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Product Low-Carbon Design using Dynamic Programming Algorithm

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Greenhouse gas emission has become a recent global concern for green manufacturing. As product low-carbon design is an essential approach to achieve low-carbon manufacturing, which has a profound effect on the product carbon footprint, many researches have been focused on it in recent years with a result of valuable contributions. This paper is devoted to presenting a dynamic programming-based approach to product low-carbon design. After product low-carbon design is characterized by a multi-stage decision process with interaction effects on each other in the product life cycle, a dynamic programming method is used to optimize the total carbon footprint of each stage while considering interaction effects of solutions at each stage in product life cycle. The low-carbon design of a cold heading machine is used to demonstrate the proposed methodology.

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NOMENCLATURE

$g_i(x_i, x_{i+1})$ = carbon footprint mapping function of x_i and x_{i+1}
$f_k(x_k)$ = sum of carbon footprint from the k_{th} to the last stage
s_{ij} = design states in set S_i
$u_i(x_i)$ = control variables of x_i at the i_{th} stage
$v_i(x_i, u_i)$ = carbon footprint under the consideration of x_i and u_i
x_i = state variables set in S_i at the i_{th} stage
x_{ij} = state variable set in the design state s_{ij}
x_{ij}^{p} = design variables of the p_{th} solution in s_{ij} at the i_{th} stage
C_i = carbon emission factor of the i_{th} activity
$D_i(x_i)$ = control variables set of x_i at the i_{th} stage
E_a = carbon footprint at the acquisition of raw materials stage
E_c = carbon footprint in the product life cycle
$E_i(s_{ik})$ = carbon footprint function of s_{ik} at the i_{th} stage
E_m = carbon footprint at the manufacturing stage
E_r = carbon footprint at the recycle and disposal stage
E_t = carbon footprint at the transportation stage
E_u = carbon footprint at the usage stage
G_k = emission of the k_{th} GHG
GWP_k = global warming potential of the k_{th} GHG
M_i = consumption of the i_{th} activity
S_i = design state set at the i_{th} stage
$T_k(x_k, u_k)$ = transition function with k stages to go
$U_k(x_k, x_{k+1})$ = control law describing x_k and x_{k+1}
X = overall state variables of different stages

1. Introduction

The emissions of greenhouse gases (GHGs), particularly carbon dioxide, will arouse global climate change.[1] As a sustainable technology,[2] the low-carbon technology[1] is stipulated to industry to reduce GHGs emissions. Derived from ecological footprint,[3] the product carbon footprint is an indicator on environmental impact for a product.[4] Generally speaking, product carbon footprint describes the sum of GHGs emissions accumulated during the entire product life cycle.[5] As a sustainable design,[6] product low-carbon design has been focused in recent years with a result of many valuable contributions.

The foundation for product low-carbon design is the calculation of product carbon footprint, which is typically estimated by activity data and the corresponding emission factors with life cycle inventory (LCI)[4]-based life cycle assessment (LCA).[7] He et al.[8] modeled carbon footprint of design solutions in conceptual design through Unascertained Theory. The essential for product low-carbon design is the reduction of GHGs emissions in its life cycle. For instance, Song et al.[9] proposed low-carbon design system based on the bill of materials (BOM) integrated with GHGs emissions data. Lee et al.[10] integrated carbon footprint in vendor selection. Su et al.[11] reduced carbon footprint through optimal structure, assembly sequence, and suppliers as well. Kuo et al.[12] developed a collaborative design framework to integrate carbon footprint with life cycle inventory database in enterprise resource planning.

Low-carbon design is a design process embedded with GHGs emissions in product life cycle. Its typical results are the design solutions with low GHGs emissions throughout the entire product life cycle. However, the current research efforts always focus on the mapping from the life cycle inventory to carbon footprint through LCA. There are several LCA optimization methods, such as multi-objective optimization,[13] productive resources maximization,[14] etc. Since the product life cycle has several stages, the low-carbon design process can be viewed as a multi-stage decision process. The main contribution of this paper is to propose a dynamic programming-based multi-stage decision process of low-carbon design for product life cycle. And in this way, it is obtained with the low-carbon solutions of the lowest carbon footprint in its entire life cycle.

The remainder of this paper is organized as follows. Section 2 described the model of product carbon footprint for product life cycle. In Section 3, after product low-carbon design is characterized by a multi-stage decision process with interaction effects on each other based on the stage of product life cycle, from which product low-carbon design is transferred to a problem to find the shortest path based on dynamic programming, a dynamic programming method is then used to optimize the total carbon footprint of these stages while considering interaction effects. The low-carbon design of a cold heading machine is used to demonstrate the proposed methodology in Section 4. And Section 5 concludes this paper.

2. Product Carbon Footprint in Product Life Cycle

Low-carbon design integrates significant GHGs emissions aspects into product design in its entire life cycle, which consists of five stages, i.e. raw materials acquisition, manufacturing, transportation, usage, and recycle and disposal stage. The typical results of low-carbon design are the generation of products with the reduction of GHGs emissions throughout the life cycle. Since the majority of carbon footprint for a product is determined at its design stage, product low-carbon design is an essential approach to achieve low-carbon manufacturing fundamentally, which has a profound effect on the product carbon footprint. According to the definition of life cycle, the overall product carbon footprint of life cycle is defined as:

$$E_c = E_a + E_m + E_t + E_u + E_r \tag{1}$$

where E_c, E_a, E_m, E_t, E_u, and E_r is carbon footprint in the product life cycle, at the acquisition of raw materials stage, the manufacturing stage, the transportation stage, the usage stage, and the recycle and disposal stage, respectively.

The calculation models are followed in detail at each stage of the product life cycle.

(1) Carbon footprint at the acquisition of raw materials stage is calculated as

$$E_a = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} M_{ij}C_{ij} + \sum_{s=1}^{n_3}\sum_{t=1}^{n_4} G_{st}GWP_t \tag{2}$$

where M_{ij} is the consumption of the j_{th} material for the i_{th} component, C_{ij} is the emission factor of the j_{th} material acquired for the i_{th} component, G_{st} is the emission of the t_{th} GHG in the acquisition of the s_{th} material, GWP_t is the global warming potential of the t_{th} GHG, $n_1 \sim n_4$ is element number.

(2) Carbon footprint at the manufacturing stage is calculated as

$$E_m = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} P_{ij}C_{ij} + \sum_{k=1}^{n_3}\sum_{l=1}^{n_4} A_{kl}C_{kl} + \sum_{m=1}^{n_5}\sum_{n=1}^{n_6} W_{mn}C_{mn} + \sum_{s=1}^{n_7}\sum_{t=1}^{n_8} G_{st}GWP_t \tag{3}$$

where P_{ij} is quantity of the j_{th} energy consumed in the i_{th} manufacturing process, A_{kl} is quantity of the l_{th} energy consumed in the k_{th} assembly process, W_{mn} is quantity of the n_{th} waste in the m_{th} manufacturing/assembly process, C_{ij} is emission factor of the j_{th} energy consumed in the i^{th} manufacturing process, C_{kl} is emission factor of the l_{th} energy consumed in the k_{th} assembly process, C_{mn} is emission factor of the n_{th} waste in the m_{th} manufacturing/assembly process, G_{st} is emission of the t_{th} GHG in the s_{th} manufacturing/assembly process, $n_1 \sim n_8$ is element number.

(3) Carbon footprint at the transportation stage is calculated as

$$E_t = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\sum_{k=1}^{n_3} Q_{ij}L_{ij}EI_{ijk}C_{ijk} + \sum_{s=1}^{n_4}\sum_{t=1}^{n_5} G_{st}GWP_t \tag{4}$$

where Q_{ij} is quantity of the j_{th} transportation object (including materials, parts, products and waste) in the i_{th} transportation mode, L_{ij} is transportation distance of the j_{th} transportation object in the i_{th} transportation mode, EI_{ijk} is energy intensity of the k_{th} energy in transporting the j_{th} object by the i_{th} transportation mode, C_{ijk} is emission factor of the k_{th} energy in transporting the j_{th} object by the i_{th} transportation mode, G_{st} is emission of the t_{th} GHG by using the s_{th} energy at the transportation stage, $n_1 \sim n_5$ is element number.

(4) Carbon footprint at the usage stage is calculated as

$$\begin{aligned} E_u = & \sum_{i=1}^{n_1}\left(\sum_{j=1}^{n_2} U_{ij}C_{ij} + \sum_{k=1}^{n_3} D_{ik}C_{ik}\right) \\ & + \sum_{m=1}^{n_4}\sum_{n=1}^{n_5}(M_{mn}C_{mn} + F_{mn}EF_{mn})\frac{L}{L_{mn}} + \sum_{s=1}^{n_6}\sum_{t=1}^{n_7} G_{st}GWP_t \end{aligned} \tag{5}$$

where U_{ij} is quantity of the j_{th} energy consumed in the i_{th} module, D_{ik} is quantity of the k_{th} energy consumed in monitoring the i_{th} module in the maintenance process, M_{mn} is quantity of the material consumed in repairing the n_{th} part of the m_{th} module, F_{mn} is quantity of the energy consumed in repairing the n_{th} part of the m_{th} module, C_{ij} is emission factor of the j_{th} energy consumed in the i_{th} module, C_{ik} is emission factor of the k_{th} energy consumed in monitoring the i_{th} module in the maintenance process, C_{mn} is emission factor of materials consumed in repairing the n^{th} part of the m_{th} module, EF_{mn} is emission factor of the energy consumed in repairing the n_{th} part of the m_{th} module, L is service life of the product, L_{mn} is service life of the n_{th} part of the m_{th} module, G_{st} is emission of the t_{th} GHG in the s_{th} module in the usage stage, $n_1 \sim n_7$ is element number.

(5) Carbon footprint at the recycle and disposal stage is calculated as

$$E_r = \sum_{i=1}^{n_1}\left(\sum_{j=1}^{n_2} D_{ij}C_{ij} + \sum_{k=1}^{n_3} W_{ik}C_{ik}\right) + \sum_{m=1}^{n_4}\left(\sum_{n=1}^{n_5} R_{mn}C_{mn} - G_m RA_m\right) + \sum_{p=1}^{n_6}\left(\sum_{q=1}^{n_7} M_{pq}C_{pq} - G_p RA_p\right) + \sum_{s=1}^{n_8}\sum_{t=1}^{n_9} G_{st}GWP_t \quad (6)$$

where D_{ij} is quantity of the j_{th} energy consumed in disassembling the i_{th} component, W_{ik} is quantity of the k_{th} energy consumed in the waste disposal of the i_{th} component, R_{mn} is quantity of the n_{th} energy consumed in recycling the m_{th} component, M_{pq} is quantity of the q_{th} energy consumed in recycling the p_{th} material, C_{ij} is emission factor of the j_{th} energy consumed in disassembling the i_{th} component, C_{ik} is emission factor of the k_{th} energy consumed in the waste disposal of the i_{th} component, C_{mn} is emission factor of the n_{th} energy consumed in recycling the m_{th} component, C_{pq} is emission factor of the q_{th} energy consumed in recycling the p_{th} material, G_m is equivalent carbon emission of the m_{th} part or component, RA_m is ratio of the m_{th} recycling part or component to the original component, G_p is equivalent carbon emission of the p_{th} material, RA_p is ratio of the p_{th} recycling material to the original material, G_{st} is emission of the t_{th} GHG in the recycle and disposal of the s_{th} material, part or component, $n_1 \sim n_9$ is element number.

3. Dynamic Programming-Based Product Low-Carbon Design

3.1 Design State in Product Low-Carbon Design at Each Stage of the Product Life Cycle

There are five stages in the product life cycle. At the i_{th} stage of product life cycle, there are some choices to be chosen and transferred to the subsequent stage of the product life cycle, denoted as design state set S_i with some design states s_{ij}.

$$S_i = \{s_{i1}, s_{i2}, s_{i3}, \ldots, s_{ij}, \ldots, s_{im}\} \quad (7)$$

where s_{ij} ($i = 0, 1, 2, 3, 4, 5$; $j = 1, 2, \ldots, m$) is a design state in design state set S_i at the i_{th} stage in product life cycle, and m is the number of choices at the i_{th} stage in product life cycle.

For instance, at the acquisition of raw materials stage, it has several choices of raw materials to achieve design requirements, which generates design state set at the acquisition of raw materials stage $S_1 = \{s_{11}, s_{12}, \ldots, s_{1j}, \ldots, s_{1a}\}$. All choices of raw materials solutions s_{1j} in S_1 might correspond with some manufacturing processes, which generates design state set at the manufacturing stage $S_2 = \{s_{21}, s_{22}, \ldots, s_{2j}, \ldots, s_{2m}\}$. After manufacturing, there are several transportation ways at the transportation stage corresponding with manufacturing process s_{2j} in S_2, which generates design state set at the transportation stage $S_3 = \{s_{31}, s_{32}, \ldots, s_{3j}, \ldots, s_{3t}\}$. After transportation, it might be used in different ways at the usage stage corresponding with above transportation ways s_{3j} in S_3, which generates design state set at usage stage $S_4 = \{s_{41}, s_{42}, \ldots, s_{4j}, \ldots, s_{4u}\}$. After the usage, there are also several solutions to recycle and disposal corresponding with above usage means s_{4j} in S_4, which generates design state set at the recycle and disposal stage $S_5 = \{s_{51}, s_{52}, \ldots, s_{5j}, \ldots, s_{5r}\}$. Since S_i is defined recursively, it is necessary to put an initiate design state $S_0 = \{s_{01}\}$ before the acquisition of raw materials stage and a final design state $S_6 = \{s_{61}\}$ after the recycle and disposal stage.

3.2 State Variables and Control Variables in Product Low-Carbon Design

The state variable set x_{ij} in the design state s_{ij} at the i_{th} stage of product life cycle is defined as:

$$x_{ij} = \{x_{ij}^1, x_{ij}^2, x_{ij}^3, \ldots, x_{ij}^p, \ldots, x_{ij}^m\} \quad (8)$$

where x_{ij}^p is defined as design variables of the p_{th} solution in design state s_{ij} at the i_{th} stage of product life cycle, m is the number of state variables in the current alterative solution.

The state variables set x_i in the design state set S_i at the i_{th} stage of product life cycle is then defined as:

$$x_i = \{x_{i1}, x_{i2}, \ldots, x_{ij}, \ldots, x_{ik}\} \quad (9)$$

where k is the number of state variables in all the solutions at the i_{th} stage of product life cycle.

Thus, overall state variables X of different stages is as follows:

$$X = \{x_1, x_2, x_3, x_4, x_5, x_6\} \quad (10)$$

The control variables $u_i(x_i)$ at the i_{th} stage of product life cycle are calculated based on the state variables set x_i at the i_{th} stage and the corresponding state variables set x_{i+1} at the next stage of product life cycle, as follows:

$$u_i \in D_i(x_i) = g_i(x_i, x_{i+1}) \quad (11)$$

where $u_i(x_i)$ is the control variables of the state variables set x_i at the i_{th} stage of product life cycle, $D_i(x_i)$ is control variables set of the state variables set x_i at the i_{th} stage of product life cycle, and $g_i(x_i, x_{i+1})$ is carbon footprint mapping function of the state variables set x_i of this stage and the corresponding state variables set x_{i+1} in the next stage.

3.3 Objective Function Calculating Approaches based on the Total Product Carbon Footprint

The product carbon footprint is shown as follows:

$$v_i(x_i, u_i) = E_i(s_{ik}) \quad (12)$$

where $v_i(x_i, u_i)$ is the step carbon footprint of at the i_{th} stage under the consideration of state variables set x_i and control variables u_i, and $E_i(s_{ik})$ is the function of carbon footprint of design state s_{ik} at the i_{th} stage. The product carbon footprint of each stage can be calculated with the equations from Eq. (2) to Eq. (6).

The total objective function M is the sum of carbon footprints in above six stages, i.e. Stage 0: Initial stage, Stage 1: Acquisition of raw materials stage, Stage 2: Manufacturing stage, Stage 3: Transportation stage, Stage 4: Usage stage, and Stage 5: Recycle and disposal stage.

$$M = \min \sum_{k=0}^{5} v_k(x_k, u_k) \quad (13)$$

The cost-to-go function $f_k(x_k)$ of the k_{th} stage is the sum of carbon footprint from this stage to the last stage. Since the search of the least optimal carbon footprint at each stage is independent of the initial stages and controls used before the current stage, the goal, to seek the minimum of the cost-to-go function of the current stage, is only related to the stage carbon footprint of the current stage and the cost-to-go function of the previous stage, as follows:

$$f_k(x_k) = \min_{u_k \in D_k(x_k)} (v_k(x_k,\ u_k) + f_{k-1}(x_{k-1})) \tag{14}$$

where $D_k(x_k)$ is the control variable set of permissible decisions.

3.4 Dynamic Programming-Based Product Low-Carbon Design Model

The dynamic programming method is applicable to the multi-stage decision process in product low-carbon design as shown in Fig. 1, in which each stage of the product life cycle might be seen as a decision stage. The product low-carbon design decision stages of dynamic programming model are linked to the above stages in product life cycle. It must be mentioned that there may be some choices without further considerations in the next stage for some reasons, named as invalid choices. For instance, the current solution can not be supported by the subsequent stage, such as with the initial choice of a certain new material, whose manufacturing process is difficult and then given up in the following process. These types of design states, such as s_{12}, s_{2p}, and s_{3i} shown in Fig. 1 are marked with dashed lines.

The design variables of different stage are used as the state variables. The control law of dynamic programming model used here is to determine the solution of the next stage based on the current stage. The control variable of each decision stage is a next stage's state variable, and therefore the domain of definition of the control variable, and therefore the domain of definition of the control variable of each decision stage is a next stage's state variable, and therefore the domain of definition of the control variable consists of the available values of the choice of the next stage. Moreover, the carbon footprint of each stage is used as the step carbon footprint of each decision stage.

The least carbon footprint decisions for different steps are to be found one stage after another stage, which are well adapted to apply the Bellman's Principle of Optimality,[15] which states that an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. The optimal product low-carbon design model of dynamic programming problem can be formulated as follows:

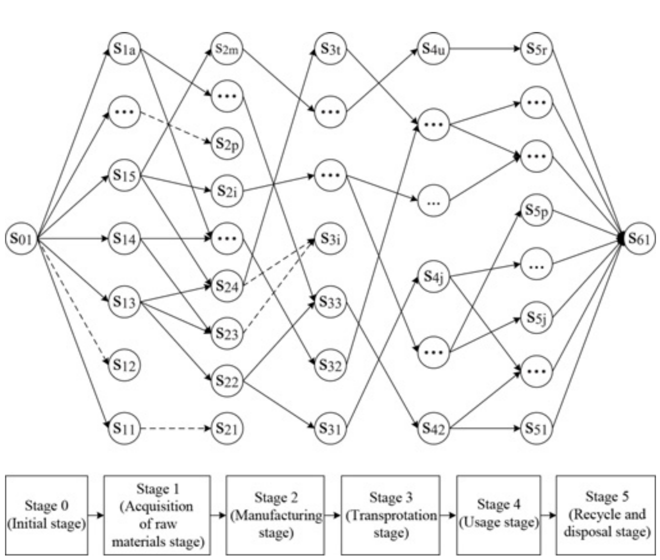

Fig. 1 Dynamic programming-based product low-carbon design

$$\begin{aligned} &v_k(x_k, u_k) = \min\{f_k(x_k, u_k) + v_{k-1}[T_k(x_k, u_k)]\} \\ &subject\ to\colon x_k \in X_k(x_k)\ ;\ u_k = U_k(x_k, x_{k+1}) \in D_k(u_k) \end{aligned} \tag{15}$$

where N is the total number of decision stages; k is the node number; $v_k(x_k, u_k)$ is carbon footprint of the k_{th} stage; x_k is state variable set; X_k is state variable set of permissible decisions; u_k is the control variable; $D_k(u_k)$ is the control variable set of permissible decisions; $T_k(x_k, u_k)$ is the transition function with k stages to go, which is an implicit function deciding the previous stage's stage varibles x_{k-1} based on the current stage' stage variables x_k and control variables u_k; $U_k(x_k, x_{k+1})$ is the control law describing the control variables' dependent relationship on the state variables of the considered stage and the previous stage. The backward optimal path is decided based on the mapping relationship T_k connecting stages and the control law U_k.

Since $v_k(x_k, u_k)$is defined recursively in terms of $v_{k1}(x_{k-1}, u_{k-1})$, in order to solve Eq. (15), it is necessary to initiate the computation by solving the stage-zero problem.[16] The stage-zero problem is not defined recursively, since there are no more stages before the initial stage of the decision process. To simplify the expression, a dummy carbon footprint is added for the first stage, and the stage-zero problem is then the following:

$$v_0(x_0,\ u_0) = 0 \tag{16}$$

3.5 Dynamic Programming Algorithm-Based Product Low-Carbon Design

The dynamic programming algorithm-based product low-carbon design is given as follows. At first, those invalid choices at each stage in product life cycle are deleted from the product low-carbon design model. Starting from the last $(N-1)_{th}$ stage, one determines the optimal carbon footprint for each state variable at the $(N-2)_{th}$ stage based on all the available control variables. Then, knowing the optimal carbon footprint of each state variable at the $(N-2)_{th}$ stage, one determines the optimal carbon footprint for each state variable at the $(N-3)_{th}$ stage, again using all the available control variables. This procedure continues recursively backward until the Stage 0, from which the optimal carbon footprint to the entire dynamic programming problem is finally obtained.

According to the dynamic programming model, the total number of decision stage $N = 6$, the optimization process starts at $k = 5$, and continues recursively down to $k = 0$ using Eq. (15), and then the optimal total objective function, i.e. the least carbon footprint in the product cycle can be obtained.

4. Application

4.1 Problem Configuration

As a case study, a cold heading machine is given as an example of low-carbon design for the product life cycle, and the result is analysed to verify the feasibility of the proposed low-carbon design model. It is an important performance for low-carbon of cold heading machine. There are choices at each stage of product life cycle proposed in detail in reference.[17]

4.2 State Variables and Control Variables

There are many choices at each stage of product life cycle. Therefore the state variables set of each decision stage, i.e. the values of the state variables, can be given by:

$x_0 \in X_0 = \{s_{01}\}$
$x_1 \in X_1 = \{s_{11},s_{12},s_{13},s_{14},s_{15},s_{16},s_{17},s_{18},s_{19}\}$
$x_2 \in X_2 = \{s_{21},s_{22},s_{23},s_{24},s_{25},s_{26},s_{27},s_{28},s_{29},s_{210}\}$
$x_3 \in X_3 = \{s_{31},s_{32},s_{33},s_{34}\}$
$x_4 \in X_4 = \{s_{41},s_{42}\}$
$x_5 \in X_5 = \{s_{51},s_{52}\}$
$x_6 \in X_6 = \{s_{61}\}$

As there are some invalid design states, such as s_{16}, s_{23} and s_{27}, after ignoring these invalid design states, the feasible values of the state variables are simplified as:

$x_0 \in X_0=\{s_{01}\}$
$x_1 \in X_1=\{s_{11},s_{12},s_{13},s_{14},s_{15},s_{17},s_{18},s_{19}\}$
$x_2 \in X_2=\{s_{21},s_{22},s_{24},s_{25},s_{26},s_{28},s_{29},s_{210}\}$
$x_3 \in X_3=\{s_{31},s_{32},s_{33},s_{34}\}$
$x_4 \in X_4=\{s_{41},s_{42}\}$
$x_5 \in X_5=\{s_{51},s_{52}\}$
$x_6 \in X_6=\{s_{61}\}$

According to the calculating approaches presented in Section 3, the step carbon footprint related to all possible control laws in each decision stage is listed as the control variables at each decision stage, i.e. the available values of control variables, are as follows:

$D_0(s_{01}) = x_1$;
$D_1(s_{11}) = \{s_{21},s_{22}\} \subseteq x_2$, $D_1(s_{12}) = \{s_{24},s_{29}\} \subseteq x_2$, $D_1(s_{13}) = \{s_{23},s_{27}\} \subseteq x_2$,
$D_1(s_{14}) = \{s_{21},s_{22}\} \subseteq x_2$, $D_1(s_{15}) = \{s_{21},s_{22},s_{26}\} \subseteq x_2$,
$D_1(s_{17}) = \{s_{25},s_{26},s_{28}\} \subseteq x_2$, $D_1(s_{18}) = \{s_{210}\} \subseteq x_2$, $D_1(s_{19}) = \{s_{24},s_{29}\} \subseteq x_2$;
$D_2(s_{21}) = \{s_{31},s_{32}\} \subseteq x_3$, $D_2(s_{22}) = \{s_{34}\} \subseteq x_3$, $D_2(s_{24}) = \{s_{31},s_{33}\} \subseteq x_3$,
$D_2(s_{25}) = \{s_{31},s_{32}\} \subseteq x_3$, $D_2(s_{26}) = \{s_{31},s_{32}\} \subseteq x_3$,
$D_2(s_{28}) = \{s_{34}\} \subseteq x_3$, $D_2(s_{29}) = \{s_{31},s_{33}\} \subseteq x_3$, $D_2(s_{210}) = \{s_{31},s_{32}\} \subseteq x_3$;
$D_3(s_{31}) = D_3(s_{32}) = D_3(s_{33}) = D_3(s_{34}) = x_4$;
$D_4(s_{41}) = D_4(s_{42}) = x_5$;
$D_5(s_{51}) = D_5(s_{52}) = x_6$.

The state variables set and control variables set are explicitly shown as follows, as the real lines connecting one design state to the next one express the available selections as the control law of each stage in Fig. 2. The step carbon footprints of all decision stages are given by:

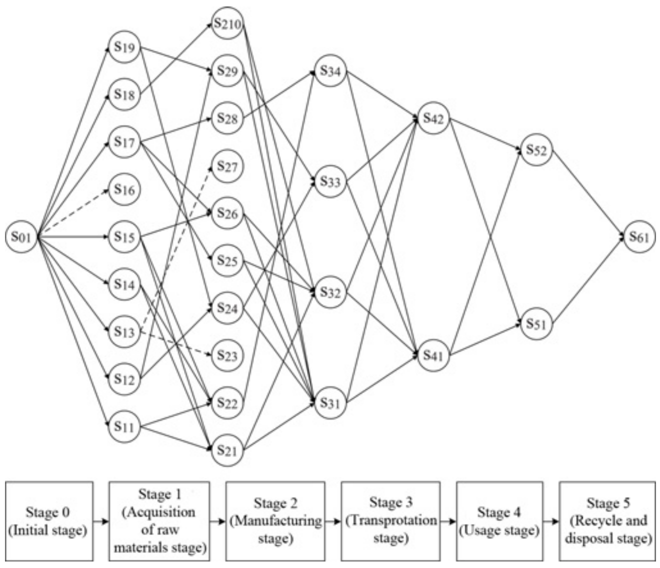

Fig. 2 Dynamic programming model-based low-carbon design for cold heading machine

$v_5\ (x_5,u_5) = f_5(x_5,u_5) + v_4[T_5(x_4,u_4)]$;
$v_4\ (x_4,u_4) = f_4(x_4,u_4) + v_3[T_4(x_2,u_2)]$;
$v_3\ (x_3,u_3) = f_3(x_3,u_3) + v_2[T_3(x_2,u_2)]$;
$v_2\ (x_2,u_2) = f_2(x_2,u_2) + v_1[T_2(x_1,u_1)]$;
$v_1\ (x_1,u_1) = f_1(x_1,u_1) + v_0[T_1(x_0,u_0)]$;
$v_0\ (x_0,u_0) = 0$.

Furthermore, it is calculated as carbon footprints (Unit: kg) from the s_{ij} to s_{pq} related to all possible control laws at each decision stage, through

Table 1 Carbon footprint (CF) from design state s_{ij} to design state s_{pq}

s_{ij}	s_{pq}	CF	s_{ij}	s_{pq}	CF	s_{ij}	s_{pq}	CF	s_{ij}	s_{pq}	CF
s_{01}	s_{11}	0	s_{13}	s_{27}	9,132	s_{22}	s_{34}	7,776	s_{31}	s_{42}	810
s_{01}	s_{12}	0	s_{14}	s_{21}	9,600	s_{24}	s_{31}	6,075	s_{32}	s_{41}	308
s_{01}	s_{13}	0	s_{14}	s_{22}	9,600	s_{24}	s_{33}	6,075	s_{32}	s_{42}	308
s_{01}	s_{14}	0	s_{15}	s_{21}	9,500	s_{25}	s_{31}	8,100	s_{33}	s_{41}	368
s_{01}	s_{15}	0	s_{15}	s_{22}	9,500	s_{25}	s_{32}	8,100	s_{33}	s_{42}	368
s_{01}	s_{16}	0	s_{17}	s_{25}	9,650	s_{26}	s_{31}	7,290	s_{34}	s_{41}	2,682
s_{01}	s_{17}	0	s_{17}	s_{26}	9,650	s_{26}	s_{32}	7,290	s_{34}	s_{42}	2,682
s_{01}	s_{18}	0	s_{17}	s_{28}	9,650	s_{28}	s_{34}	7,776	s_{41}	s_{51}	650,778
s_{01}	s_{19}	0	s_{18}	s_{210}	9,463	s_{29}	s_{31}	5,670	s_{41}	s_{52}	650,196
s_{11}	s_{21}	9,550	s_{19}	s_{24}	8,846	s_{29}	s_{33}	5,670	s_{42}	s_{51}	507,165
s_{11}	s_{22}	9,550	s_{19}	s_{29}	8,846	s_{210}	s_{31}	6,885	s_{42}	s_{52}	506,583
s_{12}	s_{24}	8,618	s_{21}	s_{31}	7,290	s_{210}	s_{32}	6,885	s_{51}	s_{61}	4,392
s_{12}	s_{29}	8,618	s_{21}	s_{32}	7,290	s_{31}	s_{41}	810	s_{52}	s_{61}	4,683
s_{13}	s_{23}	9,132									

Table 2 The design results of cold heading machine

Nodes	Descriptions of design states
s_{12}	Machine body (Resin concrete), Sliding table (Metal, plastic composite materials), Crankshaft (45CrMo), Transmission gear and transmission spur gear (POM), Flywheel (HT250), etc.
s_{29}	Machine body (The Resin concrete mixture which is constituted by fluid resin, diluent, curing agent and aggregate filler, through knock outing, loading, stiring and vibration molding. The resin concrete mixture will be made into finished product.), Sliding table (Production arrangements→Modelling and core→Pouring molten→Cleaning→The first tempering→Rough finish→The second tempering→Finish-milling (except the guideway surface)→Accurate grinding guideway surface→Clean→Plastic coated→Press hardening.), Crankshaft (Cast, clean→Normalize →Rough surfaces on both sides→Rough turning on both ends of main journal→Rough shaft diameter of rod→Finish turning on both ends of main journal→Finish turning shaft diameter of rod→Finish turning surfaces on both sides→Final inspection.), Transmission gear and transmission spur gear (Injection molding), Flywheel (Forging→Cleaning→Artificial aging→Fine cleaning→Non-machined surfaces coated with rust→Lathe →Slotting the keyway→Drill→Static balance check→Final inspection.), etc. The whole machine is assembled manually.
s_{33}	The machine is transported by highway and ship.
s_{42}	Cold heading machine runs 10 hours a day efficiently. Automatic frequency control system is used at the usage stage to achieve stepless speed, and the production speed is fast. 20 times a year during the usage stage is scheduled to detect. Parts are replaced when 50% of them are damaged. The machine is repaired 12 times a year.
s_{52}	The materials of the machine are remanufactured.

the model proposed in Section 2, listed in Table 1.

The above carbon footprints in Table 1 are caculated using the equations from Equ. 2 to Equ. 6. The caculation of carbon footprint from s_{11} to s_{21} is taken as an example to illustrate. The raw material mainly includes 440 kg steel, 7 kg aluminum and 6505 kg cast iron, and their emission factors are obtained from IPCC[1] as 1.72 kg CO_2/kg, 1.7 kg CO_2/kg, and 1.35 kg CO_2/kg. And the direct carbon footprint is ignored as it is small. Thus, the carbon footprint from s_{11} to s_{21} is caculated as 440*1.72+7*1.7+6,505*1.35 = 9,550.45 ≈ 9,550 kg CO_{2e}. In this way, all the carbon footprints can be caculated.

4.3 Results of Low-Carbon Design for Cold Heading Machine

Eventually, on the basis of the step carbon footprint in each stage obtained above, the objective function could be optimized by Eq. (15). The result is M = 0+8,618+5,670+368+506,583+4,683 = 525,922 kg CO_{2e}, with the least carbon footprint path $s_{01}\rightarrow s_{12}\rightarrow s_{29}\rightarrow s_{33}\rightarrow s_{42}\rightarrow s_{52}\rightarrow s_{61}$. The corresponding solutions are taken as the lowest-carbon design solutions for cold heading machine, with the approximate number of carbon footprint 526,000 kg CO_{2e}, the design results are shown in Table 2.

There are also several challenges of the proposed approach, as follows: it must collect many data from the industry, it also need some simplification and assumption.

5. Conclusions

Low-carbon design plays a significant role in reducing GHGs emissions for the environmental impact on the life cycle of products. This paper is devoted to a dynamic programming-based approach to product low-carbon design for product life cycle. After the product low-carbon design is characterized by a multi-stage decision process in the product life cycle, a dynamic programming method is used to optimize the total carbon footprint of each stage while considering interaction effects of solutions at each stage in product life cycle. The low-carbon design of cold heading machine is given as an example, which demonstrates that the methodology is helpful to reduce product carbon footprint in product life cycle.

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