Neural network based asynchronous synchronization for fuzzy hidden Markov jump complex dynamical networks

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Abstract

This paper investigates the drive-response synchronization problem of Takagi–Sugeno fuzzy hidden Markov jump complex dynamical networks. More precisely, a novel asynchronous synchronization control strategy is developed for coping with mismatched hidden jumping modes. Furthermore, the neural network is adopted with online learning laws for unknown function approximation. By taking advantage of Lyapunov method, sufficient conditions are established to ensure mean-square synchronization performance with disturbances. Based on the synchronization criterion, asynchronous controller gains are designed in terms of linear matrix inequalities. An illustrative example is finally given to validate the effectiveness of the proposed synchronization techniques.

Keywords Neural network-based synchronization · Asynchronous synchronization · Hidden Markov jump complex dynamical networks · Fuzzy complex dynamical networks

Introduction

Complex dynamical networks (CDNs) have been receiving more and more research attention during the past decade owing to their theoretical importance and wide applications. Well-known and typical practical implementations of CDNs can be found as communication networks, social networks, biological networks and so on [1–4]. By local interconnection via certain information exchanges, CDNs can exhibit topological and complex characteristics [5,6]. Specifically, an interesting topic is the synchronization phenomenon among numbers of dynamical nodes, where collaborative behaviors can be achieved simultaneously [7–9]. Since investigations on synchronization phenomena can provide insights into understanding inherent features of CDNs, many remarkable synchronization control methodologies have been developed in the literature [10–12]. Furthermore, it has been found that dynamics of CDNs would change with jumping features, which can be described by Markov jump models. As a result, growing research efforts have been paid to Markov jump CDNs [13–15]. Meanwhile, it is always impractical

for precise and immediate observation of true transition rates in practical applications. For hidden Markov jump systems, the mode information is more difficult to obtain. In order to overcome the resulting deficiency of mode mismatches, the asynchronous strategies have been effectively developed and several successful initial attempts have been made for asynchronous analysis and synthesis of hidden Markov jump systems [16–20]. The key idea is to utilize the observed mode information instead of true mode information to deal with control performance degradation by mismatched modes. Unfortunately, there still remain some margins for further concerns on hidden Markov jump CDNs, where dynamical nodes described by hidden Markov processes should be taken into account. This is our first motivation to shorten such a gap.

On the other hand, a significant challenge lies in the fact that exact parameters of CDNs are often difficult to acquire in real-world applications, such that only uncertain parameters can be utilized with limited prior knowledge. Fortunately, several intelligent methods have been developed against parameter uncertainties or unknown functions, which include neural network learning, fuzzy modeling and other adaptive approximating approaches [21–24]. Generally speaking, these techniques can well utilize system input and output data for training, such that exhaustive representations of true values can be obtained to a satisfied extent. In particular, neural networks (NNs) have been widely

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applied in neural network-based control designs and distinguishing advantages can be achieved by nonlinear mapping, parallel computation and learning capacity with high accuracy [25–27]. For synchronization of CDNs with uncertainties or unknown nonlinearities, it is reasonable and effective to employ NNs to cope with parameter variations. Meanwhile, it should be pointed out that Takagi–Sugeno (T–S) fuzzy models can efficiently describe complex systems and they are closely integrated with complexity and synchronicity of CDNs in practice [28–30]. Under this context, many research results on T–S fuzzy CDNs have been reported [31–33]. Nevertheless, to the authors’ best knowledge, there are few results on T–S fuzzy hidden Markov jump CDNs despite its academic significance and potential applications, which further motivates us for this study.

Inspired by aforementioned discussions, this paper aims at solving the synchronization problem of T–S fuzzy hidden Markov jump CDNs within drive–response framework. Compared with most existing works, the main novelties of our paper can be listed as follows:

- 1) Based on observed system mode information, a new asynchronous mode-dependent synchronization strategy for T–S fuzzy hidden Markov jump CDNs is proposed by utilizing the mismatched mode information between drive and response CDNs with unknown nonlinear functions.
- 2) A NN based online learning law is also integrated with synchronization controller design, such that true values of unknown nonlinear function can be estimated with desired approximation while synchronization is achieved.
- 3) A novel mode-dependent Lyapunov functional is constructed to ensure the asymptotical convergence of synchronization error in the mean-square sense and the corresponding H_∞ synchronization performance conditions are provided.

The outline of this paper is arranged by the following parts: in “Problem formulation and preliminaries”, necessary preliminaries on T–S hidden Markov jump CDNs is introduced and the asynchronous synchronization is described. “Main analysis and synthesis results” derives synchronization controller design procedure in details. In “Numerical example”, a simulation example is performed to verify the correctness of our developed approach. “Conclusions” concludes the paper with future research perspectives.

Notation: $\mathbb{R}^n$ stands for n -Euclidean space. Matrix $P \succ 0$ means that P is positive definite, $\text{tr}(P)$ represents the trace of P and $*$ represents the ellipsis parts in symmetric block matrices. $(\Omega, \mathcal{F}, \mathbb{P})$ corresponds a probability space. $\mathcal{E}$ denotes mathematical expectation. All matrices are supposed to be with compatible dimensions.

Problem formulation and preliminaries

Fuzzy hidden Markov jump complex networks

Consider the hidden Markov jump CDNs described by following IF-THEN rules:

Rule k ($k = 1, 2, \dots, r$):

IF $\vartheta_1(t)$ is $\mathcal{F}_1^k$ and $\vartheta_2(t)$ is $\mathcal{F}_2^k$ and...and $\vartheta_g(t)$ is $\mathcal{F}_r^k$,
THEN

$$\dot{x}_l(t) = A^k(\sigma_t)x_l(t) + f(x_l(t)) + \sum_{m=1}^N c_{lm}\Gamma x_m(t), \quad (1)$$

where $l = 1, 2, \dots, N$, $x_l(t) = [x_{l1}(t), x_{l2}(t), \dots, x_{ln}(t)]^T \in \mathbb{R}^n$ represents l th node’ state, $f(x_l(t)) : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a unknown nonlinear function, $\Gamma \in \mathbb{R}^{n \times n}$ denotes inner coupling matrix and $C = (c_{lm})_{N \times N} \in \mathbb{R}^{N \times N}$ corresponds outer coupling matrix describing network topology with

$$\begin{cases} c_{ll} = -\sum_{m=1, m \neq l}^N c_{lm}, l = 1, 2, \dots, N, \\ c_{lm} \neq 0, \text{ there is a connection from node } l \text{ to node } m (l \neq m), \\ c_{lm} = 0, \text{ otherwise,} \end{cases}$$

σ_t denotes a continuous-time discrete-state Markov process in $(\Omega, \mathcal{F}, \mathbb{P})$ with transition probability matrix $\Pi = (\pi_{ij})_{\mathcal{N} \times \mathcal{N}}, \forall i, j \in \mathcal{S} = \{1, \dots, \mathcal{N}\}$ with

$$\Pr(\sigma_{t+\Delta} = j | \sigma_t = i) = \begin{cases} \pi_{ij} + o(\Delta), & i \neq j, \\ 1 + \pi_{ii} + o(\Delta), & i = j, \end{cases}$$

$$\pi_{ii} = -\sum_{j=1, j \neq i}^{\mathcal{N}} \pi_{ij}.$$

Moreover, $A(\sigma_t)$ denotes a known weight matrix for certain σ_t .

Consequently, by applying centroid strategy for defuzzification, one has

$$\dot{x}_l(t) = \sum_{k=1}^r h_k(\vartheta(t)) [A^k(\sigma_t)x_l(t) + f(x_l(t)) + \sum_{m=1}^N c_{lm}\Gamma x_m(t)],$$

where $\vartheta_1(t), \vartheta_2(t), \dots, \vartheta_r(t)$ represents premise variables, r stands for number of IF-THEN rules, $\mathcal{F}_j^k(\vartheta_j(t))$ denotes the membership value of $\vartheta_j(t)$ and $h_k(\vartheta(t))$ satisfies

$$h_k(\vartheta(t)) = \frac{w_k(\vartheta(t))}{\sum_{k=1}^r w_k(\vartheta(t))},$$

$$w_k(\vartheta(t)) = \prod_{j=1}^r \mathcal{F}_j^k(\vartheta_j(t)),$$

with

$$h_k(\vartheta(t)) \geq 0, \sum_{k=1}^r h_k(\vartheta(t)) = 1.$$

NN learning-based synchronization controller

Since $f(x_l(t))$ is unknown, a dynamical NN is applied for reconstruction approximation nonlinear function. For $f(x_l(t))$, it holds that

$$f(x_l(t)) = W^* \phi(x_l(t)) + \varepsilon,$$

where ε denotes the approximate error, W^* represents optimal weight that minimizes ε and is defined by

$$W^* = \arg \min_{W(t) \in \Omega_W} \left\{ \sup_{x \in \Omega_x} \|\varepsilon\| \right\}.$$

Furthermore, denote estimation of W^* as $\hat{W}(t)$ and refer (1) as drive CDN. Then, the corresponding response CDN can be given as

$$\begin{aligned} \dot{y}_l(t) = & \sum_{k=1}^r h_k(\vartheta(t)) [A^k(\sigma_t) y_l(t) + \hat{W}(t) \phi(x_l(t)) \\ & + \sum_{m=1}^N c_{lm} \Gamma y_m(t) + u_l(t) + d_l(t)], \end{aligned} \quad (2)$$

where $y_l(t) \in \mathbb{R}^n$ represents response state, $d_l(t) \in \mathbb{R}^n$ is external disturbance and $u_l(t) \in \mathbb{R}^n$ denotes synchronization controller.

Then, the synchronization error $e_l(t) = y_l(t) - x_l(t)$, $l = 1, 2, \dots, N$ is defined under drive–response context and it follows that

$$\begin{aligned} \dot{e}_l(t) = & \sum_{k=1}^r h_k(\vartheta(t)) [A^k(\sigma_t) e_l(t) + \tilde{W}(t) \phi(x_l(t)) \\ & + \sum_{m=1}^N c_{lm} \Gamma y_m(t) + u_l(t) + \Delta_l(t)], \end{aligned} \quad (3)$$

where $\tilde{W}(t) = \hat{W}(t) - W^*$ and $\Delta_l(t) = d_l(t) - \varepsilon$. Hence, the drive–response synchronization is said to be reached, if $e_l(t)$ can be mean-square asymptotically stable, i.e.,

$$\lim_{t \rightarrow \infty} \mathcal{E}\{e_l(t)\} = 0, l = 1, 2, \dots, N.$$

Remark 1 Note that the NN learning law can be applied online during the synchronization procedure, which is applicable for unknown function approximations with desired accuracy.

As a result, in order to achieve synchronization, mode-dependent synchronization controller with asynchronous features is designed as follows:

Rule $k(k = 1, 2, \dots, r)$:

IF $\vartheta_1(t)$ is $\mathcal{F}_1^k$ and $\vartheta_2(t)$ is $\mathcal{F}_2^k$ and...and $\vartheta_g(t)$ is $\mathcal{F}_g^k$,
THEN

$$u_l = K^k(\delta_t) e_l(t), l = 1, 2, \dots, N,$$

where $K^k(\delta_t)$ represents feedback controller gain to be determined and $\delta_t \in F = \{1, \dots, F\}$ denotes another stochastic process with following conditional probability:

$$\Pr\{\delta_t = \rho | \sigma_t = i\} = \lambda_{i\rho},$$

$$\sum_{\rho=1}^F \lambda_{i\rho} = 1.$$

Remark 2 It is noted that since true mode information is always difficult to acquire, the stochastic observed mode information is utilized instead of true mode information for the synchronization controller design in practical applications.

Based on parallel distributed compensation, the resulting synchronization error dynamics can be deduced by

$$\begin{aligned} \dot{e}_l(t) = & \sum_{k=1}^r h_k(\vartheta(t)) \sum_{p=1}^r h_p(\vartheta(t)) [(A^k(\sigma_t) \\ & + K^p(\delta_t)) e_l(t) + \tilde{W}(t) \phi(x_l(t)) \\ & + \sum_{m=1}^N c_{lm} \Gamma e_m(t) + \Delta_l(t)], \end{aligned} \quad (4)$$

which can be further formulated as follows:

$$\begin{aligned} \dot{e}(t) = & \sum_{k=1}^r h_k(\vartheta(t)) \sum_{p=1}^r h_p(\vartheta(t)) [(I_N \otimes A^k(\sigma_t)) + (C \otimes \Gamma) \\ & + (I_N \otimes K^p(\delta_t)) e(t) + (I_N \otimes \tilde{W}(t)) \phi(x(t)) + \Delta(t)], \end{aligned} \quad (5)$$

where

$$e(t) = [e_1^T(t), e_2^T(t), \dots, e_N^T(t)]^T,$$

$$\Delta(t) = [\Delta_1^T(t), \Delta_2^T(t), \dots, \Delta_N^T(t)]^T,$$

$$\phi(x(t)) = [\phi^T(x_1(t)), \phi^T(x_2(t)), \dots, \phi^T(x_N(t))]^T.$$

Remark 3 In this work, the asynchronous control strategy with conditional probability is adopted for hidden Markov process observations and is more applicable than non-homogenous process modeling.

Control objective

Before proceeding further, the following definition and lemma are introduced for later use:

Definition 1 The H_∞ synchronization is said to be achieved in mean-square sense if under zero initial states there exist matrix $\Omega \succ 0$ and a constant $\gamma > 0$ such that

$$\int_0^\infty \mathcal{E}\{e^T(s)\Omega e(s)\}ds < \gamma^2 \int_0^\infty \mathcal{E}\{\Delta^T(\varphi)\Delta(\varphi)\}d\varphi.$$

Lemma 1 [34] Given real matrices $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{X}, \mathcal{W}_1, \mathcal{W}_2$ with appropriate dimensions, if there exists a matrix $\mathcal{P} \succ 0$ satisfies that

$$\begin{bmatrix} \mathcal{P}\mathcal{A}^T + \mathcal{A}\mathcal{P} + \mathcal{X} & \mathcal{B} & \mathcal{P}\mathcal{C}^T \\ * & \mathcal{W}_1 & \mathcal{W}_2 \\ * & * & \mathcal{W}_3 \end{bmatrix} < 0;$$

then there exist a matrix $\mathcal{Z} \succ 0$ and a positive scalar $\mu > 0$ such that

$$\begin{bmatrix} -\mathcal{Z} - \mathcal{Z}^T & \mathcal{Z}\mathcal{A}^T + \mathcal{P} & 0 & \mathcal{Z}\mathcal{C}^T & \mathcal{Z} \\ * & -\mu^{-1}\mathcal{P} + \mathcal{X} & \mathcal{B} & 0 & 0 \\ * & * & \mathcal{W}_1 & \mathcal{W}_2 & 0 \\ * & * & * & \mathcal{W}_3 & 0 \\ * & * & * & * & -\mu\mathcal{P} \end{bmatrix}$$

Our purpose is to design mode-dependent $K^P(\delta_i)$ with appropriate NN learning laws, such that synchronization error $e(t)$ can achieve mean-square converge and the H_∞ synchronization performance can be satisfied accordingly.

Main analysis and synthesis results

In this section, main synchronization results will be established and the synchronization controller gains will be designed accordingly.

Theorem 1 With given matrix Ω and parameter γ , the drive-response synchronization can be achieved with designed controller gain $K^P(\rho)$, $\rho \in \mathcal{F}$, $p = 1, 2, \dots, r$, if there exist mode-dependent matrices $P(i) \succ 0$, $i \in \mathcal{S}$ and parameter $\mu > 0$, such that $\Xi_{k,k}(i) < 0$ and $\Xi_{k,p}(i) + \Xi_{p,k}(i) < 0$ holds for $k, p = 1, 2, \dots, r$, $k < p$, where

$$\Xi_{k,p}(i) = \begin{bmatrix} \Xi_{1k,p}(i) & (I_N \otimes P(i)) & I \\ * & -\gamma^2 I & 0 \\ * & * & -\Omega^{-1} \end{bmatrix},$$

$$\Xi_{1k,p}(i) = 2(I_N \otimes P(i)A^k(i)) + 2(C(i) \otimes P(i)\Gamma)$$

$$+ 2 \sum_{\rho=1}^F \lambda_{i\rho} (I_N \otimes P(i)K^P(\rho)) \\ + \sum_{j=1}^{\mathcal{N}} \pi_{ij} (I_N \otimes P(j)).$$

Furthermore, the online NN learning law is updated by

$$(I_N \otimes \dot{W}(t)) = -\Phi(I_N \otimes P(i))e(t)\phi^T(x(t)).$$

Proof Denote $\sigma(t)$ and $\delta(t)$ by i, ρ indexes, and construct the following Lyapunov function:

$$V(t, i) = e^T(t)(I_N \otimes P(i))e(t) \\ + tr((I_N \otimes \tilde{W}^T(t))\Phi^{-1}(I_N \otimes \tilde{W}(t))).$$

Afterwards, define infinitesimal operator $\mathcal{L}$ for $V(t, i)$ as follows:

$$\mathcal{L}V(t, i) = \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} \{\mathcal{E}\{V(t + \Delta, i)|t\} - V(t, i)\}.$$

Then, it can be derived that

$$\begin{aligned} \mathcal{L}V(t, i) = & \sum_{k=1}^r h_k(\vartheta(t)) \sum_{p=1}^r h_p(\vartheta(t)) [2e^T(t)(I_N \otimes P(i))A^k(i) \\ & + 2e^T(t)(C \otimes P(i)\Gamma)e(t) \\ & + \sum_{j=1}^{\mathcal{N}} \pi_{ij} e^T(t)(I_N \otimes P(j))e(t) \\ & + 2 \sum_{\rho=1}^F \lambda_{i\rho} e^T(t)(I_N \otimes P(i)K^P(\rho))e(t) \\ & + 2e^T(t)(I_N \otimes P(i)\tilde{W}(t))\phi(x(t)) \\ & + 2e^T(t)(I_N \otimes P(i))\Delta(t) \\ & + 2tr((I_N \otimes \dot{W}^T(t))\Phi^{-1}(I_N \otimes \tilde{W}(t)))]. \end{aligned}$$

Furthermore, the following matrix inequality holds:

$$2e^T(t)(I_N \otimes P(i))\Delta(t) \\ \leq \frac{1}{\gamma^2} e^T(t)(I_N \otimes P(i))(I_N \otimes P(i))e(t) + \gamma^2 \Delta(t)\Delta(t).$$

Moreover, by considering the fact that

$$2e^T(t)(I_N \otimes P(i)\tilde{W}(t))\phi(x(t)) \\ \leq 2tr\{\phi(x(t))e^T(t)(I_N \otimes P(i)\tilde{W}(t))\}$$

one has

$$\begin{aligned} \mathcal{L}V(t, i) &\leq \sum_{k=1}^r h_k(\vartheta(t)) \sum_{p=1}^r h_p(\vartheta(t)) [2e^T(t)(I_N \otimes P(i))A^k(i) \\ &\quad + 2e^T(t)(C \otimes P(i))\Gamma)e(t) \\ &\quad + \sum_{j=1}^{\mathcal{N}} \pi_{ij} e^T(t)(I_N \otimes P(j))e(t) \\ &\quad + e^T(t)2 \sum_{\rho=1}^F \lambda_{i\rho}(I_N \otimes P(i)K^P(\rho))e(t) \\ &\quad + \frac{1}{\gamma^2} e^T(t)(I_N \otimes P(i))(I_N \otimes P(i))e(t) + \gamma^2 \Delta(t)\Delta(t) \\ &\quad + 2tr\{[\phi(x(t))e^T(t)(I_N \otimes P(i)) \\ &\quad + (I_N \otimes \dot{W}^T(t))\Phi^{-1}](I_N \otimes \tilde{W}(t))\}. \end{aligned}$$

Consequently, when applying the NN learning law with

$$(I_N \otimes \dot{W}(t)) = -\Phi(I_N \otimes P(i))e(t)\phi^T(x(t)),$$

it can hold that

$$\mathcal{E}\{\mathcal{L}V(t, i)\} < \mathcal{E}\{-e^T(t)\Omega e(t) + \gamma^2 \Delta(t)\Delta(t)\}$$

by

$$\tilde{\mathcal{E}}_{k,p}(i) = \begin{bmatrix} -(I_N \otimes Z) - (I_N \otimes Z^T) & \tilde{\mathcal{E}}_{1k,p}(i) & 0 & (I_N \otimes Z) & (I_N \otimes Z) \\ * & \tilde{\mathcal{E}}_{2k,p}(i) & I & 0 & 0 \\ * & * & -\Omega^{-1} & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -\mu(I_N \otimes P(i)) \end{bmatrix},$$

$$\tilde{\mathcal{E}}_{1k,p}(i) = (I_N \otimes ZA^k(i)) + (C \otimes Z\Gamma) + \sum_{\rho=1}^F \lambda_{i\rho}(I_N \otimes \tilde{K}^P(\rho)) + (I_N \otimes P(i)),$$

$$\tilde{\mathcal{E}}_{2k,p}(i) = -\mu^{-1}(I_N \otimes P(i)) + \sum_{j=1}^{\mathcal{N}} \pi_{ij}(I_N \otimes P(j))$$

$$\begin{aligned} &2(I_N \otimes P(i)A^k(i)) + 2(C \otimes P(i))\Gamma \\ &+ 2 \sum_{q=1}^F \lambda_{iq} e^T(t)(I_N \otimes P(i)K^P(\rho)) \\ &+ \frac{1}{\gamma^2} (I_N \otimes P(i))(I_N \otimes P(i)) \\ &+ \sum_{j=1}^{\mathcal{N}} \pi_{ij}(I_N \otimes P(j)) + e^T(t)\Omega e(t) < 0. \end{aligned}$$

Then, by Schur complement, one has $\sum_{k=1}^r h_k(\vartheta(t)) \sum_{p=1}^r h_p(\vartheta(t)) \tilde{\mathcal{E}}_{k,p}(i) < 0$ can ensure that $\mathcal{L}V(t, i) < -e^T(t)\Omega e(t) + \gamma^2 \Delta(t)\Delta(t)$.

Thus, one can obtain that

$$\begin{aligned} &\mathcal{E}\{V(\infty, \sigma_\infty) - V(0, \sigma_0)\} \\ &< \mathcal{E}\left\{-\int_0^\infty e^T(\varphi)\Omega e(\varphi)d\varphi + \gamma^2 \int_0^\infty \Delta(\varphi)\Delta(\varphi)d\varphi\right\}, \end{aligned}$$

which means that H_∞ synchronization can be achieved according to Definition 1 and, therefore, completes the proof. $\square$

Remark 4 The above established criteria are in the form of strict linear matrix inequalities, which can be conveniently solved by mathematical softwares. The computational complexity is related to system modes and fuzzy rules, which implies that when i or p increases, the computational complexity for solving the optimization would increase accordingly.

Remark 5 It is noteworthy that the developed synchronization conditions can be solved by linear matrix inequality method and the following theorem is established for controller calculations.

Theorem 2 With given matrix Ω and parameter γ , the drive-response synchronization can be achieved, if there exist mode-dependent matrices $P(i) > 0$, $\tilde{K}^P(\rho)$, $i \in \mathcal{S}$, $\rho \in \mathcal{F}$, matrix Z and parameter $\mu > 0$, such that $\tilde{\mathcal{E}}_{k,k}(i) < 0$ and $\tilde{\mathcal{E}}_{k,p}(i) + \tilde{\mathcal{E}}_{p,k}(i) < 0$ holds for $k, p = 1, 2, \dots, r$, $k < p$, where

and the controller gain can be designed by $K^P(\rho) = Z^{-1}\tilde{K}^P(\rho)$, $\rho \in \mathcal{F}$, $p = 1, 2, \dots, r$. Furthermore, the online NN learning law is updated by

$$(I_N \otimes \dot{W}(t)) = -\Phi(I_N \otimes P(i))e(t)\phi^T(x(t)).$$

Proof By denoting $\tilde{K}^P(\rho) = ZK^P(\rho)$ and employing Lemma 1, the proof can follow directly from Theorem 1. $\square$

Remark 6 It is noticed that the optimized minimization value of synchronization performance γ can be further obtained by solving the following optimization problem:

$$\begin{aligned} & \min \gamma, \\ \text{s.t.} \quad & \tilde{\mathcal{E}}_{k,k}(i) < 0, \\ & \tilde{\mathcal{E}}_{k,p}(i) + \tilde{\mathcal{E}}_{p,k}(i) < 0. \end{aligned}$$

Numerical example

In this section, the effectiveness of the proposed design method is verified via performed simulation results.

Consider two hidden Markov jump CDNs (1) and (2) ($N = 3$) described by following T–S fuzzy model:

$$\begin{aligned} \dot{x}_l(t) = & \sum_{k=1}^2 h_k(\vartheta(t)) [A^k(i)x_l(t) \\ & + f(x_l(t)) + \sum_{m=1}^3 c_{lm}\Gamma x_m(t)], \\ l = & 1, 2, 3, \end{aligned}$$

where

$$\begin{aligned} A^1(1) &= \begin{bmatrix} -1 & 0.1 \\ 0.3 & -4 \end{bmatrix}, \\ A^2(1) &= \begin{bmatrix} -2.1 & 0.2 \\ 0.4 & -1.1 \end{bmatrix}, \\ A^1(2) &= \begin{bmatrix} -2.3 & 0.2 \\ 0.4 & -5.5 \end{bmatrix}, \\ A^2(2) &= \begin{bmatrix} -2.3 & 0 \\ -0.3 & -2.5 \end{bmatrix}, \\ \Gamma &= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \\ C &= \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}, \end{aligned}$$

and $f(x_l(t)) = [0, (x_{l1}^2(t))]^T$. Moreover, the transition rates are supposed to be

$$\Pi = \begin{bmatrix} 1.8 & -1.8 \\ -1.5 & 1.5 \end{bmatrix},$$

and

$$\Lambda = \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{bmatrix}.$$

With given matrix $\Omega = I$ and parameters $\gamma = 10, \mu = 2$, the corresponding controller gains can be obtained by solving

Fig. 1 State trajectories of synchronization error $e_1(t)$

the conditions in Theorem 2 as follows:

$$\begin{aligned} K^1(1) &= \begin{bmatrix} -3.6593 & 0.1722 \\ 0.0550 & -7.6403 \end{bmatrix}, \\ K^2(1) &= \begin{bmatrix} -7.1722 & -0.1349 \\ -0.1776 & -7.3158 \end{bmatrix}, \\ K^1(2) &= \begin{bmatrix} -5.3117 & 0.0345 \\ 0.4456 & 4.8533 \end{bmatrix}, \\ K^2(2) &= \begin{bmatrix} -5.1505 & 0.2441 \\ 0.1717 & -14.1884 \end{bmatrix}. \end{aligned}$$

In the simulation, the parameter of NN is set by

$$\begin{aligned} \phi(x_l(t)) &= \begin{bmatrix} \frac{1}{1+e^{-x_{l1}(t)}} \\ \frac{1}{1+e^{-x_{l2}(t)}} \end{bmatrix}, \\ W(0) &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, l = 1, 2, 3, \end{aligned}$$

and the external disturbances are assumed to be $d(t) = 0.1 \sin(t)$. With random initial values $x_{l1}(t), x_{l2}(t) \in [0, 5]$ and $y_{l1}(t) = y_{l2}(t) = [0, 0]^T$, Figs. 1, 2 and 3 depict the controlled synchronization errors while Fig. 4 shows the system jumping modes. It can be seen that synchronization errors can be well converged by the designed synchronization controller despite of the controller and system mode mismatches. Figure 5 gives the NN learning errors for $f(x_l(t))$, which implies that the developed NN can adaptively approximate the unknown nonlinear dynamics of CDNs with online learning laws. Therefore, it can be observed that our developed synchronization controllers with NN learning strategy can well achieve the drive–response synchronization with desired disturbance attenuation.

Fig. 2 State trajectories of synchronization error $e_2(t)$

Fig. 3 State trajectories of synchronization error $e_3(t)$

Fig. 4 State trajectories of CDN jumping modes

Fig. 5 State trajectories of NN learning errors

Conclusions

This paper is concerned with the synchronization issue of fuzzy hidden Markov jump CDNs under drive–response context. By considering the asynchronous controller modes and unknown function, a novel learning synchronization strategy is proposed where NN is utilized to estimate the unknown function. Sufficient synchronization conditions are first derived by stochastic analysis. The NN learning law and mode-dependent controller gains are further designed accordingly. Simulation results are provided such that the usefulness of our theoretical approach is demonstrated. In our future research, one interesting extension direction would be the cases with type 2 fuzzy modeled complex networks which have more general modeling ability for complex fuzzy systems with more robustness.

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