 \times_m E_2$	u_1v_1, u_3v_1	u_2v_1, u_3v_1	u_3v_1, u_3v_2	u_1v_2, u_3v_2	u_2v_2, u_3v_2	u_2v_2, u_2v_1	u_1v_1, u_1v_2
$\mu_1^{G_1} \times_m \mu_1^{G_2}$	0.5	0.5	0.6	0.5	0.5	0.5	0.6
$\mu_2^{G_1} \times_m \mu_2^{G_2}$	0.2	0.4	0.3	0.2	0.2	0.4	0.3

Example 3.2 Consider the two intuitionistic fuzzy graphs as shown in Figs. 1 and 2 and their corresponding max product $G_1 \times_m G_2$ shown in Fig. 3.

Then, the complement of max product of G_1 and G_2 is shown in Fig. 4.

Theorem 3.1 *Let G_1 and G_2 be two regular intuitionistic fuzzy graphs. If underlying crisp graphs G_1^* and G_2^* are complete graphs and $\sigma_1^{G_1}, \sigma_2^{G_1}, \sigma_1^{G_2}, \sigma_2^{G_2}$ are constants which satisfy $\sigma_1^{G_1} \geq \mu_1^{G_2}, \sigma_2^{G_1} \leq \mu_2^{G_2}; \sigma_1^{G_2} \geq \mu_1^{G_1}, \sigma_2^{G_2} \leq \mu_2^{G_1}; \sigma_1^{G_1} > \mu_1^{G_1}$, $\sigma_2^{G_1} < \mu_2^{G_1}$ and $\sigma_1^{G_2} > \mu_1^{G_2}$, $\sigma_2^{G_2} < \mu_2^{G_2}$. Then, complement of max product of two intuitionistic fuzzy graphs G_1 and G_2 is regular intuitionistic fuzzy graph.*

Proof: Let G_1 and G_2 be two regular intuitionistic fuzzy graphs. The underlying crisp graphs G_1^* and G_2^* are complete graphs of degrees d_1 and d_2 for every vertices of V_1 and V_2.

Given that $\sigma_1^{G_1}, \sigma_2^{G_1}, \sigma_1^{G_2}$ and $\sigma_2^{G_2}$ are constants, say $\sigma_1^{G_1}(x) = c_1, \sigma_2^{G_1}(x) = c_2, \ \forall x \in V_1 \, \sigma_1^{G_2}(y) = c_3, \sigma_2^{G_2}(y) = c_4$ $\forall y \in V_2$ and $\sigma_1^{G_1} \geq \mu_1^{G_2}, \sigma_2^{G_1} \leq \mu_2^{G_2}; \sigma_1^{G_2} \geq \mu_1^{G_1}, \sigma_2^{G_2} \leq \mu_2^{G_1}$.

By theorem, max product of two regular intuitionistic fuzzy graphs is regular intuitionistic fuzzy graph.

Consider $(x_1, y_1) \in \left(\overline{\sigma_1^{G_1} \times_m \sigma_1^{G_2}}\right)$

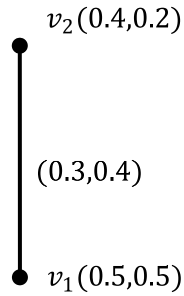

Fig. 2 Intuitionistic fuzzy graph G_2

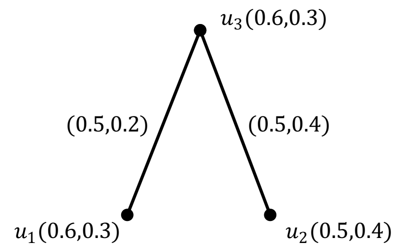

Fig. 1 Intuitionistic fuzzy graph G_1

$$
\begin{aligned}
d_1^{\overline{G_1 \times_m G_2}}(x_1, y_1) &= \sum_{(x_1,y_1)(x_2,y_2) \in E} \left(\overline{\mu_1^{G_1} \times_m \mu_1^{G_2}}\right)\left((x_1, y_1)(x_2, y_2)\right) \\
&= \sum_{x_1 = x_2,\ \ y_1 y_2 \in E_1} \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) \ - \left(\mu_1^{G_1} \times_m \mu_1^{G_2}\right)\left((x_1, y_1)(x_2, y_2)\right) \\
&\quad + \sum_{y_1 = y_2,\ \ x_1 x_2 \in E_1} \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) \ - \left(\mu_1^{G_1} \times_m \mu_1^{G_2}\right)\left((x_1, y_1)(x_2, y_2)\right) \\
&\quad + \sum_{x_1 x_2 \in E_1, y_1 y_2 \notin E_2} \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) \\
&\quad + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \in E_2} \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) \\
&\quad + \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2) \\
&\quad + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_1, y_1) \wedge \left(\sigma_1^{G_1} \times_m \sigma_1^{G_2}\right)(x_2, y_2).
\end{aligned}
$$

Since G_1^* and G_2^* are complete graphs, then

$$\begin{aligned} d_1^{\overline{G_1\times_m G_2}}(x_1,y_1) = & \sum_{x_1=x_2,\ y_1y_2\in E_2} \left(\sigma_1^{G_1}\times_m \sigma_1^{G_2}\right)(x_1,y_1)\wedge\left(\sigma_1^{G_1}\times_m \sigma_1^{G_2}\right)(x_2,y_2) \\ & -\left(\mu_1^{G_1}\times_m \mu_1^{G_2}\right)\big((x_1,y_1)(x_2,y_2)\big) \\ & +\sum_{y_1=y_2,\ x_1x_2\in E_1} \left(\sigma_1^{G_1}\times_m \sigma_1^{G_2}\right)(x_1,y_1)\wedge\left(\sigma_1^{G_1}\times_m \sigma_1^{G_2}\right)(x_2,y_2) \\ & -\left(\mu_1^{G_1}\times_m \mu_1^{G_2}\right)\big((x_1,y_1)(x_2,y_2)\big) \\ & +\sum_{x_1x_2\in E_1, y_1y_2\in E_2} \left(\sigma_1^{G_1}\times_m \sigma_1^{G_2}\right)(x_1,y_1)\wedge\left(\sigma_1^{G_1}\times_m \sigma_1^{G_2}\right)(x_2,y_2) \\ & -\left(\mu_1^{G_1}\times_m \mu_1^{G_2}\right)\big((x_1,y_1)(x_2,y_2)\big) \\ & +\sum_{x_1x_2\in E_1, y_1y_2\in E_2} \left(\sigma_1^{G_1}\times_m \sigma_1^{G_2}\right)(x_1,y_1)\wedge\left(\sigma_1^{G_1}\times_m \sigma_1^{G_2}\right)(x_2,y_2)\ . \end{aligned} \tag{5}$$

Similarly

$$\begin{aligned} d_2^{\overline{G_1\times_m G_2}}(x_1,y_1) = & \sum_{x_1=x_2,\ y_1y_2\in E_2} \left(\sigma_2^{G_1}\times_m \sigma_2^{G_2}\right)(x_1,y_1)\vee\left(\sigma_2^{G_1}\times_m \sigma_2^{G_2}\right)(x_2,y_2) \\ & -\left(\mu_2^{G_1}\times_m \mu_2^{G_2}\right)\big((x_1,y_1)(x_2,y_2)\big) \\ & +\sum_{y_1=y_2,\ x_1x_2\in E_1} \left(\sigma_2^{G_1}\times_m \sigma_2^{G_2}\right)(x_1,y_1)\vee\left(\sigma_2^{G_1}\times_m \sigma_2^{G_2}\right)(x_2,y_2) \\ & -\left(\mu_2^{G_1}\times_m \mu_2^{G_2}\right)\big((x_1,y_1)(x_2,y_2)\big) \\ & +\sum_{x_1x_2\in E_1, y_1y_2\in E_2} \left(\sigma_2^{G_1}\times_m \sigma_2^{G_2}\right)(x_1,y_1)\vee\left(\sigma_2^{G_1}\times_m \sigma_2^{G_2}\right)(x_2,y_2). \end{aligned} \tag{6}$$

Case (i) : If $\sigma_1^{G_1}(x) \leq \sigma_1^{G_2}(y)$ and $\sigma_2^{G_1}(x) \geq \sigma_2^{G_2}(y)$ for all $x \in V_1$ and $y \in V_2$.
By Eq. (5)

$$\begin{aligned} d_2^{\overline{G_1\times_m G_2}}(x_1,y_1) = & \sum_{x_1=x_2,\ y_1y_2\in E_2} \left(\sigma_2^{G_1}(x_1)\wedge\sigma_2^{G_2}(y_1)\right)\vee\left(\sigma_2^{G_1}(x_2)\wedge\sigma_2^{G_2}(y_2)\right) \\ & -\left(\mu_1^{G_1}\times_m \mu_1^{G_2}\right)\big((x_1,y_1)(x_2,y_2)\big) \\ & +\sum_{y_1=y_2,\ x_1x_2\in E_1} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right) \\ & -\left(\mu_1^{G_1}\times_m \mu_1^{G_2}\right)\big((x_1,y_1)(x_2,y_2)\big) \\ & +\sum_{x_1x_2\in E_1, y_1y_2\in E_2} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right) \\ = & \sum_{x_1=x_2,\ y_1y_2\in E_2} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right)\ . \end{aligned}$$

By Eq. (6)

$$d_2^{\overline{G_1\times_m G_2}}(x_1,y_1) = \sum_{x_1=x_2,\ y_1y_2\in E_2} \left(\sigma_2^{G_1}(x_1)\wedge\sigma_2^{G_2}(y_1)\right)\vee\left(\sigma_2^{G_1}(x_2)\wedge\sigma_2^{G_2}(y_2)\right) - \left(\overline{\mu_2^{G_1}\times_m\mu_2^{G_2}}\right)((x_1,y_1)(x_2,y_2)) + \sum_{y_1=y_2,\ x_1x_2\in E_1} \left(\sigma_2^{G_1}(x_1)\wedge\sigma_2^{G_2}(y_1)\right)\vee\left(\sigma_2^{G_1}(x_2)\wedge\sigma_2^{G_2}(y_2)\right) - \left(\overline{\mu_2^{G_1}\times_m\mu_2^{G_2}}\right)((x_1,y_1)(x_2,y_2)) + \sum_{x_1x_2\in E_1, y_1y_2\in E_2} \left(\sigma_2^{G_1}(x_1)\wedge\sigma_2^{G_2}(y_1)\right)\vee\left(\sigma_2^{G_1}(x_2)\wedge\sigma_2^{G_2}(y_2)\right).$$

Since by the definition of max product of two intuitionistic fuzzy graphs

$$\begin{aligned} d_1^{\overline{G_1\times_m G_2}}(x_1,y_1) &= \sum_{x_1=x_2,\ y_1y_2\in E_2} \sigma_1^{G_2}(y_1) - \left(\mu_1^{G_1}\times_m\mu_1^{G_2}\right)((x_1,y_1)(x_2,y_2)) \\ &+ \sum_{y_1=y_2,\ x_1x_2\in E_1} \sigma_1^{G_2}(y_1) - \left(\overline{\mu_1^{G_1}\times_m\mu_1^{G_2}}\right)((x_1,y_1)(x_2,y_2)) \\ &+ \sum_{x_1x_2\in E_1, y_1y_2\in E_2} c_3 \\ &= \sum_{x_1=x_2,\ y_1y_2\in E_2} \sigma_1^{G_2}(y_1) - \sigma_1^{G_1}(x_1)\vee\mu_1^{G_2}(y_1y_2) \\ &+ \sum_{y_1=y_2,\ x_1x_2\in E_1} \sigma_1^{G_2}(y_1) - \mu_1^{G_1}(x_1x_2)\vee\sigma_1^{G_2}(y_1) + \sum_{x_1x_2\in E_1, y_1y_2\in E_2} c_3 \\ &= \sum_{x_1=x_2,\ y_1y_2\in E_2} c_3 - \sigma_1^{G_1}(x_1) + \sum_{y_1=y_2,\ x_1x_2\in E_1} c_3 - \sigma_1^{G_2}(y_1) + c_3 d_{G_2}^*(y_1) d_{G_1}^*(x_1) \end{aligned}$$

Similarly
$$d_2^{\overline{G_1\times_m G_2}}(x_1,y_1) = (c_4-c_2)d_{G_2}^*(y_1) + c_4 d_{G_2}^*(y_1) d_{G_1}^*(x_1).$$

Since G_1 and G_2 are two regular intuitionistic fuzzy graphs & G_1^* and G_2^* are complete graphs, then $\mu_1^{G_1}$ and $\mu_1^{G_2}$ are constants say (e_1, e_2) and (e_3, e_4).

$$d_1^{\overline{G_1\times_m G_2}}(x_1,y_1) = (c_3-c_1)d_2 + c_3 d_1 d_2,$$
$$d_2^{\overline{G_1\times_m G_2}}(x_1,y_1) = (c_4-c_2)d_2 + c_4 d_1 d_2.$$

Case (ii) : If $\sigma_1^{G_1}(x) \ge \sigma_1^{G_2}(y)$ and $\sigma_2^{G_1}(x) \le \sigma_2^{G_2}(y)$ for all $x \in V_1$ and $y \in V_2$

$$\begin{aligned} d_1^{\overline{G_1\times_m G_2}}(x_1,y_1) &= \sum_{x_1=x_2,\ y_1y_2\in E_2} \sigma_1^{G_1}(x_1) - \left\{\sigma_1^{G_1}(x_1)\vee\mu_1^{G_2}(y_1,y_2)\right\} \\ &+ \sum_{y_1=y_2,\ x_1x_2\in E_1} \sigma_1^{G_1}(y_1) - \left\{\sigma_1^{G_2}(y_1)\vee\mu_1^{G_1}(x_1,x_2)\right\} \\ &+ \sum_{x_1x_2\in E_1, y_1y_2\in E_2} \sigma_1^{G_1}(x_1) \\ &= \sum_{x_1=x_2,\ y_1y_2\in E_1} c_1 - \sigma_1^{G_1}(x_1) + \sum_{y_1=y_2,\ x_1x_2\in E_2} c_1 - \sigma_1^{G_2}(y_1) + c_1 d_{G_2}^*(y_1) d_{G_1}^*(x_1) \\ &= (c_1-c_1)d_2 + (c_1-c_3)d_1 + c_1 d_1 d_2 = (c_1-c_3)d_1 + c_1 d_1 d_2 \\ &= \sum_{x_1=x_2,\ y_1y_2\in E_1} c_1 - \sigma_1^{G_1}(x_1) + \sum_{y_1=y_2,\ x_1x_2\in E_2} c_1 - \sigma_1^{G_2}(y_1) + c_1 d_{G_2}^*(y_1) d_{G_1}^*(x_1). \end{aligned}$$

Similarly

$$d_2^{\overline{G_1 \times_m G_2}}(x_1, y_1) = (c_4 - c_2)d_1 + c_4 d_1 d_2.$$

Hence, complement of max product of two regular intuitionistic fuzzy graphs is regular.

Theorem 3.2 *Let G_1 and G_2 be two regular intuitionistic fuzzy graphs of the underlying crisp graphs; G_1^* and G_2^* are regular graphs with the vertex sets; and edge sets of G_1 and G_2 are different constants which satisfies $\sigma_1^{G_1} > \mu_1^{G_2}$, $\sigma_2^{G_1} < \mu_2^{G_2}$; $\sigma_1^{G_2} > \mu_1^{G_1}$, $\sigma_2^{G_2} < \mu_2^{G_1}$; $\sigma_1^{G_1} > \mu_1^{G_1}$, $\sigma_2^{G_1} < \mu_2^{G_1}$ and $\sigma_1^{G_2} > \mu_1^{G_2}$, $\sigma_2^{G_2} < \mu_2^{G_2}$. Then, complement of the max product of two regular intuitionistic fuzzy graphs G_1 and G_2 is regular intuitionistic fuzzy graph.*

Proof: Let G_1 and G_2 be two regular intuitionistic fuzzy graphs. The underlying crisp graphs G_1^* and G_2^* are regular graphs of degrees g_1 and g_2 for every vertices in V_1 and V_2.

Given that $\sigma^{G_1}, \sigma^{G_2}$, μ^{G_1} and μ^{G_2} are constants, say $\sigma_1^{G_1}(x) = c_1, \sigma_2^{G_1}(x) = c_2$, $\forall\, x \in V_1$ $\sigma_1^{G_2}(y) = c_3, \sigma_2^{G_2}(y) = c_4$ $\forall\, y \in V_2$, $\mu_1^{G_1}(x_1y_1) = e_1, \mu_2^{G_1}(x_1y_1) = e_2$, $\mu_1^{G_2}(x_1y_1) = e_3, \mu_2^{G_2}(x_1y_1) = e_4$ and $\sigma_1^{G_1} > \mu_1^{G_2}, \sigma_2^{G_1} < \mu_2^{G_2}; \sigma_1^{G_2} > \mu_1^{G_1}, \sigma_2^{G_2} < \mu_2^{G_1}$.

Consider $(x_1, y_1) \in \left(\overline{\sigma_1^{G_1} \times_m \sigma_1^{G_2}}\right)$.

Case (i) : If $\sigma_1^{G_1}(x) \le \sigma_1^{G_2}(y)$ and $\sigma_2^{G_1}(x) \ge \sigma_2^{G_2}(y)$ for all $x \in V_1$ and $y \in V_2$

$$\begin{aligned}
d_1^{\overline{G_1 \times_m G_2}}(x_1, y_1) = & \sum_{x_1 = x_2,\ y_1y_2 \in E_2} \left(\sigma_1^{G_1}(x_1) \vee \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \vee \sigma_1^{G_2}(y_2)\right) \\
& - \left(\mu_1^{G_1} \times_m \mu_1^{G_2}\right)\left((x_1, y_1)(x_2, y_2)\right) \\
& + \sum_{y_1 = y_2,\ x_1x_2 \in E_1} \left(\sigma_1^{G_1}(x_1) \vee \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \vee \sigma_1^{G_2}(y_2)\right) \\
& - \left(\mu_1^{G_1} \times_m \mu_1^{G_2}\right)\left((x_1, y_1)(x_2, y_2)\right) \\
& + \sum_{x_1x_2 \in E_1, y_1y_2 \notin E_2} \left(\sigma_1^{G_1}(x_1) \vee \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \vee \sigma_1^{G_2}(y_2)\right) \\
& + \sum_{x_1x_2 \notin E_1, y_1y_2 \in E_2} \left(\sigma_1^{G_1}(x_1) \vee \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \vee \sigma_1^{G_2}(y_2)\right) \\
& + \sum_{x_1x_2 \notin E_1, y_1y_2 \notin E_2} \left(\sigma_1^{G_1}(x_1) \vee \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \vee \sigma_1^{G_2}(y_2)\right) \\
& + \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} \left(\sigma_1^{G_1}(x_1) \vee \sigma_1^{G_2}(y_1)\right) \wedge \left(\sigma_1^{G_1}(x_2) \vee \sigma_1^{G_2}(y_2)\right) \\
= & \sum_{x_1 = x_2,\ y_1y_2 \in E_2} \sigma_1^{G_2}(y_1) - \left\{\sigma_1^{G_1}(x_1) \vee \mu_1^{G_2}(y_1, y_2)\right\} \\
& + \sum_{y_1 = y_2,\ x_1x_2 \in E_1} \sigma_1^{G_2}(y_1) - \left\{\sigma_1^{G_2}(x_2) \vee \mu_1^{G_1}(x_1, x_2)\right\} \\
& + \sum_{x_1x_2 \in E_1,\ y_1y_2 \notin E_2} c_3 + \sum_{x_1x_2 \notin E_1,\ y_1y_2 \in E_2} c_3 + \sum_{x_1x_2 \notin E_1, y_1y_2 \notin E_2} c_3 + \sum_{x_1x_2 \in E_1, y_1y_2 \in E_2} c_3 \\
= & (c_3 - c_1)g_2 + (c_1 - c_1)g_1 + c_3 d_{G_1^*}(x_1)\left|\overline{E_2}\right| + c_3\left|\overline{E_1}\right| d_{G_2^*}(y_1) + c_3\left|\overline{E_1}\right|\left|\overline{E_2}\right| \\
& + c_3 d_{G_2^*}(x_2)\ d_{G_1^*}(x_1),
\end{aligned}$$

where $\left|\overline{E_1}\right|$ and $\left|\overline{E_2}\right|$ are the degree of vertex of complement graphs G_1^* and G_2^*.

$$d_1^{\overline{G_1\times_m G_2}}(x_1,y_1) = (c_3-c_1)g_2 + c_3 g_1\left|\overline{E_2}\right| + c_3 g_2\left|\overline{E_1}\right| + c_3\left|\overline{E_1}\right|\left|\overline{E_2}\right| + c_1 g_1 g_2.$$

Similarly

$$d_2^{\overline{G_1\times_m G_2}}(x_1,y_1) = (c_4-c_2)g_2 + c_4 g_1\left|\overline{E_2}\right| + c_4 g_2\left|\overline{E_1}\right| + c_4\left|\overline{E_1}\right|\left|\overline{E_2}\right| + c_2 g_1 g_2$$

.

This is true for all vertices of $\overline{V_1\times_m V_2}$.

Case (ii) : If $\sigma_1^{G_2}(x) \le \sigma_1^{G_1}(y)$ and $\sigma_2^{G_2}(x) \ge \sigma_2^{G_1}(y)$ for all $x \in V_1$ and $y \in V_2$.

$$\begin{aligned} d_1^{\overline{G_1\times_m G_2}}(x_1,y_1) = &\sum_{x_1=x_2,\ y_1y_2\in E_2} \sigma_1^{G_1}(x_2) - \sigma_1^{G_1}(x_1) \\ &+ \sum_{y_1=y_2,\ x_1x_2\in E_1} \sigma_1^{G_1}(x_2) - \sigma_1^{G_2}(y_1) \\ &+ \sum_{x_1x_2\in E_1,\ y_1y_2\notin E_2} \sigma_1^{G_2}(x_1) \\ &+ \sum_{x_1x_2\notin E_1,\ y_1y_2\in E_2} \sigma_1^{G_1}(x_1) \\ &+ \sum_{x_1x_2\notin E_1,\ y_1y_2\notin E_2} \sigma_1^{G_1}(x_1) \\ &+ \sum_{x_1x_2\in E_1,\ y_1y_2\in E_2} \sigma_1^{G_1}(x_1), \end{aligned}$$

where E_1 and E_2 are the degree of the vertex of complement graphs of complement graphs G_1^* and G_2^*

$$\begin{aligned} d_1^{\overline{G_1\times_m G_2}}(x_1,y_1) = &\sum_{x_1=x_2,\ y_1y_2\in E_2} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right) \\ &- \left(\mu_1^{G_1}\times_m\mu_1^{G_2}\right)\left((x_1,y_1)(x_2,y_2)\right) \\ &+ \sum_{y_1=y_2,\ x_1x_2\in E_1} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right) \\ &- \left(\mu_1^{G_1}\times_m\mu_1^{G_2}\right)\left((x_1,y_1)(x_2,y_2)\right) \\ &+ \sum_{x_1x_2\in E_1,y_1y_2\notin E_2} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right) \\ &+ \sum_{x_1x_2\notin E_1,y_1y_2\in E_2} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right) \\ &- \left(\mu_1^{G_1}\times_m\mu_1^{G_2}\right)\left((x_1,y_1)(x_2,y_2)\right) \\ &+ \sum_{x_1x_2\notin E_1,y_1y_2\notin E_2} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right) \\ &+ \sum_{x_1x_2\in E_1,y_1y_2\in E_2} \left(\sigma_1^{G_1}(x_1)\vee\sigma_1^{G_2}(y_1)\right)\wedge\left(\sigma_1^{G_1}(x_2)\vee\sigma_1^{G_2}(y_2)\right) \end{aligned}$$

$$\begin{aligned} d_1^{\overline{G_1\times_m G_2}}(x_1,y_1) &= (c_1-c_1)g_1 \\ &\quad + (c_1-c_3)g_2 + c_1 g_1\left|\overline{E_2}\right| + c_1 g_2\left|\overline{E_1}\right| + c_1\left|\overline{E_1}\right|\left|\overline{E_2}\right| + c_1 g_1 g_2 \\ &= (c_1-c_3)g_2 + c_1 g_1\left|\overline{E_2}\right| + c_1 g_2\left|\overline{E_1}\right| + c_1\left|\overline{E_1}\right|\left|\overline{E_2}\right| + c_1 g_1 g_2. \end{aligned}$$

Similarly

$$d_2^{\overline{G_1 \times_m G_2}}(x_1, y_1) = (c_2 - c_4)g_2 + c_2 g_1 \left|\overline{E_2}\right| + c_2 g_2 \left|\overline{E_1}\right| + c_2 \left|\overline{E_1}\right| \left|\overline{E_2}\right| + c_2 g_1 g_2.$$

This is true for all vertices in $\overline{V_1 \times_m V_2}$.

Hence, complement of modular product of two regular intuitionistic fuzzy graphs is regular.

Applications of max product of intuitionistic fuzzy graphs in school determination

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the universe of discourse. Let $A = \{\langle x, \mu_1^A(x), \mu_2^A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_1^B(x), \mu_2^B(x)\rangle : x \in X\}$ be two intuitionistic fuzzy sets in X. Szmidt and Kacprzyk [17, 18] proposed the following distance measure between A and B:

The Normalized Hamming Distance

$$d(A, B) = \frac{1}{2n} \sum_{i=1}^{n} \left(\left|\mu_1^A(x_i) - \mu_1^B(x)\right| + \left|\mu_2^A(x_i) - \mu_2^B(x)\right| \right).$$

Suppose that $S = \{S_1, S_2, \ldots, S_n\}$ be a set of schools, $P = \{p_1, p_2, \ldots, p_m\}$ be a set of papers, and $Q = \{q_1, q_2, \ldots, q_t\}$ be a set of students.

Let R_1 be a relation between school points and each subject paper, and relation R_2 be a relation between students and their corresponding subject entrance score.

We can describe the distance between the students and the schools using the following matrix:Equation as Image

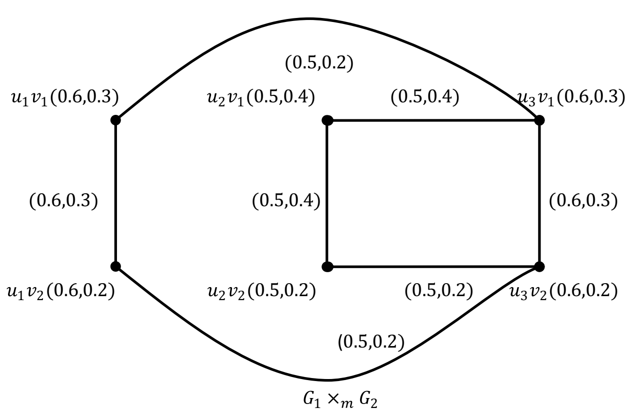

Fig. 3 Max Product $G_1 \times_m G_2$

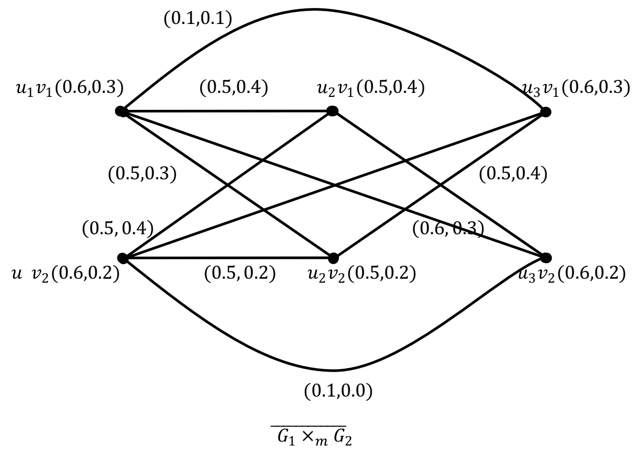

Fig. 4 Complement of max product of G_1 and G_2

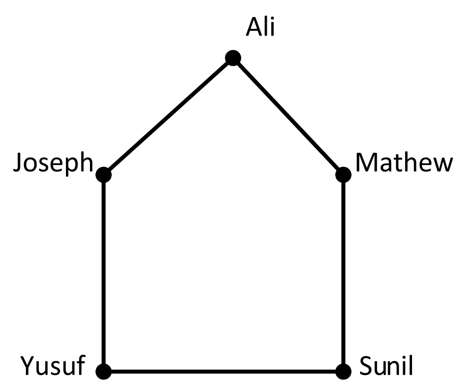

Fig. 5 Intuitionistic fuzzy graph G_1

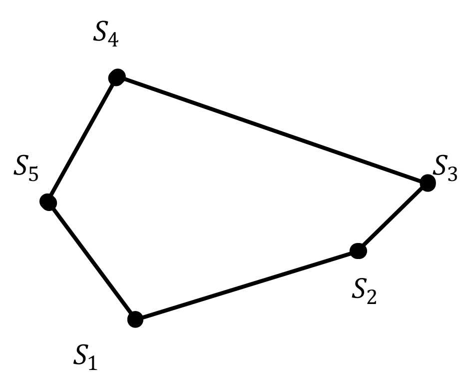

Fig. 6 Intuitionistic fuzzy graph G_2

Table 3 Relation between school points and subject papers

R_1	Tamil (Tam)	English (Eng)	Mathematics (Mat)	Science (Sci)	Social science (Sos)
S_1	(0.8, 0.1)	(0.9, 0.1)	(0.9, 0.05)	(0.7, 0.2)	(0.8, 0.2)
S_2	(0.7, 0.04)	(0.6, 0.3)	(0.95, 0)	(0.7, 0.1)	(0.6, 0.4)
S_3	(0.5, 0.02)	(0.5, 0.2)	(0.6, 0.3)	(0.6, 0.1)	(0.7, 0.1)
S_4	(0.5, 0.4)	(0.9, 0.1)	(0.8, 0.1)	(0.6, 0.2)	(0.8, 0.2)
S_5	(0.7, 0.2)	(0.8, 0.1)	(0.7. 0.2)	(0.8, 0.2)	(0.6, 0.3)

Table 4 Relation between students and their corresponding average entrance marks

R_2	Tamil	English	Mathematics	Science	Social science
Ali	(0.9, 0.1)	(0.6, 0.2)	(0.9, 0)	(0.6, 0.4)	(0.7, 0.1)
Mathew	(0.3, 0.4)	(0.7, 0.2)	(0.8, 0.2)	(0.6, 0.3)	(0.7, 0.3)
Sunil	(0.9, 0.1)	(0.8, 0.1)	(0.8, 0.2)	(0.4, 0.5)	(0.9, 0.2)
Yusuf	(0.4, 0.3)	(0.4, 0.5)	(0.6, 0.2)	(0.7, 0.1)	(0.6, 0.1)
Joseph	(0.7, 0.2)	(0.6, 0.1)	(0.9. 0.1)	(0.8, 0.2)	(0.9, 0.1)

$$
\begin{array}{c}
\\ q_1 \\ q_2 \\ . \\ . \\ . \\ q_t
\end{array}
\begin{array}{c}
\begin{array}{cccc} S_1 & S_2 & \cdots & S_n \end{array} \\
\begin{pmatrix}
d(q_1,S_1) & d(q_1,S_2) & \cdots & d(q_1,S_1) \\
d(q_2,S_1) & d(q_2,S_2) & \cdots & d(q_1,S_2) \\
. & . & \cdots & . \\
. & . & \cdots & . \\
. & . & \cdots & . \\
d(q_t,S_1) & d(q_t,S_2) & \cdots & d(q_t,S_n)
\end{pmatrix}
\end{array}
$$

Applying the fact that shortest distance between two intuitionistic fuzzy sets shows more similarity between them, therefore, it can be described that for the student q_i is to be enroll in the school corresponding to $\min_{i}\left\{d\left(q_i, S_j\right)\right\}$.

Here, we have considered a set of schools $S = \left\{S_1, S_2, S_3, S_4, S_5\right\}$, $P =${Tamil (Ta), English (Eng), Mathematics (Mat), Science (Sci), Social Science (Sos)} be a set of papers and $Q =$ {Ali, Mathew, Sunil, Yusuf, Joseph}.

In Figs. 5 and 6, we assume that $G_1 = \left(Q, \left\{N_1, N_2, N_3, N_4, N_5\right\}\right)$ is an intuitionistic fuzzy graph of the set of students and $G_2 = \left(S, \left\{N_1', N_2', N_3', N_4', N_5'\right\}\right)$ is an intuitionistic fuzzy graph of the set of schools, where $Q =$ { Ali, Mathew, Sunil, Yusuf, Joseph and $S = \left\{S_1, S_2, S_3, S_4, S_5\right\}$.

The relation between school points and subject papers and the relation between students and their corresponding average entrance marks are given in Tables 3 and 4, respectively.

In the following table, we have used intuitionistic fuzzy sets as a tool, since it incorporates the membership grades (the average marks of the questions that have been correctly answered by the student) and the non-membership grades

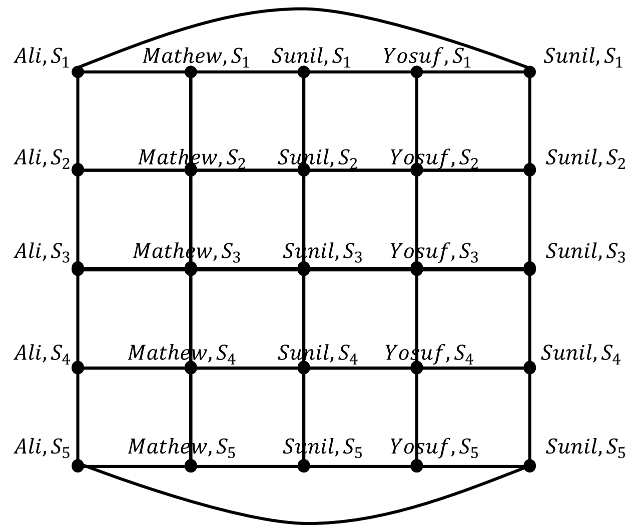

Fig. 7 Max product $G_1 \times_m G_2$

(the average marks of the questions that have been incorrectly answered by the student).

The max product of G_1 and G_2 is shown in Fig. 7.

The following decision matrix has been obtained by finding distance between each student (Table 3) and each school (Table 4) using normalized hamming distance function depending upon the their entrance marks.

	S_1	S_2	S_3	S_4	S_5
Ali	0.079167	0.084167	0.09	0.108333	0.125
Mathew	0.104167	0.1175	0.106667	0.058333	0.075
Sunil	0.0875	0.1675	0.14	0.091667	0.108333
Yusuf	0.1625	0.12583	0.08167	0.15	0.11667
Joseph	0.07083	0.1175	0.12333	0.09167	0.09167

From the above decision matrix, less distance between the student and school implies more possibility to get enrollment in the corresponding school. Therefore, the student Ali is to enroll in the school S_1, the student Mathew is to enroll in the school S_4, the student Sunil is to enroll in the school S_1, the student Yusuf is to enroll in the school S_3, and the student Joseph is to enroll in the school S_1.

Conclusion

Graph theory has numerous applications in solving various networking problems encountered in different fields such as signal processing, transportation, and error codes. In particularly, the shortest path problem is a well-known combinatorial optimization problem in graph theory. Intuitionistic fuzzy graph models are more practical and useful than fuzzy graph models as it provides membership and non-membership grades for representing imprecise information which occur in real-life situations. This paper has introduced the complement of max product of two intuitionistic fuzzy graphs. Using the max product, the different types of structural models can be combined to produce a better one. The special attention on the regularity in the complement of two intuitionistic fuzzy graphs has been given as it can be applied widely in designing reliable communication and network systems. Finally, an application of intuitionistic fuzzy graphs in decision-making concern the school determination for the students based on their entrance score has been presented. In future, we are going to extend our work to: (1) Pythagorean fuzzy graphs; (2) Interval-valued Pythagorean fuzzy graphs, and (3) Spherical fuzzy graphs.

Acknowledgements The authors are highly thankful to the Editor-in-Chief and the referees for their valuable comments and suggestions for improving the paper.

Declarations

Conflict of interest The authors declare that they have no conflict of interest regarding the publication of the article.

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