Generalized dice similarity measures for q-rung orthopair fuzzy sets with applications

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Abstract

Recently, Yager has established that the notion of q-rung orthopair fuzzy set (q-ROFS) is more accomplished than pythagorean fuzzy set (PyFS) and intuitionistic fuzzy set (IFS) to cope with awkward and complicated information in real decision theory. This notion works with yes-, no- and refusal-type fuzzy information. The constraint of q-ROFS is that the sum of n-power of the truth grade and the n-power of the falsity grade is bounded to unit interval. Generalized dice similarity measures are complimentary concepts quantifying the difference and closeness of q-ROFSs. In this paper, we suggested a number of novel dice similarity measures (DSMs) in the surroundings of the q-ROFS, and we examined some prevailing dice similarity measures and their limitations. In addition, we took the DSMs broad view to some globalized dice similarity measures (GDSMs), and we examined some of their particular cases. We employed the novel suggested GDSMs to the best selections of items on identification problems, and we analyzed their acquired consequences. There is a development of novel work in which many situations are evaluated, and from this perspective, the suggested work is changed into already prevailing work. This study also examines the merits of novel DSMs and the limitations for DSMs of IFSs and PyFSs. The comparison between established measures with existing measures is explored and their graphical interpretations are also discussed to show the reliability and effectiveness of the explored measures.

Keywords Pythagorean fuzzy sets · q-Rung orthopair fuzzy sets · Dice similarity measures · Generalized dice similarity measures

Introduction

Multi-attribute decision-making (MADM) problem is a useful technique and important part of modern decision theory. In real decision situation, because the decision-making problems are fuzzy and uncertain, the attribute values are not always shown as real numbers, and some of them are more suitable to be denoted by fuzzy numbers. So, Zadeh [1] established the framework of fuzzy set (FS) for modelling the

ambiguous dealings of real life. A fuzzy set allocates a membership grade of an element of a set in the real unit interval $[0, 1]$. The theory of a FS is extensively utilized in the field of aggregation operators [2], medical diagnosis [3] and MADM [4] problems. But in some situations, the concept of FS is failed, when a decision maker faced the human opinion in the form of yes or no. For coping such kinds of problems, Atanassov [5] established a new notion called intuitionistic fuzzy set (IFS) described by both a membership grade and a non-membership grade. This new notion has a limitation that the sum of both membership and non-membership grades could not go beyond the real unit interval; the modelling of uncertain information of IFSs has greater ability than FSs because of its improved environment. IFS has received extensive attention from a scholar and many researchers utilized it in the environment of aggregation operators [6], medical diagnosis [7] and MADM [8] problems.

When a decision maker gives information in the form of pair $(0.6, 0.5)$ for the grade of membership and non-membership, the condition of IFS cannot solve it. Yager

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Fig. 1 Comparison of spaces of IFSs, PyFSs and q-ROPFSs

[9] sorted out the following problem, where he developed the framework of Pythagorean fuzzy set (PyFS) as a generalization of IFS. The sum of both membership and non-membership grade may go above 1, in accordance with this novel prospective, but their square may not. The notion of PyFS has a number of limitations as in few cases the decision makers could not allocate values of their own choices to both membership and non-membership grades. Therefore, Yager [10] developed the concept of q -rung orthopair fuzzy sets (q-ROPFSs) that makes the field of IFSs and PyFSs to boundlessness. This sort of model is highly constructive in practical scenarios than IFSs and PyFSs. The theory of q-ROPFSs is more or less similar to the framework of intuitionistic fuzzy sets of n -th type (IFSNT) [11]. Davvaz et al. [12] have been put the graphs for IFSNT, as both concepts are the same, they can be considered as q -rung ortho pair fuzzy graph (q-ROPFG). Below in Fig. 1, the evaluation of the spaces of IFS, PyFS and q-ROPFS is illustrated. In this figure, the light turquoise color line shows the boundary space of IFSs and the light blue semi-circle represents the boundary space of PyFS; while the red curve shows the space of q-ROPFS, for $q = 7$.

A number of researchers have worked on similarity measures in different fuzzy algebraic structures and productively used to deal with problems such as pattern identification, medical judgment and multi-attribute decision-making (MADM). The notion of similarity degree between two objects begins to be evaluated in different perspectives, as FSs have been originated. In [13], Wang developed a number of novel similarity measures for FSs. Also, [14] established a novel position of axioms for similarity measures and [15] examined various pattern identification problems which build on fuzzy similarity measures. Several similarity measures of

IFSs were evaluated in [16]; whereas in [17], similarity measures employing Hausdorff distance in the surroundings of IFSs were examined. In [18], Xu together with Chen offered the general idea of the degree of similarity and distance measures between IFSs [19]. suggested a number of similarity and distance functions for hesitant fuzzy sets (HFSs) and examined their implication. Garg [20–25] has evaluated the notion of similarity of objects in IFSs and PyFSs; whereas [26, 27] employed various Pythagorean similarity measures in MADM, pattern identification and medical diagnosis issues. Further works can be found in [28–34].

Dice similarity measures (DSMs) and generalized DSMs (GDSMs) are significant notions in the theory of similarity measures. In 1948, the notion of DSMs was established and it is believed to be the most renowned similarity measure in statistics. In some fuzzy framework, the notion of DSMs has also been elaborated, see, e. g., [14–19]. Different fuzzy structures have considerably soundless literature when it comes to DSMs and GDSMs. As a result, several DSMs and GDSMs for q-ROFSs will be developed in this paper and, subsequently, for IFSs and PyFSs. These DSMs' properties have been examined and also we evaluated their implications in selections of problem identification. For some other works related to IFSs and PyFS, we refer to [7, 35–44].

The q-ROFS is a more powerful and more efficient technique to deal with uncertain and unpredictable information in real decision theory. The IFSs and PyFSs are the special cases of the q-ROFSs due to its constraint, i.e., the sum of n -powers of the membership and non-membership grades has not exceeded the unit interval. But there was still a problem, when a decision maker faced the opinion of the human being in the form of yes, abstinence, and no. The IFSs, PyFSs, and q-ROFSs cannot be described effectively. For handling such kinds of problems, the theory of neutrosophic set (NS) was introduced by Smarandache [45], which is the generalization of IFS to deal with indeterminate and inconsistency information. The NS is characterized by three functions expressing the degree of membership (MS), abstinence and non-membership (NMS). The NS is successfully applied in different areas such as distance measures [46], aggregation operators [47] and MADM [48]. Further, interval NS was pioneered by Wang et al. [49]. Broumi et al. [50] initiated the notion of rough NS. Single-valued NS (SVNS) was found by Wang et al. [51]. The constraint of NS is that the sum of MS, abstinence and NM grades is restricted to $[0 - , 3 +]$, but the constraint of SVNS is less than or equal to 3.

The concept of generalized dice similarity measure is a basic concept in human cognition. Generalized dice similarity measure plays a key role in taxonomy, recognition, case-based reasoning and other areas. There are many aspects of the concept of generalized dice similarity measure that have eluded formalization. According to the pythagorean

fuzzy formulation of a valid, general purpose definition of generalized dice similarity measure is a challenging problem. There does not exist a valid, general purpose definition of generalized dice similarity measure. There do exist many special purpose definitions which have been employed with success in cluster analysis, search, classification, recognition and diagnostics. There are several similarity measures that are proposed and used for varied purposes [14–19]. The similarity measures are classified into three categories: (1) Metric-based measures, (2) Set-theoretic-based measures and (3) Implicators-based measures. While dealing with distance-based similarity measures, examples have been constructed for perceptual similarity where every distance axiom is clearly violated by dissimilarity measures, particularly the triangle inequality [14–19] and consequently, the corresponding similarity measure disobeys transitivity. This model postulates that the perceptual distance satisfies the metric axioms, the empirical validity of which has been experimentally challenged by several researchers, particularly the triangle inequality (for details, see [14, 15]). Similarly, in case of set theoretic similarity measures, it is observed that crisp transitivity is a much stronger condition to be put upon similarity measure. Set theoretic similarity measures are further subdivided in four groups (a) generalized dice similarity measures based on crisp logic, (b) generalized dice similarity measures based on fuzzy logic, (c) generalized dice similarity measures based on intuitionistic fuzzy set, and (d) generalized dice similarity measures based on pythagorean fuzzy set.

In the generalized dice similarity measures and distance measures mentioned above, these distance measures defined based on general distance metrics have weaker discrimination capability. So, they may lead to counter-intuitive results in some special cases. Some derived measures have complex forms but without specific physical meaning. An effective distance measure with relative concise expression and clear physical meaning is desirable from both the mathematical and practical points of view. In this paper, we suggested a number of novel dice similarity measures (DSMs) in the surroundings of the q-ROFS, and we examined some prevailing dice similarity measures and their limitations. In addition, we took the DSMs broad view to some globalized dice similarity measures (GDSMs), and we examined some of their particular cases. We employed the novel suggested GDSMs to the best selections of items on identification problems, and we analyzed their acquired consequences. There is a development of novel work in which many situations are evaluated, and from this perspective, the suggested work is changed into already prevailing work. The current study also examines the merits of novel DSMs and the limitations for DSMs of IFSs and PyFSs. The comparison between established measures with existing measures are explored and their graphical

interpretations are also discussed to show the reliability and effectiveness of the explored measures.

The remainder of this paper is organized as follows. In section two, several foremost definitions of generalized fuzzy structures also with some DSMs are examined. Section three is founded on DSMs of q-ROFSs along with its features. In section four, the notion of GDSMs has been originated for q-ROFSs; also the DSMs for q-ROFSs have been globalized. In this section, the overview of these GDSMs is also presented. In section five, the four newly developed GDSMs have been used for the problem of selection of the best items and also evaluated the acquired consequences. In section six, the relationship of GDSMs for q-ROFSs is compared with the already prevailing structures, and as a result, the GDSMs for IFSs and PyFSs have been established. Consists of merits of suggested DSMs more than the ones that prevail. In the end, we summarized the whole paper and presented some significant implications.

Preliminaries

In this section, we reviewed the basic notions like IFS, PyFS, q-ROFS and their dice similarity measures and weighted dice similarity measures, which will be helpful for the proposed work in the next sections. Throughout this paper, X is represented as the finite universe, and $\eta_{\mathbb{I}}$ and $\gamma_{\mathbb{I}}$ denoted the degree of membership and the degree of non-membership in the finite universe X , respectively.

Definition 1 [5] An IFS is of the form $\mathbb{I} = \{(\eta_{\mathbb{I}}(\mathfrak{N}_i), \gamma_{\mathbb{I}}(\mathfrak{N}_i)) : \mathfrak{N}_i \in X\}$, with a condition $0 \leq \eta_{\mathbb{I}}(\mathfrak{N}_i) + \gamma_{\mathbb{I}}(\mathfrak{N}_i) \leq 1$. Moreover, the term $\mathfrak{r}_{\mathbb{I}} = 1 - (\eta_{\mathbb{I}} + \gamma_{\mathbb{I}})$ is referred to as hesitancy degree and $(\eta_{\mathbb{I}}, \gamma_{\mathbb{I}})$ is considered as intuitionistic fuzzy number (IFN).

Definition 2 [9] A PyFS is of the form $\mathbb{I} = \{(\eta_{\mathbb{I}}(\mathfrak{N}_i), \gamma_{\mathbb{I}}(\mathfrak{N}_i)) : \mathfrak{N}_i \in X\}$, with a condition $0 \leq \eta_{\mathbb{I}}^2(\mathfrak{N}_i) + \gamma_{\mathbb{I}}^2(\mathfrak{N}_i) \leq 1$. Moreover, the term $\mathfrak{r}_{\mathbb{I}} = \sqrt{1 - \eta_{\mathbb{I}}^2(\mathfrak{N}_i) - \gamma_{\mathbb{I}}^2(\mathfrak{N}_i)}$ is referred to as hesitancy degree and $(\eta_{\mathbb{I}}, \gamma_{\mathbb{I}})$ is considered as Pythagorean fuzzy number (PyFN).

Definition 3 [10] A q-ROFS is of the form $\mathbb{I} = \{(\eta_{\mathbb{I}}(\mathfrak{N}_i), \gamma_{\mathbb{I}}(\mathfrak{N}_i)) : \mathfrak{N}_i \in X\}$, with a condition $0 \leq \eta_{\mathbb{I}}^n(\mathfrak{N}_i) + \gamma_{\mathbb{I}}^n(\mathfrak{N}_i) \leq 1$. Moreover, the term $\mathfrak{r}_{\mathbb{I}} = \sqrt[n]{1 - \eta_{\mathbb{I}}^n(\mathfrak{N}_i) - \gamma_{\mathbb{I}}^n(\mathfrak{N}_i)}$ is referred to as hesitancy degree and $(\eta_{\mathbb{I}}, \gamma_{\mathbb{I}})$ is represents q-rung orthopair fuzzy number (q-ROFN).

Definition 4 [35] Let $\tilde{A} = (\eta_{\tilde{A}}, \gamma_{\tilde{A}})$ and $\mathbb{B} = (\eta_{\mathbb{B}}, \gamma_{\mathbb{B}})$ be two IFNs on X . Then, DSM for two IFNs is defined as:

$$D_{\wedge IFS}(\tilde{A}, \mathbb{B}) = \frac{1}{m} \sum_{j=1}^m \frac{2(\eta_{\tilde{A}}(\mathfrak{N}_i)\eta_{\mathbb{B}}(\mathfrak{N}_i) + \gamma_{\tilde{A}}(\mathfrak{N}_i)\gamma_{\mathbb{B}}(\mathfrak{N}_i))}{((\eta_{\tilde{A}}(\mathfrak{N}_i) + \gamma_{\tilde{A}}(\mathfrak{N}_i)) + (\eta_{\mathbb{B}}(\mathfrak{N}_i) + \gamma_{\mathbb{B}}(\mathfrak{N}_i)))}.$$

The above DSM $D_{\wedge IFS}^D(\tilde{A}, \tilde{B})$ satisfies the following properties:

1. $0 \leq D_{\wedge IFS}^D(\tilde{A}, \tilde{B}) \leq 1$,
2. $D_{\wedge IFS}^D(\tilde{A}, \tilde{B}) = D_{\wedge IFS}^D(\tilde{B}, \tilde{A})$,
3. $D_{\wedge IFS}^D(\tilde{A}, \tilde{B}) = 1$ iff $\tilde{A} = \tilde{B}$, i.e., $\eta_{\tilde{A}}(\mathfrak{K}_i) = \eta_{\tilde{B}}(\mathfrak{K}_i)$, $\gamma_{\tilde{A}}(\mathfrak{K}_i) = \gamma_{\tilde{B}}(\mathfrak{K}_i)$.

Definition 5 [35] Let $\tilde{A} = (\eta_{\tilde{A}}, \gamma_{\tilde{A}})$ and $\tilde{B} = (\eta_{\tilde{B}}, \gamma_{\tilde{B}})$ be two IFNs on X . Then, WDSM for two IFNs is defined as:

$$W_{\wedge IFS}^D(\tilde{A}, \tilde{B}) = \sum_{j=1}^m w_j \frac{2(\eta_{\tilde{A}}(\mathfrak{K}_i) \eta_{\tilde{B}}(\mathfrak{K}_i) + \gamma_{\tilde{A}}(\mathfrak{K}_i) \gamma_{\tilde{B}}(\mathfrak{K}_i))}{((\eta_{\tilde{A}}(\mathfrak{K}_i) + \gamma_{\tilde{A}}(\mathfrak{K}_i)) + (\eta_{\tilde{B}}(\mathfrak{K}_i) + \gamma_{\tilde{B}}(\mathfrak{K}_i)))}.$$

The above DSM $D_{\wedge IFS}^D(\tilde{A}, \tilde{B})$ satisfies the following properties:

1. $0 \leq W_{\wedge IFS}^D(\tilde{A}, \tilde{B}) \leq 1$,
2. $W_{\wedge IFS}^D(\tilde{A}, \tilde{B}) = W_{\wedge IFS}^D(\tilde{B}, \tilde{A})$,
3. $W_{\wedge IFS}^D(\tilde{A}, \tilde{B}) = 1$ iff $\tilde{A} = \tilde{B}$, i.e., $\eta_{\tilde{A}}(\mathfrak{K}_i) = \eta_{\tilde{B}}(\mathfrak{K}_i)$, $\gamma_{\tilde{A}}(\mathfrak{K}_i) = \gamma_{\tilde{B}}(\mathfrak{K}_i)$.

Dice similarity measure for q-rung orthopair fuzzy set

The aim of this section is to develop the new ideas of DSM and WDSM for q-ROFSs, which have four types, and to prove the conditions of DSM and WDSM for q-ROFSs. Throughout this paper, $\tilde{A} = (\eta_{\tilde{A}}, \gamma_{\tilde{A}})$ and $\tilde{B} = (\eta_{\tilde{B}}, \gamma_{\tilde{B}})$ will denote two q-ROFNs on the finite general set X .

Definition 6 The DSM for two q-ROFNs is defined as:

$$D_{\wedge q-ROFS}^1(\tilde{A}, \tilde{B}) = \frac{1}{m} \sum_{j=1}^m \frac{2(\eta_{\tilde{A}}^n(\mathfrak{K}_i) \eta_{\tilde{B}}^n(\mathfrak{K}_i) + \gamma_{\tilde{A}}^n(\mathfrak{K}_i) \gamma_{\tilde{B}}^n(\mathfrak{K}_i))}{(\eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \eta_{\tilde{B}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{K}_i))}. \quad (1)$$

The above DSM $D_{\wedge q-ROFS}^1(\tilde{A}, \tilde{B})$ satisfies the following conditions:

1. $0 \leq D_{\wedge q-ROFS}^1(\tilde{A}, \tilde{B}) \leq 1$,
2. $D_{\wedge q-ROFS}^1(\tilde{A}, \tilde{B}) = D_{\wedge q-ROFS}^1(\tilde{B}, \tilde{A})$,

3. $D_{\wedge q-ROFS}^1(\tilde{A}, \tilde{B}) = 1$ iff $\tilde{A} = \tilde{B}$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{K}_i) = \eta_{\tilde{B}}^n(\mathfrak{K}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{K}_i) = \gamma_{\tilde{B}}^n(\mathfrak{K}_i)$.

Proof 1. Consider Eq. (1)

$$D_{\wedge q-ROFS}^1(\tilde{A}, \tilde{B}) = \frac{1}{m} \sum_{j=1}^m \frac{2(\eta_{\tilde{A}}^n(\mathfrak{K}_i) \eta_{\tilde{B}}^n(\mathfrak{K}_i) + \gamma_{\tilde{A}}^n(\mathfrak{K}_i) \gamma_{\tilde{B}}^n(\mathfrak{K}_i))}{(\eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \eta_{\tilde{B}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{K}_i))}.$$

Clearly, the DSM $D_{\wedge q-ROFS}^1(S, T) \geq 0$. Then,

$$\begin{aligned} & (\eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i)) + (\eta_{\tilde{B}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{K}_i)) \\ & \geq 2(\eta_{\tilde{A}}^n(\mathfrak{K}_i) \eta_{\tilde{B}}^n(\mathfrak{K}_i) + \gamma_{\tilde{A}}^n(\mathfrak{K}_i) \gamma_{\tilde{B}}^n(\mathfrak{K}_i)) \end{aligned}$$

$$\text{Hence, } 0 \leq D_{\wedge q-ROFS}^1(\tilde{A}, \tilde{B}) \leq 1.$$

2. Further, on proving the condition two, we have

$$\begin{aligned} D_{\wedge q-ROFS}^1(\tilde{A}, \tilde{B}) &= \frac{1}{m} \sum_{j=1}^m \frac{2(\eta_{\tilde{A}}^n(\mathfrak{K}_i) \eta_{\tilde{B}}^n(\mathfrak{K}_i) + \gamma_{\tilde{A}}^n(\mathfrak{K}_i) \gamma_{\tilde{B}}^n(\mathfrak{K}_i))}{(\eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \eta_{\tilde{B}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{K}_i))} \\ &= \sum_{j=1}^m \frac{2(\eta_{\tilde{B}}^n(\mathfrak{K}_i) \eta_{\tilde{A}}^n(\mathfrak{K}_i) + \gamma_{\tilde{B}}^n(\mathfrak{K}_i) \gamma_{\tilde{A}}^n(\mathfrak{K}_i))}{(\eta_{\tilde{B}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{K}_i) + \eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i))} \\ &= D_{\wedge q-ROFS}^1(\tilde{B}, \tilde{A}). \end{aligned}$$

3. Let $\tilde{A} = \tilde{B}$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{K}_i) = \eta_{\tilde{B}}^n(\mathfrak{K}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{K}_i) = \gamma_{\tilde{B}}^n(\mathfrak{K}_i)$. Then using Eq. (1), we get $D_{\wedge q-ROFS}^1$

$$\begin{aligned} (\tilde{A}, \tilde{B}) &= \frac{1}{m} \sum_{j=1}^m \frac{2(\eta_{\tilde{A}}^n(\mathfrak{K}_i) \eta_{\tilde{B}}^n(\mathfrak{K}_i) + \gamma_{\tilde{A}}^n(\mathfrak{K}_i) \gamma_{\tilde{B}}^n(\mathfrak{K}_i))}{(\eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \eta_{\tilde{B}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{K}_i))} = \\ &= \frac{1}{m} \sum_{j=1}^m \frac{2(\eta_{\tilde{A}}^n(\mathfrak{K}_i) \eta_{\tilde{A}}^n(\mathfrak{K}_i) + \gamma_{\tilde{A}}^n(\mathfrak{K}_i) \gamma_{\tilde{A}}^n(\mathfrak{K}_i))}{(\eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i))} = \\ &= \frac{1}{m} \sum_{j=1}^m \frac{2(\eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i))}{2(\eta_{\tilde{A}}^{2n}(\mathfrak{K}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{K}_i))} = 1. \end{aligned}$$

Hence, this completes the proof.

Definition 7 The WDSM for two q-ROPFNs is defined as:

$$W_{\wedge q-ROFS}^{D^1}(\tilde{A}, B) = \sum_{j=1}^m w_j \frac{2 \left(\eta_{\tilde{A}}^n(\mathfrak{N}_i) \eta_B^n(\mathfrak{N}_i) + \gamma_{\tilde{A}}^n(\mathfrak{N}_i) \gamma_B^n(\mathfrak{N}_i) \right)}{\left(\eta_{\tilde{A}}^{2n}(\mathfrak{N}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{N}_i) + \eta_B^{2n}(\mathfrak{N}_i) + \gamma_B^{2n}(\mathfrak{N}_i) \right)}. \quad (2)$$

The above WDSM $W_{\wedge q-ROFS}^{D^1}(\tilde{A}, B)$ satisfies the following properties:

1. $0 \leq W_{\wedge q-ROFS}^{D^1}(\tilde{A}, B) \leq 1$,
2. $W_{\wedge q-ROFS}^{D^1}(\tilde{A}, B) = W_{\wedge q-ROFS}^{D^1}(B, \tilde{A})$,
3. $W_{\wedge q-ROFS}^{D^1}(\tilde{A}, B) = 1$ iff $\tilde{A} = B$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{N}_i) = \eta_B^n(\mathfrak{N}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{N}_i) = \gamma_B^n(\mathfrak{N}_i)$.

Definition 8 The WDSM for two q-ROPFNs is defined as:

$$W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) = \frac{1}{\tilde{m}} \sum_{i=1}^{\tilde{m}} \frac{2 \left(\eta_{\tilde{A}}^n(\mathfrak{N}_i) \eta_B^n(\mathfrak{N}_i) + \gamma_{\tilde{A}}^n(\mathfrak{N}_i) \gamma_B^n(\mathfrak{N}_i) + r_{\tilde{A}}^n(\mathfrak{N}_i) r_B^n(\mathfrak{N}_i) \right)}{\left(\eta_{\tilde{A}}^{2n}(\mathfrak{N}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{N}_i) + r_{\tilde{A}}^{2n}(\mathfrak{N}_i) + \eta_B^{2n}(\mathfrak{N}_i) + \gamma_B^{2n}(\mathfrak{N}_i) + r_B^{2n}(\mathfrak{N}_i) \right)}. \quad (3)$$

The above DSM $W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B)$ satisfies the following properties:

1. $0 \leq W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) \leq 1$,
2. $W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) = W_{\wedge q-ROFS}^{D^2}(B, \tilde{A})$,
3. $W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) = 1$ iff $\tilde{A} = B$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{N}_i) = \eta_B^n(\mathfrak{N}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{N}_i) = \gamma_B^n(\mathfrak{N}_i)$, $r_{\tilde{A}}^n(\mathfrak{N}_i) = r_B^n(\mathfrak{N}_i)$.

Definition 9 The WDSM for two q-ROPFNs is defined as:

$$W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) = \sum_{i=1}^m w_i \frac{2 \left(\eta_{\tilde{A}}^n(\mathfrak{N}_i) \eta_B^n(\mathfrak{N}_i) + \gamma_{\tilde{A}}^n(\mathfrak{N}_i) \gamma_B^n(\mathfrak{N}_i) + r_{\tilde{A}}^n(\mathfrak{N}_i) r_B^n(\mathfrak{N}_i) \right)}{\left(\sum_{j=1}^m \left(\eta_{\tilde{A}}^{2n}(\mathfrak{N}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{N}_i) + r_{\tilde{A}}^{2n}(\mathfrak{N}_i) \right) + \sum_{j=1}^m \left(\eta_B^{2n}(\mathfrak{N}_i) + \gamma_B^{2n}(\mathfrak{N}_i) + r_B^{2n}(\mathfrak{N}_i) \right) \right)}. \quad (4)$$

The above WDSM $W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B)$ satisfies the following properties:

1. $0 \leq W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) \leq 1$,
2. $W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) = W_{\wedge q-ROFS}^{D^2}(B, \tilde{A})$,

3. $W_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) = 1$ iff $\tilde{A} = B$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{N}_i) = \eta_B^n(\mathfrak{N}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{N}_i) = \gamma_B^n(\mathfrak{N}_i)$.

Definition 10 The DSM for two q-ROPFNs is defined as:

$$W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) = \frac{\sum_{i=1}^m 2 \left(\eta_{\tilde{A}}^n(\mathfrak{N}_i) \eta_B^n(\mathfrak{N}_i) + \gamma_{\tilde{A}}^n(\mathfrak{N}_i) \gamma_B^n(\mathfrak{N}_i) \right)}{\left(\sum_{i=1}^m \left(\eta_{\tilde{A}}^{2n}(\mathfrak{N}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{N}_i) + \eta_B^{2n}(\mathfrak{N}_i) + \gamma_B^{2n}(\mathfrak{N}_i) \right) \right)}. \quad (5)$$

The above DSM $W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B)$ satisfies the following properties:

1. $0 \leq W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) \leq 1$,
2. $W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) = W_{\wedge q-ROFS}^{D^3}(B, \tilde{A})$,
3. $W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) = 1$ iff $\tilde{A} = B$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{N}_i) = \eta_B^n(\mathfrak{N}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{N}_i) = \gamma_B^n(\mathfrak{N}_i)$.

Definition 11 The WDSM for two q-ROPFNs is defined as:

$$W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) = \frac{2 \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^n(\mathfrak{N}_i) \eta_B^n(\mathfrak{N}_i) + \gamma_{\tilde{A}}^n(\mathfrak{N}_i) \gamma_B^n(\mathfrak{N}_i) \right)}{\left(\sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^{2n}(\mathfrak{N}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{N}_i) \right) + \sum_{i=1}^m w_i^2 \left(\eta_B^{2n}(\mathfrak{N}_i) + \gamma_B^{2n}(\mathfrak{N}_i) \right) \right)}. \quad (6)$$

The above WDSM $W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B)$ satisfies the following properties:

1. $0 \leq W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) \leq 1$,
2. $W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) = W_{\wedge q-ROFS}^{D^3}(B, \tilde{A})$,
3. $W_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) = 1$ iff $\tilde{A} = B$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{N}_i) = \eta_B^n(\mathfrak{N}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{N}_i) = \gamma_B^n(\mathfrak{N}_i)$.

Definition 12 The DSM for two q-ROPFNs is defined as:

$$D_{\wedge q\text{-ROFS}}^4(\tilde{A}, \tilde{B}) = \frac{\sum_{i=1}^m 2 \left(\eta_{\tilde{A}}^n(\mathfrak{x}_i) \eta_{\tilde{B}}^n(\mathfrak{x}_i) + \gamma_{\tilde{A}}^n(\mathfrak{x}_i) \gamma_{\tilde{B}}^n(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{A}}^n(\mathfrak{x}_i) \mathfrak{r}_{\tilde{B}}^n(\mathfrak{x}_i) \right)}{\left(\sum_{i=1}^m \left(\eta_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{A}}^{2n}(\mathfrak{x}_i) \right) + \sum_{i=1}^m \left(\eta_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{B}}^{2n}(\mathfrak{x}_i) \right) \right)}. \quad (7)$$

The above DSM $D_{\wedge q\text{-ROFS}}^4(\tilde{A}, \tilde{B})$ satisfies the following properties:

1. $0 \leq D_{\wedge q\text{-ROFS}}^4(\tilde{A}, \tilde{B}) \leq 1$,
2. $D_{\wedge q\text{-ROFS}}^4(\tilde{A}, \tilde{B}) = D_{\wedge q\text{-ROFS}}^4(\tilde{B}, \tilde{A})$,
3. $D_{\wedge q\text{-ROFS}}^4(\tilde{A}, \tilde{B}) = 1$ iff $\tilde{A} = \tilde{B}$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{x}_i) = \eta_{\tilde{B}}^n(\mathfrak{x}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{x}_i) = \gamma_{\tilde{B}}^n(\mathfrak{x}_i)$.

Definition 13 The WDSM for two q-ROPFNs is defined as:

$$W_{\wedge q\text{-ROFS}}^D(\tilde{A}, \tilde{B}) = \frac{2 \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^n(\mathfrak{x}_i) \eta_{\tilde{B}}^n(\mathfrak{x}_i) + \gamma_{\tilde{A}}^n(\mathfrak{x}_i) \gamma_{\tilde{B}}^n(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{A}}^n(\mathfrak{x}_i) \mathfrak{r}_{\tilde{B}}^n(\mathfrak{x}_i) \right)}{\left(\sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{A}}^{2n}(\mathfrak{x}_i) \right) + \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{B}}^{2n}(\mathfrak{x}_i) \right) \right)}. \quad (8)$$

The above WDSM $D_{\wedge q\text{-ROFS}}^D(\tilde{A}, \tilde{B})$ satisfies the following properties:

1. $0 \leq W_{\wedge q\text{-ROFS}}^D(\tilde{A}, \tilde{B}) \leq 1$,
2. $W_{\wedge q\text{-ROFS}}^D(\tilde{A}, \tilde{B}) = W_{\wedge q\text{-ROFS}}^D(\tilde{B}, \tilde{A})$,
3. $W_{\wedge q\text{-ROFS}}^D(\tilde{A}, \tilde{B}) = 1$ iff $\tilde{A} = \tilde{B}$, i.e., $\eta_{\tilde{A}}^n(\mathfrak{x}_i) = \eta_{\tilde{B}}^n(\mathfrak{x}_i)$, $\gamma_{\tilde{A}}^n(\mathfrak{x}_i) = \gamma_{\tilde{B}}^n(\mathfrak{x}_i)$.

Generalized dice similarity measures for q-rung orthopair fuzzy sets

In the previous section, we proposed the new similarity measures, which we will generalize in this section. Also, in this section, $\tilde{A} = (\eta_{\tilde{A}}, \gamma_{\tilde{A}})$ and $\tilde{B} = (\eta_{\tilde{B}}, \gamma_{\tilde{B}})$ represent two q-ROFNs on the finite general set X. Moreover, for the following proposed concepts, the conditions of similarity measures also behave.

Definition 14 The GDSM for two q-ROPFNs is defined as:

$$\hat{G}_{\wedge q\text{-ROFS}}^{D^1}(\tilde{A}, \tilde{B}) = \frac{1}{m} \sum_{j=1}^m \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{x}_i) \eta_{\tilde{B}}^n(\mathfrak{x}_i) + \gamma_{\tilde{A}}^n(\mathfrak{x}_i) \gamma_{\tilde{B}}^n(\mathfrak{x}_i) \right)}{\left(\rho \left(\eta_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{x}_i) \right) + (1 - \rho) \left(\eta_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{x}_i) \right) \right)}. \quad (9)$$

Definition 15 The WGDSM for two q-ROPFNs is defined as:

$$W\hat{G}_{\wedge q\text{-ROFS}}^{D^1}(\tilde{A}, \tilde{B}) = \sum_{i=1}^m w_i \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{x}_i) \eta_{\tilde{B}}^n(\mathfrak{x}_i) + \gamma_{\tilde{A}}^n(\mathfrak{x}_i) \gamma_{\tilde{B}}^n(\mathfrak{x}_i) \right)}{\left(\rho \left(\eta_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{x}_i) \right) + (1 - \rho) \left(\eta_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{x}_i) \right) \right)}. \quad (10)$$

Definition 16 The GDSM for two q-ROPFNs is defined as:

$$\hat{G}_{\wedge q\text{-ROFS}}^{D^2}(\tilde{A}, \tilde{B}) = \frac{\sum_{j=1}^m \left(\eta_{\tilde{A}}^n(\mathfrak{x}_i) \eta_{\tilde{B}}^n(\mathfrak{x}_i) + \gamma_{\tilde{A}}^n(\mathfrak{x}_i) \gamma_{\tilde{B}}^n(\mathfrak{x}_i) \right)}{\left(\rho \sum_{j=1}^m \left(\eta_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{x}_i) \right) + (1 - \rho) \sum_{j=1}^m \left(\eta_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{x}_i) \right) \right)}. \quad (11)$$

Definition 17 The WGDSM for two q-ROPFNs is defined as:

$$W\hat{G}_{\wedge q\text{-ROFS}}^{D^2}(\tilde{A}, \tilde{B}) = \frac{\sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^n(\mathfrak{x}_i) \eta_{\tilde{B}}^n(\mathfrak{x}_i) + \gamma_{\tilde{A}}^n(\mathfrak{x}_i) \gamma_{\tilde{B}}^n(\mathfrak{x}_i) \right)}{\left(\rho \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{x}_i) \right) + (1 - \rho) \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{x}_i) \right) \right)}. \quad (12)$$

Definition 18 The GDSM for two q-ROPFNs is defined as:

$$\hat{G}_{\wedge q\text{-ROFS}}^{D^3}(\tilde{A}, \tilde{B}) = \frac{1}{m} \sum_{j=1}^m \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{x}_i) \eta_{\tilde{B}}^n(\mathfrak{x}_i) + \gamma_{\tilde{A}}^n(\mathfrak{x}_i) \gamma_{\tilde{B}}^n(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{A}}^n(\mathfrak{x}_i) \mathfrak{r}_{\tilde{B}}^n(\mathfrak{x}_i) \right)}{\left(\rho \left(\eta_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{A}}^{2n}(\mathfrak{x}_i) \right) + (1 - \rho) \left(\eta_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{x}_i) + \mathfrak{r}_{\tilde{B}}^{2n}(\mathfrak{x}_i) \right) \right)}. \quad (13)$$

Definition 19 The WGDSM for two q-ROPFNs is defined as:

$$\begin{aligned} & \hat{W}G_{\wedge q-ROFS}^{D^3}(\tilde{A}, \tilde{B}) \\ &= \sum_{j=1}^m w_j \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) + \tau_{\tilde{A}}^n(\mathfrak{s}_i) \tau_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(\rho \left(\sum_{i=1}^n \left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{A}}^{2n}(\mathfrak{s}_i) \right) \right) + (1-\rho) \left(\sum_{i=1}^n \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right) \right)} \end{aligned} \quad (14)$$

Definition 20 The GDSM for two q-ROPFNs is defined as:

$$\begin{aligned} & \hat{G}_{\wedge q-ROFS}^{D^4}(\tilde{A}, \tilde{B}) \\ &= \frac{\sum_{i=1}^m \left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) + \tau_{\tilde{A}}^n(\mathfrak{s}_i) \tau_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(\rho \sum_{i=1}^m \left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{A}}^{2n}(\mathfrak{s}_i) \right) + (1-\rho) \sum_{i=1}^m \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right)} \end{aligned} \quad (15)$$

Definition 21 The WGDSM for two q-ROPFNs is defined as:

$$\begin{aligned} & \hat{W}G_{\wedge q-ROFS}^{D^4}(\tilde{A}, \tilde{B}) \\ &= \frac{\sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) + \tau_{\tilde{A}}^n(\mathfrak{s}_i) \tau_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(\rho \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{A}}^{2n}(\mathfrak{s}_i) \right) + (1-\rho) \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right)} \end{aligned} \quad (16)$$

From the above analysis, it is clear that the explored measures based on q-RODS are more powerful and more superior than existing measures [14–19, 35]. When we choose the value of $w = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^B$, the Eq. (10) is reduced to Eq. (9). When we choose the value of $w = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^B$, the Eq. (12) is reduced to Eq. (11). When we choose the value of $w = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^B$, the Eq. (14) is reduced to Eq. (13). When we choose the value of $w = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^B$, the Eq. (16) is reduced to Eq. (15).

Now, some special cases and their consequences are discussed, in which the generalized DMSs (Eqs. 9–16) will be reduced to simple SMs (Eqs. 1–8).

Case 1: For $\rho = 0.5$, the GDSMs in Eqs. (9–16) are reduced to DMSs in Eqs. (1–8). For convenience, the reduction of Eq. (9) to Eq. (1) is explained below. The remaining reductions can easily be done analogously.

Consider Eq. (9).

$$\hat{G}_{\wedge q-ROFS}^{D^1}(\tilde{A}, \tilde{B}) = \frac{1}{m} \sum_{j=1}^m \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(\rho \left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{A}}^{2n}(\mathfrak{s}_i) \right) + (1-\rho) \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right)}.$$

Setting $\rho = 0.5$, we get

$$\begin{aligned} & \hat{G}_{\wedge q-ROFS}^{D^1}(\tilde{A}, \tilde{B}) \\ &= \frac{1}{m} \sum_{j=1}^m \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(0.5 \left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) \right) + (1-0.5) \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right)} \\ &= \frac{1}{m} \sum_{j=1}^m \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(0.5 \left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) \right) + \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right)} \\ &= \frac{1}{m} \sum_{j=1}^m \frac{2 \left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right)} \\ &= \hat{G}_{\wedge q-ROFS}^{D^1}(\tilde{A}, \tilde{B}) \end{aligned}$$

Case 2: For $\rho = 0, 1$, the GDSMs in Eqs. (9–16) will be reduced to asymmetric or projection SMs. For convenience, consider the following.

$$\begin{aligned} & \hat{G}_{\wedge q-ROFS}^{D^1}(\tilde{A}, \tilde{B}) \\ &= \frac{1}{m} \sum_{j=1}^m \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(\rho \left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{A}}^{2n}(\mathfrak{s}_i) \right) + (1-\rho) \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \tau_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right)} \end{aligned}$$

$$\begin{aligned} & \hat{G}_{\wedge q-ROFS}^{D^1}(\tilde{A}, \tilde{B}) \\ &= \frac{1}{m} \sum_{j=1}^m \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(0 \left(\eta_{\tilde{A}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{A}}^{2n}(\mathfrak{s}_i) \right) + (1-0) \left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right) \right)} \\ &= \frac{1}{m} \sum_{j=1}^m \frac{\left(\eta_{\tilde{A}}^n(\mathfrak{s}_i) \eta_{\tilde{B}}^n(\mathfrak{s}_i) + \gamma_{\tilde{A}}^n(\mathfrak{s}_i) \gamma_{\tilde{B}}^n(\mathfrak{s}_i) \right)}{\left(\eta_{\tilde{B}}^{2n}(\mathfrak{s}_i) + \gamma_{\tilde{B}}^{2n}(\mathfrak{s}_i) \right)}. \end{aligned}$$

Table 1 The q-ROFS data on enterprises of cars

Observation	S_1	S_2	S_3	S_4	T
X_1	(0.72, 0.79)	(0.19, 0.72)	(0.44, 0.67)	(0.23, 0.67)	(0.91, 0.7)
X_2	(0.27, 0.89)	(0.28, 0.77)	(0.61, 0.57)	(0.68, 0.45)	(0.9, 0.67)
X_3	(0.39, 0.77)	(0.82, 0.47)	(0.23, 0.47)	(0.74, 0.77)	(0.89, 0.76)
X_4	(0.29, 0.78)	(0.72, 0.38)	(0.27, 0.73)	(0.63, 0.7)	(0.8, 0.43)
X_5	(0.72, 0.76)	(0.88, 0.77)	(0.65, 0.48)	(0.39, 0.75)	(0.9, 0.7)
X_6	(0.42, 0.57)	(0.38, 0.77)	(0.71, 0.28)	(0.29, 0.63)	(0.72, 0.37)
X_7	(0.9, 0.17)	(0.17, 0.9)	(0.25, 0.84)	(0.9, 0.5)	(0.83, 0.81)

Table 2 The WGDSM of Eq. (10) and ranking order

ρ	$\dot{W}GD_{TSFS}^1(S_1, T)$	$\dot{W}GD_{TSFS}^1(S_2, T)$	$\dot{W}GD_{TSFS}^1(S_3, T)$	$\dot{W}GD_{TSFS}^1(S_4, T)$	Ranking order
0	0.416	0.511	0.319	0.435	$S_2 \geq S_4 \geq S_1 \geq S_3$
0.3	0.455	0.554	0.376	0.487	$S_2 \geq S_4 \geq S_1 \geq S_3$
0.5	0.495	0.598	0.436	0.545	$S_2 \geq S_4 \geq S_1 \geq S_3$
0.7	0.555	0.665	0.546	0.642	$S_2 \geq S_4 \geq S_1 \geq S_3$

Table 3 The WGDSM of Eq. (12) and ranking order

ρ	$\dot{W}GD_{TSFS}^2(S_1, T)$	$\dot{W}GD_{TSFS}^2(S_2, T)$	$\dot{W}GD_{TSFS}^2(S_3, T)$	$\dot{W}GD_{TSFS}^2(S_4, T)$	Ranking order
0	0.528	0.542	0.332	0.535	$S_2 \geq S_4 \geq S_1 \geq S_3$
0.3	0.566	0.595	0.414	0.610	$S_4 \geq S_2 \geq S_1 \geq S_3$
0.5	0.596	0.637	0.497	0.673	$S_4 \geq S_2 \geq S_1 \geq S_3$
0.7	0.628	0.685	0.620	0.750	$S_4 \geq S_2 \geq S_1 \geq S_3$

Applications

Similarity measure is an interesting tool in fuzzy mathematics that have been applied to many practical situations like selection of the best enterprises for business of cars, multi-attribute decision-making, Gangue cars, etc. So far, in every fuzzy algebraic structure, the theory of similarity measures has been used in practical situations, as described in the first section.

Algorithm The purpose of this application is to choose the best company for dealing of business.

Step 1: Collection the unique data of q – ROFNs.

Step 2: Used the definitions of DSM and WDSM.

Step 3: Examine the value of different enterprises.

Step 4: Ranking for all companies.

Step 5: Compared all the results.

Step 6: Choose the best one.

Step 7: End

Selection of the best companies for business of cars

In a selection of the best enterprises for business of cars problem, the aim is to classify unknown enterprises of cars to a class of known enterprises of cars. Such problems often arise

in engineering and other branches of sciences. To understand this process, we proposed the following example.

Requirements 1: Suppose that an unknown enterprise of cars T needs to be classified among some known enterprises of cars like:

S_1 : Pakistani cars company,

S_2 : Chinese cars company,

S_3 : Indian cars company,

S_4 : Malaysia cars company,

and $\{X_1, X_2, X_3, X_4, X_5, X_6, X_7\} = \{fuel, comfort, price, carsquality, carsdesign, warenti, carsensurence\}$

These known enterprises are expressed by the q-ROFNs S_i ($i = 1, 2, 3, 4$) in a feature space, which represents the information about the cars $X = \{X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$ having weight $w = (0.13, 0.16, 0.1, 0.15, 0.18, 0.08, 0.2)^T$, respectively. The information about the known enterprises and unknown enterprise is provided in Table 1.

The similarity measure of data provided in Table 1, for $\rho = 0, 0.3, 0.5$, and 0.7 is computed using WGSM proposed in Eq. (10) and ranked in Table 2.

Table 4 The WGDSM of Eq. (14) and ranking order

ρ	$\hat{W}GD_{TSFS}^3(S_1, T)$	$\hat{W}GD_{TSFS}^3(S_2, T)$	$\hat{W}GD_{TSFS}^3(S_3, T)$	$\hat{W}GD_{TSFS}^3(S_4, T)$	Ranking order
0	0.655	0.689	0.546	0.662	$S_2 \geq S_4 \geq S_1 \geq S_3$
0.3	0.639	0.666	0.517	0.645	$S_2 \geq S_4 \geq S_1 \geq S_3$
0.5	0.633	0.653	0.499	0.636	$S_2 \geq S_4 \geq S_1 \geq S_3$
0.7	0.631	0.641	0.483	0.630	$S_2 \geq S_1 \geq S_4 \geq S_3$

Table 5 The WGDSM of Eq. (16) and ranking order

ρ	$\hat{W}GD_{TSFS}^4(S_1, T)$	$\hat{W}GD_{TSFS}^4(S_2, T)$	$\hat{W}GD_{TSFS}^4(S_3, T)$	$\hat{W}GD_{TSFS}^4(S_4, T)$	Ranking order
0	0.677	0.697	0.565	0.696	$S_2 \geq S_4 \geq S_1 \geq S_3$
0.3	0.657	0.666	0.527	0.671	$S_4 \geq S_2 \geq S_1 \geq S_3$
0.5	0.645	0.647	0.505	0.655	$S_4 \geq S_2 \geq S_1 \geq S_3$
0.7	0.633	0.628	0.484	0.640	$S_4 \geq S_1 \geq S_2 \geq S_3$

The similarity measure of data provided in Table 1, for $\rho = 0, 0.2, 0.5$, and 0.7 is computed using WGSM proposed in Eq. (12) and ranked in Table 3.

The similarity measure of data provided in Table 1, for $\rho = 0, 0.2, 0.5$, and 0.7 is computed using WGSM proposed in Eq. (14) and ranked in Table 4.

The similarity measure of data provided in Table 1, for $\rho = 0, 0.2, 0.5$, and 0.7 is computed using WGSM proposed in Eq. (16) and ranked in Table 5.

From the above discussion, we noticed that the first three proposed concepts decided that S_2 , which is a Chinese cars company is the best enterprises for dealing the cars, but the forth one is S_4 , which is a Malaysia cars company. Thus, we concluded from the overall discussion that the Chinese cars company is the best for dealing with the Asian cars enterprises.

Advantages, comparative analysis, and their graphical interpretations

The purpose of this communication is to explore the novelty of the established measures and also discussed their advantages with the help of some remarks. The comparison between established measures with existing measures is to find the superiority and reliability of the explored measures in this manuscript.

Advantages of the generalized dice similarity measures based on q-ROFS

The current section investigated the relationship between DSMs suggested in this paper and the previously developed DSMs of PyFSs and IFSs. If we take $\rho = 0.5$, then GDSMs

of q-ROFSs will be reduced to DSMs, which was previously presented in Remarks 5–8. Currently, the DSMs of q-ROFSs are more productive and more globalized than the already developed DSMs of other fuzzy algebraic structures, which will be specified by us. According to the subsequent remarks, it is presented that the DSMs of q-ROFSs are modified to existing DSMs under several certain situations.

Remarks 1 The WGDSM of q-ROFSs proposed in Eq. (10) will be reduced to WGDSM of:

1. PyFSs if we take $n = 2$, and then it is given by:

$$\hat{W}G_{\wedge q-ROFS}^{D^1}(\tilde{A}, \tilde{B}) = \sum_{i=1}^m w_i \frac{(\eta_{\tilde{B}}^2(\mathfrak{N}_i) + \gamma_{\tilde{A}}^2(\mathfrak{N}_i) \gamma_{\tilde{B}}^2(\mathfrak{N}_i))}{\left(\frac{\rho(\eta_{\tilde{A}}^4(\mathfrak{N}_i) + \gamma_{\tilde{A}}^4(\mathfrak{N}_i) + (1-\rho)(\eta_{\tilde{B}}^4(\mathfrak{N}_i) + \gamma_{\tilde{B}}^4(\mathfrak{N}_i))}{\rho(\eta_{\tilde{A}}^4(\mathfrak{N}_i) + \gamma_{\tilde{A}}^4(\mathfrak{N}_i) + (1-\rho)(\eta_{\tilde{B}}^4(\mathfrak{N}_i) + \gamma_{\tilde{B}}^4(\mathfrak{N}_i))} \right)}.$$

2. IFSs if we take $n = 1$, and then it is given by:

$$\hat{W}G_{\wedge q-ROFS}^{D^1}(\tilde{A}, \tilde{B}) = \sum_{i=1}^m w_i \frac{(\eta_{\tilde{A}}^2(\mathfrak{N}_i) \eta_{\tilde{B}}^2(\mathfrak{N}_i) + \gamma_{\tilde{A}}^2(\mathfrak{N}_i) \gamma_{\tilde{B}}^2(\mathfrak{N}_i))}{\left(\frac{\rho(\eta_{\tilde{A}}^2(\mathfrak{N}_i) + \gamma_{\tilde{A}}^2(\mathfrak{N}_i) + (1-\rho)(\eta_{\tilde{B}}^2(\mathfrak{N}_i) + \gamma_{\tilde{B}}^2(\mathfrak{N}_i))}{\rho(\eta_{\tilde{A}}^2(\mathfrak{N}_i) + \gamma_{\tilde{A}}^2(\mathfrak{N}_i) + (1-\rho)(\eta_{\tilde{B}}^2(\mathfrak{N}_i) + \gamma_{\tilde{B}}^2(\mathfrak{N}_i))} \right)}.$$

Remarks 2 The WGDSM of q-ROFSs proposed in Eq. (12) will be reduced to WGDSM of:

1. PyFSs if we take $n = 2$, and then it is given by:

$$W\hat{G}_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) = \frac{\sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^2(s_i) \eta_B^2(s_i) + \gamma_{\tilde{A}}^2(s_i) \gamma_B^2(s_i) \right)}{\left(\rho \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^4(s_i) + \gamma_{\tilde{A}}^4(s_i) \right) + (1-\rho) \sum_{i=1}^m w_i^2 \left(\eta_B^4(s_i) + \gamma_B^4(s_i) \right) \right)}.$$

2. IFSs if we take $n = 1$, and then it is given by:

$$W\hat{G}_{\wedge q-ROFS}^{D^2}(\tilde{A}, B) = \frac{\sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^2(s_i) \eta_B^2(s_i) + \gamma_{\tilde{A}}^2(s_i) \gamma_B^2(s_i) \right)}{\left(\rho \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^2(s_i) + \gamma_{\tilde{A}}^2(s_i) \right) + (1-\rho) \sum_{i=1}^m w_i^2 \left(\eta_B^2(s_i) + \gamma_B^2(s_i) \right) \right)}.$$

Remarks 3 The WGDSM of q-ROFSs proposed in Eq. (14) will be reduced to WGDSM of:

1. PyFSs if we take $n = 2$, and then it is given by:

$$W\hat{G}_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) = \sum_{j=1}^m w_j \frac{\left(\eta_{\tilde{A}}^2(s_i) \eta_B^2(s_i) + \gamma_{\tilde{A}}^2(s_i) \gamma_B^2(s_i) + r_{\tilde{A}}^2(s_i) r_B^2(s_i) \right)}{\left(\rho \left(\sum_{i=1}^n \left(\eta_{\tilde{A}}^4(s_i) + \gamma_{\tilde{A}}^4(s_i) + r_{\tilde{A}}^4(s_i) \right) \right) + (1-\rho) \left(\sum_{i=1}^n \left(\eta_B^4(s_i) + \gamma_B^4(s_i) + r_B^4(s_i) \right) \right) \right)}.$$

2. IFSs if we take $n = 1$, and then it is given by:

$$W\hat{G}_{\wedge q-ROFS}^{D^3}(\tilde{A}, B) = \sum_{j=1}^m w_j \frac{\left(\eta_{\tilde{A}}^2(s_i) \eta_B^2(s_i) + \gamma_{\tilde{A}}^2(s_i) \gamma_B^2(s_i) + r_{\tilde{A}}^2(s_i) r_B^2(s_i) \right)}{\left(\rho \left(\sum_{i=1}^n \left(\eta_{\tilde{A}}^2(s_i) + \gamma_{\tilde{A}}^2(s_i) + r_{\tilde{A}}^2(s_i) \right) \right) + (1-\rho) \left(\sum_{i=1}^n \left(\eta_B^2(s_i) + \gamma_B^2(s_i) + r_B^2(s_i) \right) \right) \right)}.$$

Remarks 4 The WGDSM of q-ROFSs proposed in Eq. (16) will be reduced to WGDSM of:

1. PyFSs if we take $n = 2$, and then it is given by:

$$W\hat{G}_{\wedge q-ROFS}^{D^4}(\tilde{A}, B) = \frac{\sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^2(s_i) \eta_B^2(s_i) + \gamma_{\tilde{A}}^2(s_i) \gamma_B^2(s_i) + r_{\tilde{A}}^2(s_i) r_B^2(s_i) \right)}{\left(\rho \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^4(s_i) + \gamma_{\tilde{A}}^4(s_i) + r_{\tilde{A}}^4(s_i) \right) + (1-\rho) \sum_{i=1}^m w_i^2 \left(\eta_B^4(s_i) + \gamma_B^4(s_i) + r_B^4(s_i) \right) \right)}.$$

2. IFSs if we take $n = 1$, and then it is given by:

$$W\hat{G}_{\wedge q-ROFS}^{D^4}(\tilde{A}, B) = \frac{\sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^2(s_i) \eta_B^2(s_i) + \gamma_{\tilde{A}}^2(s_i) \gamma_B^2(s_i) + r_{\tilde{A}}^2(s_i) r_B^2(s_i) \right)}{\left(\rho \sum_{i=1}^m w_i^2 \left(\eta_{\tilde{A}}^2(s_i) + \gamma_{\tilde{A}}^2(s_i) + r_{\tilde{A}}^2(s_i) \right) + (1-\rho) \sum_{i=1}^m w_i^2 \left(\eta_B^2(s_i) + \gamma_B^2(s_i) + r_B^2(s_i) \right) \right)}.$$

All these comparisons clearly explained the generalization of GDSMs of q-ROFSs over the existing GDSMs. Further, the DSMs of q-ROFSs can be applied in problems where the information is provided in IFSs and PyFSs, but on the other hand, the DSMs of existing structures could not be applied in problems of q-ROFSs.

Comparative analysis of the generalized dice similarity measures based on q-ROFS

One of the limitations in the existing framework of DSMs is that the following notions have restricted structures that cannot analyze the data, such as q-ROFSs, which is available in more globalized frameworks. Until now, such sort of restriction is elaborated in detail in this paper. Now, with some examples, we show that the previously published papers failed to manage general information of q-ROFSs, while the suggested GDSMs of q-ROFSs can deal with the data available in the offered structures.

Table 6 reflects the information available in its entirety in the form of PyFNs for known enterprises and unknown enterprises that are listed in Table 1.

Using the approach of Example 1, the similarity degree of every S_i with T can be analyzed here, and for $n = 2$ this kind of information can be treated as q-ROFSs.

Similarly, if we look at the information provided in Table 7, all the values are IFNs which can be considered as q-ROFSs for $n = 1$, and consequently can be solved using the approach of Example 1 such sort of information could be treated as q-ROFSs.

On the other hand, if we consider Table 1 discussed in Example 1, the information is in the form of q-ROFNs for $n = 5$, which cannot be processed by the existing tools. All this clearly indicates the advantages of using GDSMs of q-ROFSs instead of using existing tools. Keeping the advantages of the explored measures in manuscript, we compared the established measures in this manuscript with the following measures whose details are given below by considering the information which are given in Tables 1, 6, and 7. We also discussed the graphical interpretation of the explored measures and existing measures. The explored measures are compared with the following existing measures, i.e., Liang

Table 6 The PyFS data on enterprises of cars

Observation	S_1	S_2	S_3	S_4	T
X_1	(0.72, 0.27)	(0.19, 0.72)	(0.44, 0.6)	(0.2, 0.67)	(0.91, 0.12)
X_2	(0.27, 0.81)	(0.28, 0.77)	(0.61, 0.57)	(0.6, 0.45)	(0.6, 0.67)
X_3	(0.39, 0.77)	(0.62, 0.47)	(0.23, 0.47)	(0.4, 0.77)	(0.32, 0.76)
X_4	(0.29, 0.78)	(0.64, 0.38)	(0.27, 0.73)	(0.39, 0.7)	(0.8, 0.3)
X_5	(0.72, 0.26)	(0.28, 0.77)	(0.5, 0.48)	(0.6, 0.5)	(0.9, 0.11)
X_6	(0.42, 0.57)	(0.8, 0.4)	(0.71, 0.28)	(0.29, 0.63)	(0.72, 0.30)
X_7	(0.6, 0.27)	(0.17, 0.82)	(0.25, 0.84)	(0.8, 0.5)	(0.23, 0.81)

Table 7 The IFS data on enterprises of cars

Observation	S_1	S_2	S_3	S_4	T
X_1	(0.2, 0.79)	(0.19, 0.62)	(0.2, 0.67)	(0.23, 0.47)	(0.91, 0.02)
X_2	(0.2, 0.8)	(0.28, 0.6)	(0.4, 0.57)	(0.68, 0.24)	(0.9, 0.04)
X_3	(0.29, 0.7)	(0.82, 0.1)	(0.23, 0.47)	(0.74, 0.13)	(0.89, 0.01)
X_4	(0.29, 0.6)	(0.72, 0.2)	(0.27, 0.53)	(0.63, 0.23)	(0.8, 0.13)
X_5	(0.2, 0.6)	(0.88, 0.11)	(0.65, 0.18)	(0.39, 0.55)	(0.9, 0.05)
X_6	(0.42, 0.5)	(0.38, 0.55)	(0.71, 0.12)	(0.29, 0.53)	(0.72, 0.13)
X_7	(0.7, 0.17)	(0.17, 0.7)	(0.25, 0.6)	(0.9, 0.0)	(0.83, 0.08)

Table 8 Comparison table for choosing the information of Table 1

Methods	Similarity values	Ranking results
Liang and Shi [28]	Cannot be classified	Cannot be classified
Wei and Wei [27]	Cannot be classified	Cannot be classified
Zeng et al. [52]	Cannot be classified	Cannot be classified
Proposed measure for $\rho = 0.5$	$\hat{W}GD_{TSFS}^4(S_1, T) = 0.645$, $\hat{W}GD_{TSFS}^4(S_2, T) = 0.647$, $\hat{W}GD_{TSFS}^4(S_3, T) = 0.505$, $\hat{W}GD_{TSFS}^4(S_4, T) = 0.655$	$S_4 \geq S_2 \geq S_1 \geq S_3$

and Shi [28] explored similarity measures based on IFS. Wei and Wei [27] explored similarity measures for PyFS. Some similarity measures based on PyFS were also explored by Zeng et al. [52] for $\rho = 0.5$. First, we considered example 1, and resolved using the established measures and existing measures, the explanation of which is given in Table 8.

The graphical representation of the information in Table 8 is discussed in Fig. 2.

Next, we considered the information which are given in Table 6, and resolved using the established measures and existing measures, the explanation of which is given in Table 9.

The graphical representation of the information in Table 9 is discussed in Fig. 3.

Lastly, we considered the information which are given in Table 7, and resolved using the established measures and existing measures, the explanation of which is given in Table 10.

The graphical representation of the information in Table 10 is discussed in Fig. 4.

The graphical interpretation of the information which are given in Tables 8, 9, and 10, are discussed in Figs. 2, 3, and

4, to show the reliability and effectiveness of the explored measures.

It is clear that the established measures in this manuscript are more powerful and more general than the existing measures due to q-ROFS, because the constraint of q-ROFS is that the sum of n-power of membership grade and the n-power of the non-membership grade is retracted to unit interval. Hence, the introduced measures in this article are more superior and more accurate than existing measures.

Conclusion

The notion of q-rung orthopair fuzzy set (q-ROFS) is more accomplished than pythagorean fuzzy set (PyFS) and intuitionistic fuzzy set (IFS) to cope with awkward and complicated information in real decision theory. This notion works with yes-, no- and refusal-type fuzzy information. The constraint of q-ROFS is that the sum of n-power of the truth grade and the n-power of the falsity grade is bounded to unit interval. Generalized dice similarity measures are complementary concepts quantifying the difference and closeness

Fig. 2 Comparison between established measures with existing measures

Fig. 3 Comparison between established measures with existing measures

Table 9 Comparison table for choosing the information of Table 6

Methods	Similarity values	Ranking results
Liang and Shi [28]	Cannot be classified	Cannot be classified
Wei and Wei [27]	$\hat{W}GD_{TSFS}^4(S_1, T) = 0.524$, $\hat{W}GD_{TSFS}^4(S_2, T) = 0.546$, $\hat{W}GD_{TSFS}^4(S_3, T) = 0.405$, $\hat{W}GD_{TSFS}^4(S_4, T) = 0.721$	$S_4 \geq S_2 \geq S_1 \geq S_3$
Zeng et al. [52]	$\hat{W}GD_{TSFS}^4(S_1, T) = 0.521$, $\hat{W}GD_{TSFS}^4(S_2, T) = 0.541$, $\hat{W}GD_{TSFS}^4(S_3, T) = 0.399$, $\hat{W}GD_{TSFS}^4(S_4, T) = 0.710$	$S_4 \geq S_2 \geq S_1 \geq S_3$
Proposed measure for $\rho = 0.5$	$\hat{W}GD_{TSFS}^4(S_1, T) = 0.742$, $\hat{W}GD_{TSFS}^4(S_2, T) = 0.749$, $\hat{W}GD_{TSFS}^4(S_3, T) = 0.701$, $\hat{W}GD_{TSFS}^4(S_4, T) = 0.822$	$S_4 \geq S_2 \geq S_1 \geq S_3$

of q-ROFSs. The generalized dice similarity measures have been examined in the current paper. Along with the DSMs of PFSs, it was noted that these tools could not analyze the data available in the form of q-ROFNs, and other tools also have several limitations. As a result, various novel similarity measures for q-ROFSs have been established which generalize each of the available tools of similarity measures. Basically, the purpose of this article is to establish four kinds of DSMs and GDSMs and their properties are also analyzed. With various explanations, we proved that the suggested approach is more effective than the available tools. The renowned problem of pattern identification is exposed by the proposed

approach and their consequences are analyzed. By employing several valuable replacement during the relative study, such sort of DSMs constructions has been established for PyFSs and IFSs, respectively. The advantages of the new approach were analyzed, as well as the limitations of the available ones.

In the future, we will try to utilize these similarity measures in the environment of complex pythagorean fuzzy sets [53], complex q-rung orthopair fuzzy sets [54, 55], Picture hesitant fuzzy sets [56, 57], and T-spherical fuzzy sets [58].

Table 10 Comparison table for choosing the information of Table 7

Methods	Similarity values	Ranking results
Liang and Shi [28]	$\hat{W}GD_{TSFS}^4(S_1, T) = 0.328$, $\hat{W}GD_{TSFS}^4(S_2, T) = 0.384$, $\hat{W}GD_{TSFS}^4(S_3, T) = 0.294$, $\hat{W}GD_{TSFS}^4(S_4, T) = 0.412$	$S_4 \geq S_2 \geq S_1 \geq S_3$
Wei and Wei [27]	$\hat{W}GD_{TSFS}^4(S_1, T) = 0.412$, $\hat{W}GD_{TSFS}^4(S_2, T) = 0.438$, $\hat{W}GD_{TSFS}^4(S_3, T) = 0.342$, $\hat{W}GD_{TSFS}^4(S_4, T) = 0.522$	$S_4 \geq S_2 \geq S_1 \geq S_3$
Zeng et al. [52]	$\hat{W}GD_{TSFS}^4(S_1, T) = 0.410$, $\hat{W}GD_{TSFS}^4(S_2, T) = 0.431$, $\hat{W}GD_{TSFS}^4(S_3, T) = 0.339$, $\hat{W}GD_{TSFS}^4(S_4, T) = 0.674$	$S_4 \geq S_2 \geq S_1 \geq S_3$
Proposed measure for $\rho = 0.5$	$\hat{W}GD_{TSFS}^4(S_1, T) = 0.839$, $\hat{W}GD_{TSFS}^4(S_2, T) = 0.848$, $\hat{W}GD_{TSFS}^4(S_3, T) = 0.794$, $\hat{W}GD_{TSFS}^4(S_4, T) = 0.944$	$S_4 \geq S_2 \geq S_1 \geq S_3$

Fig. 4 Comparison between established measures with existing measures

Compliance with ethical standards

Conflict of interest The authors declare no conflict of interest.

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