



Interdependence between transportation system and power distribution system: a comprehensive review on models and applications



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Abstract The rapidly increasing penetration of electric vehicles in modern metropolises has been witnessed during the past decade, inspired by financial subsidies as well as public awareness of climate change and environment protection. Integrating charging facilities, especially high-power chargers in fast charging stations, into power distribution systems remarkably alters the traditional load flow pattern, and thus imposes great challenges on the operation of distribution network in which controllable resources are rare. On the other hand, provided with

appropriate incentives, the energy storage capability of electric vehicle offers a unique opportunity to facilitate the integration of distributed wind and solar power generation into power distribution system. The above trends call for thorough investigation and research on the interdependence between transportation system and power distribution system. This paper conducts a comprehensive survey on this line of research. The basic models of transportation system and power distribution system are introduced, especially the user equilibrium model, which describes the vehicular flow on each road segment and is not familiar to the readers in power system community. The modelling of interdependence across the two systems is highlighted. Taking into account such interdependence, applications ranging from long-term planning to short-term operation are reviewed with emphasis on comparing the description of traffic-power interdependence. Finally, an outlook of prospective directions and key technologies in future research is summarized.

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1 Introduction

Urban commute vehicles and coal-fired/gas-fired power generators are considered as major fossil fuel consumers and carbon dioxide emitters. To cut down fossil fuel dependence, many countries have set clear goals for promoting renewable energy generation [1, 2] and electric vehicles (EVs) [3], and promulgated market incentives to achieve their targets.

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Thanks to the development of battery technology, the driving ranges of recent EV models have been significantly improved. At current stage, standard EV models have a driving range of 100–250 km, while some elite ones can reach 300–500 km without charging [4]. However, the lack of public charging facility brings some inconvenience to EV owners. To elevate the usage of EVs in modern smart cities, adequate charging facilities are indispensable. There are two different charging modes: slow charging and fast charging. The former one usually takes place at home. Because the driving patterns of most citizens are regular, the residential charging power demand is predictable. The latter one happens during travel, and the power demand forecast for fast charging stations could be more difficult, because it is influenced by many factors such as the vehicular flow distribution, road congestion, electricity price, etc. Today, the widely used charger CHAdeMO (for fast charging) has a rated power of 50 kW, and that of Tesla Supercharger is 120 kW [4]. If a fast charging station serves dozens of EVs, the total demand easily reaches 1 MW or above. Meanwhile, dispatchable resources (especially large generators) in the companion power distribution network (PDN) are rare. It has been widely acknowledged that distribution system operator should take effective measures to resolve the impact of massive charging of EV fleets [5–7].

Existing research on domestic EV charging scheduling is relatively mild, because the daily driving pattern is regular. This paper will mainly focus on the on-road fast charging station, which creates a bilateral tie between the transportation system and the power distribution system. From the transportation system side, EVs plan their routes according to road congestion pattern and locations of fast charging stations. The number of EVs receiving service determines the electric power demand at a fast charging station. In traditional power system research, to acquire the power demand, vehicle arrival rates are exogenously given as constants or probability distributions, or developed from queueing theory [8]. This assumption is acceptable when the penetration of EVs is low and charging stations are distant from each other; otherwise, spatial correlation could appear: if long queue occurs at one charging station, drivers would probably seek service elsewhere. From the power system side, a charging station has certain capacity and should also obey system security constraints, such as bus voltage limits. Unexpected outage may influence traffic condition. According to [9], on 19 May 2018, in the city of Shenzhen in south China, more than 500 charging poles in 5 charging stations were out of service due to power system maintenance. As a result, nearly 2700 electric taxis failed to get recharged in time, and long queues emerged in adjacent charging stations. This event evidently showed the interdependence between transportation and power

distribution infrastructures. In the future, if EVs account for a large portion of traffic flow, it is also possible to actively influence the traffic condition by elaborately announcing difference electricity prices at individual charging stations. Such interdependence phenomenon and synergetic potential have been reported in [10].

To better understand traffic–power interdependence, this paper launches an in-depth survey in this line of research. In Section 2, respective network flow models of the transportation system and the power distribution system are presented; then the interactive model, namely the network equilibrium, is introduced. In Section 3, applications found in the current literature are reviewed, mainly from the aspects of system planning and operation. Section 4 envisions prospective topics in this young and active research field. Section 5 concludes this paper.

2 Network flow models

Network flow here refers to the steady-state distributions of vehicular flow on each road in the transportation network and bus voltage/line power flow in the distribution network, which are respectively determined by a traffic assignment problem (TAP) and an optimal power flow (OPF) problem introduced in Section 2.1 and Section 2.2. Their connection is elaborated in Section 2.3.

2.1 Transportation network model

Similar to the power flow problem, it is important for the transportation authority to know road traffic flow distributions in steady state. Such an analog in transportation engineering is called TAP. However, unlike a power system where an operator can directly control generators, the traffic flow distribution in a transportation system spontaneously reaches a steady state owing to the rationality of individual travelers. The classic model in [11] will be introduced in this section.

A transportation network can be described as a connected graph $G_T = (T_N, T_A)$, where T_N denotes the set of nodes, including origins and destinations of vehicles, as well as intersections in the physical system; T_A is the set of arcs, representing roads in the physical system. The connection topology is described by node–link incidence matrix A , whose dimension is $|T_N| \times |T_A|$. An element of A is described as (1). Each column of A is associated with a link with 1 (or -1) at the entry corresponding to the entrance (or exit) node.

$$A_{ij} = \begin{cases} +1 & \text{node } i \text{ is the head of link } j \\ -1 & \text{node } i \text{ is the tail of link } j \\ 0 & \text{node } i \text{ and link } j \text{ are disconnected} \end{cases} \quad (1)$$

Let (r, s) be a pair of origin and destination (O-D). Traffic demand q^{rs} is defined as the quantity of vehicles travelling from origin node r to destination node s , and the $|T_N| \times |T_N|$ matrix Q is the system O-D demand matrix, which can be non-symmetric. A path consists of connected arcs between (r, s) . All available paths connecting O-D pair (r, s) are denoted by K^{rs} , and each path is indexed by $k \in K^{rs}$. f_k^{rs} is the traffic flow carried by path k between (r, s) . The sum of path flows must meet the traffic demand, i.e.

$$\sum_{k \in K^{rs}} f_k^{rs} = q^{rs} \quad \forall (r, s) \quad (2)$$

Topological relations between paths and links are depicted via a link-path incidence matrix $A = (A^{rs})$, $\forall (r, s)$, where the sub-matrix A^{rs} with a dimension of $|T_A| \times |K^{rs}|$ corresponds to certain O-D pair (r, s) , and its element is defined as:

$$\delta_{ak}^{rs} = \begin{cases} 1 & \text{arc } a \text{ belongs to path } k \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

With the help of link-path incidence matrix A , the traffic flow x_a on arc a is a linear function of path flow f_k^{rs} :

$$x_a = \sum_{(r,s)} \sum_k f_k^{rs} \delta_{ak}^{rs} \quad \forall a \quad (4)$$

It should be mentioned that the vehicular flow in traffic assignment is non-atomic: the flow is a real number; the system impact of a single vehicle is negligible.

For most drivers, travel time is the primary concern. Let t_a be the travel time on link a ; it only depends on x_a in the simplest case (classic model). Traffic assignment aims to identify a reasonable outcome of traffic flow distribution from feasible set (2)-(4). Two criteria have been introduced:

- 1) Social optimum (SO). Traffic flow pattern is said to be social optimal if the total travel time $\sum_a x_a t_a$ reaches minimum. In this setting, a central operator determines travel plans on behalf of all travelers, and travelers are willing to cooperate in order to use the entire transportation system in the most effective way.
- 2) User equilibrium (UE). Travelers determine their routes individually in order to minimize their own travel times. In this setting, a stable state emerges if no one is willing to alter his current route.

SO is an ideal concept in theoretical research; UE is likely to happen because it captures the selfish behavior of motorists. Hereinafter, any reasonable outcome of traffic flow distribution will be referred to as a traffic assignment, including SO and UE. In this sense, traffic assignment has a broader content than UE. Nonetheless, because UE better fits the reality, we will mainly discuss UE in the rest of this paper.

2.1.1 Beckmann model

To quantify road travel time, a natural assumption is that the travel time t_a on arc a solely depends on x_a and is independent of the condition in remaining parts of the transportation system. A commonly used candidate is the bureau of public road (BPR) function in [12]:

$$t_a(x_a) = t_a^0 \left[1 + 0.15 \left(\frac{x_a}{c_a} \right)^4 \right] \quad \forall a \quad (5)$$

where t_a^0 is the free travel time; c_a is sometimes called the capacity of arc a . However, it is not a mandatory upper bound of x_a ; instead, the travel time t_a quickly grows when $x_a > c_a$, preventing further congestion. To model a strict capacity limit c_a , the Davidson function has been proposed in [13]:

$$t_a(x_a) = t_a^0 \left(1 + \frac{Jx_a}{c_a - x_a} \right) \quad \forall a \quad (6)$$

where J is a parameter which should be calibrated from real data.

Provided with arc travel time, the total travel time c_k^{rs} perceived by a single traveler on path k between (r, s) is given by

$$c_k^{rs} = \sum_{a \in T_A} t_a(x_a) \delta_{ak}^{rs} \quad \forall k, \forall (r, s) \quad (7)$$

Apparently, when someone can shorten travel time by using an alternative path, the traffic flow pattern will not be stable. The UE pattern occurs only if travel times on all used paths are equal. This is similar to the equal incremental cost criterion in power system economic dispatch, and is known as the Wardrop principle in transportation engineering, which is formally stated as follows [11, 14].

Wardrop principle: the traffic flow reaches a UE if travel times on all used paths between any O-D pair are equal, and no greater than those which would be perceived on any unused path.

The above condition has a logic form: $\exists u^{rs}$,



$$\begin{cases} c_k^{rs} = u^{rs} & k \in K^{rs} \text{ and } f_k^{rs} > 0, \forall (r, s) \\ c_k^{rs} > u^{rs} & k \in K^{rs} \text{ and } f_k^{rs} = 0, \forall (r, s) \end{cases} \quad (8)$$

where u^{rs} is the minimal travel time between (r, s) . These logical conditions can be easily interpreted by a nonlinear complementarity problem (NCP):

$$\begin{cases} 0 \leq f_k^{rs} \perp (c_k^{rs} - u^{rs}) \geq 0 & k \in K^{rs}, \forall (r, s) \\ \text{s.t. (2), (4), (5), (7)} \end{cases} \quad (9)$$

where notation $0 \leq a \perp b \geq 0$ stands for $a \geq 0, b \geq 0$, and $ab = 0$, or in other words, at most one of a and b can take a strictly positive value. However, solving NCP (9) for a large-scale system is still challenging due to its non-convexity introduced by the first complementarity and slackness condition.

Fortunately, it turns out that (9) exactly constitutes the Karush-Kuhn-Tucher (KKT) optimality conditions of the following convex optimization problem [11, 15]:

$$\begin{cases} \min \sum_{a \in T_A} \int_0^{x_a} t_a(\theta) d\theta \\ \text{s.t. (2), (4), } f_k^{rs} \geq 0 & \forall k, \forall (r, s) \end{cases} \quad (10)$$

where the arc travel time t_a can be BPR function (5), Davidson function (6), or other empirical function $t_a(x_a)$ which is increasing. To see the convexity of the objective function, its second-order derivative is calculated:

$$\frac{d^2}{dx_a^2} \left(\int_0^{x_a} t_a(\theta) d\theta \right) = \frac{dt_a(x_a)}{dx_a} \quad (11)$$

Because arc travel time t_a must be increasing in x_a , the second-order derivative is always positive, and thus the objective function is strictly convex. Therefore, problem (10) can be easily solved by a general-purpose nonlinear program (NLP) solver. The optimal solution x_a^* uniquely determines traffic flow distribution at UE pattern. Please note that Davidson function actually imposes bounds on x_a , so problem (10) may become infeasible if the traffic demand is too high.

2.1.2 Nesterov model

The intuition of latency function $t_a(x_a)$ in Beckmann model is apparent: the more vehicles travelling on a road, the more it will be congested. This is reasonable if the traffic flow x_a varies in a certain range. If we dig into its physical reality, inconsistency would emerge when x_a keeps growing, which has been reported in [16, 17]. Traffic flow is the number of vehicles passing through a cross-section per unit time, i.e.: flow equals speed multiplied by density. When x_a grows large, neither the vehicle speed nor density can take a very small value. However, every

vehicle must keep a safety distance with adjacent ones, and the safety distance increases with the growth of speed, resulting in a drop in vehicle density. Consider an extreme case: the safety distance is a constant and the vehicle density is maximum regardless of vehicle speed. Under this situation, in order to increase x_a , every vehicle must move faster, which in turn leads to a drop (not rise) of the travel time. To reconcile this conflict, a new traffic assignment model has been developed in [16, 17] based on weaker assumptions without relying on a specific latency function t_a .

Assumption 1 Arc flow cannot exceed its capacity:

$$x_a \leq c_a \quad \forall a \quad (12)$$

Here the capacity c_a may have a different value compared to that in Beckmann model.

Assumption 2 Arc travel time is equal to t_a^0 if arc flow is less than its capacity; delay occurs only if arc flow reaches its capacity, i.e.

$$\begin{cases} t_a = t_a^0 & x_a \leq c_a \\ t_a \geq t_a^0 & x_a = c_a \end{cases} \quad (13)$$

Define vectors $\mathbf{x} = (x_a), \mathbf{c} = (c_a), \mathbf{t}^0 = (t_a^0), \forall a$. The Nesterov SO model is cast as a linear program (LP) as follows:

$$\min_{\mathbf{x}, \mathbf{f}} \mathbf{x}^T \mathbf{t}^0 \quad (14)$$

$$\text{s.t. } \mathbf{x} - \mathbf{A}\mathbf{f} = \mathbf{0} \quad (15)$$

$$\mathbf{E}\mathbf{f} = \mathbf{q}\mathbf{f} \geq \mathbf{0} \quad (16)$$

$$\mathbf{x} \leq \mathbf{c} : \boldsymbol{\lambda} \quad (17)$$

where (15) and (16) are the compact forms of (4) and (2), respectively; \mathbf{A} is the node-link incidence matrix; \mathbf{f} is the vector of path flow variables; \mathbf{E} is a zero-one matrix corresponding to the coefficients of (2). As long as the capacity is adequate, congestion will not happen in the SO model; otherwise, problem (14)-(17) will be infeasible. According to the physical interpretation of Lagrangian dual multipliers, $\boldsymbol{\lambda} = (\lambda_a), \forall a \in T_A$ corresponding to the flow capacity constraint (17) represents the delay that a single user would perceive when travelling through a congested arc. Let \mathbf{x}^* and $\boldsymbol{\lambda}^*$ be the optimal primal and dual variables, an important conclusion drawn in [16, 17] is: traffic assignments $(\mathbf{x}^*, \mathbf{t}^0)$ and $(\mathbf{x}^*, \mathbf{t}^0 + \boldsymbol{\lambda}^*)$ reach SO and UE, respectively.

In Beckmann model, road travel time $t_a(x_a)$ only depends on x_a . However, in Nesterov model, the delay on each road is determined by the utilization of the whole

network. The two models are comprehensively compared on some benchmark instances and real-world transportation networks in Switzerland [18], demonstrating that both models provide similar congestion patterns. However, the travel times offered by two models are generally incomparable, because they rest on different assumptions of arc latency. Nesterov model is an LP, and can be embedded into another high-level optimization problem more conveniently in form of KKT optimality condition or primal-dual optimality condition.

An implicit assumption in setting up problems (10) and (14) is that the path sets $K^{rs}, \forall(r, s)$ are available. This can be done by enumerating all usable paths K^{rs} for every O-D pair in advance. Nevertheless, such an enumeration could be time-consuming and also unnecessary for large-scale systems, as most paths existing in theory will never be used in practice. To vanquish this difficulty, a reasonable mean is to start with a restricted traffic assignment problem based on subsets of $K^{rs}, \forall(r, s)$ in which the paths are most likely to be visited, and then test whether a new path will be activated according to the congestion pattern at the restricted UE. This procedure is mathematically streamlined in [19]. An adaptive path generation algorithm is developed, in which path generation is formulated as an MILP based on \mathcal{A} .

2.1.3 Other UE models

The basic UE models presented above capture congestion effect, the foremost concern in traffic engineering. For more extensive discussions about UE models considering elastic traffic demands, travel pattern and destination choices, as well as link interactions, please refer to [11]. In addition to the Davidson function method, tailored UE models with arc capacity constraints can be found in [19–22]. The core idea is to introduce a generalized path travel cost, which introduces a penalty whenever $x_a > c_a$. The cost can be extracted from KKT condition of (10) with an additional constraint $x_a \leq c_a$.

Some earlier EV models have limited driving ranges. Researchers have developed distance-aware UE models which highlight the special route selection behaviors of EVs. For example, length limit is incorporated in the model proposed in [23], which is further generalized in [24] to include trip chains.

In the near future, gasoline vehicles (GVs) will still account for the majority part of traffic demand; GV and EVs will co-exist in the next a few decades. It is essential to investigate UE models with both kinds of vehicles. Along this line, a mixed UE model is developed in [25]; two algorithms are devised to solve the model efficiently. A more comprehensive UE model that jointly considers

destination, route, and parking location choices of GV and EVs subject to driving distances is proposed in [26].

To mimic drivers' charging decision meticulously, en-route energy consumption should be simulated in the UE model. Based on different assumptions on the relations among battery state-of-charge (SoC), recharging time and road traffic flows, three sophisticated UE models are expounded in [27]. Two of them are cast as convex traffic assignment problems, and the third one yields an NCP. Similar work has been found in [28, 29] considering battery swapping stations. Trip chains of EVs are incorporated in the UE model studied in [30]. Limited driving ranges and recharging needs of EVs are main concerns. A UE model with charging-in-motion lanes are portrayed in [31]. A mixed UE model with GV and EVs as well as autonomous-vehicle-only lanes is formulated in [32]. A special UE model is put forward in [33] to investigate the impact of travel time display on driver route choices.

Above work originating from transportation research community puts more emphasis on the traffic flow distribution. Operation of charging stations and its connection with power systems are neglected, but deserve more attentions.

2.2 PDN model

Economical power system operation entails solving an OPF problem [34]. The most well-known OPF model renders a non-convex quadratically constrained program with the traditional bus-injection model (BIM). The active power flow can be approximated via the direct-current OPF (DCOPF) [34], which is an LP. However, the charging facilities in urban areas are connected to a PDN; distribution lines possess higher resistance to reactance ratios, bringing tight correlation among power flow variables. The traditional lossless DCOPF which assumes constant bus voltage and neglects reactive power is no longer satisfactory for modelling a PDN. In contrast to a transmission network which is usually meshed in topology, a PDN is often intentionally operated with radial topology. In such circumstance, the power flow equation can be recursively constructed via the branch flow model (BFM) developed in [34–37]:

$$P_{ij}^l - r_{ij}^l I_{ij}^l + P_j^N = \sum_{k \in c(j)} P_{jk}^l \quad \forall j \quad (18)$$

$$Q_{ij}^l - x_{ij}^l I_{ij}^l + Q_j^N = \sum_{k \in c(j)} Q_{jk}^l \quad \forall j \quad (19)$$

$$U_j = U_i - 2(P_{ij}^l r_{ij}^l + Q_{ij}^l x_{ij}^l) + (z_{ij}^l)^2 I_{ij}^l \quad \forall l \quad (20)$$

$$U_i I_{ij}^l = (P_{ij}^l)^2 + (Q_{ij}^l)^2 \quad \forall l \quad (21)$$

where $c(j)$ is the set of child buses of bus j ; P_{ij}^l , Q_{ij}^l , and I_{ij}^l are active power, reactive power and squared current in



line l between its head bus i and tail bus j , respectively; r_{ij}^l, x_{ij}^l and $z_{ij}^l = r_{ij}^l + jx_{ij}^l$ are resistance, reactance, and impedance of line l , respectively; P_j^N and Q_j^N are nodal power injections; P_{jk}^l and Q_{jk}^l are power flows in downstream lines connected to bus j ; $U_j = V_j^2$ is the squared bus voltage magnitude. Equations (18) and (19) depict nodal power balance; (20) is voltage drop equation; (21) defines apparent power injected into the head bus of each line.

BFM can be derived from BIM, so they actually offer the same power flow solutions in terms of bus voltage and line flow. However, BFM (18)-(21) is more tractable than BIM, because (18)-(20) are linear, and only (21) is non-linear. Another advantage of BFM is that it allows an efficient convex relaxation by replacing (21) with a rotated second-order cone:

$$U_i I_{ij}^l \geq (P_{ij}^l)^2 + (Q_{ij}^l)^2 \quad (22)$$

So the relaxed model yields a second-order cone program (SOCP). For radial networks, this relaxation is shown to be exact under mild conditions [37–40], and is more tractable than the semidefinite program relaxation for BIM-based OPF problems. For meshed networks, because bus angle variables are neglected, BFM and BIM may no longer be consistent.

If we neglect the lossy items in BFM, (18)-(20) again define a proper power flow solution; (21) and squared current I_{ij}^l variable can be neglected, leading to the linearized BFM:

$$P_{ij}^l + P_j^N = \sum_{k \in c(j)} P_{jk}^l \quad \forall j \quad (23)$$

$$Q_{ij}^l + Q_j^N = \sum_{k \in c(j)} Q_{jk}^l \quad \forall j \quad (24)$$

$$U_j = U_i - 2(P_{ij}^l r_{ij}^l + Q_{ij}^l x_{ij}^l) \quad \forall l \quad (25)$$

Since reactive power and bus voltage are taken into account, model (23)-(25) is still more competent than the traditional DC power flow model.

If we assume that all bus voltages are close to that at the reference bus, $(V_i - V_0)^2 \approx 0$ holds, implying $U_j = 2V_0 V_j - U_0$. Substituting it into (25), we obtain:

$$V_j = V_i - \frac{P_{ij}^l r_{ij}^l + Q_{ij}^l x_{ij}^l}{V_0} \quad \forall l \quad (26)$$

Equations (25) and (26) are interchangeably used in the existing literature. Linearized BFM is shown to perform well in distribution system related studies [40–43].

2.3 Interdependent network model

2.3.1 Coupling points

Charging facilities tie the two traditionally isolated systems. The typical connection is shown in Fig. 1. In the OPF problem, equality constraints correspond to the power flow model, while inequality constraints include variable lower and upper bounds as well as (22). The UE problem is associated with (10). The interaction between two systems is described as follows.

The traffic flow forms a UE in the transportation system. Suppose a fast charging station rests on arc a where the traffic flow is x_a . If N_E vehicles receive battery charging service during a period of Δt , the flow at the charging station link is $x_a^e = N_E / \Delta t$. Suppose the average energy demand of each EV is E_B , then the total electric power demand P_j^{dc} at charging station a connected with bus j is:

$$P_j^{dc} = \frac{N_E E_B}{\Delta t} = \phi(x_a^e) = x_a^e E_B \quad (27)$$

In the existing literature, there are two representative methods to model electric vehicular flow x_a^e .

- 1) Proportional assumption (Ass-P for short). The simplest way is to assume $x_a^e = k_0 x_a$, where k_0 is a constant, as in [43–46]. This assumption is more suitable for wireless charging road: charging facilities are paved under the surface of roadway segments, and EVs are charged in motion [45, 47]. However, it may be difficult to determine the value of k in practice, because it could change over time, and may be different for each road. It is also worth mentioning that not all EVs have to recharge on travel. Only those in need of the service are included in x_a^e .
- 2) Independent modelling of vehicle types (Ass-I for short). Another way is to treat GV flow x_a^g and EV flow x_a^e separately, and $x_a = x_a^g + x_a^e$. This needs a particular route choice model for EVs, which can take more factors such as electricity prices and queuing into

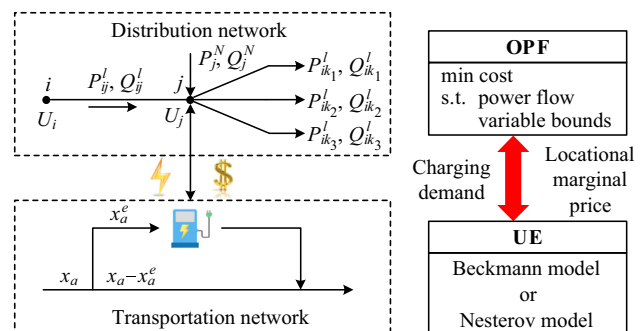


Fig. 1 Coupling between the two systems

account. This paradigm is used in [19]. If EV charging demands are not the same, they can be categorized into several clusters, and each cluster shares the same parameter [48].

The charging load in (27) is added with other loads in the PDN. In non-deregulated case, the system power flow is determined by BFM. The electricity price at charging stations may influence the route choices of EVs (this requires a special treatment in the UE model, as explained previously), if power system is allowed to announce different prices for individual charging stations, then charging loads will spontaneously respond to the price signal, offering additional flexibility to system operation. This dimension of flexibility could be significant because dispatchable resource in distribution system is often rare. In this context, it is more interesting and promising to envision deregulated power markets in distribution system. In particular, two pricing schemes were found in existing work.

- 1) Retail price (RTP). The distribution system can directly set electricity prices at individual fast charging stations, as in [46]. To ensure fairness, the price maker must obey certain pricing policy, such as limits on maximum price in each period and daily average price.
- 2) Locational marginal price (LMP), which is similar to the wholesale power market. Given demand bidding at all buses, the distribution market is cleared according to an OPF problem, as illustrated in Fig. 1, where the equality collects active power balance condition in (18) or (23), and inequality encapsulates remaining constraints in BFM as well as variable lower and upper bounds. LMP reflects the marginal cost of supplying per unit incremental load at particular charging station, and has been adopted in [19, 44].

Mature LMP markets rely on DCOPF model, which is appropriate for transmission power grid. As distribution systems possess high resistance to reactance ratios, the market clearing model should take into account network losses. Fortunately, BFM-based OPF formulation admits exact SOCP relaxation. LMPs can be extracted from dual variable μ , which is supported in MOSEK solver. Alternatively, LMP can be obtained by solving the dual problem (an SOCP) or the KKT condition with the given primal optimal solution (a set of linear equations).

An LP market clearing problem can be obtained by applying polyhedral approximation technique [49] on second-order cone constraint (22). It provides a more convenient computational tool for distribution market studies, because the KKT or primal-dual optimality condition can be easily linearized, and the market clearing problem can be embedded in another optimization problem that mimic the strategic decision of market participants. If the loads in

three phases are unbalanced, the equivalent network topology is no longer radial, the three-phase distribution OPF model in [50] and semidefinite programming relaxation method can provide an OPF solution.

2.3.2 Coupled networks

As shown in Fig. 1, the traffic assignment problem and OPF problem are independently solved; an equilibrium in the coupled networks will emerge when no one has the incentive to change its strategy unilaterally.

Mathematically, such an equilibrium is usually described by a fixed-point problem: suppose the OPF solution is $y^*(x)$, where x corresponds to the UE pattern in the transportation system; LMP can be determined from a mapping $\mu^* = LMP(y^*)$; $x^*(\mu)$ is the UE pattern with a given LMP μ^* . A fixed point is reached if (28) is satisfied, inspiring an iterative method to identify the network equilibrium [19], described as follows.

$$\mu^* = LMP(y^*(x^*(\mu^*))) \quad (28)$$

Step 1: Initiate an arbitrary LMP μ .

Step 2: Solve the traffic assignment problem (referring to the UE models in this paper).

Step 3: Update charging demands; solve the market clearing problem and calculated LMP.

Step 4: If the change of LMP in two consecutive iterations is small enough, terminate; otherwise, return to *Step 2*.

We choose to initiate LMP because it does not impact feasibility. If the equilibrium exists and is stable, this iterative algorithm is likely to converge. It relies on convex optimization and is thus preferred in many applications.

Another viable option is to simultaneously solve the optimality conditions of traffic assignment problem and OPF problem. For the former one, the KKT condition is like (9); the convex objective can be approximated by a piece-wise linear function, and (9) can be linearized by the integer programming method [51]. For the latter one, we recommend linearizing the market clearing problem by performing polyhedral approximation [49] on constraint (22), because KKT condition of an LP is a linear complementarity problem, and can be linearized by the same technique in [51]. The advantage of KKT system method stems from its compactness: the network equilibrium is expressed by closed-form constraints.

The above modelling framework has been reported in [19]. It also discusses the existence and stability of the network equilibrium and provided some intuitive explanations. To our knowledge, reference [44] may be the first work that sheds light on this equilibrium problem in the coupled transportation and power systems, in which DCOPF is used to clear the power market.



3 Applications

Since EVs have shown their advantages in low environmental impact and great market potential, the research on integration of EVs in transportation system and power system has been a hot topic for more than a decade. Among transportation research community, traffic assignment models with EVs received most attentions. However, the charging station and its impact on the power system is usually simplified or neglected. Among power system research community, there have been extensive publications on various EV charging schemes and their system impact. Nevertheless, the driving patterns of EVs are assumed to be given in either a deterministic or stochastic manner. This section aims to bridge the gap: we give a comprehensive review on interdisciplinary research which considers the reaction from the traffic side (although some neglects the power grid), covering the main topics of system planning and operation. The models of respective system and the mathematical formulation of the whole problem will be compared.

3.1 System expansion planning

The task of system expansion planning aims to determine the construction strategies on one or more system components including charging stations/lanes, roadway segments, distribution lines, generators, etc. Charging facility planning has received most attentions up to now, amid the research on new UE models.

Reference [30] proposes a tour-based UE model with several classes of EVs; the charging station planning problem is then formulated as a bi-level program (BLP), and solved by a genetic-algorithm-based procedure. Reference [31] studies optimal deployment of charging lanes under an energy-aware UE model. The planning problem is cast as a mathematical program with complementarity constraints (MPCC), which is solved by an active-set algorithm. Above work does not consider the PDN.

Reference [44] presents the first study on interdependent transportation and power infrastructure from a network equilibrium perspective. The UE model captures destination choice via a multinomial logit function. The network equilibrium consists of KKT conditions of UE and DCOPF, and the charging station allocation problem is posed as an MPCC. An active-set algorithm combining with NLP solvers is used to solve the problem. A multi-objective charging station planning model is developed in [52], which maximizes the vehicle flows captured by charging stations and compromises network losses and bus voltage deviations. Collaborative planning of distribution system and charging stations is studied in [53], where Beckmann

UE model captures traffic flow distributions. Similar framework is also adopted in [54]; however, queuing time and tie line allocation are taken into account. In aforementioned work, the UE appears in the form of KKT condition, and the nonlinear BPR function (3) makes the complementarity condition hard to tackle. Reference [55] borrows the Nesterov UE model, and suggests a mixed-integer linear program (MILP) model for coordinate expansion planning, including roadway segments in the transportation system, generators and distribution lines in the PDN, as well as charging facilities which couple the two systems.

Researchers also propose alternative transportation system models for charging station planning problem. A capacitated flow-refueling location model is reported in [56] which mainly concerns limited driving ranges of EVs, instead of congestion, and the fast charging station planning problem yields an MILP. Notably, the PDN model is the linearized power flow model in [57]. In [58], the capacitated flow-refueling location model is improved by considering heterogeneous EV driving ranges and stochastic arriving rates, and the power system model is also replaced with the BFM which is more suitable for distribution systems. As a result, the planning problem comes down to a mixed-integer SOCP (MISOCP). Photovoltaic (PV) generation is growing fast in distribution systems. However, the output of PV power plant is volatile. A two-stage stochastic MISOCP model is offered in [59] to determine the optimal size of charging stations and PV generators. In practice, an entirely accurate probability distribution of uncertain factor is difficult to acquire. To vanquish this difficulty, PV generation is represented via ambiguous distribution in [60]. Traffic demands on a highway network are obtained via Monte Carlo simulation. A data-driven robust stochastic program is proposed in [60] for siting and sizing stand-alone renewable powered charging stations.

All literatures mentioned above are summarized and compared in Table 1, where the symbol “ \surd ” (or “ \times ”) signifies that the listed item is (or is not) considered; ACOPF is the abbreviation of alternating current optimal power flow.

3.2 Coordinated operation

With a high penetration of EVs, the interactions between transportation system and power distribution system impose challenges on real-time operation, as the decision space grows higher. On the other hand, the synergetic potential also creates opportunity to ameliorate operating conditions of both systems through proper price signals, calling for careful coordination between two systems. A basic model is the network equilibrium elaborated in

Table 1 Comparison of models in system expansion planning

Reference	Network model		Vehicular flow		Components for expansion	Final model
	Transportation network	PDN	GV	EV		
[30]	Tour-based UE	×	✓	✓	Charging stations	BLP
[31]	UE (SoC, recharge rate)	×	×	✓	Charging lanes	MPCC
[44]	UE (destination choice)	DCOPF	✓	✓	Charging stations	MPCC
[52]	Shortest path	ACOPF (BIM)	×	✓	Charging stations	Multi-objective NLP
[53]	UE (Beckmann model)	ACOPF (BIM)	Ass-P + (27)	Ass-P + (27)	Charging facilities, feeders, substations	Multi-objective NLP
[54]	UE (Beckmann model)	ACOPF (BIM)	Ass-P + (27)	Ass-P + (27)	Charging stations, tie lines	NLP
[55]	UE (Nesterov model)	Linearized BFM (23)-(25)	Ass-P + (27)	Ass-P + (27)	Charging facilities, local generators, distribution lines, roadway segments	MILP
[56]	Capacitated flow-refueling location model	Linearized power flow model in [57]	×	✓	Charging stations	MILP
[58]	Modified capacitated flow-refueling location model	BFM (18)-(21)	×	✓	Charging stations	MISOCP
[59]	Modified capacitated flow-refueling location model	BFM (18)-(21)	×	✓	Charging stations, PV power plants	Two-stage stochastic MISOCP
[60]	Monte Carlo simulation	×	×	✓	Sites of charging station, sizes of PV panels, size of energy storage unit	Data-driven robust stochastic program

Section 2.3.2. This equilibrium state is often described by complementarity constraints, as in many publications reviewed in Section 3.1. In operation related studies, locations and capacities of charging facilities are fixed; electricity price is an effective mean to influence traffic flows. As charging stations mainly demand active power, generators in the distribution systems also appear to be important dispatchable resources.

Along this line, coordinated system operation with road/electricity pricing is studied in [45, 46, 61]. Beckmann UE model or its modification is adopted for traffic flow, and congestion toll is imposed on each roadway segment, while DCOPF/BFM is employed for power flow. The system operation models in [46, 61] render MPCCs which can be solved by a manifold optimization algorithm. Because the solution is often suboptimal, derivative-free search algorithms are suggested in [45, 46] for cases with only a few pricing variables. MISOCP can be solved by off-the-shelf solvers such as CPLEX, GUROBI and MOSEK. To consider traffic demand uncertainty, a two-stage robust optimization (TSRO) model is proposed in [62]. By solving traffic assignment problem (10) in extreme scenarios, traffic demand variation is mapped into power demand uncertainty. The optimal solution of TSRO is

identified by iteratively solving an SOCP master problem and a max-min SOCP subproblem. By dualizing the inner SOCP, a bi-convex program arises, which is further transformed into an MISOCP. Reference [63] applies the interdependence model in the unit commitment problem, and proposes to use EV fleets as a battery storage unit.

Above work assumes that there is a central agency that can anticipate the network flow in both systems. However, if the two systems are independently operated, or the privacy of system data should be preserved, new decision-making methods are needed. Reference [47] considers the coordinated system operation as a decentralized optimization (DO) problem, and develops an algorithm based on alternative direction method of multiplier. A bilevel game model is proposed in [64] to coordinate power and traffic flows, shave peak loads and smooth load ramps in both systems. The upper level designs the network equilibrium (a welfare equilibrium in the sense of [65]) between the two systems; in the lower level, a quasi-dynamic UE model is incorporated to simulate time-varying demands and spontaneous delay of departure time. GVs and EVs are separately treated in [19, 48]; charging stations are represented by virtual nodes and arcs in an extended transportation network. Network equilibrium is introduced in



Table 2 Comparison of models in system operation

Reference	Network model		Vehicular flow		Pricing scheme		Final model
	TN	PDN	GV	EV	TN	PDN	
[61]	UE (Beckmann model)	DCOPF	Ass-P + (27)	Ass-P + (27)	Road toll	LMP	MPCC
[45]	UE (Beckmann model)	BFM (18)-(21)	Ass-P + (27)	Ass-P + (27)	Road toll	×	MISOCP
[46]	UE (destination choice)	BFM (18)-(21)	Ass-P + (27)	Ass-P + (27)	Road toll	Retail price	MPCC
[62]	UE (Beckmann model)	BFM (18)-(21)	Ass-P + (27)	Ass-P + (27)	×	×	TSRO
[63]	UE (Beckmann model)	DCOPF	Ass-P + (27)	Ass-P + (27)	×	×	MILP
[47]	Modified UE	DCOPF	Ass-P + (27)	Ass-P + (27)	×	LMP	DO
[64]	UE (Beckmann model)	Unbalanced BFM	Ass-P + (27)	Ass-P + (27)	×	Welfare equilibrium [65] equivalent to LMP	Bilevel game
[48]	SO traffic assignment	DCOPF	✓	✓	Road toll	LMP	LP
[19]	UE (GVs + EVs)	BFM (18)-(21)	✓	✓	×	LMP	Fixed-point problem
[66]	Joint transportation and charging scheduling	×	×	✓	×	Price function	Normalized Nash equilibrium
[67]	Dynamic routing for individual vehicles	DCOPF	×	✓	×	LMP	Fixed-point problem

[19]. Collaborative pricing schemes are suggested in [48] to help each network achieve SO while preserving the privacy of operational data.

Some literature studies inter-system collaboration through more tailored models. For example, a joint transportation and charging scheduling model is portrayed in [66], and is formulated as a normalized Nash equilibrium problem. The distribution system is omitted; the LMP is approximated by a power function in nodal demand. In [67], a routing optimization algorithm is presented to navigate individual vehicles; vehicle routing and distribution system will reach a fixed point linked by LMP. All literatures mentioned above are summarized and compared in Table 2.

3.3 Other applications

Besides the aforementioned interdependence model, researchers have proposed alternative ways to forecast loads of fast charging stations. In [68], EV arrival rate is derived from a fluid dynamic model, and then the charging load is predicted by queueing theory. The spatial and temporal distribution of EV charging demands are estimated by Monte Carlo simulation method in [69]. Geographical movements of EVs are indicated by twenty-four O-D matrices with a time granularity of one hour. BIM of

power flow is solved by fast-decoupled Newton-Raphson method.

The impact of wireless charging roads on electricity market is investigated in [70]. EV arrival rate at each street is assumed to follow Poisson process, and thus transportation network model is not considered. Power market is cleared according to DCOPF. A retail pricing mechanism is recommended for load-serving entities to reduce LMP variation and improve the social welfare.

Vulnerability of interdependent transportation and power distribution systems is studied in [71]. Traffic flow is determined from Beckmann UE model, where road capacity c_a also acts as a variable. An MILP model is proposed to identify the most vulnerable arcs, whose degradation have the worst impact on total vehicle travel time. The distribution system is modeled via linearized BFM (23), (24), (26). It is found that road capacity degradation may cause over load in some charging stations, which may trigger security issues in the power grid. In [72], traffic demand is described by hourly O-D matrices, and the impact of road block on the coupled networks is investigated via simulation. Resilience of the coupled systems in extreme events is studied in [73]. A dynamic UE model is used to capture time-varying traffic flows after a disruption occurs, taking into account traffic signal control, which is shown to be effective in traffic flow management [74, 75]. Distribution system is modeled by linearized BFM

(23), (24), (26). A min-max-min problem is set forth to enhance system resilience by strategically hardening lines and placing DGs.

4 Future research directions

With the increasing penetration of EVs, the system-wide interdependence across transportation and power distribution infrastructures will become more prominent. The topic reviewed in this paper is a very young and active research field. Authors believe the following directions deserve more attentions in the future.

4.1 Traffic assignment models

4.1.1 Dynamic traffic assignment

The UE model introduced in Section 2.1 assumes all trips are completed instantly. In fact, not all vehicles enter the transportation system and begin to travel at the same time. To simulate spatial and temporal distributions of traffic flows more precisely, dynamic traffic assignment models should be used. Indeed, as the power system load and electricity price are time-varying, it is always more appropriate to use dynamic traffic assignment in the study of system interdependence. However, dynamic traffic assignment theory is much more complicated than the static one. There is no universal model; existing ones usually require more sophisticated mathematical tools. There have been extensive publications in this area during the past decades. A review on some earlier dynamic traffic assignment (DTA) models can be found in [76], and an up-to-date survey can be found in [77]. This section only sheds light on some most representative ones that might be combined with power system research.

Continuous-time traffic flow can be described by kinematic wave equation, or namely, the Lighthill-Whitham-Richards (LWR) model [78, 79]. Cell transmission model (CTM) proposed in [80, 81] is a discrete-time approximation of the LWR model. It simulates traffic flow and density on a corridor (represented by a finite number of segments) over a certain period. Discrete-time and continuous-time dynamic user equilibrium (DUE) models based on CTM is developed in [82, 83], respectively. Because system resilience entails time elapse, the discrete-time DUE model is employed in [73] as complementarity constrains [84]. Several other discrete-time DTA models can be found in [84–88]. Discrete-time DTA models are more likely to be used in the interdependence research, because mainstream power system dispatch and power market models give rise to discrete-time optimization problems.

Continuous-time DTA models have been proposed in form of optimal control [89], differential game [90, 91], differential variational inequality [92], differential complementarity problem [93] and partial-differential complementarity problem [94]. Meanwhile, continuous-time power system dispatch methods are also developed in [95] for unit commitment and in [96] for power market. Combining dynamic models of transportation system and power system, it is more convenient to investigate the transient behavior and stability of network flow evolution. However, the computational tractability may be a main issue for practical usage.

4.1.2 Ride-sharing UE

In light of the fast development of information and communication technology, as well as the proliferation of smart phones, ride-sharing has become a prevalent trend in recently years, and related services have witnessed rapid growth.

Beckmann UE model is revamped in [97] by combining a ride-sharing demand function with elastic traffic demands. The model can be utilized to analyze ride-sharing price, driver's willingness to share, as well as their interaction with road congestion level. In [98], travelers are categorized into solo/ride-sharing drivers and passengers; a mixed complementarity program is devised for the ride-sharing UE over an extended network. Path-flow based ride-sharing UE is formulated as NCP in [99], and is improved to a link-node based one in [100]. The latter one drastically reduces problem size and computational burden, and also allows problem decomposition for large-scale systems. An alternative method is suggested in [101] to consider ride-sharing behaviors in Beckmann UE model with travel mode choice. Waiting time that depends on the number of available drivers is taken into account.

4.2 Future distribution systems

Due to the constantly increasing penetration of renewable energy and distributed generation [102, 103], the distribution system itself is undergoing rapid change. Power flow could be reversed depending on renewable power output; a fraction of demand such as EV charging becomes deferrable and more flexible; energy storage devices are deployed to facilitate system operation; above changes pave the way to the so-called active distribution network [104, 105], and call for innovative ideas, technologies and initiatives to support such a transition. To our knowledge, energy storage sharing and its business model have already attracted much attention [105–108]. In a broader sense, prosumers [109] and distribution power market [50, 110] are playing increasingly important roles in



the operation of distribution systems. In the energy internet era [111], we believe new business models will emerge, such as sharing economy [112], multi-energy market [113, 114], peer-to-peer exchange [114–117], and so on. In such circumstances, EVs and charging stations have better opportunities and stronger incentives to take part in future distribution power markets.

4.3 Prospective applications

Provided with more advanced UE models, some interesting directions are open for future research.

4.3.1 Demand response

Because controllability (or flexibility) of a power distribution system is not as high as a transmission system, it is very promising to ameliorate system operating condition by leveraging demand response potentials of private electric cars and electric buses. Please bear in mind that private cars, buses and metro trains have different driving patterns: buses cannot alter their routes, while metro train will not suffer congestions. Electric buses and metro trains are good providers of demand response, because they are regularly scheduled by a transportation authority. In the future, private EVs and fast charging stations are likely to join in demand response programs in the presence of appropriate economic incentives. This entails accurate time-dependent DTA models with multiple travel mode choices to capture the reaction from the transportation system in response to the price signal from the power grid, as well as thorough investigation of market stability using dynamic interdependence models.

4.3.2 Shared EVs

Shared EVs which resemble shared bikes are developing rapidly in recent years. Unlike fast charging stations which draw electricity from the power system, unused EVs in a parking lot can act as an energy storage unit that can either consume or provide electricity. However, the usage of shared EVs depends on the service price and congestion condition in the transportation system. An interesting topic for the EV-sharing platform is to determine real-time service prices which encourage commuters to avoid traffic-peak hours, as well as charging-discharging schedules which bring the platform additional benefits via providing auxiliary service to the power grid or participating demand response programs. The strategic behaviors of EV-sharing platforms need careful investigation using the time-varying traffic flow patterns in the transportation network, calling for thorough study on modelling dynamic interdependence.

4.3.3 System resilience

Transportation system and power distribution system are critical infrastructures in modern society. It is important to study the resilience of such infrastructures under extreme weather conditions and natural disasters, activating fast plans for system restoration and repairing, and emergency evacuation. In the case of a major blackout, EVs can provide valued backup of electric energy. In addition, contingency in one system can propagate to the other, such as the event in the city of Shenzhen mentioned in Section 1. Unlike power system blackout which rarely happens, traffic accidents and traffic flow control take place more frequently. It is also essential to study how traffic flow and power flow are influenced after a contingency, so as to identify vulnerable components that need upgrade or expansion.

4.3.4 Robust system operation

Finally, uncertainty in such coupled infrastructures is ubiquitous: distributed renewable generation, power demand, traffic demand, various contingencies, and so on. Robust system operation still deserves research attention, especially in a decentralized manner.

5 Conclusion

The penetration of EVs in modern smart cities is growing fast, and the interaction between transportation and power distribution systems are becoming more evident. This paper provides an up-to-date survey on modelling and applications of such interdependent infrastructures, and envisions prospective research directions in the near future. The power flow model of distribution system is relatively mature, while the user equilibrium model of the transportation system varies significantly under different assumptions, especially in the presence of EVs. System-level coordination also yields very different problems with/without a central authority. We believe the distributed mode and network equilibrium are more practical in the future. In conclusion, this direction is still in its infancy stage. With further proliferation of EVs and the advent of sharing economy, the benefits gained from traffic-power coordination for enhancing system reliability, efficiency and resilience will be prominent.

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