Operation of natural gas and electricity networks with line pack

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Abstract This paper addresses the coordinated operation of the electricity and natural gas networks considering the line pack flexibility in the natural gas pipelines. The problem is formulated as a mixed integer linear programming problem. The objective is to minimize the operation cost of the electricity and natural gas networks considering the price of the natural gas supply. Benders decomposition is used to solve the formulated problem. The master problem minimizes the startup and shutdown costs as well as the operation cost of the thermal units other than the gas-fired generation units in the electricity network. The first subproblem validates the feasibility of the decisions made in the master problem in the electricity network. And if there is any violation, feasibility Benders cut is generated and added to the master problem. The second subproblem ensures the feasibility of the decisions of the master problem in the natural gas transportation network considering the line pack constraints. The last subproblem ensures the optimality of the natural gas network operation problem considering the demand of the gas-fired generation units and line pack. The nonlinear line pack and flow constraints in the feasibility and optimality subproblems of natural gas transportation network are linearized using Newton-Raphson technique. The presented case study shows the

effectiveness of the proposed approach. It is shown that leveraging the stored gas in the natural gas pipelines would further reduce the total operation cost.

Keywords Benders decomposition, Line pack flexibility, Natural gas transportation network, Unit commitment

1 Introduction

The environmental considerations to reduce the greenhouse gas generation, the reduction in building time and investment cost of combined-cycle generation units, the increase in the installed capacity of renewable generation, and the emergence of shale gas promote gas-fired generation (GFG) technology in the bulk power networks [1, 2]. In some countries, natural gas remains as the primary source for the electricity generation [3]. GFG is expected to provide 33% and 34% of the total energy demand in the U.S. in 2018 and 2019, respectively [4]. The increase in the installed capacity of this technology highlights the essence of capturing the interdependence among the electricity and natural gas networks.

Considering the interconnection among the electricity and natural gas networks, the deficiency in natural gas supply could impose risks on the electricity supply adequacy in the bulk power networks. The outages in natural gas pipelines and severe weather conditions could impede the normal operation of the natural gas network and further jeopardize the security and reliability of the electricity networks by mitigating the available GFG capacity. Effective coordination among the electricity and natural gas networks, diversifying the fuel resources, and incorporating effective load shedding strategies could improve

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the reliability and security of the electricity network when exposed to such contingencies.

Several research works focused on the coordinated operation of the electricity and natural gas networks. The proposed short-term operation framework in [5] addresses the interdependence among the electricity and natural gas networks by incorporating the natural gas network constraints into the security-constrained unit commitment problem. A two-stage nonlinear optimization model for the integrated operation of natural gas and the electricity networks is proposed in [6]. At the first stage, a mixed integer linear programming (MILP) problem is formulated to determine the direction of the natural gas flow. And in the second stage, the maximum electric power generated by GFG units in the bulk power network is determined by formulating a nonlinear programming problem knowing the direction of the natural gas flow from the first stage. The intra-day interaction among the electricity and natural gas networks is addressed in [7] by developing a continuous time optimal power flow formulation that incorporates the dynamic optimal gas flow as well as the real-time GFG consumption in the natural gas transportation network. The electricity and natural gas flows are optimized considering the time-variable gas demand. While such research efforts were focused on the steady-state and dynamic operations of the natural gas network, limited attention is dedicated to the line pack flexibility in the natural gas pipelines. Line pack allows for temporarily storing natural gas in the pipeline by regulating the pressure at the input and output of the pipeline. It is discussed in [8] that the line pack contributes to the reduction of the system operation cost as natural gas could be stored at periods with lower price and utilized once the price increases. Furthermore, the line pack provides significant value to GFGs by leveraging the difference between the spatial and temporal prices of natural gas supply and provide the flexibility for the natural gas transportation networks [9].

The line pack in pipelines could be used as a tool by the gas transmission system operator (GTSO) to improve the reliability and security of the natural gas transportation network as the stored gas in the pipelines could compensate for the disturbance in the natural gas injection and withdrawal balance [10]. In [11], the security-constrained optimal power and gas flow is formulated as an MILP problem to determine the stabilized operation of the interconnected network exposed to contingencies. While the storage is considered as an asset of the natural gas network, the line pack is ignored in the presented formulation. Benders decomposition and linearization techniques are used in [12] to solve the security constrained unit commitment considering the dynamic natural gas constraints. However, the economic benefits of line pack flexibility in the natural gas network are ignored and the approximations are used to calculate the stored mass of natural gas in the pipeline.

While the compressibility and gas travel velocity is considered in [13], linearization techniques are used to solve the integrated operation of the electricity and natural gas networks while ignoring the impact of intake natural gas compression of GFG units on their generated output power and the operation cost of the electricity network. The contributions of this paper are as follows:

The coordination among natural gas and the electricity is presented in the day-ahead operation in which Benders decomposition is used to decompose the problem into feasibility and optimality subproblems for the electricity and natural gas networks.

The impact of line pack in the natural gas pipelines on the operation cost of the natural gas and the electricity network is addressed.

Newton-Raphson (NR) technique is used to handle the nonlinear line pack and natural gas flow constraints in the natural gas feasibility and optimality subproblems.

The paper is organized as follows. Section 2 presents the integrated electricity and natural gas operation problem which includes the unit commitment (UC), economic dispatch (ED) and optimal flow in the natural gas network. Section 3 provides a solution algorithm that captures the autonomous operation of the electricity and natural gas networks. Benders decomposition and NR linearization techniques are used to solve the feasibility and optimality of the electricity and natural gas network operation. Section 4 presents the case study to validate the proposed solution methodology and evaluate the impact of line pack flexibility on the electricity and natural gas network operation. Section 5 summarizes the outcomes and presents the conclusion of the paper.

2 Problem formulation

The problem formulation is shown in (1)–(49). The short-term operation problem in the bulk power network is presented as UC and ED problems. As the electricity network is coupled with the natural gas network, the proposed problem should further capture the natural gas transmission network constraints. The objective function (1) minimizes the operation cost of the electricity and natural gas transportation networks.

$$\begin{aligned} \min_{P, u, \pi, v, L, H} Z = & \sum_i \sum_t [SU_{i,t} + SD_{i,t}] \\ & + \sum_c \sum_t [SU_{c,t} + SD_{c,t} + F_{c,t}(P_{c,t}) \cdot cp_{c,t}] \\ & + \sum_s \sum_t v_{s,t} \cdot gp_{s,t} \end{aligned} \quad (1)$$

where P, u, π, v, L, H are the vectors of generation unit

dispatch, generation unit commitment, nodal gas pressure, gas supply volume, gas load, and compressor power consumption, respectively; $SU_{i,t}$ and $SD_{i,t}$ are the startup and shutdown costs of the gas-fired units at time t , respectively; $SU_{c,t}$ and $SD_{c,t}$ are the startup and shutdown costs of the coal-fired units at time t , respectively; $P_{c,t}$ is the generation dispatch for a coal-fired unit at time t ; $F_{c,t}$ is the required thermal energy for a coal-fired unit (MBTU) at time t ; $cp_{c,t}$ is the fuel price for coal-fired unit; and $gp_{s,t}$, $v_{s,t}$ are the gas price and gas supply volume of gas supplier s at time t , respectively. The constraints include the generation unit constraints, the electricity network constraints and the natural gas network constraints [14]. The demand and supply balance in the electricity network is enforced by (2).

$$\sum_i P_{i,t} + \sum_c P_{c,t} = De_t \quad (2)$$

where $P_{i,t}$ is the generation dispatch for a GFG unit at time t ; and De_t is the total the electricity demand of the system at time t . The constraints of GFG units are shown in (3)–(15).

$$F_{i,t}(P_{i,t}) = a_i P_{i,t}^2 + b_i P_{i,t} + c_i \quad (3)$$

$$P_{\min,i} u_{i,t} \leq P_{i,t} \leq P_{\max,i} u_{i,t} \quad (4)$$

$$y_{i,t} - z_{i,t} = u_{i,t} - u_{i,(t-1)} \quad (5)$$

$$SU_{i,t} = csu_i \cdot y_{i,t} \quad (6)$$

$$SD_{i,t} = csd_i \cdot z_{i,t} \quad (7)$$

$$0 \leq su_{i,t} \leq MN_i \cdot u_{i,t} \quad (8)$$

$$(MN_i + 1)u_{i,t} - MN_i \leq su_{i,t} - su_{i,(t-1)} \leq 1 \quad (9)$$

$$su_{i,t} \geq T_{on,i} \cdot z_{i,t} \quad (10)$$

$$0 \leq sd_{i,t} \leq MF_i (1 - u_{i,t}) \quad (11)$$

$$1 - (MF_i + 1) \cdot u_{i,t} \leq sd_{i,t} - sd_{i,(t-1)} \leq 1 \quad (12)$$

$$sd_{i,t} \geq T_{off,i} y_{i,(t+1)} \quad (13)$$

$$P_{i,t} - P_{i,(t-1)} \leq UR_i (1 - y_{i,t}) + P_{\min,i} y_{i,t} \quad (14)$$

$$P_{i,(t-1)} - P_{i,t} \leq DR_i (1 - z_{i,t}) + P_{\min,i} z_{i,t} \quad (15)$$

The thermal energy required to produce the electricity by a GFG unit is determined using (3), where a_i , b_i , c_i are the empirical parameters of gas-fired unit. The power dispatch limits for the GFG unit is imposed by (4), where $P_{\min,i}$, $P_{\max,i}$ are the minimum and maximum capacity of gas-fired unit, respectively. The relationship between startup and shutdown indicators and commitment states are shown in (5), where $y_{i,t}$, $z_{i,t}$ are the startup and shutdown indicators of a gas-fired unit at time t , respectively; and $u_{i,t}$ is the commitment status of a gas-fired unit at time t . The startup and shutdown costs for a

GFG unit are formulated as (6) and (7), where csu_i and csd_i are the startup and shutdown costs, respectively. The number of hours that a GFG unit is on or off is determined using (8)–(13), where, $su_{i,t}$, $sd_{i,t}$ are the startup and shutdown time of gas-fired unit; MN_i is the minimum hours that a gas-fired unit is on; MF_i is the maximum hours that a gas-fired unit is off; $T_{off,i}$ is the minimum off time of gas-fired units; and $T_{on,i}$ is the minimum on time of gas-fired units. The minimum up time and down time are enforced by (10) and (13), respectively. The limits for ramping of a GFG unit are enforced by (14) and (15), where UR_i is the ramping up rate limit, and DR_i is the ramping down rate limit of the gas-fired unit. Similar constraints are considered for the coal-fired units as shown in (16)–(28).

$$F_{c,t}(P_{c,t}) = a_c P_{c,t}^2 + b_c P_{c,t} + c_c \quad (16)$$

$$P_{\min,c} u_{c,t} \leq P_{c,t} \leq P_{\max,c} u_{c,t} \quad (17)$$

$$y_{c,t} - z_{c,t} = u_{c,t} - u_{c,(t-1)} \quad (18)$$

$$SU_{c,t} = csu_c \cdot y_{c,t} \quad (19)$$

$$SD_{c,t} = csd_c \cdot z_{c,t} \quad (20)$$

$$0 \leq su_{c,t} \leq MN_c \cdot u_{c,t} \quad (21)$$

$$(MN_c + 1)u_{c,t} - MN_c \leq su_{c,t} - su_{c,(t-1)} \leq 1 \quad (22)$$

$$su_{c,t} \geq T_{on,c} z_{c,t} \quad (23)$$

$$0 \leq sd_{c,t} \leq MF_c \cdot (1 - u_{c,t}) \quad (24)$$

$$1 - (MF_c + 1)u_{c,t} \leq sd_{c,t} - sd_{c,(t-1)} \leq 1 \quad (25)$$

$$sd_{c,t} \geq T_{off,c} y_{c,(t+1)} \quad (26)$$

$$P_{c,t} - P_{c,(t-1)} \leq UR_c (1 - y_{c,t}) + P_{\min,c} y_{c,t} \quad (27)$$

$$P_{c,(t-1)} - P_{c,t} \leq DR_c (1 - z_{c,t}) + P_{\min,c} z_{c,t} \quad (28)$$

where a_c , b_c , c_c are the empirical parameters of coal-fired units; $P_{\min,c}$, $P_{\max,c}$ are the minimum and maximum capacity of coal-fired units, respectively; $y_{i,t}$, $z_{i,t}$ are the startup and shutdown indicators of a coal-fired unit at time t , respectively; and $u_{c,t}$ is the commitment status of a coal-fired unit at time t ; csu_c and csd_c are the startup and shutdown costs of coal-fired units, respectively; $su_{c,t}$, $sd_{c,t}$ are the startup and shutdown time of coal-fired units; MN_c is the minimum hours that a gas-fired unit is on; MF_c is the maximum hours that a coal-fired unit is off; $T_{off,c}$ is the minimum off time of a coal-fired unit; and $T_{on,c}$ is the minimum on time of a coal-fired unit; UR_c is the ramping up rate limit of coal-fired units, and DR_c is the ramping down rate limit of the gas-fired unit.

The nodal power balance is enforced by (29).

$$E_{p,b} \cdot p_{b,t}^f = C_{p,i} P_{i,t} + C_{p,c} P_{c,t} - D_{p,l} P_{l,t} \quad (29)$$

where $C_{b,i}$ is the element of bus-gas fired generator incidence matrix; $C_{b,c}$ is the element of bus-coal fired generator incidence matrix; $D_{p,l}$ is the element of the bus-load incidence matrix; $E_{p,b}$ is the element of the bus-line incidence matrix; $p_{b,t}^f$ is the power flow in line b at time t ; and $P_{l,t}$ is the demand of the electricity load l .

The power flow in each line is limited by (30).

$$p_{b,t}^{f,\min} \leq p_{b,t}^f \leq p_{b,t}^{f,\max} \quad (30)$$

where $p_{b,t}^{f,\min}$ and $p_{b,t}^{f,\max}$ are the minimum and maximum power transmitted through line b .

The power flow in a line is determined by the difference between the voltage angles at the connected buses at two sides of the line and the impedance of the line as shown in (31).

$$p_{b,t}^f = \frac{\theta_{p,t} - \theta_{q,t}}{x_{p,q}^b} \quad (31)$$

where $\theta_{p,t}$ is the voltage angle at bus p at time t ; and $x_{p,q}^b$ is the inductive reactance of line b between buses p and q . The voltage angle for the reference bus is zero as enforced by (32).

$$\theta_{slack} = 0 \quad (32)$$

The natural gas network constraints are presented in (33)–(49).

$$\begin{aligned} h_m(\pi, v, L, H) = & \sum_s A_{m,s} v_{s,t} - \sum_j B_{m,j} L_{j,t} - \sum_{n \in GC(m)} (f'_{m,n,t} \\ & - f''_{m,n,t}) - \sum_r F_{m,r} G_r(H_{r,t}) \\ = & 0 \end{aligned} \quad (33)$$

$$V_{lp_{m,n,t}} = V_{lp_{m,n,t-1}} + f'_{m,n,t} - f''_{m,n,t} \quad (34)$$

$$f_{m,n,t} = \frac{f'_{m,n,t} + f''_{m,n,t}}{2} \quad (35)$$

$$V_{lp_{m,n,t}} = C_{m,n}'' \frac{2}{3} \left(\frac{\pi_{m,t}^3 - \pi_{n,t}^3}{\pi_{m,t}^2 - \pi_{n,t}^2} \right) \quad (36)$$

$$V_{stock_{m,n,t}} = V_{lp_{m,n,t}} - V_{lp_{m,n,t-1}} \quad (37)$$

$$f_{m,n,t} = \text{sgn}(\pi_{m,t}, \pi_{n,t}) C_{m,n}' \sqrt{|\pi_{m,t}^2 - \pi_{n,t}^2|} \quad (38)$$

$$\text{sgn}(\pi_{m,t}, \pi_{n,t}) = \begin{cases} 1 & \pi_{m,t} \geq \pi_{n,t} \\ -1 & \pi_{m,t} < \pi_{n,t} \end{cases} \quad (39)$$

$$f_{n,t} = \text{sgn}(\pi_{m,t}, \pi_{n,t}) \frac{H_{r,t}}{k_{2,r} - k_{1,r} \left[\frac{\max(\pi_{m,t}, \pi_{n,t})}{\min(\pi_{m,t}, \pi_{n,t})} \right]^{\alpha_r}} \quad (40)$$

$$v_s^{\min} \leq v_{s,t} \leq v_s^{\max} \quad (41)$$

$$\pi_m^{\min} \leq \pi_{m,t} \leq \pi_m^{\max} \quad (42)$$

$$H_r^{\min} \leq H_{r,t} \leq H_r^{\max} \quad (43)$$

$$R_r^{\min} \leq \frac{\max(\pi_{m,t}, \pi_{n,t})}{\min(\pi_{m,t}, \pi_{n,t})} \leq R_r^{\max} \quad (44)$$

$$G_r(H_r) = a_r^2 H_{r,t} + b_r H_{r,t} + c_r \quad (45)$$

$$L_{j,t} = \rho G_{j,i} F_{i,t}(P_{i,t}) \quad (46)$$

$$f_{m,n,t} = -f_{n,m,t} \quad (47)$$

$$f'_{n,m,t} = -f''_{n,m,t} \quad (48)$$

$$f'_{m,n,t} = -f''_{m,n,t} \quad (49)$$

The natural gas flow in the pipelines is dependent on the length and diameter of the pipeline, temperature, the pressure of the nodes, the type of natural gas, altitude change and the friction of pipelines [13]. However, most of the above parameters are fixed in the daily operation of natural gas transportation network. The nodal supply and demand balance in natural gas network is enforced by (33), where $A_{m,s}$ is the element of node-gas supplier incidence matrix; $B_{m,j}$ is the element of node-gas load incidence matrix; $f'_{m,n,t}$ is the natural gas inflow from node m to n at time t ; $f''_{m,n,t}$ is the natural gas outflow from node m to n at time t ; $F_{m,r}$ is the element of the node-gas compressor incidence matrix; $L_{j,t}$ is the gas volume of gas load j at time t ; and $H_{r,t}$ is the power output of the compressor r at time t . The line pack is the amount of natural gas in a pipeline. The natural gas pipelines can store natural gas and the difference between the mass of natural gas in two consecutive periods is considered as the stored gas. The relationship between in-flow and out-flow at each node of a pipeline and the line pack is shown in (34), where $V_{lp_{m,n,t}}$ is the mass of gas in the pipeline between nodes m and n at time t . The flow of natural gas in the pipeline is calculated using (35), where $f_{m,n,t}$ is the natural gas flow between nodes m and n at time t . The line pack in a pipeline is determined using (36), while the volume of stored natural gas in the pipeline is calculated using (37). Here, $\pi_{m,t}$ and $\pi_{n,t}$ are the natural gas pressure at nodes m and n at time t ; $V_{stock_{m,n,t}}$ is the change in mass of gas in the pipeline between nodes m and n at time t ; and $C_{m,n}''$ is the pipeline constant for line pack from nodes m to n and is dependent on the geometrical volume and the temperature of the pipeline [10].

The gas flow in the pipelines without compressor is determined using (38) and (39), where $C_{m,n}'$ is the pipeline constant for gas flow from nodes m to n which is dependent on the operating temperature, length, diameter, roughness of the pipeline as well as the gas type. Note that the limitation on the nodal pressure would limit the natural gas

flow in the pipelines. In the natural gas transportation network, the pressure will drop due to the pipeline resistance, and in order to compensate for the pressure loss, the compressors are used. The gas flow through the compressor is given by (40), where $k_{1,r}$, $k_{2,r}$ and α_r are the empirical parameters of compressor r . The volume of the supplied natural gas is limited by (41), the nodal pressure in the natural gas network is limited by (42), and the power output of the compressor is limited by (43), where $v_s^{\min}$ is the minimum output volume; $v_s^{\max}$ is the maximum output volume of supplier s ; $\pi_m^{\min}$, $\pi_m^{\max}$ are the minimum and maximum pressures at node m , respectively; and $H_r^{\min}$, $H_r^{\max}$ are the minimum and maximum power output of compressor r , respectively.

Furthermore, the pressure ratio is limited by (44), where $R_r^{\max}$, $R_r^{\min}$ are the maximum and minimum pressure ratios of compressor r , respectively. The volume of consumed natural gas to provide the required pressure by the compressor is determined by (45), where a_r, b_r, c_r are the parameters for gas consumption function of compressor r . The volume of natural gas consumed by the GFG unit is determined by (46), where $G_{j,i}$ is the element of gas load GFG unit incidence matrix. The in-flow and out-flow of natural gas in a pipeline satisfy (47)–(49). The in-flow, out-flow, and the flow of natural gas in the pipeline is shown in Fig. 1, where $f_{m,n,t}$ represents the average flow in pipeline between nodes m and n which could be calculated by in-flow (the natural gas flow that enters) and out-flow (the natural gas flow that exits) at each node of the pipeline. As shown in Fig. 1, $f'_{m,n,t}$ is the in-flow of the pipeline m to n ; $f'_{n,m,t}$ is the in-flow of the pipeline n to m ; $f''_{m,n,t}$ is the out-flow of the pipeline m to node n , and $f''_{n,m,t}$ is the out-flow of the pipeline n to node m . All variables related to the flow of natural gas are positive.

3 Solution framework

While the proposed mathematical problem in the previous section captures the electricity and natural gas constraints, the information from each infrastructure system may not be readily available to the other system operator.

Fig. 1 In-flow, out-flow and gas flow in a natural gas pipeline

Fig. 2 Proposed solution methodology

For example, the electricity network operator may not have access to the natural gas network data to consider the natural gas network constraints. Therefore, the proposed mathematical programming problem is decomposed using Benders decomposition to capture the interaction among the electricity and natural gas system operators. Figure 2 shows the flowchart of the presented optimization framework for the short-term operation of the electricity and natural gas networks. The steps taken are as follows.

3.1 Master problem (UC and ED)

The master problem is formulated by the electricity network operator. Here the objective is to minimize the operation cost of the coal generation units as well as the startup and shutdown costs of coal and GFG units. The objective function is shown in (50) which is limited by (51).

$$\min Z \quad (50)$$

$$Z \geq \sum_i \sum_t [SU_{i,t} + SD_{i,t}] + \sum_c \sum_t [SU_{c,t} + SD_{c,t} + F_{c,t}(P_{c,t}) \cdot cp_{c,t}] \quad (51)$$

The constraints in this problem include (51) and (2)-(28). More details of these constraints are presented in [14]. The solution to this problem yields the commitment and dispatch of the coal-fired and GFG units. The solution is passed to the subproblem of electricity network feasibility check in the next step.

3.2 Subproblem of electricity network feasibility check

After solving the master problem (UC and ED), the subproblem of the electricity network feasibility check is formulated to check for any network constraint violation caused by the solution of the master problem (UC and ED). The feasibility check subproblem is formulated as (52)-(54) and as (30)-(32), where the objective is to minimize the mismatch in the nodal supply and demand subjected to DC power flow constraints (30)-(32), and the nodal electricity demand and supply balance (53) and (54). If the value of the objective function is zero, the solution of the master problem satisfies the electricity network constraints. Otherwise, feasibility Benders cut (55) is generated and added to the master problem.

$$\min \omega(\hat{P}) = \sum_t \sum_p (S_{1,p,t} + S_{2,p,t}) \quad (52)$$

s.t.

$$E_{p,b} \cdot p_{b,t}^f = C_{p,i} P_{i,t} - D_{p,l} P_{l,t} + S_{1,p,t} - S_{2,p,t} \quad (53)$$

$$P_{i,t} = \hat{P}_{i,t} \quad : \mu_{1,i,t} \quad (54)$$

where $S_{1,p,t}$, $S_{2,p,t}$ are positive slack variable, and $\mu_{1,i,t}$ is the dual variable corresponding to the equality constraint (54).

$$\omega(\hat{P}) + \sum_i \sum_t \mu_{1,i,t} (P_{i,t} - \hat{P}_{i,t}) \leq 0 \quad (55)$$

where $\hat{P}$ represents the vector of generation dispatch of all generation units determined by solving the master problem (UC and ED); and $\omega(\hat{P})$ is the value of the objective function (52). Similarly, the solution of the master problem will be checked again using the subproblem of the electricity network feasibility check until the values of all slack variables are zero and the provided solution leads to no network violation. At this stage, the solution of the master problem is passed to subproblem of the natural gas network feasibility check.

3.3 Subproblem of natural gas network feasibility check

In this step, the feasibility of the natural gas network constraints is checked with the solution passed from the master problem. The solution to the subproblem of natural gas network feasibility check generates infeasibility cuts (in case of violation) to ensure that the proposed solution by the master problem satisfies the natural gas network constraints. The feasibility check subproblem is formulated as (56), (34)-(49) and (57). The objective function is to minimize the nodal demand and supply mismatch in the natural gas network as shown in (56) and (57), subjected to the natural gas network constraints (34)-(49).

$$\min \omega(\hat{L}) = \sum_t \sum_{j \in GF} S_{j,t} \quad (56)$$

$$\begin{aligned} & \sum_s A_{m,s} V_{s,t} - \sum_j B_{m,j} L_{j,t} - \sum_j B_{m,j \in GF} S_{j,t} - \sum_{n \in GC(m)} (f'_{m,n,t} \\ & - f''_{m,n,t}) - \sum_r F_{m,r} G_r(H_{r,t}) \\ & = 0 \end{aligned} \quad (57)$$

where $S_{j,t}$ is positive slack variable and GF is the set of nodes with gas-fired generation units. Since the constraints (36), (38), (40), (45) and (57) include nonlinear terms, successive linearization techniques using NR method is used to solve the feasibility check subproblem iteratively.

The developed NR algorithm is as follows:

Step 1: Initiate the nodal pressure $\pi_{m,t}^0$, gas supplier volume $v_{s,t}^0$ and power output of the compressor $H_{r,t}^0$ and set iteration index $k = 0$.

Step 2: Calculate the elements of the Jacobian matrix $[J_{\pi,t}, J_{v,t}, J_{L,t}, J_{H,t}]$, using (58)-(62) where the required elements are calculated using (63)-(69) and ε, M are a small and relatively large numbers, respectively, to avoid numerical instability. Here the diagonal and off-diagonal elements of $J_{\pi,t}$ are calculated using (60) and (61), respectively. The derivatives of the natural gas flow with respect to the nodal pressure in the pipelines without the compressor are formulated in (63) and (64). Similarly, the derivative of the line pack with respect to the nodal pressure is formulated in (67) and (68). For the lines with the compressor, the partial derivative of the natural gas flow with respect to the nodal pressure is calculated using (65) and (66). Note that the line pack is ignored for the pipelines with the compressor. The elements of $J_{v,t}$, $J_{L,t}$ and $J_{H,t}$ are calculated by (58), (59) and (62), respectively.

$$\frac{\partial h_m}{\partial v_{s,t}} = A_{m,s} \quad (58)$$

$$\frac{\partial h_m}{\partial L_{j,t}} = -B_{m,j} \quad (59)$$

$$\frac{\partial h_m}{\partial \pi_{m,t}} = -\sum_n \frac{\partial(f'_{m,n,t} - f''_{m,n,t})}{\partial \pi_{m,t}} = -\sum_n \frac{\partial\left(f_{m,n,t} + \frac{V_{stock_{m,n,t}}}{2}\right)}{\partial \pi_{m,t}} \quad (60)$$

$$\frac{\partial h_m}{\partial \pi_{n,t}} = -\frac{\partial\left(f_{m,n,t} + \frac{V_{stock_{m,n,t}}}{2}\right)}{\partial \pi_{n,t}} \quad (61)$$

$$\frac{\partial h_m}{\partial H_{r,t}} = -\sum_n \frac{\partial f_{m,n,t}}{\partial H_{r,t}} - F_{m,r}(2a_r + b_r) \quad (62)$$

$$\frac{\partial f_{m,n,t}}{\partial \pi_{m,t}} = \begin{cases} M & |\pi_{m,t}^2 - \pi_{n,t}^2| \leq \varepsilon \\ \frac{C'_{m,n} \cdot \pi_{m,t}}{\sqrt{|\pi_{m,t}^2 - \pi_{n,t}^2|}} & |\pi_{m,t}^2 - \pi_{n,t}^2| > \varepsilon \end{cases} \quad (63)$$

$$\frac{\partial f_{m,n,t}}{\partial \pi_{n,t}} = \begin{cases} -M & |\pi_{m,t}^2 - \pi_{n,t}^2| \leq \varepsilon \\ -\frac{C'_{m,n} \cdot \pi_{n,t}}{\sqrt{|\pi_{m,t}^2 - \pi_{n,t}^2|}} & |\pi_{m,t}^2 - \pi_{n,t}^2| > \varepsilon \end{cases} \quad (64)$$

$$\frac{\partial f_{m,n,t}}{\partial \pi_{m,t}} = \begin{cases} \alpha_r H_{r,t} k_{1,r} \frac{\pi_{n,t}^{\alpha_r}}{\pi_{m,t}^{\alpha_r+1}} & \pi_{n,t} \geq \pi_{m,t} \\ \frac{\left[k_{1,r} \left(\frac{\pi_{n,t}}{\pi_{m,t}}\right)^{\alpha_r} - k_{2,r}\right]^2}{\alpha_r H_{r,t} k_{1,r} \frac{\pi_{m,t}^{\alpha_r-1}}{\pi_{n,t}^{\alpha_r}}} & \pi_{n,t} < \pi_{m,t} \end{cases} \quad (65)$$

$$\frac{\partial f_{m,n,t}}{\partial \pi_{n,t}} = \begin{cases} -\frac{\alpha_r H_{r,t} k_{1,r} \frac{\pi_{n,t}^{\alpha_r-1}}{\pi_{m,t}^{\alpha_r}}}{\left[k_{1,r} \left(\frac{\pi_{n,t}}{\pi_{m,t}}\right)^{\alpha_r} - k_{2,r}\right]^2} & \pi_{n,t} \geq \pi_{m,t} \\ -\frac{\alpha_r H_{r,t} k_{1,r} \frac{\pi_{m,t}^{\alpha_r}}{\pi_{n,t}^{\alpha_r+1}}}{\left[k_{1,r} \left(\frac{\pi_{m,t}}{\pi_{n,t}}\right)^{\alpha_r} - k_{2,r}\right]^2} & \pi_{n,t} < \pi_{m,t} \end{cases} \quad (66)$$

$$\frac{\partial V_{stock_{m,n,t}}}{\partial \pi_{m,t}} = \frac{2}{3} C''_{m,n} \left(1 - \frac{\pi_{n,t}^2}{(\pi_{m,t} + \pi_{n,t})^2}\right) \quad (67)$$

$$\frac{\partial V_{stock_{m,n,t}}}{\partial \pi_{n,t}} = \frac{2}{3} C''_{m,n} \left(1 - \frac{\pi_{m,t}^2}{(\pi_{m,t} + \pi_{n,t})^2}\right) \quad (68)$$

$$\frac{\partial f_{m,n,t}}{\partial H_{r,t}} = \begin{cases} \frac{1}{k_{1,r} \left(\frac{\pi_{n,t}}{\pi_{m,t}}\right)^{\alpha_r} - k_{2,r}} & \pi_{n,t} \geq \pi_{m,t} \\ -1 & \pi_{n,t} < \pi_{m,t} \end{cases} \quad (69)$$

Step 3: Solve (70)–(76) to determine $\Delta \pi_t^k$, Δv_t^k , ΔL_t^k , ΔH_t^k . If the elements of vectors $\Delta \pi_{m,t}^k \in \Delta \pi_t^k$, $\Delta v_{s,t}^k \in \Delta v_t^k$, $\Delta L_{j,t}^k \in \Delta L_t^k$, $\Delta H_{r,t}^k \in \Delta H_t^k$ are less than the threshold ε_1 then go to *Step 4* otherwise go to *Step 5*. Note that $\Delta L_{j,t}^k$ represents the changes in natural gas volume of load j from iteration $(k-1)$ to k at time t ; $\Delta v_{s,t}^k$ represents the changes in natural gas volume of supplier s from iteration $(k-1)$ to k at time t ; and $\Delta \pi_{m,t}^k$ represents the changes in nodal pressure from iteration $(k-1)$ to k at node m at time t .

$$\min \omega^k = \sum_{j \in GF} \sum_t (-\Delta L_{j,t}^k) \quad (70)$$

$$[J_{\pi,t} J_{v,t} J_{L,t} J_{H,t}] \begin{bmatrix} \Delta \pi_t^k \\ \Delta v_t^k \\ \Delta L_t^k \\ \Delta H_t^k \end{bmatrix} = -h(\pi_{m,t}^k, v_{s,t}^k, \hat{L}_{j,t}, H_{r,t}^k) \quad (71)$$

$$\pi_{\min} \leq \Delta \pi_{m,t}^k + \pi_{m,t}^k \leq \pi_{\max} \quad (72)$$

$$v_{\min} \leq \Delta v_{s,t}^k + v_{s,t}^k \leq v_{\max} \quad (73)$$

$$-\hat{L}_{j,t} \leq \Delta L_{j,t}^k \leq 0 \quad \forall j \in GF \quad (74)$$

$$H_r^{\min} \leq H_{r,t} + \Delta H_{r,t}^k \leq H_r^{\max} \quad (75)$$

$$R_r^{\min} \leq \frac{\Delta \pi_{m,t}^k + \pi_{m,t}^k}{\Delta \pi_{n,t}^k + \pi_{n,t}^k} \leq R_r^{\max} \quad (76)$$

Step 4: The value of objective function (70) is checked. If the value of the objective function (70) i.e. ω^k is positive, feasibility Benders cuts (77) are generated. Here, $\hat{L}_{j,t}$ is the value of the gas load j of GFG units from the master problem. The Benders cut (70) is transformed to

$$\hat{\Delta P}_{i,t}^k + \sum_m \mu_{2,m,t} B_{m,j \in GF} G_{j,i} (P_{i,t} - \hat{P}_{i,t}) \leq 0$$

using (3) and (46), where $\mu_{2,m,t}$ is the dual variable of the equality constraint (71). The natural gas network feasibility cut is sent to the master problem. If the value of the objective function is zero, the solution of the master problem is feasible for the natural gas network and the process ends. At this stage, the solution of the master problem is passed to the natural gas transmission optimality subproblem.

$$\widehat{\Delta L}_{j,t}^k + \sum_m \mu_{2,m,t} B_{m,j} (L_{j,t} - \hat{L}_{j,t}) \leq 0 \quad \forall j \in GF \quad (77)$$

where $\widehat{\Delta L}_{j,t}^k$ represents the difference between gas load of GFG units that could be served by the natural gas transportation network at node j and the natural gas demand provided by the master problem. If $\widehat{\Delta L}_{j,t}^k$ is negative, this means that part of the natural gas demand imposed by the GFG units could not be served.

Step 5: The vector of variables $[\pi_t^k \ v_t^k \ L_t^k \ H_t^k]^T$ is updated using (78). Increase the iteration index by one, ($k = k + 1$); and go to *Step 2* to calculate the Jacobian matrix.

$$\begin{bmatrix} \pi_{m,t}^{k+1} \\ v_{s,t}^{k+1} \\ L_{j,t}^{k+1} \\ H_{r,t}^{k+1} \end{bmatrix} = \begin{bmatrix} \Delta \pi_{m,t}^k \\ \Delta v_{s,t}^k \\ \Delta L_{j,t}^k \\ \Delta H_{r,t}^k \end{bmatrix} + \begin{bmatrix} \pi_{m,t}^k \\ v_{s,t}^k \\ L_{j,t}^k \\ H_{r,t}^k \end{bmatrix} \quad (78)$$

3.4 Subproblem of natural gas network optimality

In this problem, the operation cost of the natural gas network is minimized. Similar to the feasibility check subproblem, this problem is solved by the natural gas network operator. The solution to the natural gas network optimality subproblem yields optimality cuts to increase the lower bound of the objective function in the master problem and to incorporate the operation cost of the natural gas network. The problem is formulated as (79), (33)-(49). The objective function is to minimize the total cost of acquiring the natural gas from the suppliers in the natural gas network.

$$\min \sum_s \sum_t g p_{s,t} \cdot v_{lp_{s,t}} \quad (79)$$

Similar to the subproblem of natural gas feasibility check, as the constraints (33), (36), (38), (40), and (45) include nonlinear terms, successive linearization using NR method is used to solve this problem. The solution algorithm is as follows.

Step 1: Initiate the nodal pressure $\pi_{m,t}^0$, gas supplier volume $v_{s,t}^0$ and power output of the compressor $H_{r,t}^0$ and set iteration index $k = 0$.

Step 2: Calculate the elements of the Jacobian matrix $[J_{\pi,t} \ J_{v,t} \ J_{L,t} \ J_{H,t}]$, using (58)-(62) where the required elements are calculated using (63)-(69).

Step 3: Solve (79), (71)-(73), (75), (76), (80), to determine $\Delta \pi_t^k$, Δv_t^k , ΔL_t^k , ΔH_t^k . If the elements of vectors $\Delta \pi_{m,t}^k \in \Delta \pi_t^k$, $\Delta v_{s,t}^k \in \Delta v_t^k$, $\Delta L_{j,t}^k \in \Delta L_t^k$, $\Delta H_{r,t}^k \in \Delta H_t^k$ are less than threshold ϵ_1 , go to *Step 4*. Otherwise, go to *Step 5*. Note that as the feasibility of the natural gas network is

guaranteed by enforcing feasibility cuts, the natural gas demand for GFG units at each iteration is enforced by (80).

$$L_{j,t}^k + \Delta L_{j,t}^k = \hat{L}_{j,t} \quad : \mu_{3,j,t} \quad (80)$$

Step 4: Calculate $\hat{z}_1$ using (81) and $\hat{z}_2 = \hat{z}_1 + g p_{s,t} \cdot \hat{v}_{lp_{s,t}}$ where $\hat{v}_{lp_{s,t}}$ is the solution to subproblem of the natural gas network optimality. If (82) is satisfied, then the process ends. Otherwise, generate the optimality cut formulated as (83) and send it to the master problem. Here, ϵ is the threshold value and $\mu_{3,j,t}$ is the dual variable of constraint (80). Note that the optimality Benders cut is reformulated as

$$Z \geq \hat{z}_2 + \sum_t \sum_i \mu_{3,j,t} \cdot G_{j,i} (P_{i,t} - \hat{P}_{i,t})$$

considering (3) and (46).

$$\begin{aligned} \hat{z}_1 = & \sum_i \sum_t [\widehat{S}U_{i,t} + \widehat{S}U_{i,t}] \\ & + \sum_c \sum_t [\widehat{S}U_{c,t} + \widehat{S}D_{c,t} + F_{c,t}(\hat{P}_{c,t}) \cdot c p_c] \end{aligned} \quad (81)$$

$$2|\hat{z}_2 - \hat{z}_1|/(\hat{z}_1 + \hat{z}_2) \leq \epsilon \quad (82)$$

$$Z \geq \hat{z}_2 + \sum_t \sum_j \mu_{3,j,t} (L_{j,t} - \hat{L}_{j,t}) \quad \forall j \in GF \quad (83)$$

Step 5: The vector of variables $[\pi_t^k \ v_t^k \ L_t^k \ H_t^k]^T$ is updated using (80). Increase the iteration index by one, ($k = k + 1$) and go to *Step 2* to calculate the Jacobian matrix.

4 Case study

4.1 6-bus electricity network with seven-node natural gas network

The electricity network and natural gas network topologies are shown in Figs. 3 and 4, respectively. Two generation units G1 and G2 are GFG units, and G3 is coal-fired generation unit. The characteristics of the units and

Fig. 3 6-bus electricity network

Fig. 4 7-node natural gas network

the transmission lines are in [15]. Here, 1 kcf of natural gas provides 1.037 MBTU of energy in G1 and G2. The total peak demand of the system is 256 MW at hour 17 and the hourly total demand profile is shown in Fig. 5. The characteristics of the natural gas pipelines are in [15]. The gas load consists of two residential gas loads D3 and D4; and the demand for G1 and G2. To highlight the impact of line pack and stored natural gas in the pipelines, the prices of natural gas for two suppliers are changing every four hours as shown in Fig. 5. The simulation is performed on a PC with Intel Core i5 processor 3.4 GHz and 8 GB of memory and CPLEX 12.7.1 is used as the MILP solver in this paper. The following cases are considered:

- 1) Case 1: UC and ED without the line pack
- 2) Case 2: UC and ED with the line pack in the natural gas network.
- 3) Case 3: UC and ED with the line pack and congestion in the natural gas network

4.1.1 Case 1: UC and ED without line pack

The hourly commitment of the generation units ignoring the electricity network constraints is shown in Table 1. Once the electricity network feasibility check is considered, the commitments of the generation units are shown in Table 2. The impact of considering the natural gas network constraints on the solution of the master problem is shown in Table 3. Figure 6a shows the dispatch of G1-G3 in the

Fig. 5 Hourly total electricity demand and price of natural gas

operation horizon without considering the electricity and natural gas network constraints. Figure 6b shows the dispatch of G1-G3 considering the electricity network constraints. Figure 6c shows the dispatch of G1-G3 considering the electricity and natural gas network constraints. Comparing Table 1 with Table 2, the commitment of G3 is changed from hour 13-19 to 11-22 because of the congestion in the electricity network. Here, without considering the electricity network constraints, transmission line L2 carries at least 102.3 MW at hours 10-23 which exceeds its maximum limit of 100 MW. Therefore, the dispatch of G1 is reduced at hours 10-23 as shown in Fig. 6b. Table 3 shows that considering the natural gas network constraints impacts the commitment of the generation units, and ignoring the natural gas network constraints will result in a violation of the natural gas flow in pipeline P1. Here, the natural gas flow in pipeline P1 exceeds the limits at hours 10-22. For instance, by ignoring the network constraints, the supplied gas by pipeline P1 to node 1 is 7001 kcf at hour 10; however, considering the pressure limits at the end nodes, the limit for gas flow in pipeline P1 is 6765 kcf which is less than the demand at node 1. In order to eliminate this violation, the natural gas demand of G1 is reduced and the dispatch of G3 is increased at hours 9-23 to compensate for the shortage in generation as shown in Fig 6c. As shown in Fig. 6c, G1, which is the least expensive unit in the electricity network, provides its maximum generation capacity at hours 10-22. The rest of the demand is served by dispatching G2 and G3 as more expensive units. In order to serve the load at hours 12-21, the dispatch of G2 will reach its maximum and G3 further compensates for the unserved demand. Dispatching G1 and G2 will further impact the demand in the natural gas network. Here, the generation dispatches at hour 12 are 189.9 MW, 20 MW and 26.3 MW for G1, G2 and G3, respectively. Therefore, the total natural gas demands at nodes 1 and 3 are 6737.0 kcf and 2494.0 kcf gas, respectively. Considering the natural gas optimality subproblem, the total production costs for G1, G2 and G3 in the operation horizon are \$241075, \$43389, \$113803, respectively. The total operation cost in this case is \$855613. The total operation cost of the natural gas network in this case is \$741810.

4.1.2 Case 2: UC and ED with line pack in the natural gas network

In this case, except the pipelines with the compressors, the line pack for all pipelines is considered. Therefore, for these pipelines, the inflow of the natural gas pipeline is not equal to its outflow. Fig. 7 shows the line pack for pipelines P1 and P3. The line pack is dependent on the average nodal pressure at two sides of the pipeline. The

Table 1 Hourly unit commitment without electricity network feasibility check

Unit	Time (hour)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0

Table 2 Hourly unit commitment with electricity network feasibility check

Unit	Time (hour)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0

Table 3 Hourly unit commitment with natural gas feasibility check

Unit	Time (hour)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

line pack of P3 is larger than P1 as the average pressure of nodes 5 and 2 is larger than the average pressure of nodes 2 and 1. As the direction of flow is from the nodes with higher pressure to the nodes with the lower pressure, the direction of the natural gas flow is from node 5 to node 2 and from node 2 to node 1. Table 4 presents the in-flow, out-flow, line pack and the volume of stored natural gas in P1. The volume of stored natural gas in P1 is calculated by the difference among the line pack in two consecutive periods. Therefore, the flexibility of pipelines to serve the natural gas load is determined by the volume of stored natural gas in the pipeline. As show in Table 4, at hours 1-4, P1 stores natural gas because the prices of natural gas at hours 1-4 are 1.89 \$/MBTU and 3.16 \$/MBTU for suppliers 1 and 2, respectively. These prices are the lowest in the operation period and therefore, the stored level of natural gas in the system reaches its maximum. At hours 6-9 when the price of natural gas increases, the stored gas in P1 is consumed as shown in Table 4. The total volume of stored gas in P1 at hours 10-20 is zero. The stored gas at hours 1-4 and 23-24 is used at hours 6-9. At hours 10-20, P1 does not have enough capacity to store natural gas as the in-flow and out-flow limits reach. In this case, the natural gas operator can store 9.514 kcf of gas in P1 at hours 1-4 and use it at hours 6-9. Using such storage capacity will reduce the operation cost to \$855606 from \$855613 in

Case 1. Although the savings is \$7 which is small compared to the total operation cost, the savings will increase as the limitation on the nodal pressure is further relaxed. For instance, if the maximum nodal pressure increases by 2 times and the minimum nodal pressure is reduced by half, the savings will increase to \$54. It is worth noting that the pipelines do not always store gas as they may reach their maximum flow limits in the peak periods. The differences between the natural gas price in two successive periods contributes to the savings in the operation horizon. The savings from line pack is not significant in this case. However, with the increase in the size of the system and larger fluctuation of natural gas prices, the savings will become considerable in the operation horizon. In this case, subproblem of the electricity network feasibility check is solved twice and the natural gas feasibility subproblem is solved twice. The natural gas optimality subproblem is solved once and the solution time is 29.77 seconds. In the first iteration without any optimality cuts, the number of iterations for NR method used in the first and second iterations of natural gas feasibility check subproblem are 10 and 12, respectively. The number of NR iteration for subproblem of the natural gas network optimality is 10. After generating the optimality cut, in the second iteration, the number of iterations for NR method used in subproblem of the natural gas network feasibility check is 14 and

(a) Ignoring electricity network constraints

(b) Considering electricity network constraints

(c) Considering natural gas and electricity network constraints

Fig. 6 Generation dispatch

the solution is feasible for the natural gas network. Therefore, no feasibility cut is generated in this iteration. The number of NR iterations for the natural gas network optimality subproblem is 14. Figure 8 shows the objective of the subproblem of electricity network feasibility check in the first iteration of the algorithm without any optimality cuts. As shown in this figure, with generation of feasibility cuts, the value of the objective function of the electricity network feasibility check subproblem decreases and

Fig. 7 Line pack for P1 and P3 with and without congestion in the natural gas network**Table 4** Inflow, outflow, line pack, and stored energy in P1

Time	Inflow	Outflow	Line pack	Stored NG
1	6570.1	6566.6	323.4	3.470
2	6433.0	6430.0	326.4	3.007
3	6339.4	6337.4	328.4	1.983
4	6288.4	6287.4	329.4	1.057
5	6288.7	6288.7	329.4	0
6	6364.4	6366.0	327.8	-1.606
7	6538.8	6542.0	324.0	-3.795
8	6635.8	6637.3	321.9	-2.162
9	6729.8	6732.0	319.7	-2.185
10	6726.0	6726.0	319.8	0.114
11	6728.5	6728.5	319.7	-0.059
12	6736.8	6737.0	319.5	-0.197
13	6743.1	6743.2	319.4	-0.145
14	6745.0	6745.0	319.3	-0.045
15	6751.8	6751.9	319.1	-0.160
16	6760.1	6760.3	319.0	-0.195
17	6763.1	6763.2	318.9	0
18	6749.5	6749.2	319.2	0.325
19	6749.7	6749.7	319.2	0
20	6738.8	6738.5	319.5	0.260
21	6738.5	6738.5	319.5	0
22	6729.4	6729.2	319.7	0.213
23	6645.3	6643.3	321.7	1.965
24	6719.4	6721.1	319.9	-1.753

eventually reaches zero which indicates the feasible solution for the electricity network. Furthermore, adding the Benders cut will increase the lower bound of the objective function and increase the total operation cost. It is worth noting that the increase in the objective function with no feasibility cut from the electricity network is due to the feasibility cut generated for the natural gas network in the second subproblem.

Fig. 8 Objective of electricity network feasibility check subproblem in the first iteration of the algorithm without any optimality cuts

4.1.3 Case 3: UC and ED with line pack and congestion in natural gas network

In this case, a congestion is considered in P3 between nodes 2 and 5, the gas constant and line pack constant decrease into 25% and 57% of their values in previous case, respectively [10]. In this case, even if the pressures at nodes 2 and 5 reach their limits, the flow rate in Case 2 could not be attained. Therefore, the natural gas supply to G1 and G2 is restricted by the capacity of P3. For instance, the volume of natural gas supplied to G1 at hour 1 is 6370 kcf which is lower than that in Case 2 (i.e. 6568 kcf). After solving the subproblem of the natural gas feasibility check, the coal-fired unit G3 is committed at hours 1–24 as shown in Table 5. The generation dispatch of G1 is reduced dramatically in the operation period as shown in Fig. 9, and the coal-fired unit G3 produces more energy. The total generated energy for G3 is 1230 MWh compared to that in Case 2 which is 491 MWh. As a result, the total operation cost increases to \$966341. Here, although the operation cost of the natural gas network decreases to \$695743, the operation cost of coal-fired generation G3 is much higher than the operation cost of G1 and G2. Therefore, the total operation cost of the electricity and natural gas networks increases. It is worth noting that the outage in P3 between nodes 2 and 5 will result in a deficiency in the natural gas supply for G1 and consequently the infeasibility of the electricity network problem as no load shedding is allowed in this case. Fig. 7 shows the line pack of P3 with congestion. As shown in this figure, the line pack is lower than

that in Case 2 as the line pack constant is decreased by 57%. Here, the natural gas flow in P3 will reach its maximum limit of 1339 kcf/h which is lower than that in Case 2 (i.e. 5356 kcf/h). Therefore, the congestion in the natural gas pipeline will reduce the natural gas supply for the GFG units and could lead to the increase in the operation cost of the system and the deficiency in the electricity supply.

4.2 30-bus electricity network with 12-node natural gas network

The 30-bus electricity network and 12-node natural gas network are shown in Fig. 10 and Fig. 11, respectively. G1, G2, G3 and G4 are GFG units. Two coal-fired units G5 and G6 are connected to buses 11 and 12, respectively. The characteristics of the generation units are presented in [16]. The peak load is 414 MW that occurs at hour 17. The characteristics of the natural gas pipelines are presented in [16]. The gas load is composed of four residential loads D5–D8 and four GFG loads D1–D4 for G1–G4, respectively. The price of natural gas for the sources connected to nodes 1 and 4 are the same as that of supplier 2 in the previous case study. The price of natural gas supply at node 9 is the same as that of supplier 1 in the previous case study.

Fig. 9 Dispatch of G1 with/without natural gas network congestion

Table 5 Hourly power units' dispatch after processing gas transmission feasibility check problem

Unit	Time (hour)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

4.2.1 Case 1: UC and ED without line-pack

The commitment of the generation units is shown in Table 6 and the dispatch of GFG units (G1-G4) and coal units (G5-G6) are shown in Figs. 12(a) and 12(b) respectively. Here, the marginal costs of the coal-fired generation units are larger than those of the GFG units. Therefore, the demand is served by GFG units as they reach their maximum limits at hours 10-23, and coal-fired generation serve the rest. The operation costs of G1, G2, G3 and G4 are \$13835, \$11511, \$56558 and \$9540, respectively. The total operation cost is \$1116742 and the total operation cost of the natural gas network is \$1109379. It is worth noting that the operation cost of the natural gas network captures the cost of supplying natural gas loads in the network as well as the cost of providing natural gas to the GFG units.

4.2.2 Case 2: UC and ED with line-pack in the natural gas network

In this case, the line-pack is considered in every natural gas pipeline. Fig. 13 shows the line-pack for P4, P6 and P10. In this case, at hours 1-4, the natural gas network stored 2634 kcf of gas as the pipelines reach their maxi-

Fig. 10 30-bus electricity network

Fig. 11 12-node natural gas network

um capacity for storing natural gas. At hours 6-13, the stored gas is used as the price of natural gas increases. Using such storage capacity will reduce the operation cost from \$989865 to \$988174, where storing natural gas in the pipelines can save \$1691. In this case, the sub-problem of electricity network feasibility check is solved once and the subproblem of natural gas feasibility check is solved twice. The subproblem of natural gas optimality is solved once and the total solution time is 49.37 seconds. In the first iteration without any optimality cuts, the number of iterations for NR method used in the subproblem of natural gas feasibility check is 8. The number of NR iteration for the natural gas network optimality subproblem is 19. After generating the optimality cut, the number of iterations for NR method used in the subproblem of natural gas network feasibility check is 23. In this iteration, no feasibility cut is generated for the natural gas network. The number of NR iteration for the subproblem of natural gas network optimality is 29.

4.2.3 Case 3: UC and ED with line-pack and congestion in natural gas network

In this case, a congestion is considered in P9. Therefore, the pipeline constant and the line-pack constant decreases

Fig. 12 Generation dispatch of units in Case 1

Fig. 13 Line-pack of pipelines in Case 2

Fig. 14 Gas flow in P8 with and without congestion in natural gas network

into 20% and 52.5% of those in Case 2, respectively. Here, the natural gas supplied from supplier 3 through P9 decreases and suppliers 1 and 2 will serve D4 and D8 by increasing the flow in P8 as shown in Fig. 14. At peak hour 17, the natural gas flow in P8 increases to 3831 kcf/h from 2393 kcf/h in Case 2. In contrary, the natural gas flow in some pipelines (e.g. P9) is reduced as shown in Fig. 15. At hour 17, the natural gas flow in P9 is 1605 kcf/h which is lower than that in Case 2 (i.e. 3042 kcf/h). In this case, supplier 3, which is the cheapest supplier, serves less hourly load. Here, supplier 3 serves 9636 kcf at peak hour 17 which is less than that in Case 2 (i.e. 10640 kcf). The total operation cost is \$1134176, which is higher than that in Case 2 (\$988174).

5 Conclusion

This paper presents the coordinated operation of the electricity and natural gas network considering the line pack in the natural gas transportation network. Benders decomposition is used to decompose the problem into a

Fig. 15 Gas flow in P9 with and without congestion in natural gas network

master problem solved by the electricity network operator and several subproblems solved by the electricity and natural gas network operators. Here, the master problem addresses the commitment of the generation units while the first subproblem handled by the electricity network operator ensures the feasibility of the provided solution for the electricity network. Once the solution is feasible for the electricity network, the natural gas network operator will ensure the feasibility of the solution for the natural gas network by solving the feasibility subproblem for the natural gas network. Feasibility Benders cuts are generated and added to the master problem in case of any violation. Once the solution is feasible for the natural gas network, optimality subproblem is solved and optimality Benders cuts are generated and added to the master problem. The solution process stops once there is no improvement to the lower bound of the objective function. The merit of the proposed method is that the natural gas network data is not shared with the electricity network operator, and the Benders cut includes the required information passed from the natural gas transportation network to the electricity network operator. Furthermore, constraints of nonlinear natural gas transportation network are captured in the natural gas feasibility and optimality subproblems. NR technique as a successive linearization method is used to solve the feasibility and optimality subproblems iteratively. The value of line pack is shown by the presented case study. It is shown that line pack could contribute to the stored volume of natural gas in the pipelines and further reduces the operation cost of the natural gas network. Furthermore, the congestion in natural gas transportation network would increase the total operation cost as it limits the supply of cheaper GFG units. The congestion in natural gas transportation network could further jeopardize the security of the electricity network.

Table 6 Hourly commitment of generation units in Case 1

Unit	Time (hour)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
G1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
G2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
G3	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
G4	0	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
G5	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
G6	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0

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