



Implications of social network structures on socially influenced decision-making

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Abstract Individuals often make decisions in a social environment where social influence can impact on people’s decision-making domains such as online purchasing, political voting and voluntary vaccine uptake. Social influence can be recognised as the intentional or unintentional change in an individual’s belief, perception, or behaviours caused by an information diffusion process embedded in a social network. However, there is limited research on how this diffusion process is shaped by the topology or structure of the social network. This work provides an exploratory and systematic analysis of how decision-making outcomes in a population can be affected by both the structure of the social network and the starting node of where new information starts to diffuse. Simulation results considering three common network structures highlight how social networks with clear community structures lead to a larger absolute impact on decision-making outcomes and networks where the social connections follow a preferential attachment rule show the largest relative impact than the others. The results also suggest scenarios in which introducing new pieces of information to the social network can facilitate the information diffusion process and produce a more significant impact in terms of the overall population decision-making process.

Keywords Decision-making · Social network · Network structure · Network analysis · Information diffusion

Introduction

Human beings make decisions all the time, each decision being the final product of a particular psychological/cognitive decision-making process of an individual or a group. The decision-making process can be defined as ‘*a set of action and dynamic factors that begin with the identification of a stimulus for actions and end with a specific commitment to action*’ (Mintzberg et al. 1976). This decision-making process is often accompanied by uncertainty, which is known as the feeling of being unsure of what will happen in the future. Uncertainty can come from various sources, such as limited or incalculable information about decision criteria and anticipated decision outcomes (Huettel et al. 2005). For example, Pavlou et al. (2007) highlighted uncertainties in the e-commerce context stemming from concerns about seller opportunism, information privacy issues, information security and which discourage consumers from committing to the decision of an online purchase. Thus, in order to diminish these uncertainties decision-makers turn to additional information obtained from the social environment, and this interaction that may affect decision-makers is referred to as social influence (Flache et al. 2017).

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Research on socially influenced decision-making can be applied to a wide range of real-world problems (Frederiks et al. 2015; Li et al. 2023; Matthews et al. 2007; Ni et al. 2021; Prasath and Yoganathen 2018; Sul et al. 2017). In marketing and advertising, such research can help marketers better understand consumer behaviour and design more effective marketing campaigns that resonate with consumers (Kim and Srivastava 2007). Similarly, it can help policymakers understand how social influence affects public opinion and their decision-making (Challa et al. 2018), thus leading to more effective public policies. However, the research from Du et al. (2019) can also show that ineffective use of decision analysis (such as ignoring decision bias) can have negative implications, for example, altering a firms' equilibrium strategies and profits.

Socially influenced decision-making has also been studied considering the embedding of the information in a social network (Berger et al. 2014; Jackson et al. 2017; Li et al. 2019; Ni et al. 2021). Here, social influence can be directly associated with the interactions (links or edges) among individuals (nodes) conforming a social influence network (Zhang et al. 2018). In this context, studies in economics (Jackson et al. 2017), finance (Allen et al. 2018; Li et al. 2019), healthcare (Ni et al. 2021; Xie et al. 2020), and information systems (Berger et al. 2014; Hong et al. 2017; Misllove 2009) have found that properties of the social network structures (patterns or regularities in relationships between individuals obtained directly from the network structural summaries) can be associated with changes on the decision-making behaviour. For example, Engel et al. (2021) constructed a network containing different economic actors (nodes) and where a link establishes whether an actor holds shares of another economic actor. This study used this shareholding network to illustrate the importance of understanding the ownership structure for investment decisions in a diversified portfolio. Ben-Shimon et al. (2007) formed a network of customers' online friendships and concluded that a product recommendation system informed by the preferences of a customer's personal social network can more effectively affect their online shopping decisions. However, existing research on socially influenced decision-making exhibits fragmentation across diverse domains as exemplified earlier. Therefore, there is a need for research that

comprehensively examines the role of network structures in shaping decision-making processes across diverse contexts.

The primary objective of this paper is to illustrate the significant effect of well-known mathematical network model structures on the general context of decision-making through an evidence-based socially influenced decision-making model. Particularly, this work focuses in three types of social networks that are associated with (a) independence, (b) preferential attachment, and (c) community structure. The analytical framework considers (I) the decision problem formulation, (II) the individuals' decision-making and (III) the process by which individuals are socially influenced and make decisions in the context of social networks. In more detail, for a given observed social network, the social influence to which individuals are exposed can be manifested through information diffusion. Therefore, the potential paths of information diffusion can reflect the process of individuals' opinion dynamics related to decision-making. In this work, the understanding of a decision problem for each individual is represented by a belief structure that takes into account uncertainty, rendering our analytical framework to be more applicable to real-world situations than traditional probabilistic representations. After receiving information from connected neighbours, individuals can update their belief structures on the decision problem based on a weighted Dempster's rule of combination. Lastly, our framework is applied considering a Monte Carlo simulation to further demonstrate the impact of network structures on socially influenced decision-making.

The paper is organised as follows. Section 2 reviews background and related literature in decision-making, followed by a further description of information diffusion among individuals and the description of the different network models considered. The overall population based socially influenced decision-making framework is developed in Sect. 3, which is divided and explained in three main steps. Section 4 validates the theoretical framework through a simulation-based analysis of the information diffusion process, and the impact of network structures is highlighted by comparing the outcomes of decision-making at the population level. Finally, the conclusions are discussed in Sect. 5 along with future research directions.

Literature review

Information diffusion is the process by which information is communicated through certain channels over time among the members of a social system. Here the social system refers to the patterned network of interrelationships existing in a coherent whole between individuals, groups, and institutions (Parsons and Shils 2017). Information diffusion therefore can reflect the opinion dynamics of individuals (Zhu et al. 2021). In social networks, individuals are now no longer simple receivers, they are also influencers (Razaque et al. 2019). Therefore, individuals demonstrate their important role in the information diffusion process over social networks by exchanging information with their neighbours. The social interactions that individuals form gradually develop over time, eventually gathering and leading to a large and complex social network (Al-Taie et al. 2017).

A network can be mathematically represented by a graph $G(\mathcal{V}, \mathcal{E})$, where \mathcal{V} is the vertex set whose elements can be called vertices or nodes, and \mathcal{E} is the set of interactions, links or edges. Depending on the nature of the relationship, networks or graphs can be directed or undirected. A graph is undirected when it contains relations that do not distinguish between where the edges originate. In contrast, a directed graph consists of relations between pairs of nodes where the start and end nodes are relevant to the considered interaction (Yang et al. 2016). An example of a directed social influence network is shown in Fig. 1, where the connections are not necessarily reciprocated. For example, node 3 influences node 1, but node 1 does not influence node 3. Many real-world relationships indeed form directed networks, such as seeking advice from someone or passing a message to others. In this way, the node sending the information can be called the sender or source node, and the node receiving the information can be called the receiver or target node.

In addition to edge directions, weights are also a frequently discussed property in real-world networks. Edges are usually associated with weights that illustrate the power, strength or capacity of the interaction (Barrat et al. 2004; Horvath 2011). Weighted networks have been widely studied because they can be used to demonstrate a diversity of phenomena (Amano et al. 2018). For example,

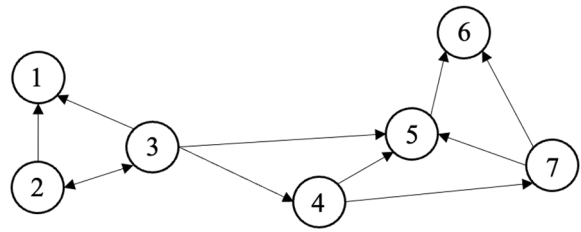


Fig. 1 An illustrative directed network

in infrastructure and information networks, weights represent the flow of materials and information such as energy, goods, and communications along that edge (Barrat et al. 2004; De Montis et al. 2007). In social networks, Granovetter (1973) argued that a weight can represent the strength of the social connections such as emotional intensity, closeness, or a quantification of the exchange of services. In addition, studies have also used weights to represent the degree of trust between nodes in the context of information diffusion through social networks (Arnaboldi et al. 2017; Li et al. 2017; Liu et al. 2019).

In large networks, summary statistics could be obtained to provide meaningful insight into the structure and dynamics of the network. The most common summary statistics are ‘degree’ (i.e., the number of links attached to a node), ‘betweenness’ (i.e., the number or proportion of times a node lies on the shortest path between all other nodes), and ‘density’ (i.e., the number of edges present divided by the total number of potential edges in the network). Summary statistics of the network structure provide indications of the degree to which individuals are interconnected. The pattern of these connections reflects the strength and boundaries of information flow (Himmelboim et al. 2017). For example, Zubcsek et al. (2014) noted that information is more likely to spread among closely connected individuals than loosely connected individuals. Himmelboim et al. (2017) further explained that the rate of interconnection among individuals and groups characterises networks and highlights the beneficial differences among them.

Individuals' decision dynamics in the context of information diffusion

Individuals spread and receive information in social networks. The effect of social influence occurs at the step where individuals gather information for decision-making (Lunenburg 2010). Each node (also called individual) is potentially influenced by its connected nodes.

In the decision-making process, uncertainty is an unavoidable characteristic (Fattahi and Govindan 2022; Hofstra et al. 2022). For example, Simon (1955) proposed the 'bounded rationality', which refers to the fact that individuals' rationality is constrained by some natural factors such as their intelligence and time. The unreliability of information and data is also a source that contributes to uncertainty in decision-making. This indicates that some critical information may not be available within a given time frame, while the collection of irrelevant or misleading information can also increase uncertainty in the decision-making process (Koppenjan et al. 2004). Therefore, when considering the formulation of the decision-making process, it is crucial to incorporate uncertainty in order to accurately reflect real-world decision situations.

Researchers have proposed a wide variety of frameworks for representing uncertainty in decision-making (Shafer 1976; Sugeno 1993; Yager et al. 1994). The choice of uncertainty representation indeed impacts the expressiveness and tractability of the decision models (Huynh and Yan 2021). Among all theories that have been proposed the more relevant ones are Probability Theory (DeGroot and Schervish 1975), Possibility Theory (Zadeh 1979), and Evidence Theory (Shafer 1976). Evidence theory is a more general framework that combines probability and possibility theory. It deals with situations where the available information is uncertain, and where there may be conflicts or contradictions in the evidence (Ferson et al. 2003). In contrast, probability theory and possibility theory are more restricted methods with roots in measure theory and fuzzy set theory (Sentz and Ferson 2002).

In the decision-making process, individuals' perceptions of decision problems are often reflected through belief structures, which are defined as a distributed assessment with beliefs (Shafer 1976). Then, in the information diffusion process,

individuals' belief structures are updated according to their received influences through the network. However, referring to the action of making a decision, the concept of belief boundary is presented as an evidence threshold value where once enough evidence is accumulated such that it surpasses the boundary, a corresponding decision will take place. This process is also the basic principle of the most widely used psychological description of decision-making (Drugowitsch et al. 2012; Li et al. 2022), i.e. once the degree of belief exceeds the boundary, individuals make a decision (Huang and Rao 2013; Thura et al. 2014; Zhao et al. 2016).

Information diffusion among individuals

Information diffusion in the context of social networks occurs through the interaction between individuals. Here, the interaction extends beyond traditional social ties such as friendship and occupational relationships because with the development of digital networked communications methods, the interactions may also take the form of links or associations between users on the Internet service platforms like Facebook, Twitter and others. Furthermore, additional relationships have emerged that are not bounded by traditional social factors, such as relationships that can be expressed as a parasocial; i.e. a one-sided relationship that an ordinary individual engages in with for example famous actors and athletes (Giles 2002). However, regardless of the type of relationships, social networks will always embody individuals' social relations in an integrated way (Baek et al. 2013). Hence, the 'interaction' stated in this study is regarded as the integrated connection between individuals in the context of information diffusion, which can capture different kinds of relationships.

Information diffusion models are developed to understand the mechanisms and factors that affect the spread of information, ideas, opinions, and behaviours in social networks. Two main categories of models have been proposed in the literature to explain the way information spreads in real-world scenarios. On one hand, predictive models have been applied to predict how a specific diffusion process would unfold in a given social network. The Independence Cascade model and the Linear Threshold model (Kempe et al. 2003) are two typical models in this category.

These models have been applied in the context of information diffusion in Twitter on the basis of users' past activity and evaluate the efficiency in information spreading (Morales et al. 2014). Similarly, epidemic models such as Susceptible-Infectious-Susceptible (SIS), Susceptible-Infectious-Recovered (SIS), and Susceptible-Infectious-Recovered-Susceptible (SIRS) are also models in this first category that were originally introduced to simulate the diffusion of disease, but different variations have been proposed to model information diffusion under different social environments (Leskovec et al. 2007; Liu et al. 2016; Nandi and Medal 2016). On the other hand, explanatory models have been developed to retrace the spreading path of information in social networks, and elucidate the factors that affect information diffusion. Importantly, influence models were typically developed in this category to analyse individual influences and community influences, in order to solve the practical problem of influence maximisation (Cosley et al. 2010; Kuikka 2018; Li et al. 2018).

Social network structure in information diffusion

Social networks provide an infrastructure for information diffusion. Researchers pay particular attention to the network topology analysis which investigates the structural properties of the network. For example, Sueur et al. (2011) showed that heterogeneous groups, characterised by high inter-individual variations in network measures, are less efficient to spread information than networks that leaned towards social connections of homogeneous groups. He et al. (2019) introduced a balanced structure on directed networks that could promote the magnitude and speed of information diffusion, eliminate path dependence, and lead to polarisation.

Research on the influence of social network structures in the information diffusion process also has many practical implications. For example, evaluating users' position and relevance within the network structure could identify influential spreaders (Cataldi et al. 2013; Kitsak et al. 2010), and this process could also be used as part of the solution to the influence maximisation problem mentioned in Sect. 2.2 (Kempe et al. 2003). Similarly, another practical application can be seen in source detection which could help prevent outbreaks of pandemics

(Shelke and Attar 2019; Zhu and Ying 2014) and trace the rumour source in social networks (Gundecha et al. 2013; Varshney and Vishwakarma 2021).

Different social network structures can be represented by various stochastic network models (de Arruda et al. 2018). In this work, we consider three well-known network models that reproduce the following social characteristics: independence, preferential attachment, and community structure. These models are:

(1) Erdős–Rényi model

This model considers a network where the probability of the connection between any two distinct nodes is always constant, i.e. the presence or absence of an interaction does not depend on any other interaction. Specifically, for any pair of nodes in the network, they are connected with probability p and are not connected with probability $1 - p$. The expected number of connections for each node is $p * (N - 1)$ where N is the total number of nodes (Erdos and Rényi 1960).

(2) Barabási–Albert model

Barabási and Albert (1999) introduced a growing network model to explain the presence of power-law (or scale-free) degree distribution in multiple real-world networks by considering growth and preferential attachment in the network formation process. Growth means new nodes incrementally join the already existing network over time, and the preferential attachment can be explained as the preference of new nodes to connect to nodes that already have a large number of links to others. In this model, $P(k)$ is the probability of any node having degree k and it is directly proportional to $k^{-\gamma}$, where γ is a quantity characterising the network topology. In the case of the Barabási–Albert model, the degree distribution follows a power law with degree exponent $\gamma = 3$. Generalisations of this model for other values of the power exponent γ are also available depending on different cases (Barabási and Albert 1999). In general, multiple real-world networks, in addition to social networks, have been found to follow scale-free models, such as biological networks (Koutrouli et al. 2020), financial networks (Souza and Aste 2019) and World Wide Web-based networks (Albert et al. 2000).

In this work, the direction of each edge is selected at random.

(3) Stochastic block model

The stochastic block model was first introduced in 1983 in the field of social networks by Holland et al. (1983). It can be considered as a two-step Erdős–Rényi model. Firstly, nodes are assigned to communities, and then edges between two communities and within communities are created in an Erdős–Rényi random manner with probabilities given by an edge probability matrix associated with the communities considered (Holland et al. 1983). This model aims to partition the set of nodes in the network in such a way that the proportion of edges between nodes in the same block/community can be higher than the proportion of edges connecting nodes in different blocks (Ludkin et al. 2018).

Research methodology

This work aims to better understand the effect of the social network structure on decision-making by simulating the information diffusion process at the population level under social networks. The population-level analysis involves characterising interactions between individuals in the social network. There are three key steps for analysing socially influenced decision-making: (1) problem formulation and decision modelling, (2) information diffusion in the network, and (3) consolidation of individuals' belief structures via information aggregation mechanisms. A summary of these steps and ideas is given in Fig. 2. The main steps of the analytical framework are explained in detail and in concrete terms in this section.

Decision problem formalisation and modelling

Evidence theory also referred to as Dempster–Shafer (D–S) theory (Shafer 1976), takes into account the uncertainty in the decision-making process. It is used to address the decision-making problem in this work. The belief function is used to represent an individual's attitudes towards different outcomes of a decision. Regardless of the type of initial data and the complexity of the decision problem, this theory can easily assign probabilities by using a basic probability

assignment (BPA), which is often interpreted in a broad sense as a piece of confidence associated with a certain hypothesis or class (Xu et al. 2013). The mathematical theory of evidence can be founded axiomatically on the notion of belief functions or on the allocation of belief masses to subsets of a frame of discernment (Kohlas and Monney 1994). Wang and Song (2018) also claim that evidence theory has broader applicability as it does not require prior knowledge in uncertainty reasoning.

Take for example the binary decision problem of whether to vaccinate or not, the frame of discernment is thus given as $\Omega = \{\text{accept}, \text{reject}\}$, 2^Ω is the power set of Ω which stands for the set of all subsets of Ω including the empty set \emptyset , and which elements correspond to all possible states of an individual decision:

$$2^\Omega = \{\{\text{accept}\}, \{\text{reject}\}, \{\text{yet to decide}(\Omega)\}, \{\emptyset\}\}. \quad (1)$$

Here, the uncertainty is included in the mathematical expression as a decision outcome 'yet to decide'(Ω). A belief mass $m \in [0, 1]$ is then assigned to each element of 2^Ω , such that the following conditions are met,

$$m(\emptyset) = 0, \quad \sum_{x \in 2^\Omega} m(x) = 1. \quad (2)$$

Information propagation through network

In the context of the social network, the connections and interactions between individuals provide the path of information diffusion. The information diffusion process considered for a directed network can be illustrated in Fig. 3, if it is assumed that the information propagation starts when a new piece of information is introduced to the social network, for example, node 3 in Fig. 3.

The propagation process considers the following two steps: (A) A node that receives information from any neighbouring node(s) tagged as a source node and updates its own information accordingly (the updating process is described in the next section). (B) Any tagged node propagates the updated information to all of its neighbours. These two steps are repeated until a stop criterion is met. In this work, the process is stopped when all nodes that can be reached from the node that started the propagation process have

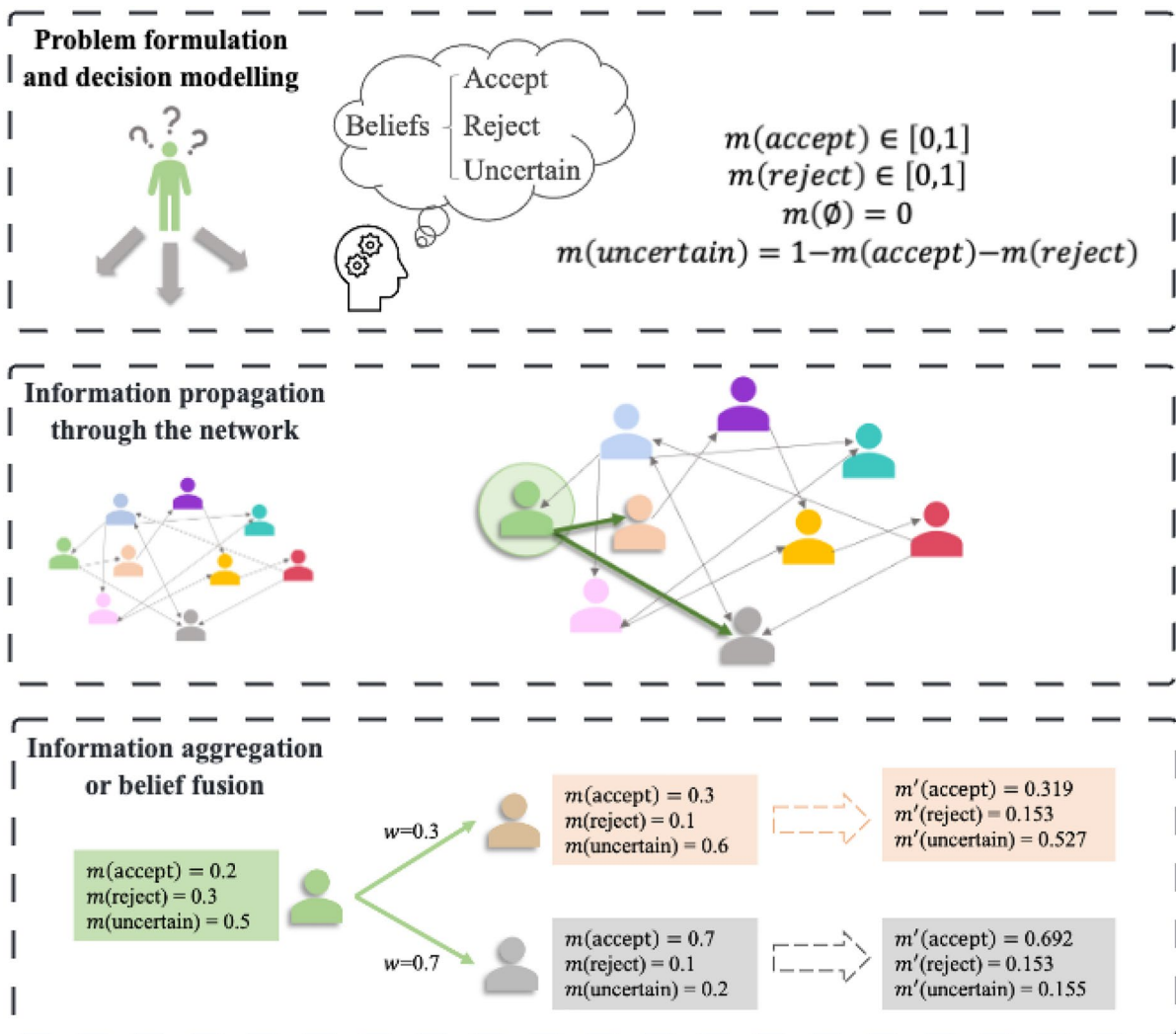


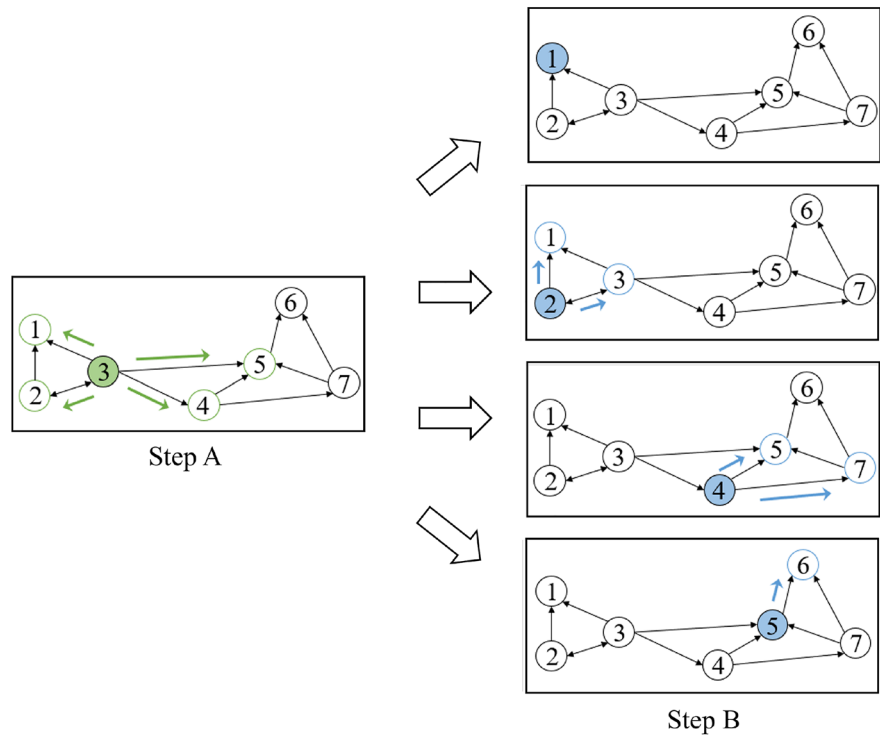
Fig. 2 Flowchart of the research methodology

received new information. In this process, starting from the initial source node, every iteration of the propagation is considered as a round which refers to one complete propagation step followed by one complete aggregation step.

Figure 3 shows just one iteration or one round. In Fig. 3, node 3 (highlighted in green) in step A is taken as the source node to start the information diffusion process. Node 3 thus influences nodes 1, 2, 4, and 5 (highlighted in blue). In this case, the beliefs of nodes 1, 2, 4, and 5 will be updated according to the received information (updating rule described

in the next section). Those four updated nodes then diffuse their beliefs to their target nodes in step B simultaneously. Note that in this example node 5 is influenced twice by this stage by node 3 first and now by node 4. Even though the information passed from node 4 to node 5 was already influenced by node 3, node 4 with updated beliefs is still considered as new information from node 5’s perspective. The diffusion process continues, and the influenced nodes become the new source nodes to spread the information to their target nodes.

Fig. 3 (Left) The initial propagation step from a unique starting source node (node 3). (Right) The subsequent propagation of information from new source nodes (i.e. neighbours of node 3)



Information aggregation or belief fusion between individuals

Here, individuals’ beliefs are considered to update according to the newly obtained evidence following

$$\begin{aligned}
 m_j^i(\text{accept}) &= m_i(\text{accept}) \cdot w_{ij}, \\
 m_j^i(\text{reject}) &= m_i(\text{reject}) \cdot w_{ij}, \\
 m_j^i(\text{uncertain}) &= m_i(\text{uncertain}) + (1 - w_{ij})(m_i(\text{accept}) + m_i(\text{reject})),
 \end{aligned}
 \tag{3}$$

Dempster’s rule of combination (Sentz and Ferson 2002). Given a weighted directed social network, with weights in $[0, 1]$ assessing the strength of the social influence, the multivariate mass functions for the decision states on nodes i and j are given by m_i

and m_j , respectively, and satisfy the conditions in Eq. (2). Due to the existence of weighted edges (w_{ij}), any evidence transmitted through the edge from node i and j is weighted. Thus, the correspondent weighted evidence mass functions sent by node i to node j are:

where $w_{ij} \in [0, 1]$ is the weight of the edge e_{ij} between node i and j . The extended Dempster’s rule used in this work could mitigate the deficiencies of the D-S theory in the combination of conflicting beliefs and which is criticised by Zadeh (1979).

The updated belief structure of node j after receiving information from node i following the extended Dempster’s rule is obtained by the two sets of masses m_j^i and m_j as

$$m'_j(x_1) = m_j(x_1) \oplus m_j^i(x_1) = \frac{1}{1-K} \sum_{x_2 \cap x_3 = x_1 \neq \emptyset} m_j(x_2) * m_j^i(x_3),$$

$$K = \sum_{x_2 \cap x_3 = \emptyset} m_j(x_2) * m_j^i(x_3),$$
(4)

where $x_1, x_2, x_3 \subseteq \{\text{accept, reject, uncertain}\}$, \oplus denotes the combination operator. Furthermore, it should be noted that Dempster’s rule of combination is an associated operator, thus leading to the same updated values of information in the target node regardless of the order in which the information is combined from multiple information sources (i.e. neighbours).

In order to better demonstrate the process of belief fusion, a numerical example of belief combination for node 5 from the illustrative network (Fig. 3) is shown in Fig. 4, in which node 5 receives weighted influences from nodes 3, 4 and 7, and its belief structure is updated to $\{m'_5(\text{ accept}), m'_5(\text{ reject}), m'_5(\text{ uncertain})\}$.

Simulation design

Our simulation setup considers two main elements: (1) considerations regarding the generation of random networks and (2) the overall simulation pipeline of the information diffusion process. The whole simulation design is conducted through R 4.3.1 with some particular packages such as igraph 2.0.2 network 1.18.2, DirichletReg 0.7–1, ggplot2 3.5.0, and dst 1.5.2. R itself and all dependent packages used

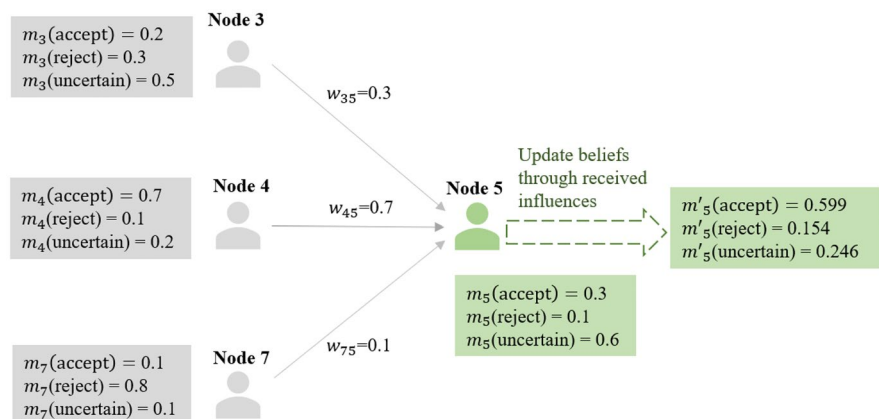
are available from the Comprehensive Archive Network (CRAN) at <https://CRAN.R-project.org>.

Networks generation

Three types of directed networks are used to evaluate the impact of the network structure on the overall decision outcome after the complete propagation of information through the generated social network. As mentioned in Sect. 2.3 the network models used are the Erdős–Rényi (ER) network, preferential attachment (PA) network, and stochastic block model (SBM) network. To ensure comparability of results, all generated networks have the same number of nodes, 1000, and have the same fixed average degree (the average number of edges per node in the network), 6.

The value of the average degree used in the network generations was based on observed values in real-world datasets such as Taro gift exchange (Schwimmer 1973), YouTube friendships (Guimera et al. 2003; Yang and Leskovec 2012), FilmTrust trust (Guo et al. 2016), and Hamsterster households (Kunegis 2013). Most related network datasets have an average degree between 4 and 8 with node sizes between 30 and 2000. Therefore, this work uses an average degree of 6.

Fig. 4 A numerical example of belief fusion



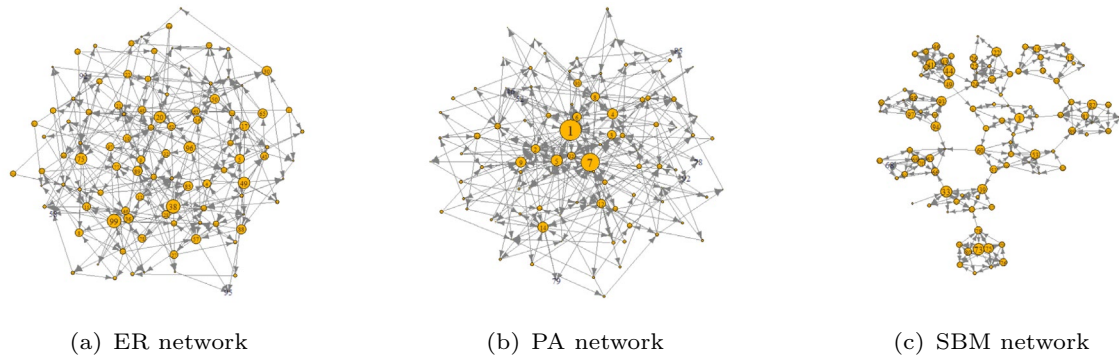


Fig. 5 Visualisation of three illustrative social network structures with 100 nodes

For the stochastic block model, due to its special community structure, two additional parameters need to be fixed, the number of communities/groups and the probability of creating edges between distinct communities. 10 communities were initially selected to ensure a moderate number of groups, neither too small nor too large. The number of edges between communities is thus fixed to be 5% of the total number of edges in the network. A value of 5% is used as it aligns with values seen in real-world datasets, such as friendship data (Choukas-Bradley et al. 2015) and Zachary's karate club (Girvan and Newman 2002).

It should be noted that the group sizes in the SBM are all the same in order to avoid the introduction of another feature apart from the group presence into the generated networks.

A visualisation of one generation of the three types of network structures is shown in Fig. 5.

In networks shown in Fig. 5, the yellow circles stand for nodes or individuals, and the links between nodes represent individuals' interactions in terms of sending or receiving information and influence. The arrow implies the direction to which information and influence flow, and the size of nodes is determined by the number of outward links (i.e., out-degree). Here, different characteristics of networks can also be observed. Specifically, in the ER network where there is no obvious pattern; the PA showing a preference of highly connected nodes at the centre and which act as the pivot of the whole network; and the SBM network which shows a clear view of 10 highly connected groups or communities. The simulation of the diffusion process for any of the networks is shown below.

Simulation of the information diffusion process

The main purpose of this paper is to find the general influence of social network structure on decision-making. Thus, the social network applied in this work is not constrained to physical or online interactions, it is instead considered to be a representation of general relationships among individuals. Therefore, a wide range of situations can be fit into our work albeit with specific considerations. For example, the weight distribution of each edge can be changed to introduce different types of relationships. Alternatively, the initial belief structure can adapt to the specific case and align with the given community structure. Moreover, individuals' belief boundaries can be personalised according to the different characteristics of nodes given by additional information.

The decision scenario applied in this simulation is a binary decision problem, for example, whether to vaccinate or not. This experimental study will start to simulate the information diffusion process under a social network generated from one of the three network models. The basic assumption of information diffusion through these networks is that each node of the network is influenced by its neighbours, and the belief fusion is performed through a consistent aggregation mechanism (the weighted Dempster's combination rule). The flow chart of the entire simulation of the information diffusion process under a given network is thus shown in Fig. 6.

Once a network is generated, the information propagation simulation starts by randomly assigning initial belief values that follow a Dirichlet distribution to all the nodes so that the sum of values of mass

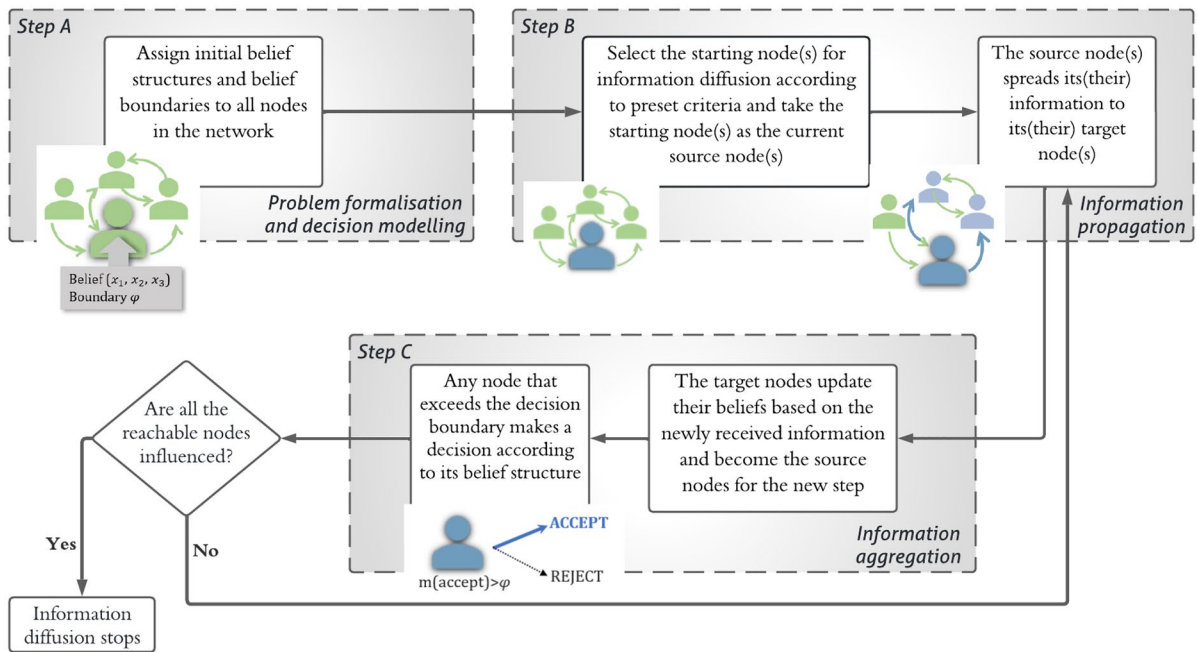


Fig. 6 Flowchart of the simulation process under a network

functions satisfies the condition of summing to one. Each node is also randomly assigned a unique belief boundary which is uniformly distributed between 0.6 and 1.

In the information diffusion process, the starting node is selected at random (although two additional situations are also used, and described below) and assigned a piece of positive information with a belief structure (1,0,0). The starting node, as the current source node, spreads the positive information to its target nodes, which then update their beliefs based on the newly received information and become the new source nodes. During the process, nodes (individuals) make a decision when their belief boundaries are exceeded. Diffusion of information continues until all the reachable nodes from the starting node are influenced at least once. It should be noted that the belief updating process is not a fully instantaneous process, and it is an evolving process where the beliefs of each node are constantly influenced throughout the network until the diffusion stops.

In the above diffusion process described, two other additional simulations are considered with respect to the selection of the starting node. The first additional

case selects the starting node as the most central node according to betweenness centrality (betweenness-based selection), and the second additional case selects the node with the highest number of outer links to other nodes in the network (out-degree-based selection).

During the information diffusion process, the decision outcomes of each node are recorded. A node that has already decided on ‘accept’ will not change the outcome of that decision regardless of how its belief structure is affected later. This diffusion process, as given in Fig. 6, is repeated 1000 times and each time with a new generated network, initial belief structures and belief boundaries are generated in order to remove any potential bias introduced by the initial observed graph, belief structures or belief boundaries assignment. The result is assessed by comparing the average percentage of outcomes on decisions ‘Accept’, ‘Reject’ and ‘Yet to decide’ after 1000 simulations and their converging trends on different decision outcomes over time.

From these processes, we obtained results for a total of 9 scenarios composed of the combinations of three network structures and three starting node selection criteria.

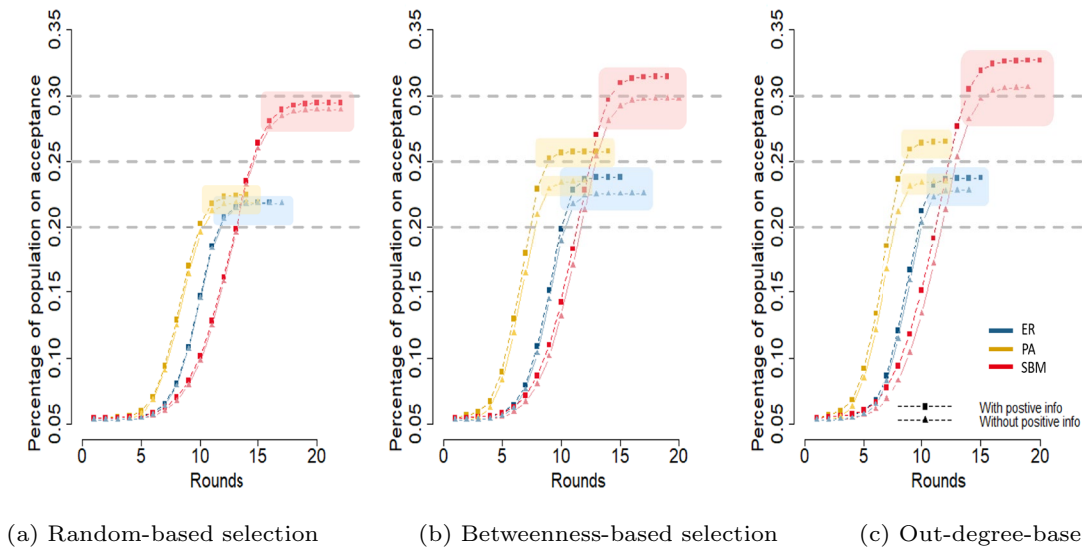


Fig. 7 The average percentage (%) of decision outcomes on acceptance in each round for different networks with three scenarios of the selected starting node. The shape represents decision outcomes on whether to include the initial positive information intervention. The plateau behaviour shown in

the shaded area does not represent decision convergence, but rather represents the current value of acceptance rates. Horizontal lines stand for the acceptance rate of 0.2, 0.25 and 0.3

Analytical results

The simulation described above follows a binary decision problem in, for example, the context of vaccination where the general outcome is whether to vaccinate or not. Individuals who initially fail to make a firm decision can receive evidence from their social neighbours as well as diffuse their beliefs to others via social connections. The average of 1000 simulations of the percentage acceptance outcome (vaccinate) on the different network structures and starting nodes are shown in Fig. 7. Here, the x-axis stands for the diffusion rounds, and the y-axis stands for the average acceptance (vaccinate) rate. Figure 7 also shows the average acceptance rates when there is no intervention on the starting node (triangles), i.e. the expected outcome that would be seen if the starting node keeps its own belief structure instead of the assigning (1,0,0) as its initial belief. Lastly, it should also be noted that the plateau behaviour, highlighted by the enclosed regions in Fig. 7, does not represent decision convergence but rather represents a stable value of acceptance rates of the diffusion process when it reaches the stopping criteria. Note that as the simulation considers multiple replicates, each

replicate may reach the stopping criteria at a different round. This means that if the propagation of influence were to continue, even after all reachable nodes in the networks were influenced at least once, the acceptance rate could continue rising according to its previous trend for an additional number of rounds.

As the simulation stops when all reachable nodes have received new information, instead of when the system reaches a steady state, the final average percentages of decision outcomes on acceptance for three network structures are roughly between 22 and 33%. These values show, in addition and as expected, a higher acceptance rate than when no positive information is introduced on the starting node. For example, for ER-random starting point, the ratio of acceptance (Fig. 8) changes by 1%. In contrast, for PA-highest out-degree base selection, the ratio of acceptance changes by 12 percentage points.

Overall for a fixed number of rounds, Fig. 7 shows that the PA networks obtain a higher population acceptance outcome quicker than both ER and SBM networks. PA networks as well as ER networks, show a similar and much faster (8–16 rounds) initial spread of the information across the networks than SBM networks (14–24 rounds) regardless of the

Fig. 8 The ratio in the average acceptance rate of simulations with positive intervention over no intervention. Selection scenarios on the starting node are **a** Random-based selection, **b** Betweenness-based selection, and **c** Out-degree-based selection

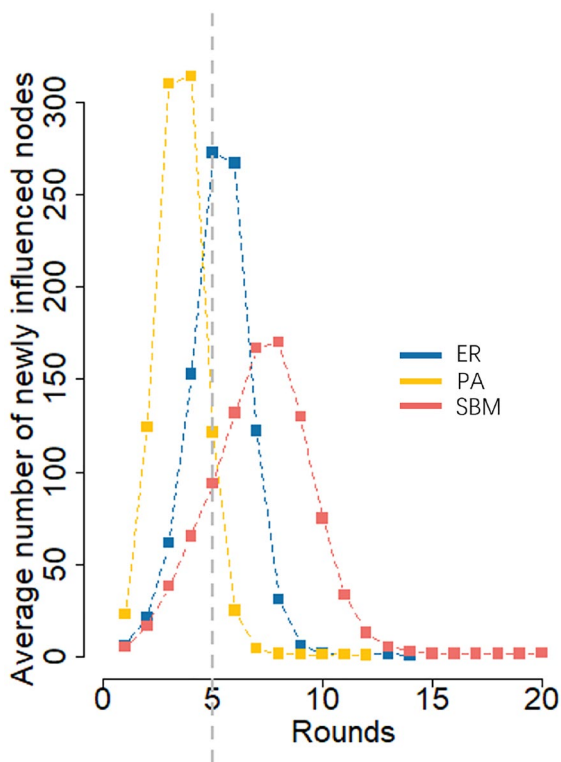
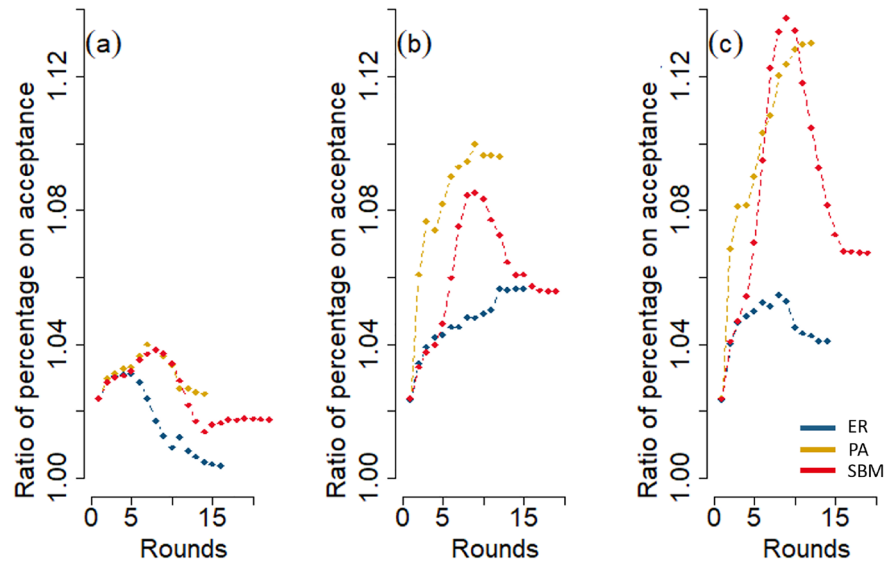


Fig. 9 Average number of newly influenced nodes in each round for different network structures with the starting node chosen at the highest betweenness. The grey vertical dashed lines represent the result on round 5. Similar figures are found for the other scenarios

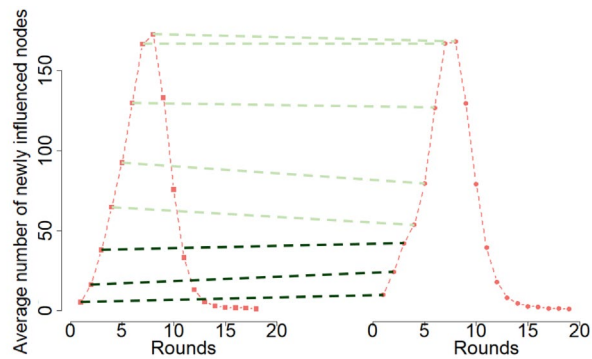
starting node considered. These outcomes show that social networks with a predominant preferential attachment structure lead to a larger and faster impact from new information coming into the system than social networks with a strong community structure component or with a completely at random structure.

Among the 9 simulation scenarios, networks with community structures (SBM) showed the slowest diffusion of new positive information and highlighted the difficulty in reaching a larger overall outcome than the other structures on a fixed number of rounds (Figs. 7 and 9).

From both Figs. 7 and 8, it can be seen that the network structure plays an important role in the outcome and overall process of socially influenced decision-making. Furthermore, both figures, and particular Fig. 8 also highlight the impact of the way new information is introduced. Selecting a random node to introduce the information, leads to the smaller impact in the overall decision outcome across networks compared with selecting the most central individual (Fig. 8b) or the individual with the largest outward social connections in the networks (Fig. 8c).

Figure 8 also provides an important distinction on the diffusion process occurring on the social networks with strong community structure (i.e. SBM networks). As it can be seen, the round times for SBM networks achieve a peak acceptance ratio similar to PA networks. However, in contrast to the two other social network structures, this peak rapidly

Fig. 10 Average number of newly influenced nodes in each round for SBM networks with the selected starting node based on **a** the highest betweenness and **b** the highest out-degree. The dashed lines connect the average number of newly influenced nodes in both selections from round 1 to round 8



(b) Betweenness-based selection

(c) Out-degree-based selection

decreases in SBM networks, and thus hints towards a moment in the diffusion process where either the new information has been diluted and therefore is not able to drive a significant change on the belief structures of the different communities; or alternative belief structures in communities not yet influenced could have gain enough strength (aided by potential echo chambers that are product of the community structure) to oppose the new information.

Discussion

The results from this simulation analysis although constrained in their generalisation can nonetheless provide potential practical real-world applications. For example, in the process of promoting vaccination in social environments, the proposed framework could evaluate population-level outcomes with respect to vaccination campaigns (new information), and to evaluate the most favourable method and platform of dissemination of the vaccination campaign. Furthermore, the approach becomes even more relevant when considering promoting vaccination information on social media. In this case, both the urgency and the budget of the campaign can be jointly considered. In terms of urgency, public health officials can identify the top influencers (accounts) with many followers (nodes with the highest out-degree) to achieve the maximum effect of promotion in the shortest time. However, as top influencers could be inevitably costly, accounts (nodes) with high betweenness centrality, such as accounts that have joined many Facebook groups can be selected to promote vaccination information at a slightly slower pace but still achieve maximum influence (Fig. 10) and possibly using a smaller budget.

Conclusion

This work analysed the independent impact of the network structure on socially influenced decision-making. An analytical framework from problem formulation to empirical simulation was proposed to model the information diffusion process under three potential social network structures. In the framework, individuals update their beliefs towards decisions based on the evidence they receive from their social neighbours. The uncertainty present in socially influenced decision-making and the subjective judgements of individuals are all included and represented by belief functions. In order to establish how the information from a source individual changes the information or beliefs of another, the weighted Dempster's rule of combination is used to implement evidence aggregation under uncertainty. Thus, this framework considers an individual's decision-making process in the population as a dynamic process, starting from belief formation and going through a series of belief adjustments and evidential reasoning, and finally leading to a decision if there is sufficient evidence.

The simulation-based experiment conducted in this work reveals the effect of three network structures on socially influenced decision outcomes at the population level and includes insights regarding the speed of the information diffusion or the effectiveness of the information in a given structure. In addition, this work highlights relevant differences with respect to the selection of the starting node and the consequences for the 'speed' of the information diffusion process.

The structural characteristics of social networks can be further modelled and parameterised to solve

practical problems, such as influence maximisation and recommendation systems. In more detail, active action could be taken to enhance the structure of real networks in key strategic places (e.g. bridging of communities) in order to obtain either a higher impact or faster propagation of the information. In addition, real-world networks can be applied to illustrate the efficacy of the proposed theoretical framework in the future work.

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Declarations

Conflict of interest The authors declare no Conflict of interest.

Appendix 1

See Figs. 11, 12 and 13

Fig. 11 Average number of times that nodes are influenced in each round for different network structures with the remaining two scenarios of the selected starting node. The grey vertical dashed lines represent the result on rounds 10 and 15

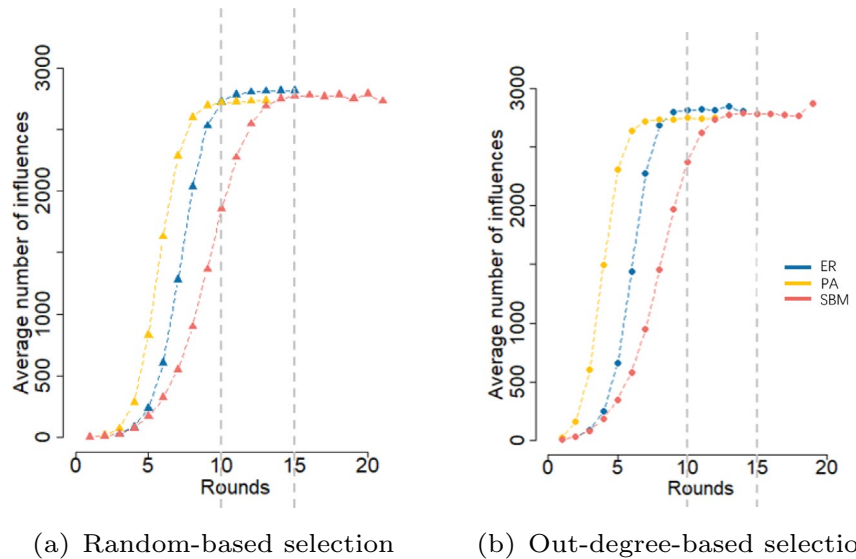
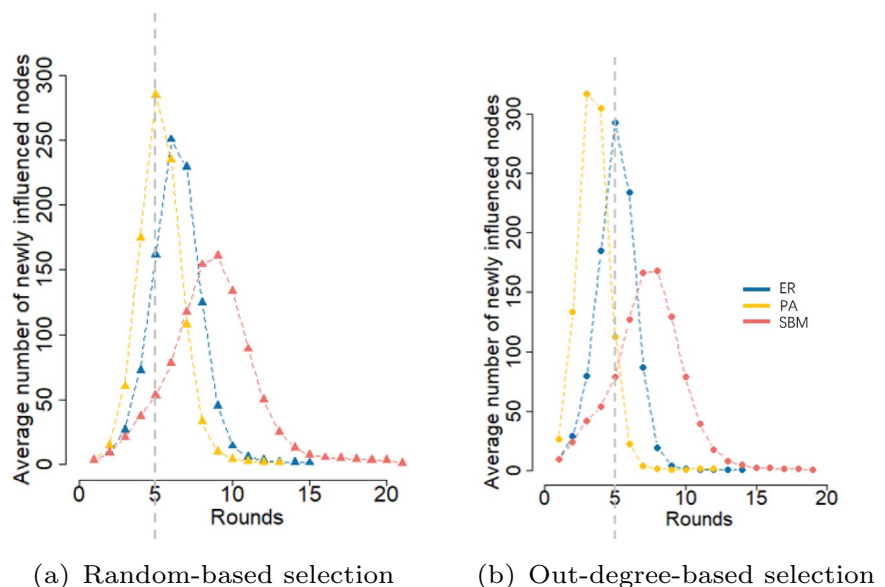


Fig. 12 Average number of newly influenced nodes in each round for different network structures with the remaining two scenarios of the selected starting node. The grey vertical dashed lines represent the result on round 5



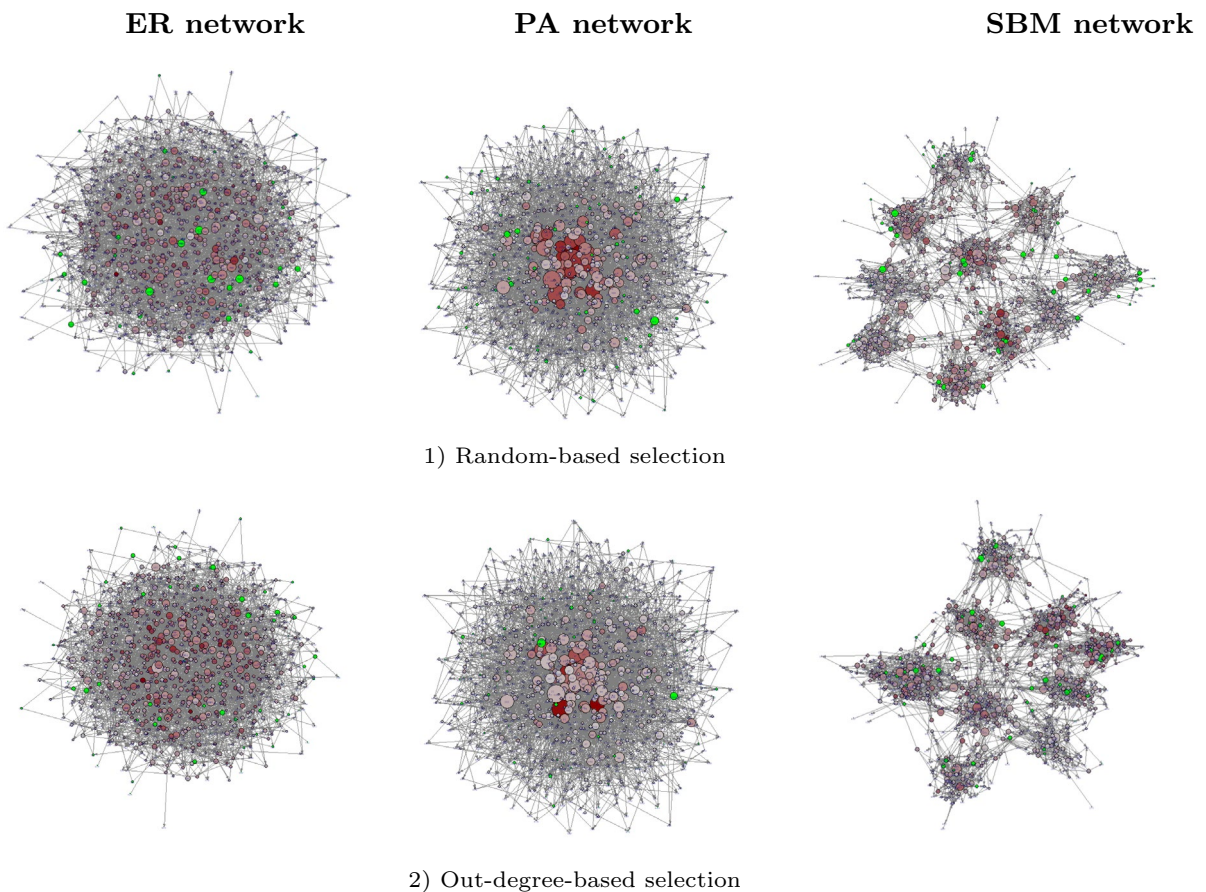


Fig. 13 Visualisation of the diffusion in different social networks with the remaining two scenarios of the selected starting node. The size of nodes is determined by their out-degree

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