SHORT COMMUNICATION

Estimation of wave runup on shorelines based on long-term variation of wave conditions

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Abstract   This article provides a simple analytical method which can be used to give estimates of the wave runup on shorelines based on long-term variation of wave conditions. This is achieved by providing bivariate distribution of significant wave height with the wave runup. This wave runup is defined in terms of significant wave height and spectral peak period in deep water. This article presents the mean value and the standard deviation, that is, more precisely the conditional expected value and the conditional variance of the wave runup for given significant wave height. Examples of results corresponding to typical field conditions are also given. Based on, for example, global wave statistics, the present analytical results can be used to make estimates of the wave runup.

Keywords   Wave runup · Significant wave height · Bivariate distributions · Coastal protection work

1 Introduction

The present work addresses random wave runup on beaches and coastal structures such as breakwaters, seawalls and artificial reefs. For beaches and such structures it is essential to make reliable assessments of the maximum wave runup to be able to take necessary protection measures. The recent focus on extreme phenomena induced by climate change has generated new interest in the analysis of wave runup on shorelines [see, e.g., de la Pena et al. (2014)].

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The wave runup height is defined as the vertical difference between the highest point of wave runup and the still water level. Here, the runup is given by two components; the wave setup and the swash. The wave setup is the mean water elevation level (e.g., referring to the deep water level) and is caused by the radiation stress [see, e.g., Dean and Dalrymple (1984)]. The swash motion oscillates from the wave setup and is taken as the interception between the sea and the beach or structure [see de la Pena et al. (2014) for more details]. Due to the stochastic nature of waves, many of the commonly used design formulas use the runup height (R_2), which is the runup height exceeded by 2 % of the runup maxima at the toe of the beach or structure. de la Pena et al. (2014) presented a new formulation for the wave runup (R_2) on a shoreline in terms of the significant wave height (H_s) in deep water, the slope of the beach, and the surf parameter defined in terms of H_s and the spectral peak period (T_p) in deep water. de la Pena et al. (2014) included also a literature review as well as a summary of previous wave runup formulations.

The purpose of this study is to present a simple analytical method which can be used to give estimates of the wave runup R_2 based on long-term variation of wave conditions of (H_s, T_p) or (H_s, T_z), where T_z is the mean zero-crossing wave period. This is obtained by providing parametric models of joint distributions of (H_s, R_2). This is achieved by adopting de la Pena et al.'s (2014) runup formulation and by transforming the parametric models of the joint (H_s, T_p) and (H_s, T_z) distributions fitted to field data by Moan et al. (2005) and Bitner-Gregersen and Guedes Soares (2007), respectively. Examples of calculating the conditional mean values and conditional standard deviations of R_2 given H_s corresponding to typical field conditions are also provided to demonstrate the application of the method. Thus, it is demonstrated how joint (H_s, T_p) or (H_s, T_z) distributions can be used to provide estimates of wave runup, and how,

for example, global wave statistics can be used to calculate this.

The long-term variation of wave conditions referred to in this paper, and as presented in Moan et al. (2005) and Bitner-Gregersen and Guedes Soares (2007), is expressed in terms of H_s, T_p (or T_z) assuming stationary sea state conditions with a duration of, e.g., 3 h based on a 20 min time series of the free surface elevation. The number of occurrences of each sea state is described by a frequency table or scatter diagram representing a discrete (long-term) joint H_s, T_p (or T_z) distribution of the wave conditions at the location considered.

2 Background

Several laboratory and field experiments have been performed to study extreme runup events, and equations for the estimation of the 2 % exceedance value of runup maxima, R_2, have been provided. de la Pena et al. (2014) have recently given a review of various formulations (see their Table 1), which can be represented by

$$\frac{R_2}{H_s} = K\xi_p \tag{1}$$

where K is a coefficient, and H_s is the significant wave height in deep water (i.e., representing one storm condition with a duration of, e.g., 3 h and with a return period specified by the user). Moreover, ξ_p is the surf parameter defined as

$$\xi_p = m\left(\frac{H_s}{\frac{g}{2\pi}T_p^2}\right)^{-1/2} \tag{2}$$

where $m = \tan\alpha$ is the slope with an angle α with the horizontal, T_p is the spectral peak period, and g is the acceleration due to gravity. Thus, this surf parameter is defined in terms of sea state parameters in deep water. Substitution of Eq. (2) in (1) can be rearranged to give (by defining R as a mathematical help)

$$R \equiv \frac{R_2}{Km\sqrt{g/2\pi}} = T_p\sqrt{H_s} \tag{3}$$

If, e.g., the mean zero-crossing wave period is given, then relationships between T_p and T_z exist, for example, given by

Table 1 Weibull parameters for H_s, see Eq. (7)

BGGS07	s	r	t
Dataset 1	3.104	1.357	0.906
Dataset 2	2.848	1.419	1.021
Dataset 3	2.939	1.240	0.896
Dataset 4	2.857	1.449	0.838
Dataset 5	2.420	1.169	1.258

$T_p = cT_z$ where c is a coefficient. Here, Eq. (3) is rearranged to

$$R = \frac{R_2}{cKm\sqrt{g/2\pi}} = T\sqrt{H_s} \tag{4}$$

where T represents T_p or T_z. Thus, R (and R_2) is defined in terms of the sea state parameters H_s and T (i.e., T_p or T_z) in deep water [i.e., representing a sea state where each pair of H_s, T_p (or T_z) represent one storm condition with a duration of, e.g., 3 h]. Different parametric models for the joint probability density function (pdf) of H_s and T_p or H_s and T_z are given in the literature. Examples are Moan et al. (2005) (hereafter referred to as MGAU05) and Haver (1985) for H_s and T_p, and Bitner-Gregersen and Guedes Soares (2007) (hereafter referred to as BGGS07) and Mathisen and Bitner-Gregersen (1990) for H_s and T_z. In the present paper, the statistical properties of R are exemplified using the joint pdf of H_s and T_p fitted by MGAU05 to 29 years of wave data from the Northern North Sea using the joint Haver (1985) model, as well as the joint pdf of H_s and T_z fitted by BGGS07 to five data sets from the North Atlantic using the joint Bitner-Gregersen (1988) model. These pdfs are given as

$$p(H_s, T) = p(T|H_s)p(H_s) \tag{5}$$

where $p(H_s)$ is the marginal pdf of H_s, which for the MGAU05 distribution is given by the following combined lognormal and Weibull distributions

$$p(H_s) = \begin{cases} \frac{1}{\sqrt{2\pi}\kappa H_s}\exp\left[-\frac{(\ln H_s - \theta)^2}{2\kappa^2}\right], & H_s \leq 3.25 \text{ m} \\ \beta\frac{H_s^{\beta-1}}{\zeta^\beta}\exp\left[-\left(\frac{H_s}{\zeta}\right)^\beta\right], & H_s > 3.25 \text{ m} \end{cases} \tag{6}$$

Here, $\theta = 0.801$, $\kappa^2 = 0.371$ are the mean value and the variance, respectively, of $\ln H_s$ and $\zeta = 2.713$, $\beta = 1.531$ are the Weibull parameters.

For the BGGS07 distribution, $p(H_s)$ is given by the following three-parameter Weibull distribution

$$p(H_s) = \frac{r}{s}\left(\frac{H_s - t}{s}\right)^{r-1}\exp\left[-\left(\frac{H_s - t}{s}\right)^r\right], \quad H_s \geq t \tag{7}$$

where r, s and t are the Weibull parameters given in BGGS07, see Table 1.

$p(T|H_s)$ is the conditional pdf of T given H_s, which for both distributions is given by the lognormal distribution

$$p(T|H_s) = \frac{1}{\sqrt{2\pi}\sigma T}\exp\left[-\frac{(\ln T - \mu)^2}{2\sigma^2}\right] \tag{8}$$

where μ and σ^2 are the mean value and the variance, respectively, of $\ln T$. For the MGAU05 distribution, $T = T_p$ and (Gao 2007)

Table 2 Mean value of $\ln T_z$, see Eq. (11)

BGGS07	a_1	a_2	a_3
Dataset 1	1.350	0.366	0.392
Dataset 2	1.365	0.375	0.453
Dataset 3	0.790	0.805	0.292
Dataset 4	0.835	1.139	0.119
Dataset 5	1.952	0.168	0.499

Table 3 Standard deviation of $\ln T_z$, see Eq. (11)

BGGS07	b_1	b_2	b_3
Dataset 1	0.020	0.165	−0.166
Dataset 2	0.033	0.285	−0.752
Dataset 3	0.055	0.195	−0.269
Dataset 4	0.140	0.030	−0.958
Dataset 5	0.070	0.066	−0.081

$$\mu = a_1 + a_2 H_s^{a_3}, \quad (a_1, a_2, a_3) = (1.780, 0.288, 0.474) \quad (9)$$

$$\sigma^2 = b_1 + b_2 e^{b_3 H_s}, \quad (b_1, b_2, b_3) = (0.001, 0.097, -0.255) \quad (10)$$

For the BGGS07 distribution, $T = T_z$ and

$$\mu = a_1 + a_2 H_s^{a_3}, \quad \sigma = b_1 + b_2 H_s^{b_3} \quad (11)$$

where the parameters in μ, σ are given in BGGS07, see Tables 2 and 3. All these data represent wave conditions in the North Atlantic. Data sets 1, 2 and 3 are numerically generated wave data taken from global databases representing 44 years (1958–2004) at 59°00′N, 19°00′W. Data set 4 refers to Global Wave Statistics (GWS) zone 9 (the zone located south of Iceland and west of UK) representing visual observations collected from ship in normal service all over the world in the period 1949–1986. Data set 5 refers to Juliet shipborne wave recorder (SBWR) representing data registered at the Ocean Weather Station Juliet during 13 years since 1952 at 52°00′N, 20°00′W. More details are given in Bitner-Gregersen and Guedes Soares (2007).

3 Statistical properties of runup

Statistical properties of R (from which the statistical properties of R_2 can be obtained) can be derived using the joint pdf of H_s and T, e.g., giving the joint pdf of R and H_s. This is obtained from Eq. (4) by a change of variables from (H_s, T) to (H_s, R), which takes the form

$$p(H_s, R) = p(R|H_s) p(H_s) \quad (12)$$

It should be noted that this change of variables only affects $p(T|H_s)$ since $T = R/\sqrt{H_s}$, yielding a lognormal pdf of R given H_s in the form (by using the Jacobian $|\partial R/\partial T| = \sqrt{H_s}$)

$$p(R|H_s) = \frac{1}{\sqrt{2\pi}\sigma_R R} \exp\left[-\frac{(\ln R - \mu_R)^2}{2\sigma_R^2}\right] \quad (13)$$

where μ_R and σ_R^2 are the conditional mean value and the conditional variance, respectively, of $\ln R$, given by

$$\mu_R = \mu + \frac{1}{2}\ln H_s; \ \sigma_R^2 = \sigma^2 \quad (14)$$

where μ and σ^2 are given in Eqs. (9) and (10) for the MGAU05 distribution and in Eq. (11) for the BGGS07 distribution.

The cumulative distribution function (cdf) of R given H_s is obtained from

$$P(R|H_s) = \Phi\left[\frac{\ln R - \mu_R}{\sigma_R}\right] \quad (15)$$

where Φ is the standard Gaussian cdf given by

$$\Phi(v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{v} e^{-t^2/2} dt \quad (16)$$

The expected value of R given H_s is given by (Bury 1975)

$$E[R|H_s] = \exp\left(\mu_R + \frac{1}{2}\sigma_R^2\right) \quad (17)$$

The standard deviation of R given H_s is given by (Bury 1975)

$$\sigma[R|H_s] = \left[(e^{\sigma_R^2} - 1)\exp(2\mu_R + \sigma_R^2)\right]^{1/2} \quad (18)$$

4 Examples of results

Here, examples of results are given by adopting the new formulation proposed by de la Pena et al. (2014) given by Eq. (1) by taking

$$K = 4m^{0.3} \quad (19)$$

and valid for $\xi_p < 0.6$. Their results are based on physical model experiments with a sand seabed using two grain sizes performed for the estimation of extreme runup for the three beach slopes $m = 1/50, 1/30, 1/20$ [see de la Pena et al. (2014) for more details]. Thus, Eq. (4) takes the form

$$R \equiv \frac{R_2}{c \cdot 4m^{1.3}\sqrt{g/2\pi}} = T\sqrt{H_s} \quad (20)$$

Moreover, $c = 1$ for $T = T_p$ and $c = 1.28$ for $T = T_z$; where the latter is taken from Myrhaug and Kjeldsen (1987, Fig. 11) where $T_p = 1.28 T_z$ for a JONSWAP spectrum with

Table 4 Example of results for wave runup on shorelines based on the de la Pena et al. (2014) formulation and the MGAU05 and BGGS07 distribution results for $H_s = 7.5$ m and slope $m = 1/20$

Distribution	$E[R_2 \mid H_s = 7.5$ m$]$ (m)	$\sigma[R_2 \mid H = 7.5$ m$]$ (m)
MGAU05	3.52	0.44
BGGS07		
Dataset 1	3.12	0.47
Dataset 2	3.57	0.34
Dataset 3	3.40	0.58
Dataset 4	3.53	0.51
Dataset 5	4.01	0.51

peakedness factor $\gamma = 3.3$. However, it should be noted that this relationship between T_p and T_z is not necessarily valid for the data sets used here. They might contain mixed swell and wind sea for which other relationships exist.

The given flow conditions are (e.g., representing one storm condition with a duration of 3 h):

- Significant wave height in deep water, $H_s = 7.5$ m
- Slope of beach, $m = 1/20$

Since the de la Pena et al. (2014) formulation is valid for $\xi_p < 0.6$, it is required that $T_p < 26.3$ s and $T_z < 20.5$ s.

For the runup it follows from Eqs. (20) and (17) that

$$E[R_2 \mid H_s = 7.5 \text{ m}] = c 4 m^{1.3} \sqrt{\frac{g}{2\pi}} E[R \mid H_s = 7.5 \text{ m}] \quad (21)$$

and from Eqs. (20) and (18) that

$$\sigma[R_2 \mid H_s = 7.5 \text{ m}] = c 4 m^{1.3} \sqrt{\frac{g}{2\pi}} \sigma[R \mid H_s = 7.5 \text{ m}] \quad (22)$$

The MGAU05 distribution results are obtained by combining Eqs. (21) and (22) for $c = 1$ with Eqs. (14), (9) and (10). The BGGS07 distribution results are obtained by combining Eqs. (21) and (22) for $c = 1.28$ with Eqs. (14) and (11). The results are presented in Table 4, showing that the mean values of R_2 are in the range 3–4 m with standard deviations in the range 0.3–0.5 m depending on the distribution and data set considered. More explicitly this means that, e.g., based on the MGAU05 distribution results, the expected maximum runup is 3.52 m with a standard deviation of 0.44 m in a sea state (one storm condition) with $H_s = 7.5$ m and a duration of, e.g., 3 h. The actual duration of the sea state can only be obtained if duration statistics are available.

5 Comments

Finally, some comments are given on the present method versus common practice in coastal engineering. For assess-

ment of, e.g., maximum runup height on beaches and coastal structures common practice would be to start from available data on joint statistics of H_s and T_p (or T_z) within directional sectors at a nearby offshore location; then to transform these using a wave simulation model to obtain joint statistics of H_s and T_p (or T_z) at the relevant near-shore location; then to use this information as input for the assessment of maximum wave runup height. Alternatively, the present method provides a simple analytical tool giving first estimates of maximum runup on beaches and coastal structures for given values of H_s, T_p (or T_z). Such estimates are useful for comparison and verification of more complete computational methods, as well as in situations when time and access to computational resources are limited (under, e.g., field conditions). Moreover, it might also serve as a first inexpensive estimate of the quantities of interest before eventually applying more work-intensive computational tools. Although the present results are valid for the specifically chosen wave runup formulation and joint distributions of (H_s, T_p) and (H_s, T_z) it gives an analytically based method which can be used for other wave runup formulations and joint distributions of H_s and T_p (or T_z).

6 Summary

A simple analytical method which can be used to give estimates of the wave runup on shorelines based on statistics of long-term observation of wave conditions is provided. This is achieved by providing bivariate distribution of H_s with the wave runup. The wave runup formulation by de la Pena et al. (2014) defined in terms of H_s and T_p is adopted. This bivariate distribution of H_s and wave runup is obtained by transformations of joint distributions of (H_s, T_p) and (H_s, T_z) based on fitting to data from the Northern North Sea and the North Atlantic, respectively. The wave runup parameters such as the conditional expected values and the conditional variances for given significant wave height as well as examples of results corresponding to typical field conditions are presented. The present analytical method can be used to calculate estimates of wave runup on shorelines based on, for example, global wave statistics.

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