Triangular interval type-2 fuzzy soft set and its application

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Abstract

Decision-making is an essential task in Science and Engineering. Since most of the real-world problems have uncertainty in nature, making the decision is challengeable one for the decision makers. Soft set has the advantage of free from the deficiency of the parameterization tools of existing theories, namely probability, fuzzy theory and the theory of rough sets. Linguistic terms mean different things to different people, so variability in expert's acceptance degree is possible. Here usage of type-1 fuzzy leads to noisy and uncertain, and the parameters also may be noisy and hence type-2 fuzzy sets may be used to address the mentioned issues. Therefore, a triangular interval type-2 fuzzy soft set has been considered in the present work by combining triangular interval type-2 fuzzy set and soft set. In this paper, a triangular interval type-2 fuzzy soft weighted arithmetic operator (TIT2FSWA) has been proposed with its desired mathematical properties; also applied the proposed methodology in a decision-making problem for profit analysis. Further comparative analysis has been made with the existing methods to show the effectiveness of the proposed method.

Keywords Triangular interval type-2 fuzzy soft set · Aggregation operators · Properties · Decision-making problem

Introduction

Decision-making is a very important process for leadership and management. There are processes and techniques available for decision-making and improve the quality of the decision as well. This process involves numerous information, and the collected information need to be aggregated to find the desired result. Hence, aggregation operators are playing a vital role especially in the decision-making process. Triangular inequalities were protracted by the theoretical

concept of triangular norms which are introduced from the scope of prospect metric [1]. To date, different operators have been used under different set environments. As the real-world problems contain uncertainty we need to use the concept of fuzzy and its extensions. Soft set theory is a general mathematical tool to handle with uncertainty of the real-world problems. Some of the operations on soft sets have been introduced and applied in various fields. The application of fuzzy soft set in decision-making problem has received more attention among the researchers. It has been applied in the field of engineering, economics and all the environmental areas as it deals with uncertainties successfully than other set environments by getting free from difficulties [16, 19].

Most of the objectives in real-time applications are communicated in linguistic terms, but a concise mathematical formula is not applicable in management decisions. If the objective of the decision is correctable, then the constraints of the decision may be flexible. Modeling decision-making process using soft set is more realistic and applied fruitfully to various problems. Representation of a soft set is described by the soft matrix and has many advantages in collecting the information and applying matrices. An algebraic structure of soft set theory has two types of soft sets like a soft set with a fixed set of parameters and with different sets of parameters with new operations, and this may be

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either similar or different [20–27, 52]. Soft set theory normally solves the problem using rough and fuzzy soft sets. Many of the conventional techniques for explicit modeling, logic, and calculations are crisp, precise and acceptable. But the data obtained from real-world problems are not always crisp. Because of this situation, one faces uncertainty in the real problems. The available theories, namely the theory of FSs, IFs, vague sets, interval mathematics, rough sets are playing as the mathematical tool to handle the uncertainties. But still, all of these theories have their complication due to the insufficient parameterization mechanism of the mentioned theories. Hence, Molodtsov introduced the notion of soft theory to clear the impreciseness which is exempted from these kinds of difficulties.

A fuzzy set is the principle idea of fuzzy logic. It contributes a lot in dealing with uncertainties by including some impreciseness corresponding to the membership functions. Fuzzy sets are also called type-1 fuzzy sets (T1FSs) [17]. General additional operations in the interval $[0, 1]$ are t norms and t conorms called triangular norms ever-present in the theory and applications of Type-2 FS (T2FSs). Generalization of T1FS to interval-valued FSs (IVFSs) can be done by generalizing the triangular norms to interval-based cases [18]. Many of the real-time problems are described by the flexibility of the constraint. Particularly in the decision-making process, this kind of flexibility could accompany workable solutions, where the aim and conditions identified by various parties convoluted in the decision-making are determined to one another and delighted to different degrees. This kind of soft constraints can be modeled by fuzzy sets.

Fuzzy set theory contributes to a methodology of representing and handling flexible or soft constraints. In many cases, there is imperfect knowledge of the data and hence, uncertainties should be taken care of in many dimensions and kinds. Choosing membership functions of linguistic terms is the essential one for dealing with uncertainty [5]. In T1FSs, the membership grade of the element is taken from the unit interval $[0, 1]$ and is crisp. T2FSs are generally used for modeling imprecision and uncertainty in an enhanced way with the membership value itself is fuzzy. It is characterized by upper and lower membership functions. The interval between these membership functions represents the footprint of uncertainty, which is used to define the uncertainty level. In T2FSs, two membership functions (MFs) are available, namely primary and secondary membership functions. For all the values of the primary variable x on the universal set X , the function has membership rather than a characteristic value. Secondary membership function provides three-dimensional T2FS, where the third dimension produces a certain degree of freedom to deal with uncertainties. Also, this set can provide more parameters, so that more uncertainties can be handled [8, 9, 11, 42, 67, 68].

The generalized type-2 fuzzy set has more computational complexity for defuzzification and hence, it is necessary to use interval type-2 fuzzy sets. Interval type-2 fuzzy sets have been used in traffic control management, image processing, image extraction, control system, and pattern recognition. At the beginning stage of fuzzy sets, the analysis was made regarding the fact that the membership function of a conventional fuzzy set has no ambiguity connected with it which contradicts the word fuzzy though it implies lots of uncertainty. Hierarchical type-2 fuzzy logic control design has been proposed for autonomous mobile robots that express the efficiency of type-2 fuzzy sets than type-2 fuzzy sets. Due to the computational complexity of general type-2 fuzzy sets, interval type-2 fuzzy sets have been used. All of the results that are needed to implement an interval type-2 fuzzy set can be obtained using type-2 fuzzy mathematics. All of the results that are required to implement an interval type-2 fuzzy logic system can be acquired using type-1 fuzzy set. An interval type-2 fuzzy system has the benefit of direct and indirect approaches. In the direct method, rules are developed through the extraction of knowledge and in the latter case, through historical data. From this concept, it is concluded that interval type-2 fuzzy set is a hybrid approach of these two approaches.

A system with type-2 fuzzy sets allows modeling the uncertainties between the rules and parameters related to data analysis [23, 24, 43]. The concept of interval-valued fuzzy soft set or interval type-2 fuzzy soft set is the combination of interval type-2 fuzzy sets and soft set [25]. Generally, real-world problems are described by huge levels of numerical and linguistic uncertainties. Since the type-1 fuzzy system cannot handle these huge levels of uncertainties completely available in real-world applications, the type-2 fuzzy set has been used to overcome this issue and getting successful results [26].

Many aggregation operators have been introduced for aggregating the information so far. Different types of uncertainties are represented and solved by various methods. In the decision-making process, information is collected from several experts by preparing a survey using linguistic terms where uncertainty naturally exists as different words mean different things. This can be solved by T2 fuzziness, where the FSs have degrees of membership that themselves fuzzy rather than type-1 fuzzy sets [21, 58, 62].

If there are multiple numbers of inputs, then they can be accumulated into interval type-2 input MFs using the methodologies [31, 39]. T2FS is an expansion of T1FSs. The mathematical functions which are applied to combine the information are called aggregation operators. They play a vital aspect in the field of computational intelligence due to their capacity for combining pieces of linguistic information. Fuzzy logic is an engineering mechanism and is explained in a broad sense and is a multi-valued function [33, 34]. Some of the well-known fuzzy numbers are triangular, trapezoidal,

right and left shoulder and piecewise linear functions. Fuzzy logic contributes to compositional calculation of degrees of truth. The set operations namely union, intersection, complement, and implication are working on the case of fuzzy using t-norm and t-conorm [15, 38, 41, 47, 49].

Decision-making process using fuzzy logic is a red-hot area for the research community. It contains the method of choosing the best alternatives from possible alternatives in consideration of many attributes, where the information of the decision is generally with fuzziness is contributed by several experts in the fuzzy setting. Recently, many methods have been established for decision-making problem. Fuzzy soft games for two persons and the work has been extended to n-persons. Also probabilistic equilibrium solution has been provided [44, 57]. The aggregation operators under interval type-2 fuzzy environment are based on the theorem of alpha cut decomposition and derived from the fuzzy extension principle; whereas, the operators under other fuzzy settings are derived by triangular norms. Further decision-making problem has been solved under interval-valued fuzzy soft sets based on the concept of level soft sets. Robust aggregation operators have been proposed with their mathematical properties under an intuitionistic fuzzy soft set environment and applied in decision-making problem.

Uncertainty of the real world with its related problems can be dealt with well using soft set theory [29, 40, 61]. Since there is an increasing difficulty in the real-world environment and the inadequate data of the problem, decision-makers may give their preferences about alternatives in the form of interval-valued fuzzy numbers. Matrices are playing a crucial role in the wide area of engineering and science [35, 37]. The conventional theory of matrices could not solve the problems which contain impreciseness. The matrix portrayal of the fuzzy soft set employed in decision-making problem [30, 50, 56, 60].

Fuzzy soft sets on strong pre-continuity have been introduced in later decay. Some of the comments have been made on interval-valued hesitant fuzzy soft sets and generalized trapezoidal fuzzy soft sets in medical diagnosis. Patients' prioritization has been analyzed under a fuzzy soft environment. New aggregations operators are applied in the decision making problem under triangular interval type-2 fuzzy soft set environment. Also, bipolar FPSS-theory has been introduced and applied in a decision-making problem. Theoretical concepts like convex and concave sets based on fuzzy set and fuzzy soft set environments [69–79]. In type-1 fuzzy logic systems, uncertainty may exist due to the following reasons: words are foul-mouthed different things to different people, knowledge may be drawn out from a group of experts who do not agree cooperatively, measurements may be uncertain, and noisy tuning parameters. Hence, fuzzy set membership functions are uncertain. Conventional fuzzy

sets, where membership functions are crisp, cannot deal with such uncertainties precisely.

Type-2 fuzzy sets are the sets whose membership functions are fuzzy and take values from the real unit interval [0, 1]. It enhances the inference better than conventional fuzzy with increasing uncertainty, imprecision, and obscurity of information and, hence, acquiring more popularity. In many of the real-time applications, evaluation of parameters is uncertain due to the usage of linguistic terms and represented by fuzzy sets instead of exact numerical values, interval numbers, and fuzzy numbers under intuitionistic and interval-valued intuitionistic environment. Also, it is very difficult for the conventional soft set and its extensions to deal with the above issue and hence, it is mandatory that soft set theory entertains the situation in which the evaluation of parameters is fuzzy. As a type-2 fuzzy set can be applied directly to express the fuzziness for the same, we used a triangular interval type-2 fuzzy soft set in this paper.

The rest of the paper is organized as follows. In section “Literature review”, a literature review is presented related to the proposed work. In section “Basic concepts”, some of the basic concepts are given for a better understanding of the work. In section “Proposed methodology”, a new aggregation operator called a triangular interval type-2 fuzzy soft weighted arithmetic operator is proposed with its desired properties. In Sect. Proposed algorithm, a new algorithm is proposed based on the proposed aggregation operator and profit analysis has been examined using the proposed algorithm. In section “Comparative analysis”, comparative analysis has been made with the existing methods to show the potential of the proposed algorithm. In section Conclusion”, a conclusion is given with the future direction.

Literature review

Gupta and Qi [1] introduced the theory of t-norms and fuzzy inference methods. John [2] examined the theory type-2 fuzzy sets and their applications. Karthik and Mendel [3] proposed the operations of type-2 fuzzy sets. Maji et al. [4] introduced a soft set theory. Garibaldi and John [5] introduced the way of choosing membership functions of linguistic terms. Hagra [6] proposed the architecture of a hierarchical type-2 fuzzy logic and applied for autonomous mobile robots. Mendel et al. [7] explained how an interval type-2 fuzzy logic system is made simple. Castillo et al. [8] introduced the theory and applications of type-2 fuzzy logic. Castillo and Melin [9] introduced the interval type-2 fuzzy logic intelligent system.

Coupland and John [10] introduced a fast geometric method for defuzzification under the type-2 fuzzy environment. Zarandi et al. [11] analyzed stock price using the model of type-2 rule-based expert system. Mendel [12] explained

the way of how to learn type-2 fuzzy sets and systems. Yang et al. [13] introduced interval-valued fuzzy soft set. Ahmad and Kharal [14] proposed the concepts of fuzzy soft sets. Rickard et al. [15] proposed type-2 fuzzy trust aggregation. Feng et al. [16] applied interval-valued fuzzy soft sets in a decision-making process. Chetia and Das [17] dealt with medical diagnosis using interval-valued fuzzy soft sets. Cagman and Enginoglu [18] applied the theory of soft matrix in decision-making. Min [19] summarized soft topological spaces. Ali et al. [20] proposed the structure of the soft set and its operations. Qin et al. [21] introduced an adjustable approach and applied in a decision-making process under interval-valued intuitionistic fuzzy soft environment. Alkhazaleh et al. [22] introduced a parameterized interval-valued fuzzy soft set.

Zarandi et al. [23] proposed the hybrid approach for developing an interval type-2 fuzzy logic system. Zarandi et al. [24] predicted for carbon monoxide concentration in megacities using an interval type-2 fuzzy expert system. Min [25] proposed the concept of similarity in soft set theory. Hagrais and Wagner [26] analyzed the widespread of the type-2 fuzzy logic system in real-time applications. Zhang and Zhang [27] solved a decision-making problem using the type-2 fuzzy soft set. Borah et al. [28] examined some of the operations of fuzzy soft sets. Rajarajeswari and Dhanalakshmi [29] applied a similarity measure of fuzzy soft set based on distance in a decision-making problem. Basu et al. [30] determined the best alternative using various types of fuzzy soft set matrices. Wang et al. [31] proposed interval-valued intuitionistic fuzzy aggregation operators. Zhang and Zhang [32] applied a type-2 fuzzy soft set in a decision-making problem. Wang and Liu [27] proposed intuitionistic fuzzy aggregation operators using Einstein operations.

Cagman and Deli [33] introduced products of FP-soft sets and applied in a decision-making problem and the same authors [34] proposed means of FP-soft sets and applied in a decision-making problem. Hernandez et al. [35] analyzed the role of t-norms on type-2 fuzzy sets. Bobillo and Straccia [36] introduced aggregation operators for fuzzy ontologies. Mondal and Roy [37] projected the theory of fuzzy soft matrix and its application in a decision-making process. Xie et al. [38] applied the concept of fuzzy soft sets in medical diagnosis for gray relational analysis. Muthumeenakshi and Muralikrishna [39] examined SFPM analysis using a fuzzy soft set. Liang et al. [40] proposed new aggregation operators under triangular intuitionistic fuzzy numbers and applied in a decision-making problem. Qin et al. [41] proposed novel aggregation operators for triangular interval type-2 fuzzy set using Frank triangular norms. Rajarajeswari and Dhanalakshmi [42] introduced theoretical concepts of interval-valued intuitionistic fuzzy soft matrix. Chen et al. [43] analyzed dynamic decision-making using interval-valued triangular fuzzy soft set. Castillo et al. [44] commented on interval

type-2 fuzzy sets and intuitionistic fuzzy sets. Sola et al. [45] described the relationship between interval type-2 fuzzy sets and generalization of interval-valued fuzzy sets. Alcántud [46] introduced a novel alternative approach and applied in a decision-making problem. Tripathy and Sooraj [47] solved a decision-making problem using interval-valued fuzzy soft sets. Sellappan et al. [48] evaluated the risk priority number in design failure mode and examined the effects using factor analysis. Hernandez et al. [49] proposed the model of type-2 fuzzy sets. Deli and Cagman [50] introduced two person fuzzy soft games and extended to n -person fuzzy soft games. Deli and Cagman [51] contributed a probabilistic equilibrium solution of soft games. Bajestani et al. [52] analyzed the role of interval type-2 fuzzy regression model with crisp inputs and type-2 fuzzy outputs for TAIEX forecasting. Sudharsan and Ezhilmaran [53] proposed a new aggregation operator for interval-valued intuitionistic fuzzy numbers and applied in a decision-making problem. Shenbagavalli et al. [54] determined attribute weights for fuzzy soft set and applied in a decision-making problem. Selvachandran and Sallah [55] introduced interval-valued complex fuzzy soft sets. Senthilkumar [56] solved a decision-making problem using a weighted fuzzy soft matrix. Nagarajan et al. [57] proposed the methodology for image extraction for the DICOM image using the type-2 fuzzy set. Anusuya and Nisha [58] introduced a type-2 fuzzy soft set with distance measure. Lathamaheswari et al. [59] made a review of applications of type-2 fuzzy logic in the field of biomedicine. Dinagar and Rajesh [60] introduced a new approach on the aggregation of interval-valued fuzzy soft matrix and applied in a multi-criteria decision-making problem. Qin et al. [61] proposed a new method for a decision-making problem under interval-valued intuitionistic fuzzy environment. Lathamaheswari et al. [62] made a review of applications of the type-2 fuzzy controller. Nagarajan et al. [63] applied the concept of type-2 fuzzy in image edge detection. Selvachandran and Singh [64] proposed the theoretical concepts of interval-valued complex fuzzy soft set and applied in a real-time application. Mahmooda et al. [65] introduced lattice ordered intuitionistic fuzzy soft sets. Arora and Garg [66] proposed robust aggregation operators for multi-criteria decision-making problem. Alcántud and Torrecillas [67] introduced the intertemporal choice of fuzzy soft sets. Smarandache [68] introduced extended versions of the soft set. Nagarajan et al. [69] analyzed traffic control management using interval type-2 fuzzy sets and interval neutrosophic sets. Nagarajan et al. [70] examined the stability of an intelligent system using a type-2 fuzzy controller. Shi and Fan [71] introduced fuzzy soft sets as L-fuzzy sets. Hassan and Al-Qudah [72] proposed fuzzy parameterized complex multi-fuzzy soft sets. Cakalli et al. [73] defined strong pre-continuity with fuzzy soft sets. Khali et al. [74] appraised interval-valued hesitant fuzzy soft sets and generalized trapezoidal fuzzy soft sets. Khan and Zhu

Fig. 1 Triangular interval type-2 fuzzy set

[75] introduced a new methodology for parameter reduction of fuzzy soft set. Deli and Karaaslan [76] proposed bipolar FPSS-theory and applied in a decision-making problem. Deli [77] introduced convex and concave sets under soft sets and fuzzy soft sets environments. Nagarajan et al. [78] proposed a new aggregation operator using Frank triangular norms for interval-valued triangular fuzzy soft numbers and applied in a decision-making problem. Rahimi [79] applied a fuzzy soft set in patients' prioritization.

From this literature review, to the best of our knowledge, there is no contribution of research for profit analysis in decision-making process using triangular interval type-2 fuzzy soft numbers under triangular interval type-2 fuzzy soft environment. Also, this is the first study to apply the decision-making process for analyzing profit using triangular interval type-2 fuzzy soft numbers.

Basic concepts

In this section, some of the basic concepts are presented for the better understanding of the work.

Definition 3.1 [39] Let $(M_\alpha)_{\alpha \in [0, 1]}$ be a group of aggregation operators (AOs) which is non-decreasing. If A is an AO, then

$$M_A : \bigcup_{n \in N} [0, 1]^n \rightarrow [0, 1].$$

Definition 3.2 [53] Let $\bar{M}$ be the triangular interval type-2 fuzzy set and is defined in Fig. 1.

The membership functions (MFs) are developed using the triangular fuzzy number in interval type-2 fuzzy set which is called TIT2FS. In IT2FS, upper and lower membership

functions are represented by a triangular fuzzy number $\bar{M} = \langle [l_M, \bar{l}_M], c_M, [r_M, \bar{r}_M] \rangle$, and are defined by

$$\text{LMF}_{\bar{M}}(x) = \begin{cases} \frac{x - \bar{l}_M}{c_M - \bar{l}_M}, & \bar{l}_M \leq x < c_M \\ 1, & x = c_M \\ \frac{x - r_M}{c_M - r_M}, & c_M \leq x < r_M \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

$$\text{UMF}_{\bar{M}}(x) = \begin{cases} \frac{x - l_M}{c_M - l_M}, & l_M \leq x < c_M \\ 1, & x = c_M \\ \frac{x - \bar{r}_M}{c_M - \bar{r}_M}, & c_M \leq x < \bar{r}_M \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

where $l_M, \bar{l}_M, c_M, r_M, \bar{r}_M$ are the reference points on TIT2FS satisfying the condition $0 \leq l_M \leq \bar{l}_M \leq c_M \leq r_M \leq \bar{r}_M \leq 1$. If we consider X as a set of real numbers, a TIT2FS in X is called TIT2FN. The FOU is the area between lower and upper membership functions in Fig. 1. If $l_M = \bar{l}_M, r_M = \bar{r}_M$, then $\text{UMF}_{\bar{M}}(x) = \text{LMF}_{\bar{M}}(x)$ for all the values of x in X , then the TIT2FS will become Type-1 case. Here, FOU is the footprint of Uncertainty.

Definition 3.3 [4] Let $\bar{M} = \langle [l_M, \bar{l}_M], c_M, [r_M, \bar{r}_M] \rangle$ be a triangular interval type-2 fuzzy number (TIT2FN) in Fig. 1. The score function for TIT2FN is defined as follows:

$$\text{SF}(\bar{M}) = \left(\frac{l_M + \bar{r}_M}{2} + 1 \right) \frac{l_M + \bar{l}_M + r_M + \bar{r}_M + 4c_M}{8}. \quad (3)$$

Definition 3.4 [39] Let $P_i = \langle [l_{P_i}^-, l_{P_i}^+], m_{P_i}, [r_{P_i}^-, r_{P_i}^+] \rangle$, $i = 1, 2, 3, \dots, n$ be a set of triangular interval type-2 fuzzy soft numbers (TIT2FSNs) and if TIT2FSWA: $\Omega^n \rightarrow \Omega$, then triangular interval type-2 fuzzy soft weighted arithmetic (TIT2FSWA) operator and triangular interval type-2 fuzzy soft weighted geometric (TIT2FSWG) operator are defined by

$$\begin{aligned} &\text{TIT2FSWA}_\xi(P_1, P_2, \dots, P_n) \\ &= \xi_1 \cdot P_1 \oplus \xi_2 \cdot P_2 \oplus \dots \oplus \xi_n \cdot P_n \quad (\text{TIT2FSWA}), \end{aligned} \quad (4)$$

$$\begin{aligned} &\text{TIT2FSWG}_\xi(P_1, P_2, \dots, P_n) \\ &= P_1^{\xi_1} \otimes P_2^{\xi_2} \otimes \dots \otimes P_n^{\xi_n} \quad (\text{TIT2FSWG}), \end{aligned} \quad (5)$$

where $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ is the weight vector and $\xi_i \geq 0$. If $\xi = (1/n, 1/n, \dots, 1/n)^T$, then TIT2FSWA and TIT2FSWG operators become triangular interval type-2 fuzzy soft averaging operators.

Definition 3.5 [41] Let U be the universe of discourse, $\wp(U)$ be the power set of U and P a set of attributes. Then the pair

(F, U) , where $F : P \rightarrow \wp(U)$ is called a soft set over U . In other words, a soft set over U is a parameterized family of subsets of the universe. For $\varepsilon \in P$, $F(\varepsilon)$ is regarded as the set of ε -approximate elements of the soft set (F, U) .

Definition 3.6 [41] Let U be the universe of discourse, $\wp(U)$ be the power set of U and P a set of attributes. Then the pair (F, U) , where $F : P \rightarrow \wp(U)$ is called a soft set over U . A triangular-valued fuzzy soft set is a parameterized family of triangular-valued fuzzy soft of U and therefore its inverse is the set of all triangular-valued fuzzy sets of U i.e., $\wp(U)$. Hence triangular-valued fuzzy soft set is a special case of a fuzzy soft set.

Let $A \subseteq E$, then (F, A) is a triangular interval type-2 fuzzy soft set of U , where F is a mapping given by $F : A \rightarrow \text{TIT2FS}(U)$. Here, $\text{TIT2FS}(U)$ is the set of all triangular interval type-2 fuzzy sets of U and the membership degree that object x holds parameter e , where $x \in U$ and $e \in A$, then $F(e) = \left\{ \left\langle x, \left[\mu_{F(e)}^-(x), \mu_{F(e)}^+(x) \right] \right\rangle / x \in U \right\}$. And for the triangular interval type-2 fuzzy soft set, $F(e) = \left\{ \left\langle x, s_{F(e)}(x) \right\rangle / x \in U \right\}$, where $s_{F(e)}(x)$ is the triangular interval type-2 fuzzy number of x in $F(e)$.

Definition 3.7 [41] If

$$s = \left\{ \begin{array}{l} (a^-, b, c^-) \\ (a^+, b, c^+) \end{array} \right\}, \quad 0 \leq a^- \leq a^+ \leq b \leq c^- \leq c^+,$$

then s is called triangular interval type-2 fuzzy number.

Suppose $s = [(a^-, a^+); b; (c^-, c^+)]$ and $t = [(k^-, k^+); l; (m^-, m^+)]$ are the triangular interval type-2 fuzzy numbers, then

$$s \cap t = [\min(a^-, k^-), \min(a^+, k^+); \min(b, l); \min(c^-, m^-), \min(c^+, m^+)], \quad (6)$$

$$s \cup t = [\max(a^-, k^-), \max(a^+, k^+); \max(b, l); \max(c^-, m^-), \max(c^+, m^+)], \quad (7)$$

$$r s = [(1 - (1 - a^-)^r, 1 - (1 - a^+)^r); 1 - (1 - b)^r; (1 - (1 - c^-)^r, 1 - (1 - c^+)^r)], \quad r > 0, \quad (8)$$

And the complementary set of t is defined as

$$t^c = [(1 - m^+, 1 - m^-); 1 - l; (1 - k^+, 1 - k^-)]. \quad (9)$$

Proposed methodology

In this section, a new aggregation operator namely triangular interval type-2 fuzzy soft weighted arithmetic (TIT2FSWA) operator is proposed with their desired mathematical properties in detail.

Definition 4.1 (Aggregation Operators for Triangular Interval Type-2 Fuzzy Soft Numbers (TIT2FSNs))

Let $P_{k_{ij}} = \left(\left(r_{ij}^-, r_{ij}^+ \right), s_{ij}, \left(t_{ij}^-, t_{ij}^+ \right) \right)$, $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$ be the TIT2FSNs and χ_j, ξ_i be the weight vectors of the parameters k_j 's and experts x_i , respectively, then $\text{TIT2FSWA}(P_{k_{11}}, P_{k_{12}}, \dots, P_{k_{nm}}) = \bigoplus_{j=1}^m \chi_j \left(\bigoplus_{i=1}^n \xi_i F_{k_{ij}} \right)$ and satisfying the following conditions: $\sum_{j=1}^m \chi_j = 1$ and $\sum_{i=1}^n \xi_i = 1$, $\chi_j > 0$, $\xi_i > 0$.

We used the notations for $\prod_j = \wp_j$ and $\prod_i = \mathbb{P}_i$ throughout the paper.

Theorem 4.2 Let $P_{k_{ij}} = \left(\left(r_{ij}^-, r_{ij}^+ \right), s_{ij}, \left(t_{ij}^-, t_{ij}^+ \right) \right)$, $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$ be the set of TIT2FSNs, the aggregated value using TIT2FSWA operator is also TIT2FSN and is given by

$$\begin{aligned} \text{TIT2FSWA}(P_{e_{11}}, P_{e_{12}}, \dots, P_{e_{nm}}) \\ = \left[\left(1 - \left(\wp_j \left(\mathbb{P}_i \left(1 - k_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - k_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \right), \right. \\ \left. 1 - \wp_j \left(\left(\mathbb{P}_i \left(1 - b_{ij} \right)^{\xi_i} \right)^{\chi_j} \right), \right. \\ \left. \left(1 - \left(\wp_j \left(\mathbb{P}_i \left(1 - c_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right), 1 - \left(\wp_j \left(\mathbb{P}_i \left(1 - c_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \right) \right]. \end{aligned} \quad (10)$$

Proof For $n = 1$, $\xi_1 = 1$:

Using the operational law,

$$c \cdot P = \left[[1 - (1 - r^-)^c, 1 - (1 - r^+)^c], 1 - (1 - s)^c, [1 - (1 - t^-)^c, 1 - (1 - t^+)^c] \right],$$

$$\begin{aligned} \text{TIT2FSWA}(P_{k_{11}}, P_{k_{12}}, \dots, P_{k_{1m}}) \\ = \bigoplus_{j=1}^m \chi_j P_{k_{1j}} \\ = \left[\left(1 - \left(\wp_j \left(1 - r_{1j}^- \right)^{\chi_j} \right), 1 - \left(\wp_j \left(1 - r_{1j}^+ \right)^{\chi_j} \right) \right), \right. \\ \left. 1 - \left(\wp_j \left(1 - s_{1j} \right)^{\chi_j} \right), \right. \\ \left. \left(1 - \left(\wp_j \left(1 - t_{1j}^- \right)^{\chi_j} \right), 1 - \left(\wp_j \left(1 - t_{1j}^+ \right)^{\chi_j} \right) \right) \right]. \end{aligned}$$

For $m = 1$, $\xi_1 = 1$

$$\begin{aligned} \text{TIT2FSWA}(P_{k_{11}}, P_{k_{21}}, \dots, P_{k_{n1}}) = \bigoplus_{i=1}^n \xi_i P_{k_{i1}} \\ = \left[\left(1 - \left(\wp_i \left(1 - r_{i1}^- \right)^{\xi_i} \right), 1 - \left(\wp_i \left(1 - r_{i1}^+ \right)^{\xi_i} \right) \right), \right. \\ \left. 1 - \left(\wp_i \left(1 - s_{i1} \right)^{\xi_i} \right), \left(1 - \left(\wp_i \left(1 - t_{i1}^- \right)^{\xi_i} \right), \right. \right. \\ \left. \left. 1 - \left(\wp_i \left(1 - t_{i1}^+ \right)^{\xi_i} \right) \right) \right]. \end{aligned}$$

Hence, the result is true for $n = 1$, $m = 1$.

Consider the result is true for $m = q_1 + 1$, $n = q_2$ and $m = q_1$, $n = q_2 + 1$

Now,

$$\begin{aligned} & \bigoplus_{j=1}^{q_1+1} \chi_j \left(\bigoplus_{i=1}^{q_2} \xi_i P_{kij} \right) \\ &= \left\langle \left[\left(1 - \wp_j \left(\mathbb{P}_i \left(1 - r_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - r_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \right], \right. \\ & \left. \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - b_{ij} \right)^{\xi_i} \right)^{\chi_j} \right), \right. \\ & \left. \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - t_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - t_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \right] \right\rangle. \end{aligned}$$

And

$$\begin{aligned} & \bigoplus_{j=1}^{q_1} \chi_j \left(\bigoplus_{i=1}^{q_2+1} \xi_i P_{kij} \right) \\ &= \left\langle \left[\left(1 - \wp_j \left(\mathbb{P}_i \left(1 - r_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - r_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \right], \right. \\ & \left. \left[\left(1 - \wp_j \left(\mathbb{P}_i \left(1 - t_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - t_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \right] \right\rangle. \end{aligned}$$

Therefore, it holds for $m = q_1 + 1$, $n = q_2 + 1$.

Hence by method of induction, the result is true for all the values of m , $n \geq 1$. Since, $0 \leq r_{ij}^- \leq 1$,

$$\begin{aligned} & \Leftrightarrow 0 \leq \mathbb{P}_i \left(1 - r_{ij}^- \right)^{\xi_i} \leq 1 \Leftrightarrow 0 \leq \wp_j \left(\mathbb{P}_i \left(1 - r_{ij}^- \right)^{\xi_i} \right)^{\psi_j} \leq 1, \\ & \Leftrightarrow 0 \leq \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - r_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right) \leq 1. \end{aligned}$$

Similarly,

$$\begin{aligned} 0 &\leq \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - r_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \leq 1, \\ 0 &\leq \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - s_{ij} \right)^{\xi_i} \right)^{\chi_j} \right) \leq 1, \\ 0 &\leq \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - t_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right) \leq 1, \\ 0 &\leq \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - t_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \leq 1. \end{aligned}$$

For $m = 1$,

$$\begin{aligned} & \text{TIT2FSWA}(P_{k11}, P_{k21}, \dots, P_{kn1}) \\ &= \left\langle \left[\left(1 - \mathbb{P}_i \left(1 - r_{i1}^- \right)^{\xi_i} \right), \left(1 - \mathbb{P}_i \left(1 - r_{i1}^+ \right)^{\xi_i} \right) \right], \right. \\ & \left. \left(1 - \mathbb{P}_i \left(1 - b_{i1} \right)^{\xi_i} \right), \right. \\ & \left. \left(1 - \mathbb{P}_i \left(1 - t_{i1}^- \right)^{\xi_i} \right), \left(1 - \mathbb{P}_i \left(1 - t_{i1}^+ \right)^{\xi_i} \right) \right] \right\rangle. \end{aligned}$$

$\Rightarrow$ the aggregation operator defined under TIT2FS environment is considered as a special case of the proposed operator.

Theorem 4.3 If $P_{kij} = P_k = \langle (r^-, r^+), s, (t^-, t^+) \rangle$, $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$, then

$$\text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{knm}) = P_k. \quad (11)$$

Proof Since all $P_{kij} = P_k = \langle (r^-, r^+), s, (t^-, t^+) \rangle$,

$$\begin{aligned} & \text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{knm}) \\ &= \left\langle \left[\left(1 - \left(\wp_j \left(\mathbb{P}_i \left(1 - k_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i \left(1 - k_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \right] \right], \right. \\ & \left. 1 - \wp_j \left(\mathbb{P}_i \left(1 - b_{ij} \right)^{\xi_i} \right)^{\chi_j}, \right. \\ & \left. \left[1 - \left(\wp_j \left(\mathbb{P}_i \left(1 - c_{ij}^- \right)^{\xi_i} \right)^{\chi_j} \right), 1 - \left(\wp_j \left(\mathbb{P}_i \left(1 - c_{ij}^+ \right)^{\xi_i} \right)^{\chi_j} \right) \right] \right] \right\rangle \\ &= \left\langle \left[1 - \left(\left(1 - r_{ij}^- \right)^{\sum_{i=1}^n \xi_i} \right)^{\sum_{j=1}^m \chi_j}, 1 - \left(\left(1 - r_{ij}^+ \right)^{\sum_{i=1}^n \xi_i} \right)^{\sum_{j=1}^m \chi_j} \right], \right. \\ & \left. 1 - \left(\left(1 - s_{ij} \right)^{\sum_{i=1}^n \xi_i} \right)^{\sum_{j=1}^m \chi_j}, \left[1 - \left(\left(1 - t_{ij}^- \right)^{\sum_{i=1}^n \xi_i} \right)^{\sum_{j=1}^m \chi_j}, \right. \right. \\ & \left. \left. 1 - \left(\left(1 - t_{ij}^+ \right)^{\sum_{i=1}^n \xi_i} \right)^{\sum_{j=1}^m \chi_j} \right] \right] \right\rangle \\ &= \langle [1 - (1 - r^-), 1 - (1 - r^+)], \\ & 1 - (1 - s), [1 - (1 - t^-), 1 - (1 - t^+)] \rangle \\ &= \langle (r^-, r^+), s, (t^-, t^+) \rangle = P_k. \end{aligned}$$

Hence the proof.

Theorem 4.4 If $P_{kij}^- = \langle [\min_j \min_i \{r_{ij}^- \}, \min_j \min_i \{r_{ij}^+ \}], \min_j \min_i \{s_{ij} \}, [\min_j \min_i \{t_{ij}^- \}, \min_j \min_i \{t_{ij}^+ \}] \rangle$ and $P_{kij}^+ = \langle [\max_j \max_i \{r_{ij}^- \}, \max_j \max_i \{r_{ij}^+ \}], \max_j \max_i \{s_{ij} \}, [\max_j \max_i \{t_{ij}^- \}, \max_j \max_i \{t_{ij}^+ \}] \rangle$, then

$$P_{kij}^- \leq \text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{knm}) \leq P_{kij}^+. \quad (12)$$

Proof Since

$$\begin{aligned} & P_{kij} = \langle (r_{ij}^-, r_{ij}^+), s_{ij}, (t_{ij}^-, t_{ij}^+) \rangle, \\ & \min_j \min_i \{r_{ij}^- \} \leq r_{ij}^- \leq \max_j \max_i \{r_{ij}^- \}, \end{aligned}$$

$$\begin{aligned} & \min_j \min_i \{r_{ij}^+ \} \leq r_{ij}^+ \leq \max_j \max_i \{r_{ij}^+ \}, \min_j \min_i \{s_{ij} \} \\ & \leq s_{ij} \leq \max_j \max_i \{s_{ij} \}, \end{aligned}$$

$$\min_j \min_i \{t_{ij}^-\} \leq t_{ij}^- \leq \max_j \max_i \{t_{ij}^-\},$$

$$\min_j \min_i \{t_{ij}^+\} \leq t_{ij}^+ \leq \max_j \max_i \{t_{ij}^+\}.$$

Consider,

$$\begin{aligned} \min_j \min_i \{r_{ij}^-\} &\leq r_{ij}^- \leq \max_j \max_i \{r_{ij}^-\} \\ \Leftrightarrow 1 - \max_j \max_i \{r_{ij}^-\} &\leq 1 - r_{ij}^- \leq 1 - \min_j \min_i \{r_{ij}^-\} \\ \Leftrightarrow 1 - \max_j \max_i \{r_{ij}^-\}^{\xi_i} &\leq (1 - r_{ij}^-)^{\xi_i} \\ &\leq 1 - \min_j \min_i \{r_{ij}^-\}^{\xi_i} \\ \Leftrightarrow \left(1 - \max_j \max_i \{r_{ij}^-\}\right)^{\sum_{j=1}^m \chi_j} &\leq \mathbb{P}_i (1 - r_{ij}^-)^{\xi_i} \\ &\leq \left(1 - \min_j \min_i \{r_{ij}^-\}\right)^{\sum_{j=1}^m \chi_j} \\ \Leftrightarrow \left(1 - \max_j \max_i \{r_{ij}^-\}\right) &\leq \wp_j \left(\mathbb{P}_i (1 - r_{ij}^-)^{\xi_i}\right)^{\chi_j} \\ &\leq \left(1 - \min_j \min_i \{r_{ij}^-\}\right) \\ \Leftrightarrow \min_j \min_i \{r_{ij}^-\} &\leq 1 - \wp_j \left(\mathbb{P}_i (1 - r_{ij}^-)^{\xi_i}\right)^{\chi_j} \\ &\leq \max_j \max_i \{r_{ij}^-\}. \end{aligned}$$

Similarly,

$$\begin{aligned} \Leftrightarrow \min_j \min_i \{r_{ij}^+\} &\leq 1 - \wp_j \left(\mathbb{P}_i (1 - r_{ij}^+)^{\xi_i}\right)^{\chi_j} \leq \max_j \max_i \{r_{ij}^+\} \\ \Leftrightarrow \min_j \min_i \{s_{ij}\} &\leq 1 - \wp_j \left(\mathbb{P}_i (1 - s_{ij})^{\xi_i}\right)^{\chi_j} \leq \max_j \max_i \{s_{ij}\} \\ \Leftrightarrow \min_j \min_i \{t_{ij}^-\} &\leq 1 - \wp_j \left(\mathbb{P}_i (1 - t_{ij}^-)^{\xi_i}\right)^{\chi_j} \leq \max_j \max_i \{t_{ij}^-\} \\ \Leftrightarrow \min_j \min_i \{t_{ij}^+\} &\leq 1 - \wp_j \left(\mathbb{P}_i (1 - t_{ij}^+)^{\xi_i}\right)^{\chi_j} \leq \max_j \max_i \{t_{ij}^+\}. \end{aligned}$$

$$\text{Let } \beta = \text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{knm}) = \left\langle (r_\beta^-, r_\beta^+), s, (t_\beta^-, t_\beta^+) \right\rangle.$$

Then, from the above results,

$$\begin{aligned} \Leftrightarrow \min_j \min_i \{r_{ij}^-\} &\leq r_\beta^- \leq \max_j \max_i \{r_{ij}^-\} \\ \Leftrightarrow \min_j \min_i \{r_{ij}^+\} &\leq r_\beta^+ \leq \max_j \max_i \{r_{ij}^+\} \\ \Leftrightarrow \min_j \min_i \{s_{ij}\} &\leq s_\beta \leq \max_j \max_i \{s_{ij}\} \\ \Leftrightarrow \min_j \min_i \{t_{ij}^-\} &\leq t_\beta^- \leq \max_j \max_i \{t_{ij}^-\} \\ \Leftrightarrow \min_j \min_i \{t_{ij}^+\} &\leq t_\beta^+ \leq \max_j \max_i \{t_{ij}^+\}. \end{aligned}$$

Thus, by the definition of a score function

$$\begin{aligned} S(\beta) &= \left(\frac{r_{ij}^- + t_{ij}^+}{2} + 1 \right) \times \frac{r_{ij}^- + r_{ij}^+ + t_{ij}^- + t_{ij}^+ + 4s_{ij}}{8} \\ &\leq \left(\frac{\max_j \max_i \{r_{ij}^-\} + \max_j \max_i \{t_{ij}^+\}}{2} + 1 \right) \\ &\quad \times \frac{\max_j \max_i \{r_{ij}^-\} + \max_j \max_i \{r_{ij}^+\} + \max_j \max_i \{t_{ij}^-\} + \max_j \max_i \{t_{ij}^+\} + 4 \max_j \max_i \{s_{ij}\}}{8} = S(P_{kij}^+) \\ S(\beta) &= \left(\frac{r_{ij}^- + t_{ij}^+}{2} + 1 \right) \times \frac{r_{ij}^- + r_{ij}^+ + t_{ij}^- + t_{ij}^+ + 4s_{ij}}{8} \geq \left(\frac{\min_j \min_i \{r_{ij}^-\} + \min_j \min_i \{t_{ij}^+\}}{2} + 1 \right) \\ &\quad \times \frac{\min_j \min_i \{r_{ij}^-\} + \min_j \min_i \{r_{ij}^+\} + \min_j \min_i \{t_{ij}^-\} + \min_j \min_i \{t_{ij}^+\} + 4 \min_j \min_i \{s_{ij}\}}{8} \\ &= S(P_{kij}^-). \end{aligned} \tag{13}$$

If $S(P_{kij}) \leq S(P_{kij}^+) \& S(P_{kij}) \geq S(P_{kij}^-)$, then by the comparison law between TIT2FSNs, we get

$$P_{kij}^- \leq \text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{knm}) \leq P_{kij}^+.$$

Hence the theorem.

Theorem 4.5 If $P_k = \langle (r^-, r^+), s, (t^-, t^+) \rangle$ is TIT2FSN, then

$$\begin{aligned} \text{TIT2FSWA}(P_{k11} \oplus P_k, P_{k12} \oplus P_k, \dots, P_{knm} \oplus P_k) \\ = \text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{knm}) \oplus P_k. \end{aligned} \quad (14)$$

Proof Since P_k and P_{kij} are TIT2FSNs,

$$\begin{aligned} P_k \oplus P_{kij} = & \left\langle \left[1 - (1 - r^-)(1 - r_{ij}^-), 1 - (1 - r^+)(1 - r_{ij}^+) \right], \right. \\ & 1 - (1 - s)(1 - r_{ij}), \left[1 - (1 - t^-)(1 - t_{ij}^-), \right. \\ & \left. \left. 1 - (1 - t^+)(1 - t_{ij}^+) \right] \right\rangle, \end{aligned}$$

$$\text{TIT2FSWA}(P_{k11} \oplus P_k, P_{k12} \oplus P_k, \dots, P_{knm} \oplus P_k)$$

$$\begin{aligned} & \bigoplus_{j=1}^m \chi_j \left(\bigoplus_{i=1}^n \xi_i (P_{kij} \oplus P_k) \right) \\ & = \left\langle \left[\left(1 - \wp_j \left(\mathbb{P}_i (1 - r_{ij}^-)^{\xi_i} (1 - r^-)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i (1 - r_{ij}^+)^{\xi_i} (1 - r^+)^{\xi_i} \right)^{\chi_j} \right) \right], \left(1 - \wp_j \left(\mathbb{P}_i (1 - s_{ij})^{\xi_i} (1 - s)^{\xi_i} \right)^{\chi_j} \right), \right. \\ & \quad \left. \left[\left(1 - \wp_j \left(\mathbb{P}_i (1 - t_{ij}^-)^{\xi_i} (1 - t^-)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i (1 - t_{ij}^+)^{\xi_i} (1 - t^+)^{\xi_i} \right)^{\chi_j} \right) \right] \right\rangle \\ & = \left\langle \left[\left(1 - (1 - r^-) \wp_j \left(\mathbb{P}_i (1 - r_{ij}^-)^{\xi_i} \right)^{\chi_j} \right), \left(1 - (1 - r^+) \wp_j \left(\mathbb{P}_i (1 - r_{ij}^+)^{\xi_i} \right)^{\chi_j} \right) \right], \left(1 - (1 - s) \wp_j \left(\mathbb{P}_i (1 - s_{ij})^{\xi_i} \right)^{\chi_j} \right), \right. \\ & \quad \left. \left[\left(1 - (1 - t^-) \wp_j \left(\mathbb{P}_i (1 - t_{ij}^-)^{\xi_i} \right)^{\chi_j} \right), \left(1 - (1 - t^+) \wp_j \left(\mathbb{P}_i (1 - t_{ij}^+)^{\xi_i} \right)^{\chi_j} \right) \right] \right\rangle \\ & = \left\langle \left[\left(1 - \wp_j \left(\mathbb{P}_i (1 - r_{ij}^-)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i (1 - r_{ij}^+)^{\xi_i} \right)^{\chi_j} \right) \right], \left(1 - \wp_j \left(\mathbb{P}_i (1 - s_{ij})^{\xi_i} \right)^{\chi_j} \right), \right. \\ & \quad \left. \left[\left(1 - \wp_j \left(\mathbb{P}_i (1 - t_{ij}^-)^{\xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i (1 - t_{ij}^+)^{\xi_i} \right)^{\chi_j} \right) \right] \right\rangle \\ & \oplus \langle (r^-, r^+), s, (t^-, t^+) \rangle \\ & = \text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{knm}) \oplus P_k. \end{aligned}$$

Hence the theorem.

Theorem 4.6 For any real number $c > 0$, we have

$$\begin{aligned} \text{TIT2FSWA}(c \cdot P_{k11}, c \cdot P_{k12}, \dots, c \cdot P_{knm}) \\ = c \cdot \text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{knm}). \end{aligned} \quad (15)$$

Proof By the operational law of TIT2FSN,

$$\begin{aligned} c \cdot P_{kij} = & \left\langle \left[1 - \left(1 - r_{ij}^- \right)^c, 1 - \left(1 - r_{ij}^+ \right)^c \right], \right. \\ & 1 - \left(1 - s_{ij} \right)^c, \left[1 - \left(1 - t_{ij}^- \right)^c, 1 - \left(1 - t_{ij}^+ \right)^c \right] \rangle, \end{aligned}$$

$$\begin{aligned}
& \text{TIT2FSWA}(c \cdot P_{k11}, c \cdot P_{k12}, \dots, c \cdot P_{k_{nm}}) \\
&= \left\langle \left[\left(1 - \wp_j \left(\mathbb{P}_i (1 - r_{ij}^-)^{c \cdot \xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i (1 - r_{ij}^+)^{c \cdot \xi_i} \right)^{\chi_j} \right) \right], \left(1 - \wp_j \left(\mathbb{P}_i (1 - s_{ij})^{c \cdot \xi_i} \right)^{\chi_j} \right), \right. \\
&\quad \left. \left[\left(1 - \wp_j \left(\mathbb{P}_i (1 - t_{ij}^-)^{c \cdot \xi_i} \right)^{\chi_j} \right), \left(1 - \wp_j \left(\mathbb{P}_i (1 - t_{ij}^+)^{c \cdot \xi_i} \right)^{\chi_j} \right) \right] \right\rangle \\
&= \left\langle \left[\left(1 - \left\{ \wp_j \left(\mathbb{P}_i (1 - r_{ij}^-)^{\xi_i} \right)^{\chi_j} \right\}^c \right), \left(1 - \left\{ \wp_j \left(\mathbb{P}_i (1 - r_{ij}^+)^{\xi_i} \right)^{\chi_j} \right\}^c \right) \right], \left(1 - \left\{ \wp_j \left(\mathbb{P}_i (1 - s_{ij})^{\xi_i} \right)^{\chi_j} \right\}^c \right), \right. \\
&\quad \left. \left[\left(1 - \left\{ \wp_j \left(\mathbb{P}_i (1 - t_{ij}^-)^{\xi_i} \right)^{\chi_j} \right\}^c \right), \left(1 - \left\{ \wp_j \left(\mathbb{P}_i (1 - t_{ij}^+)^{\xi_i} \right)^{\chi_j} \right\}^c \right) \right] \right\rangle \\
&= c \cdot \text{TIT2FSWA}(P_{k11}, P_{k12}, \dots, P_{k_{nm}}).
\end{aligned}$$

Hence the theorem.

Proposed algorithm

In this section, an efficiency of the proposed operators has been proved by the practical example for a decision-making problem. Consider a set of d different alternatives, $V = \{V_1, V_2, \dots, V_d\}$ which will be evaluated by the set of n experts $w_1, w_2, \dots, w_n$ under the parameters $K = \{k_1, k_2, \dots, k_m\}$ and the weight vectors are $\chi = (\chi_1, \chi_2, \dots, \chi_n)^T$ and $\xi = (\xi_1, \xi_2, \dots, \xi_m)^T$, respectively. Also, $\xi_i \in (0, 1]$, $\chi_j \in (0, 1]$, $\sum_{j=1}^m \chi_j = 1$ and $\sum_{i=1}^n \xi_i = 1$. Here, the experts/decision makers give their preferences values in terms of TIT2FSNs $P_{kij} = \left\langle (r_{ij}^-, r_{ij}^+), s_{ij}, (t_{ij}^-, t_{ij}^+) \right\rangle$, $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$. The overall collective decision matrix is represented by $S = (P_{kij})_{n \times m}$. According to the expert's preference values, the aggregated value of TIT2FSN γ_h , ($h = 1, 2, \dots, d$) is calculated for the alternatives using the proposed aggregation operator. The score function of the aggregated TIT2FSN P_{hd} ($h = 1, 2, \dots, d$) is used to rank the alternatives.

The above acknowledged methodology has been summarized as follows.

Step 1: Gather the information associated with all the alternatives under various criteria/parameters in the form of Triangular Interval Type-2 Fuzzy Soft matrix (TIT2FSM) S is defined by,

$$S = \left\langle (r_{ij}^-, r_{ij}^+), s_{ij}, (t_{ij}^-, t_{ij}^+) \right\rangle_{n \times m}.$$

Step 2: Normalize the matrix S by converting the rating values of the cost parameters into benefit type by applying the following formula.

$$R_{ij} = \begin{cases} P_{kij}, & \text{for benefit type parameters} \\ P_{kij}^c, & \text{for cost type parameters,} \end{cases}$$

where $P_{kij}^c = \left\langle (1 - t_{ij}^-, 1 - t_{ij}^+), 1 - s_{ij}, (1 - r_{ij}^-, 1 - r_{ij}^+) \right\rangle$ is the complement of P_{kij} .

If all the parameters are of the same type, then normalization is not necessary.

Step 3: Aggregate the TIT2FSNs P_{kij} for all the alternatives $V = \{V_1, V_2, \dots, V_d\}$ into the collective decision matrix γ_h using proposed TIT2FSWA operator.

Step 4: Calculate the score value of all the alternatives

Step 5: Using score value of the alternatives, select the best one.

Step 6: End.

Application for financial gain analysis using the proposed algorithm

Consider a decision-making problem for profit analysis with different alternatives. The board of four experts d_1, d_2, d_3, d_4 whose weight vector is $\xi = (0.3, 0.2, 0.1, 0.4)^T$ will award their preference values V_1, V_2, V_3, V_4 under some criteria/parameters $K = \{\text{"High production"} (k_1), \text{Number of clients} (k_2), \text{Number of products} (k_3), \text{Duration} (k_4)\}$ with the weight vector $\chi = (0.35, 0.25, 0.25, 0.15)^T$. Using the proposed algorithm, profit analysis is determined as follows.

Step 1: The four experts d_i will measure profit for four alternatives in terms of TIT2FSNs. Parameters and their values of rating are summarized as below (Tables 1, 2, 3, 4).

Step 2: Here all the parameters are in the same type, there is no necessary for normalization.

Step 3: The different opinions of the experts for each alternative are aggregated using Eq. (10).

Table 1 Triangular interval type-2 fuzzy soft matrix for V_1

$\mathbb{U}$	k_1	k_2	k_3	k_4
d_1	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.6, 0.7), 0.8, (0.8, 0.9) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$
d_2	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.6, 0.7) \rangle$	$\langle (0.6, 0.7), 0.8, (0.8, 0.9) \rangle$
d_3	$\langle (0.3, 0.4), 0.5, (0.5, 0.6) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$
d_4	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$

Table 2 Triangular interval type-2 fuzzy soft matrix for V_2

$\mathbb{U}$	k_1	k_2	k_3	k_4
d_1	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$
d_2	$\langle (0.6, 0.7), 0.8, (0.8, 0.9) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$
d_3	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$
d_4	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$

Table 3 Triangular interval type-2 fuzzy soft matrix for V_3

$\mathbb{U}$	k_1	k_2	k_3	k_4
d_1	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$
d_2	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$
d_3	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$	$\langle (0.6, 0.7), 0.8, (0.8, 0.9) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.4, 0.5), 0.6, (0.7, 0.8) \rangle$
d_4	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$

Table 4 Triangular interval type-2 fuzzy soft matrix for V_4

$\mathbb{U}$	k_1	k_2	k_3	k_4
d_1	$\langle (0.3, 0.4), 0.5, (0.5, 0.6) \rangle$	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.7) \rangle$	$\langle (0.2, 0.3), 0.4, (0.5, 0.6) \rangle$
d_2	$\langle (0.5, 0.6), 0.7, (0.8, 0.9) \rangle$	$\langle (0.4, 0.5), 0.6, (0.6, 0.7) \rangle$	$\langle (0.4, 0.5), 0.5, (0.5, 0.6) \rangle$	$\langle (0.3, 0.4), 0.5, (0.6, 0.8) \rangle$
d_3	$\langle (0.4, 0.5), 0.6, (0.6, 0.7) \rangle$	$\langle (0.7, 0.8), 0.8, (0.8, 0.9) \rangle$	$\langle (0.6, 0.7), 0.7, (0.8, 0.9) \rangle$	$\langle (0.1, 0.2), 0.3, (0.4, 0.5) \rangle$
d_4	$\langle (0.5, 0.6), 0.6, (0.7, 0.8) \rangle$	$\langle (0.6, 0.7), 0.7, (0.7, 0.8) \rangle$	$\langle (0.5, 0.6), 0.6, (0.7, 0.8) \rangle$	$\langle (0.3, 0.4), 0.5, (0.5, 0.7) \rangle$

Table 5 Comparative analysis

Existing methods	Score values of the parameters				Ranking
	γ_1	γ_2	γ_3	γ_4	
[27]	0.3219	0.3042	0.2507	0.3460	$SV(\gamma_4) > SV(\gamma_3) > SV(\gamma_2) > SV(\gamma_1)$
[41]	1.2027	1.2719	0.9022	0.8821	$SV(\gamma_2) > SV(\gamma_1) > SV(\gamma_3) > SV(\gamma_4)$
[56]	0.1274	0.04	0.07	0.15	$SV(\gamma_4) > SV(\gamma_1) > SV(\gamma_3) > SV(\gamma_2)$
[66]	0.2338	0.2621	0.1895	0.2428	$SV(\gamma_2) > SV(\gamma_4) > SV(\gamma_1) > SV(\gamma_3)$
[78]	1.0073	0.9597	0.8414	0.9527	$SV(\gamma_1) > SV(\gamma_2) > SV(\gamma_4) > SV(\gamma_3)$

The aggregated value of the four alternatives is,

$$\gamma_1 = \langle (0.4354, 0.5422), 0.6271, (0.6971, 0.8066) \rangle,$$

$$\gamma_2 = \langle (0.4009, 0.5044), 0.6019, (0.6965, 0.8082) \rangle,$$

$$\gamma_3 = \langle (0.3480, 0.4018), 0.5130, (0.6445, 0.7565) \rangle,$$

$$\gamma_4 = \langle (0.4448, 0.5481), 0.6090, (0.6518, 0.7660) \rangle.$$

Step 4: Using Eq. (13), the obtained score values of the four alternatives are

$$SV(\gamma_1) = 1.0126,$$

$$SV(\gamma_2) = 0.9663,$$

$$SV(\gamma_3) = 0.8156,$$

$$SV(\gamma_4) = 0.9726,$$

and the ranking is $SV(\gamma_1) > SV(\gamma_2) > SV(\gamma_4) > SV(\gamma_3)$, the symbol ‘>’ represents, superior to.

Step 5: Hence, high production is the best alternative for getting more profit.

Comparative analysis

This section provides a comparative study of the proposed method with the existing method. A comparison of the results between existing and new techniques is shown in Table 5.

It is observed that high production and duration have more possibilities for profit analysis. These results overlap the proposed result. Hence, the proposed methodology can be utilized using the concepts of the triangular interval type-2 fuzzy soft set to solve the decision-making problem suitably in comparison with the existing methods.

Conclusion

Aggregation operator plays a vital role in decision-making as there is a need for aggregating multi attributes to decide the best one. Though many aggregation operators are available, a new aggregation operator namely a triangular interval type-2 fuzzy soft weighted arithmetic (TIT2FSWA) operator has been proposed with its desired mathematical properties in detail. Since it is the combination of fuzzy soft set and triangular interval type-2 fuzzy set, more uncertainties due to the usage of linguistic terms can be addressed well. Also, profit analysis has been done by choosing the best alternative among the alternatives among the different alternatives using the proposed aggregation operator and it is found that the first alternative high production is the best alternative for getting more profit. The present work may be extended under hyper soft set, whole hyper soft set and plithogenic hyper soft set environments.

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Compliance with Ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval The article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

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References

- Gupta MM, Qi J (1991) Theory of T-norms and fuzzy inference methods. *Fuzzy Sets Syst* 40:431–450
- John R (1998) Type-2 fuzzy sets: an appraisal of theory and applications. *Int J Uncertain Fuzziness Knowl Based Syst* 6(6):563–576
- Karnik NN, Mendel JM (2001) Operations on type-2 fuzzy sets. *Fuzzy Sets Syst* 122:327–348
- Maji PK, Biswas R, Roy AR (2003) Soft set theory. *Comput Math Appl* 45(4–5):555–562
- Garibaldi JM, John RI (2003) Choosing membership functions of linguistic terms. In: *The 12th IEEE international conference on fuzzy systems (FUZZ'2003)*, vol 1, pp 578–583
- Hagras HA (2004) A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots. *IEEE Trans Fuzzy Syst* 12(4):524–539
- Mendel JM, John RI, Liu F (2006) Interval Type-2 fuzzy logic systems made simple. *IEEE Trans Fuzzy Syst* 14(6):808–821
- Castillo O, Melin P, Kacprzyk J and Pedrycz W (2007) Type-2 fuzzy logic: theory and applications. In: *2007 IEEE international conference on granular computing (GRC 2007)*, pp 145–150. <https://doi.org/10.1109/grc.2007.118>
- Castillo O, Melin P (2008) Intelligent systems with interval type-2 fuzzy logic. *Int J Innov Comput Inf Control* 4(4):771–783
- Coupland S, John R (2008) A fast geometric method for defuzzification of type-2 fuzzy sets. *IEEE Trans Fuzzy Syst* 16(4):929–941
- Zarandi MHF, Rezaee B, Turksen IB, Neshat E (2009) A type-2 rule-based expert system model for stock price analysis. *Expert Syst Appl* 36:139–154
- Mendel JM (2009) Type-2 fuzzy sets and systems: how to learn about them. *IEEE SMC eNewslett* 27:1–8
- Yang X, Lin TN, Yang J, Li Y, Yu D (2009) Combination of interval-valued fuzzy set and soft set. *Comput Math Appl* 58(3):521–527
- Ahmad B, Kharal A (2009) On fuzzy soft sets. *Adv Fuzzy Syst* 2009:1–6
- Rickard JT, Hamilton H, Hamilton W (2009) Type-2 fuzzy trust aggregation. In: *IFSA-EUSFLAT 2009 conference*, pp 70–75. ISBN: 978-989-95079-6-8
- Feng F, Li Y, Fotea VL (2010) Application of level soft sets in decision making based on interval-valued fuzzy soft sets. *Comput Math Appl* 60:1756–1767
- Chetia B, Das PK (2010) An application of interval-valued fuzzy soft sets in medical diagnosis. *Int J Contemp Math Sci* 5(38):1887–1894
- Cagman N, Enginoglu S (2010) Soft matrix theory and its decision making. *Comput Math Appl* 59:3308–3314
- Min WK (2011) A note on soft topological spaces. *Comput Math Appl* 62:3524–3528
- Ali MI, Muhammad S, Naz M (2011) Algebraic structures of soft sets associated with new operations. *Comput Math Appl* 61:2647–2654
- Qin H, Ma X, Herawan T, Zain JM (2011) An adjustable approach to interval-valued intuitionistic fuzzy soft sets based decision making. *ACIIDS 2011. LNAI 6592*:80–89
- Alkhazaleh S, Salleh AR, Hassan N (2011) Fuzzy parameterized interval-valued fuzzy soft set. *Appl Math Sci* 5(67):3335–3346
- Zarandi MHF, Sedehizadeh S, Turksen IB (2012) A hybrid approach to develop an interval type-2 fuzzy logic system. In: *2012 Annual meeting of the North American Fuzzy Information Processing Society (NAFIPS)*, pp 1–5. <https://doi.org/10.1109/nafigs.2012.6290971>
- Zarandi MHF, Faraji MR, Karbasian M (2012) Interval type-2 fuzzy expert system for prediction of carbon monoxide concentration in mega-cities. *Appl Soft Comput* 12:291–301
- Min WK (2012) Similarity in soft set theory. *Appl Math Lett* 25:310–314
- Hagras H, Wagner C (2012) Towards the wide spread use of type-2 fuzzy logic systems in real world applications. *IEEE Comput Intell Mag* 7(3):14–24
- Zhang Z, Zhang S (2012) Type-2 fuzzy soft sets and their applications in decision making. *J Appl Math* 2012:1–35
- Borah M, Neog TJ, Sut DK (2012) A study on some operations of fuzzy soft sets. *Int J Mod Eng Res* 2(2):219–225
- Rajarajeswari P, Dhanalakshmi P (2012) An application of similarity measure of fuzzy soft set based on distance. *IOSR J Math* 4(4):27–30
- Basu TM, Mahapatra NK, Mondal SK (2012) Different types of matrices in fuzzy soft set theory and their application in decision making problems. *Int Res Assoc Comput Sci Technol* 2(3):389–398
- Wang W, Liu X, Qin Y (2012) Interval-valued intuitionistic fuzzy aggregation operators. *J Syst Eng Electron* 23(4):574–580
- Wang W, Liu X (2012) Intuitionistic fuzzy information aggregation using Einstein operations. *IEEE Trans Fuzzy Syst* 20(5):923–938
- Cagman N, Deli I (2012) Products of FP-soft sets and their applications. *Hacet J Math Stat* 41(3):365–374
- Cagman N, Deli I (2012) Means of FP-soft sets and their applications. *Hacet J Math Stat* 41(5):615–625
- Hernandez P, Cubillo S, Torres C (2013) About T-norms on type-2 fuzzy sets. *Eur Soc Fuzzy Logic Technol* 2013:171–178
- Bobillo F, Straccia U (2013) Aggregation operators for fuzzy ontologies. *Appl Soft Comput* 13:3816–3830
- Mondal JJ, Roy TK (2013) Theory of fuzzy soft matrix and its multi criteria in decision making based on three basic T-norms operator. *Int J Innov Res Sci Eng Technol* 2(10):5715–5723
- Xie N, Wen G, Li Z (2014) A method for fuzzy soft sets in decision making based on grey relational analysis and D-S theory of evidence: application to medical diagnosis. *Comput Math Methods Med* 2014:1–12
- Muthumeenakshi M, Muralikrishna P (2014) A study of SFPM analysis using fuzzy soft set. *Int J Pure Appl Math* 94(2):207–213
- Liang C, Zhao S, Zhang J (2014) Aggregation operators on triangular intuitionistic fuzzy numbers and its application to multi-criteria decision making problems. *Found Comput Decis Sci* 39:189–208
- Qin J, Liu X (2014) Frank aggregation operators for triangular interval type-2 fuzzy set and its application in multiple attribute group decision making. *J Appl Math* 2014:1–24
- Rajarajeswari P, Dhanalakshmi P (2014) Interval-valued intuitionistic fuzzy soft matrix theory. *Int J Math Arch* 5(1):152–161
- Chen X, Du H, Yang Y (2014) The interval-valued triangular fuzzy soft set and its method of dynamic decision making. *J Appl Math*, 1–12
- Castillo O, Melin P, Tsvetkov R, Atanassov K (2014) Short remark on interval type-2 fuzzy sets and intuitionistic fuzzy sets. In: *18th International conference on intuitionistic fuzzy sets. Notes on Intuitionistic Fuzzy Sets*, vol 20, no 2, pp 1–5
- Sola HB, Fernandez J, Hagras H, Herrera F, Pagola M, Barrenechea E (2015) Interval type-2 fuzzy sets are generalization of interval-valued fuzzy sets: toward a wider view on their relationship. *IEEE Trans Fuzzy Syst* 23(5):1876–1882
- Alcantud JCR (2015) Fuzzy soft set based decision making: a novel alternative approach. In: *9th Conference of the European Society for Fuzzy Logic and Technology*, pp 106–111. <https://doi.org/10.2991/ifsa-eusflat-15.2015.18>
- Tripathy BK, Sooraj TR (2015) On interval valued fuzzy soft sets and their application in group decision making, national confer-

- ence on analysis and applications. In project: decision making and interval valued intuitionistic fuzzy soft set, pp 1–5
48. Sellappan N, Nagarajan D, Palanikumar K (2015) Evaluation of risk priority number (RPN) in design failure modes and effects analysis (DFMEA) using factor analysis. *Int J Appl Eng Res* 10(14):34194–34198
 49. Hernandez P, Cubillo S, Torres-Blanc C (2015) Norms for type-2 fuzzy sets. *IEEE Trans Fuzzy Syst* 23(4):1155–1163
 50. Deli I, Cagman N (2015) Fuzzy soft games. *Filomat* 29(9):1901–1917
 51. Deli I, Cagman N (2016) Probabilistic equilibrium solution of soft games. *J Intell Fuzzy Syst* 30(3):2237–2244
 52. Bajestani NS, Kamyad AV, Zare A (2016) An interval type-2 fuzzy regression model with crisp inputs and type-2 fuzzy outputs for TAIEX forecasting. In: International conference on information and automation, pp 681–685
 53. Sudharsan S, Ezhilmaran D (2016) Weighted arithmetic average operator based on interval-valued intuitionistic fuzzy values and their application to multi criteria decision making for investment. *J Inf Optim Sci* 37(2):247–260
 54. Shenbagavalli R, Balasubramanian G, Solairaju A (2017) Attributes weight determination for fuzzy soft multiple attribute group decision making problems. *Int J Stat Syst* 12(3):517–524
 55. Selvachandran G, Sallah AR (2017) Interval-valued complex fuzzy soft sets. In: Fourth international conference on mathematical sciences. AIP conference proceedings 1830:070009-1-070009-8
 56. Senthilkumar S (2017) Solving a decision making problem using weighted fuzzy soft matrix. *Int J Sci Res Eng Technol* 6(1):12–16
 57. Nagarajan D, Lathamaheswari M, Kavikumar J, Hamzha (2018) A type-2 fuzzy in image extraction for DICOM image. *Int J Adv Comput Sci Appl* 9(12):352–362
 58. Anusuya V, Nisha B (2018) Type-2 fuzzy soft set with distance measure. *Int J Math Trends Technol* 54(2):186–192
 59. Lathamaheswari M, Nagarajan D, Udayakumar A, Kavikumar K (2018) Review on type-2 fuzzy in biomedicine. *Indian J Public Health Res Dev* 9(12):322–326 ([55] made a review of applications of type-2 fuzzy logic in the field of biomedicine)
 60. Dinagar DS, Rajesh A (2018) A new approach on aggregation of interval-valued fuzzy soft matrix and its applications in MCDM. *Int J Math Arch* 9(6):16–22
 61. Qin Y, Liu Y, Liu J (2018) A novel method for interval-value intuitionistic fuzzy multicriteria decision-making problems with immediate probabilities based on OWA distance operators. *Math Probl Eng*, 1–11
 62. Lathamaheswari M, Nagarajan D, Kavikumar K, Phang C (2018) A review on type-2 fuzzy controller on control system. *J Adv Res Dyn Control Syst* 10(11):430–435
 63. Nagarajan D, Lathamaheswari M, Sujatha R, Kavikumar J (2018) Edge detection on DICOM image using triangular norms in type-2 fuzzy. *Int J Adv Comput Sci Appl* 9(11):462–475
 64. Selvachandran G, Singh PK (2018) Interval-valued complex fuzzy soft set and its application. *Int J Uncertain Quantif* 8(2):101–117
 65. Mahmooda T, Alib MI, Malika MA, Ahmeda W (2018) On lattice ordered intuitionistic fuzzy soft sets. *Int J Algebra Stat* 7(1–2):46–61
 66. Arora R, Garg H (2018) Robust aggregation operators for multi-criteria decision-making with intuitionistic fuzzy soft set environment. *Sci Iran* 25(2):931–942
 67. Alcantud JCR, Torrecillas MJM (2018) Intertemporal choice of fuzzy soft sets. *Symmetry* 10(9):371
 68. Smarandache F (2018) Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic Sets Syst* 22:168–170
 69. Nagarajan D, Lathamaheswari M, Broumi S, Kavikumar J (2019) A new perspective on traffic control management using triangular interval type-2 fuzzy sets and interval neutrosophic sets. *Oper Res Perspect*. <https://doi.org/10.1016/j.orp.2019.100099>
 70. Nagarajan D, Kavikumar J, Lathamaheswari M, Broumi S (2019) Intelligent system stability using type-2 fuzzy controller. *Int J Integr Eng* 11(1):270–282
 71. Shi FG, Fan CZ (2019) Fuzzy soft sets as L-fuzzy sets. *J Intell Fuzzy Syst* 37:5061–5066
 72. Hassan N, Al-Qudah Y (2019) Fuzzy parameterized complex multi-fuzzy soft set. *Journal of Physics: conference series*, 14th international symposium on geometric function theory and applications, vol 1212, p 012016
 73. Cakalli H, Acikgoz A, Esenbel F (2019) On strong pre-continuity with fuzzy soft sets. *AIP Conf Proc* 2183(1):030007
 74. Khali AM, Li SG, Li HX, Zhang HD (2019) Comment on “Corrigendum to ‘On interval valued hesitant fuzzy soft sets’” and “Generalized trapezoidal fuzzy soft set and its application in medical diagnosis”. *Math Probl Eng* 2019:1–6
 75. Khan A, Zhu AY (2019) A novel approach to parameter reduction of fuzzy soft set. *IEEE Access*. <https://doi.org/10.1109/access.2019.2940484>
 76. Deli I, Karaaslan F (2019) Bipolar FPSS-theory with applications in decision making. *Afr Mat*. <https://doi.org/10.1007/s13370-019-00738-4>
 77. Deli I (2019) Convex and concave sets based on soft sets and fuzzy soft sets. *J New Theory* 29:101–110
 78. Nagarajan D, Lathamaheswari M, Broumi S, Smarandache F, Kavikumar J (2020) An interval valued triangular fuzzy soft sets and its application in decision making process using new aggregation operator. In: Dash S, Lakshmi C, Das S, Panigrahi M (eds) *Artificial intelligence and evolutionary computations in engineering systems*. *Advances in intelligent systems and computing*, vol 1056, pp 493–503
 79. Rahimi SA (2020) Application of fuzzy soft set in patients’ prioritization. In: *Hospital management and emergency medicine*. <https://doi.org/10.4018/978-1-7998-2451-0.ch019>

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