Consensus of cyber-physical power systems based on multi-agent systems with communication constraints

Chi HUANG^{1,2} , Chengli FENG³, Jinde CAO²

Abstract The consensus protocol of cyber-physical power systems is proposed based on fractional-order multi-agent systems with communication constraints. It aims to enable each generator to reach a time-varying common rotor angle and rotor speed. Communication constraints including event-triggered sampling and partial information transmission are considered to render the consensus protocol more realistic. The Zeno behavior is excluded during the system sampling process. A sufficient condition is derived to solve the consensus problem. The effectiveness of the proposed consensus protocol is demonstrated by a numerical example.

Keywords Cyber-physical power system, Fractional-order system, Multi-agent system, Communication constraint

1 Introduction

The concept of cyber-physical power systems (CPPSs) is first proposed in [1] as a dedicated case of a cyber-physical system in a power system. It is composed of a large number of computing devices (servers, computers, and embedded computing devices), data acquisition devices (sensors, phasor measurement unit, and embedded data acquisition equipment), and physical devices (large-scale generator set, distributed power supply, and load). These devices are connected through communication and transmission networks. CPPSs combine the advantages of cyber-physical systems and power systems. However, it also experiences many technical and security challenges [2, 3]. Owing to the ability to collect and handle massive information compared with traditional power systems, studies on CPPSs such as traffic networks [4] and aviation electric power systems [5] have been performed. Among these, the stability of CPPSs has been emphasized, because an unstable power supply may lead to voltage collapse and breakdown of the entire power system [6, 7]. Most studies on stability in the existing literature are focused on the supply-demand balance and rotor angle stability. Specifically, the target of the supply-demand balance is to continuously match the production and consumption of electricity across networks [8]. The rotor angle stability [9] enables the CPPS to operate smoothly and efficiently by allowing each generator to reach a time-varying common rotor angle and speed, separately. Both of them can be regarded as a consensus perspective. Thus, many studies use the consensus theory to solve stability problems in CPPSs [10, 11].

A multi-agent system (MAS) model has been extensively studied as an effective method for the consensus problem [12, 13]. It is a cooperative system whereby agents

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can share their information to communicate with each other. The MAS has many significant real-world applications, such as sensor networks [14], unmanned air vehicles formations [15, 16], and microgrids [17]. Furthermore, many studies have applied the MAS to a CPPS [18–21]. In [18], a distributed multi-agent-based load shedding algorithm that can yield an efficient load shedding decision based on the discovered global information was proposed. In [20], an innovative approach was proposed to use real-time scheduling techniques for the automation of electric loads in CPPSs. In [21], a distributed control method to enhance power system rotor angle stability based on the second-order MAS consensus was proposed, where each generator can be regarded as an agent to communicate with each other. Nevertheless, most of the studies on CPPSs are based on integer-order MASs. In fact, real-world processes are generally (or most likely) fractional-order systems [22, 23]. Many physical systems exhibit fractional dynamical behavior owing to their special materials and chemical properties [24–27]. Thus, it is highly desirable to study CPPSs with fractional-order MASs.

Communication constraints that are crucial factors to the system performance cannot be ignored in practice. Among them, the partial information transmission is significant. That is to say, not all of the information can be transmitted to their neighbors perfectly [28–30]. Energy saving is another important topic. Considering the massive communicating and computing work in CPPSs, sampling would be an efficient method to reduce resources. Traditional periodic sampling techniques would typically waste much energy and consequently shorten the lifespan of the system to a certain degree [31]. To address this problem, event-triggered control has attracted significant attention from research into the consensus of MASs [32–37]. The Zeno behavior must be excluded in the event sampling process; otherwise, continuous communication will be required again [38]. However, among the existing literatures, the proofs absent of the Zeno behavior are not presented (or are naturally avoided), because events can only occur at sampling time instants that are a multiple of some given sampling period [34, 39]. Therefore, it is necessary to propose a more specific proof when the Zeno behavior is excluded.

Our current study is of practical significance because it can effectively reduce the gap between the physical and cyber worlds. However, few results have been found in the existing literature. Motivated by the discussions above, we herein consider the consensus of CPPSs based on fractional-order MASs with communication constraints. Our objective is to ensure that each generator achieves a time-varying common rotor angle and rotor speed. The contribution of this paper can be summarized in three aspects: 1) a fractional-order MAS modeling with communication

constraints is established for the consensus of the CPPS; 2) based on communication constraints, a distributed event condition is introduced for energy conservation, and consensus criteria are obtained that can be applied to more practical systems; 3) a finite number of broadcasts and updates exist by each agent in any finite time period, thus ensuring that the Zeno behavior does not occur in the CPPS. The effectiveness of the proposed consensus criteria is illustrated with a numerical example.

2 Model description and preliminaries

In this section, some basic notions and properties of the algebraic graph theory are introduced.

Each agent can be abstracted as a node. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ be a weighted directed network of order N , with the set of nodes (vertices) $\mathcal{V} = \{1, 2, \dots, N\}$, the set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A = (a_{ij})_{N \times N}$. A directed edge in graph $\mathcal{G}$ is denoted by the ordered pairs of nodes $\mathcal{E}_{ij} = (i, j)$, where i and j are called the terminal and initial nodes, respectively. If edge $(j, i) \in \mathcal{E}$, then node j is called a neighbor of node i , implying that node i can receive information from node j . Therefore, $a_{ij} > 0$; otherwise, $a_{ij} = 0$. The neighborhood index set of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$, while we indicate with $|\mathcal{N}_i|$ its cardinality (the number of neighbors of node i). $\|\cdot\|$ and $\|\cdot\|_1$ represent the Euclidean norm and one norm for a vector or a matrix, respectively. I_n and O_n express the identity matrix and zero matrix with dimension n , respectively. D_+^α represents the fractional-order Dini (upper right) derivative. Matrix $A > \mathbf{0}$ denotes that A is a positively defined matrix, and $\text{diag}\{\cdot\}$ denotes a diagonal matrix.

2.1 Fractional calculus and preliminary lemmas

Definition 1 The fractional integral with non-integer order $\alpha > 0$ of function $p(\cdot)$ is defined as follows:

$$D_{t_0, t}^{-\alpha} p(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t - \tau)^{\alpha-1} p(\tau) d\tau$$

where $\Gamma(\cdot)$ is the gamma function $\Gamma(t) = \int_0^\infty t^{\alpha-1} e^{-t} dt$.

Several definitions exist that refer to the fractional derivative of order $\alpha > 0$. Compared with other definitions, the Caputo derivative has a unique advantage; it only claims the initial conditions provided as the integer-order derivative of $p(\cdot)$ and the function itself. Therefore, our consideration is a fractional-order MAS with a Caputo derivative, whose definition is given below.

Definition 2 The Caputo derivative of fractional order $\alpha > 0$ of function $p(\cdot)$ is defined as follows:

$$\begin{aligned} {}^C D_t^\alpha p(t) &= D_{t_0,t}^{-(n-\alpha)} p^{(n)}(t) \\ &= \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t (t-\tau)^{n-\alpha-1} p^{(n)}(\tau) d\tau \end{aligned}$$

where $n-1 < \alpha \leq n \in \mathbf{Z}^+$.

For simplicity, $D_{0,t}^{-\alpha}, {}^C D_t^\alpha$ are rewritten as $D^{-\alpha}$ and D^α , respectively. Some properties of these two operators are introduced.

Lemma 1 [40] If $p(t), g(t) \in C^n[0, +\infty)$ and $n-1 < \alpha, \beta \leq n \in \mathbf{Z}^+$, then:

- 1) $D^{-\alpha} D^{-\beta} p(t) = D^{-(\alpha+\beta)} p(t), \alpha, \beta > 0.$
- 2) $D^\alpha D^{-\alpha} p(t) = p(t), n = 1.$
- 3) $D^{-\alpha} D^\alpha p(t) = p(t) - \sum_{j=0}^{n-1} (t^j/j!) p^{(j)}(0), \alpha \geq 0.$
- 4) $D^\alpha(p(t) \pm g(t)) = D^\alpha p(t) \pm D^\alpha g(t).$

where $C^n(M, N)$ denotes the space consisting of n -order continuous differentiable functions from M into N .

Lemma 2 [25] Let $\mathbf{x}(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbf{R}^n$ be a differential vector-value function. Then, for any time instant $t \geq t_0$,

$${}^C D_t^\alpha [\mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t)] \leq \mathbf{x}^T(t) \mathbf{P} {}^C D_t^\alpha \mathbf{x}(t) + \left({}^C D_t^\alpha \mathbf{x}^T(t) \right) \mathbf{P} \mathbf{x}(t)$$

Therefore,

$${}^C D_t^\alpha [\mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t)] \leq 2\mathbf{x}^T(t) \mathbf{P} {}^C D_t^\alpha \mathbf{x}(t)$$

where $\mathbf{P} \in \mathbf{R}^{n \times n}$ is a symmetric positive definite matrix, $\alpha \in (0, 1)$.

2.2 Network model

In this section, we will discuss the stability of the rotor angle. The CPPS can be modeled by the formulas as follows [41].

$$\dot{\delta}_i = \omega_i \quad (1)$$

$$\begin{cases} \dot{\omega}_i = -\frac{D_i}{M_i} \omega_i + \frac{1}{M_i} (P_{mi} - P_{ei}) \\ \dot{\mathbf{E}}'_i = \mathbf{F}_i \mathbf{E}'_i + \mathbf{L}_i(\delta_i) \mathbf{V}_i + \mathbf{H}_i \mathbf{E}_{fi} \\ P_{ei} = \mathbf{V}_i^T [\mathbf{R}_i(\delta_i) \mathbf{V}_i + \mathbf{T}_i(\delta_i) \mathbf{E}'_i] \\ P_{mi} = \frac{B_{mi} p}{A_{mi} p} u_{mi} \\ E_{fi} = \frac{B_{fi} p}{A_{fi} p} u_{fi} \\ \mathbf{V} = [\mathbf{Y} - \mathbf{R}(\boldsymbol{\delta})]^{-1} \mathbf{T}(\boldsymbol{\delta}) \mathbf{E}' \end{cases} \quad (2)$$

where δ_i is the rotor angle; ω_i is the rotor speed; D_i is the damping coefficient; M_i is the inertia constant; P_{mi} is the mechanical power; P_{ei} is the electrical power; $\mathbf{E}'_i$ is the internal bus voltage vector; $\mathbf{V}$ is the generator terminal voltage vector. $\mathbf{R}(\boldsymbol{\delta})$ and $\mathbf{T}(\boldsymbol{\delta})$ are matrices comprised of elements $\mathbf{R}_i(\delta_i)$ and $\mathbf{T}_i(\delta_i)$, respectively. $\mathbf{R}_i(\delta_i), \mathbf{T}_i(\delta_i) \mathbf{F}_i$, and $\mathbf{H}_i$ are described in [41] in detail.

A classical coherency criterion states that a generator pair (i, j) is considered coherent if

$$\begin{cases} \lim_{t \rightarrow \infty} \|\delta_i - \delta_j\| = 0 \\ \lim_{t \rightarrow \infty} \|\omega_i - \omega_j\| = 0 \end{cases} \quad (3)$$

which can be considered a consensus problem of a second-order multi-agent system. It appears that we can use a multi-agent system to study the rotor angle stability of the CPPS.

From (1) and (2), if the controller input is chosen in the following form:

$$\tilde{P}_{mi} = M_i u'_i + D_i \omega_i + P_{ei} \quad (4)$$

where u'_i is the new input to be determined, then the differential equations of the rotor angle and rotor speed in the CPPS is described as follows:

$$\begin{cases} \dot{\delta}_i = \omega_i \\ \dot{\omega}_i = u'_i \end{cases} \quad (5)$$

Thus, the design of u'_i is important.

Next, we discuss the rotor angle stability of the CPPS with a multi-agent system model.

The fractional-order calculation is more general, because it is an expansion of the integer-order calculation. We consider the fractional-order multi-agent system as follows:

$$\begin{cases} D^\beta \mathbf{x}_i(t) = \mathbf{v}_i(t) \\ D^\beta \mathbf{v}_i(t) = \mathbf{u}_i(t) \end{cases} \quad i = 1, 2, \dots, N \quad (6)$$

where $\beta \in (0, 1]; \mathbf{x}_i(t) \in \mathbf{R}^n, \mathbf{v}_i(t) \in \mathbf{R}^n$ are the position and speed information of the i th agent, respectively; $\mathbf{u}_i(t)$ is the control input.

To achieve a common position and speed for the MAS based on the fractional-order MAS (6), we consider the following control protocol:

$$\begin{aligned} \mathbf{u}_i(t) &= \mathbf{f}(\mathbf{x}_i(t), \mathbf{v}_i(t), t) - \sum_{j=1, j \neq i}^N a_{ij} [\mathbf{H}_{ij}(\mathbf{x}_i(t) - \mathbf{x}_j(t)) \\ &\quad + \mathbf{H}_{ij}(\mathbf{v}_i(t) - \mathbf{v}_j(t))] \end{aligned} \quad (7)$$

where $\mathbf{A} = [a_{ij}]_{N \times N}$ is the weighted adjacency matrix, which represents the connection among agents. Considering the factors of the MAS and the influence of the external

environment, the nonlinear term [42] is added to the control model of the system. $\mathbf{f} : \mathbf{R}^n \times \mathbf{R}^n \times \mathbf{R}^+ \rightarrow \mathbf{R}^n$ is a continuously differentiable vector-valued nonlinear function. $\mathbf{H}_{ij} = \text{diag}\{h_{ij}^1, h_{ij}^2, \dots, h_{ij}^n\}$ $h_{ij}^k = 0$ or 1 , for each $k = 1, 2, \dots, n$. It is called the channel matrix [28], owing to the imperfect signal transmission in a communications network.

Let $\mathbf{R}_{ij} = a_{ij}\mathbf{H}_{ij} = \text{diag}\{r_{ij}^1, r_{ij}^2, \dots, r_{ij}^n\}$, for $i, j = 1, 2, \dots, N$ and $\mathbf{R}_{ii} = -\sum_{j=1, j \neq i}^N \mathbf{R}_{ij}$.

The control input of the partial information transmission for the consensus of the MAS (7) can be rewritten in the following form:

$$\mathbf{u}_i(t) = \mathbf{f}(\mathbf{x}_i(t), \mathbf{v}_i(t), t) - \sum_{j=1, j \neq i}^N \mathbf{R}_{ij}[(\mathbf{x}_i(t) - \mathbf{x}_j(t)) + (\mathbf{v}_i(t) - \mathbf{v}_j(t))] \quad (8)$$

for any $i, j = 1, 2, \dots, N$. For mathematical derivation, we define the channel Laplacian matrix as follows:

$$\mathbf{R}_k = \begin{bmatrix} r_{11}^k & r_{12}^k & \cdots & r_{1N}^k \\ r_{21}^k & r_{22}^k & \cdots & r_{2N}^k \\ \vdots & \vdots & & \vdots \\ r_{N1}^k & r_{N2}^k & \cdots & r_{NN}^k \end{bmatrix} \quad k \in \{1, 2, \dots, n\}$$

It is clear that the matrices $\mathbf{R}_k$ are decomposed from $\mathbf{R}_{ij}$. Moreover, $R_k(i, j)$ is the k^{th} diagonal element of $\mathbf{R}_{ij}$ for any $i, j = 1, 2, \dots, N$. Obviously, $\mathbf{R}_k$ has the zero row-sum property. Herein, we always assume that $\mathbf{R}_k$ is irreducible. Suppose $\xi_k = [\xi_{k1}, \xi_{k2}, \dots, \xi_{kN}]^T \in \mathbf{R}^N$ satisfying $\sum_{i=1}^N \xi_{ki} = 1$ is the normalized left eigenvector of matrix $\mathbf{R}_k$ corresponding to the eigenvalue 0. From [43], $\xi_{ki} > 0$ holds for all $i \in \{1, 2, \dots, N\}$ and $k \in \{1, 2, \dots, n\}$. Setting $\Phi_i = \text{diag}\{\xi_{1i}, \xi_{2i}, \dots, \xi_{ni}\}$, we can obtain $\sum_{i=1}^N \Phi_i \mathbf{R}_{ij} = \mathbf{0}$.

Owing to the massive distributed components and the large number of data in the CPPS, energy saving appears particularly important. Compared with traditional period sampling, the event-trigger sampling strategy is more advantageous. Therefore, we adopt the event-trigger sampling method. The control input $\mathbf{u}_i(t)$ is converted into

$$\begin{cases} \mathbf{u}_i(t) = \mathbf{f}(\mathbf{x}_i(t), \mathbf{v}_i(t), t) - \sum_{j=1, j \neq i}^N [\mathbf{R}_{ij}(\mathbf{x}_i(t_k^i) - \mathbf{x}_j(t_{k'}^j)) + \mathbf{R}_{ij}(\mathbf{v}_i(t_k^i) - \mathbf{v}_j(t_{k'}^j))] & t \in [t_k^i, t_{k+1}^i) \\ k'(t) = \arg \min_{l \in N, t > t_l^j} \{t - t_l^j\} \end{cases} \quad (9)$$

where t_k^i is the sampling instant that is analytically determined by the following individual event condition:

$$t_{k+1}^i = \inf\{t > t_k^i, E_i(\mathbf{x}(t_k^i), \mathbf{v}(t_k^i), \mathbf{x}_i(t), \mathbf{v}_i(t)) > 0\}$$

where

$$\begin{aligned} E_i(\mathbf{x}(t_k^i), \mathbf{v}(t_k^i), \mathbf{x}_i(t), \mathbf{v}_i(t)) &= \|\mathbf{e}_{xi}(t)\|_1 + \|\mathbf{e}_{vi}(t)\|_1 \\ &\quad - \varphi_1 \sum_{j \in N_i} [\|\mathbf{R}_{ij}(\mathbf{x}_i(t_k^i) - \mathbf{x}_j(t_{k'}^j))\|_1 \\ &\quad + \|\mathbf{R}_{ij}(\mathbf{v}_i(t_k^i) - \mathbf{v}_j(t_{k'}^j))\|_1] - \varphi_2 e^{-\gamma(t-t_0)}, \\ \mathbf{e}_{xi}(t) &= \mathbf{x}_i(t_k^i) - \mathbf{x}_i(t), \mathbf{e}_{vi}(t) = \mathbf{v}_i(t_k^i) - \mathbf{v}_i(t), \\ t &\in [t_k^i, t_{k+1}^i), i = 1, 2, \dots, N \end{aligned}$$

are the position measurement errors and speed measurement errors.

The following assumption, definitions, and lemmas will be used to derive our primary results. Thus, they are given as below:

Assumption 1 Assume that two positive constants l_1, l_2 , exist such that the vector-valued nonlinear function $\mathbf{f}(\cdot)$ satisfies

$$\|\mathbf{f}(\mathbf{x}_1, \mathbf{x}_2, t) - \mathbf{f}(\mathbf{y}_1, \mathbf{y}_2, t)\| \leq \sum_{i=1}^2 l_i \|\mathbf{x}_i - \mathbf{y}_i\|$$

where $\mathbf{x}_i, \mathbf{y}_i \in \mathbf{R}^n$; $t \geq 0$.

Definition 3 The consensus in MAS (6) is said to be achieved, if for any initial states,

$$\begin{aligned} \lim_{t \rightarrow \infty} \|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| &= 0 \\ \lim_{t \rightarrow \infty} \|\mathbf{v}_i(t) - \mathbf{v}_j(t)\| &= 0 \end{aligned}$$

Definition 4 System (9) does not exhibit the Zeno behavior, if $\inf_k \{t_{k+1}^i - t_k^i\} > 0$, for all i . In other words, there is no system trajectory with an infinite number of events in a finite time.

Definition 5 [44] The two parameters form of the Mittag-Leffler function is defined as follows:

$$E_{\alpha, \beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

Lemma 3 [45] For two vectors $\mathbf{x}(t)$ and $\mathbf{y}(t) \in \mathbf{R}^{Nn}$,

$$\begin{aligned} \mathbf{x}(t) &= [\mathbf{x}_1^T(t), \mathbf{x}_2^T(t), \dots, \mathbf{x}_N^T(t)]^T \\ \mathbf{y}(t) &= [\mathbf{y}_1^T(t), \mathbf{y}_2^T(t), \dots, \mathbf{y}_N^T(t)]^T \end{aligned}$$

we have the following properties:

- 1) $\sum_{i=1}^N \|\mathbf{x}_i(t)\|_1 = \|\mathbf{x}(t)\|_1.$
- 2) $\sum_{i=1}^N \sum_{j=1}^N \|\mathbf{x}_i(t)\|_1 \|\mathbf{y}_j(t)\|_1 = \|\mathbf{x}(t)\|_1 \|\mathbf{y}(t)\|_1.$
- 3) $\|\mathbf{x}(t)\|_1 \|\mathbf{x}(t)\|_1 \leq Nn \|\mathbf{x}(t)\|^2 = Nn \mathbf{x}^T(t) \mathbf{x}(t).$

Lemma 4 (Schur complement) The following linear matrix inequality

$$\begin{bmatrix} \mathbf{Q}(x) & \mathbf{S}(x) \\ \mathbf{S}^T(x) & \mathbf{R}(x) \end{bmatrix} > \mathbf{0}$$

where $\mathbf{Q}(x) = \mathbf{Q}^T(x)$ and $\mathbf{R}(x) = \mathbf{R}^T(x)$ is equivalent to one of the following conditions.

- 1) $\mathbf{Q}(x) > \mathbf{0}, \mathbf{R}(x) - \mathbf{S}^T(x) \mathbf{Q}(x)^{-1} \mathbf{S}(x) > \mathbf{0}.$
- 2) $\mathbf{R}(x) > \mathbf{0}, \mathbf{Q}(x) - \mathbf{S}(x) \mathbf{R}(x)^{-1} \mathbf{S}^T(x) > \mathbf{0}.$

Lemma 5 [46] The Mittag-Leffler function generalizes the simple exponential function (recovered for $\beta = 1$) and, if $0 < \beta < 1$, it interpolates the positive real axis to a stretched exponential and a power law according to

$$E_\beta[-(\tau/\tau_0)^\beta] \sim \begin{cases} \exp[-(\tau/\tau_0)^\beta / \Gamma(1 + \beta)] & \tau/\tau_0 \rightarrow 0^+ \\ (\tau/\tau_0)^{-\beta} / \Gamma(1 - \beta) & \tau/\tau_0 \rightarrow \infty \end{cases}$$

2.3 Primary results

In this section, we shall obtain the consensus condition of the fractional-order MAS with communication constraints.

Let

$$\begin{aligned} \bar{\mathbf{x}}(t) &= \sum_{k=1}^N \mathbf{\Phi}_k \mathbf{x}_k(t), \quad \bar{\mathbf{v}}(t) = \sum_{k=1}^N \mathbf{\Phi}_k \mathbf{v}_k(t) \\ \tilde{\mathbf{x}}_i(t) &= \mathbf{x}_i(t) - \bar{\mathbf{x}}(t), \quad \tilde{\mathbf{v}}_i(t) = \mathbf{v}_i(t) - \bar{\mathbf{v}}(t) \end{aligned}$$

where $\bar{\mathbf{x}}(t)$ is the time-varying common position and $\bar{\mathbf{v}}(t)$ is the time-varying common speed for each agent; $\tilde{\mathbf{x}}(t), \tilde{\mathbf{v}}(t)$ are the errors between the current state information and the common state information. Subsequently, the consensus problem can be translated into a stable issue.

Because

$$\tilde{\mathbf{x}}_i(t) = \mathbf{x}_i(t) - \sum_{k=1}^N \mathbf{\Phi}_k \mathbf{x}_k(t), \quad \tilde{\mathbf{v}}_i(t) = \mathbf{v}_i(t) - \sum_{k=1}^N \mathbf{\Phi}_k \mathbf{v}_k(t)$$

we can obtain

$$\begin{aligned} \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \mathbf{\Phi}_i &= \sum_{i=1}^N \left(\mathbf{x}_i^T(t) - \sum_{k=1}^N \mathbf{x}_k^T(t) \mathbf{\Phi}_k \right) \mathbf{\Phi}_i \\ &= \sum_{i=1}^N \mathbf{x}_i^T(t) \mathbf{\Phi}_i - \sum_{i=1}^N \sum_{k=1}^N \mathbf{x}_k^T(t) \mathbf{\Phi}_k \mathbf{\Phi}_i, \\ \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \mathbf{\Phi}_i &= \sum_{i=1}^N \left(\mathbf{v}_i^T(t) - \sum_{k=1}^N \mathbf{v}_k^T(t) \mathbf{\Phi}_k \right) \mathbf{\Phi}_i \\ &= \sum_{i=1}^N \mathbf{v}_i^T(t) \mathbf{\Phi}_i - \sum_{i=1}^N \sum_{k=1}^N \mathbf{v}_k^T(t) \mathbf{\Phi}_k \mathbf{\Phi}_i. \end{aligned}$$

From $\sum_{i=1}^N \mathbf{\Phi}_i = \mathbf{I}_n$, we obtain

$$\sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \mathbf{\Phi}_i = \mathbf{0}, \quad \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \mathbf{\Phi}_i = \mathbf{0}.$$

Subsequently, we can obtain the following error dynamical system

$$\left\{ \begin{aligned} D^\beta \tilde{\mathbf{x}}_i(t) &= \tilde{\mathbf{v}}_i(t) \\ D^\beta \tilde{\mathbf{v}}_i(t) &= \mathbf{u}_i(t) - \sum_{k=1}^N \mathbf{\Phi}_k \mathbf{u}_k(t) \\ &= \mathbf{f}(\mathbf{x}_i(t), \mathbf{v}_i(t), t) - \sum_{k=1}^N \mathbf{\Phi}_k \mathbf{f}(\mathbf{x}_k(t), \mathbf{v}_k(t), t) \\ &\quad + \sum_{j=1}^N \mathbf{R}_{ij} \mathbf{e}_{xj}(t) + \sum_{j=1}^N \mathbf{R}_{ij} \tilde{\mathbf{x}}_j(t) \\ &\quad + \sum_{j=1}^N \mathbf{R}_{ij} \mathbf{e}_{vj}(t) + \sum_{j=1}^N \mathbf{R}_{ij} \tilde{\mathbf{v}}_j(t) \end{aligned} \right. \quad (10)$$

Based on the analysis above, we can obtain the condition that the agents cooperate to achieve the time-varying common position and speed in the MAS.

Theorem 1 Suppose that Assumption 1 holds and $t_0^i = t_0 = 0, i = 1, 2, \dots, N$ under the event condition. $\mathbf{R}_k$ is irreducible. Therefore, the consensus in network (6) is achieved under the designed control input (9) if the scalar $\alpha > 0$ exists that satisfies the following LMIs:

$$\begin{aligned} \mathbf{Q}_k^T (\mathbf{\Xi}_k \mathbf{R}_k + \mathbf{R}_k^T \mathbf{\Xi}_k + \alpha \mathbf{I}_N) \mathbf{Q}_k &< \mathbf{0} \\ \mu_1 \mathbf{I}_n - (l_1^2 + l_1 l_2 + \frac{1}{2}) \mathbf{\Phi}_i &> \mathbf{0} \\ \mu_2 \mathbf{I}_n - (l_2^2 + l_1 l_2 + \frac{3}{2}) \mathbf{\Phi}_i &> \mathbf{0} \\ \alpha \mathbf{I}_n - \mathbf{\Phi}_i &> \mathbf{0} \end{aligned}$$

where

$$\begin{aligned}\mu_1 &= \frac{\alpha}{2} - 2M\phi_1 Nn - \frac{M^2\phi_2^2 Nn}{4\kappa^2}, \\ \mu_2 &= \frac{\alpha}{2} - 2M\phi_1 Nn - \frac{M^2\phi_2^2 Nn}{4\sigma^2}, \\ \Xi_k &= \text{diag}\{\zeta_{k1}, \zeta_{k2}, \dots, \zeta_{kN}\}, \\ \mathbf{Q}_k &= \begin{bmatrix} & & & \mathbf{I}_{N-1} \\ -\frac{\zeta_{k1}}{\zeta_{kN}} & -\frac{\zeta_{k2}}{\zeta_{kN}} & \dots & -\frac{\zeta_{kN-1}}{\zeta_{kN}} \end{bmatrix}, \\ M &= \max_{1 \leq i,j,p,q \leq N} (\Pi_{ij})_{pq}, \quad \phi_1 = \frac{\phi_1 N_M N \bar{r}}{1 - \phi_1 N_M N \bar{r}}, \\ \phi_2 &= \frac{\phi_2 N}{1 - \phi_1 N_M N \bar{r}}, \quad 0 \leq \phi_1 < \frac{1}{N_M N \bar{r}}, \quad \phi_2 \geq 0.\end{aligned}$$

Proof Consider the following Lyapunov function candidate:

$$V(t) = \frac{1}{2} \sum_{i=1}^N (\tilde{\mathbf{x}}_i^T(t), \tilde{\mathbf{v}}_i^T(t)) \begin{bmatrix} \alpha \mathbf{I}_n & \Phi_i \\ \Phi_i & \Phi_i \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_i(t) \\ \tilde{\mathbf{v}}_i(t) \end{bmatrix} \quad (11)$$

Let $\mathbf{y}_i^T(t) = (\tilde{\mathbf{x}}_i^T(t), \tilde{\mathbf{v}}_i^T(t))^T$, then

$$V(t) = \frac{1}{2} \sum_{i=1}^N \mathbf{y}_i^T(t) \begin{bmatrix} \alpha \mathbf{I}_n & \Phi_i \\ \Phi_i & \Phi_i \end{bmatrix} \mathbf{y}_i(t)$$

By Lemma 4, $\begin{bmatrix} \alpha \mathbf{I}_n & \Phi_i \\ \Phi_i & \Phi_i \end{bmatrix} > \mathbf{0}$ is equivalent to $\Phi_i > \mathbf{0}$ and $\alpha \mathbf{I}_n - \Phi_i > \mathbf{0}$. Then, $V(t) \geq 0$. $V(t) = 0$ if and only if $\mathbf{y}(t) = \mathbf{0}$. By calculating the time derivative of the $V(t)$ along the trajectory of the error system (10), the following equation can be obtained:

$$\begin{aligned}D^\beta V(t) &\leq \sum_{i=1}^N \mathbf{y}_i^T(t) \begin{bmatrix} \alpha \mathbf{I}_n & \Phi_i \\ \Phi_i & \Phi_i \end{bmatrix} D^\alpha \mathbf{y}_i(t) \\ &= \sum_{i=1}^N (\tilde{\mathbf{x}}_i^T(t), \tilde{\mathbf{v}}_i^T(t)) \begin{bmatrix} \alpha \mathbf{I}_n & \Phi_i \\ \Phi_i & \Phi_i \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{v}}_i(t) \\ \mathbf{u}_i(t) - \sum_{k=1}^N \Phi_k \mathbf{u}_k(t) \end{bmatrix} \\ &= \alpha \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{v}}_i(t) + \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \tilde{\mathbf{v}}_i(t) + \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \\ &\quad \cdot [\mathbf{f}(\mathbf{x}_i(t), \mathbf{v}_i(t), t) - \mathbf{f}(\bar{\mathbf{x}}(t), \bar{\mathbf{v}}(t), t)] \\ &\quad + \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i [\mathbf{f}(\mathbf{x}_i(t), \mathbf{v}_i(t), t) - \mathbf{f}(\bar{\mathbf{x}}(t), \bar{\mathbf{v}}(t), t)] \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \mathbf{R}_{ij} \mathbf{e}_{xj}(t) + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \mathbf{R}_{ij} \tilde{\mathbf{x}}_j(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \mathbf{R}_{ij} \mathbf{e}_{vj}(t) + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \mathbf{R}_{ij} \tilde{\mathbf{v}}_j(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \mathbf{R}_{ij} \mathbf{e}_{xj}(t) + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \mathbf{R}_{ij} \tilde{\mathbf{x}}_j(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \mathbf{R}_{ij} \mathbf{e}_{vj}(t) + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \mathbf{R}_{ij} \tilde{\mathbf{v}}_j(t)\end{aligned} \quad (12)$$

Then, (12) can be rewritten as

$$D^\beta V(t) \leq I_1 + I_2 + I_3 + I_4$$

where

$$\begin{aligned}I_1 &= \alpha \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{v}}_i(t) + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \mathbf{R}_{ij} \tilde{\mathbf{v}}_j(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \mathbf{R}_{ij} \tilde{\mathbf{x}}_j(t) \\ I_2 &= \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \tilde{\mathbf{v}}_i(t) + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \mathbf{R}_{ij} \tilde{\mathbf{x}}_j(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \mathbf{R}_{ij} \tilde{\mathbf{v}}_j(t) \\ I_3 &= \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \mathbf{R}_{ij} \mathbf{e}_{xj}(t) + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i \\ &\quad \cdot \mathbf{R}_{ij} \mathbf{e}_{vj}(t) + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \mathbf{R}_{ij} \mathbf{e}_{xj}(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \mathbf{R}_{ij} \mathbf{e}_{vj}(t) \\ I_4 &= \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \Phi_i [\mathbf{f}(\mathbf{x}_i(t), \mathbf{v}_i(t), t) - \mathbf{f}(\bar{\mathbf{x}}(t), \bar{\mathbf{v}}(t), t)] \\ &\quad + \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i [\mathbf{f}(\mathbf{x}_i(t), \mathbf{v}_i(t), t) - \mathbf{f}(\bar{\mathbf{x}}(t), \bar{\mathbf{v}}(t), t)] \\ I_1 &= \alpha \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{v}}_i(t) + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^n \tilde{\mathbf{x}}_i(t) \zeta_{ki} r_{ij}^k \tilde{\mathbf{v}}_{jk}(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^n \tilde{\mathbf{v}}_i(t) \zeta_{ki} r_{ij}^k \tilde{\mathbf{x}}_{jk}(t) \\ &= \alpha \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{v}}_i(t) + \frac{1}{2} \sum_{k=1}^n \delta \tilde{\mathbf{x}}_k^T(t) (\Xi_k \mathbf{R}_k + \mathbf{R}_k^T \Xi_k) \\ &\quad \cdot \delta \tilde{\mathbf{v}}_k(t) + \frac{1}{2} \sum_{k=1}^n \delta \tilde{\mathbf{v}}_k^T(t) (\Xi_k \mathbf{R}_k + \mathbf{R}_k^T \Xi_k) \delta \tilde{\mathbf{x}}_k(t)\end{aligned} \quad (13)$$

where

$$\delta \tilde{\mathbf{x}}_k = [\tilde{x}_{1k}, \tilde{x}_{2k}, \dots, \tilde{x}_{Nk}]^T, \quad \delta \tilde{\mathbf{v}}_k = [\tilde{v}_{1k}, \tilde{v}_{2k}, \dots, \tilde{v}_{Nk}]^T.$$

According to the definition of $\mathbf{Q}_k$, we conclude that the columns of $\mathbf{Q}_k$ form a basis of the subspace $\mathbf{E}_k$, where $\mathbf{E}_k = \{\mathbf{z} = [z_1, z_2, \dots, z_N]^T \in \mathbf{R}^N : \sum_{i=1}^N \zeta_{ki} z_i = 0\}$. Therefore, we obtain $\delta \tilde{\mathbf{v}}_k, \delta \tilde{\mathbf{x}}_k \in \mathbf{E}_k$. Hence, there exists $\delta \hat{\mathbf{v}}_k, \delta \hat{\mathbf{x}}_k \in \mathbf{R}^{N-1}$, such that $\delta \tilde{\mathbf{v}}_k = \mathbf{Q}_k \delta \hat{\mathbf{v}}_k$ and $\delta \tilde{\mathbf{x}}_k = \mathbf{Q}_k \delta \hat{\mathbf{x}}_k$. Thus, (13) follows that

$$\begin{aligned}
 I_1 &\leq \alpha \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{v}}_i(t) - \frac{1}{2} \sum_{k=1}^n \hat{\delta} \mathbf{x}_k^T \mathbf{Q}_k^T \alpha \mathbf{Q}_k \hat{\delta} \mathbf{v}_k(t) \\
 &\quad - \frac{1}{2} \sum_{k=1}^n \hat{\delta} \mathbf{v}_k^T \mathbf{Q}_k^T \alpha \mathbf{Q}_k \hat{\delta} \mathbf{x}_k(t) \\
 &= \alpha \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{v}}_i(t) - \frac{\alpha}{2} \sum_{k=1}^n \tilde{\delta} \mathbf{x}_k^T(t) \tilde{\delta} \mathbf{v}_k(t) \\
 &\quad - \frac{\alpha}{2} \sum_{k=1}^n \tilde{\delta} \mathbf{v}_k^T(t) \tilde{\delta} \mathbf{x}_k(t) \\
 &= \alpha \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{v}}_i(t) - \frac{\alpha}{2} \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{v}}_i(t) \\
 &\quad - \frac{\alpha}{2} \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \tilde{\mathbf{x}}_i(t) \leq 0
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 I_2 &= \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \tilde{\mathbf{v}}_i(t) + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^n \tilde{x}_{ik}(t) \xi_{ki} r_{ij}^k \tilde{x}_{jk}(t) \\
 &\quad + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^n \tilde{v}_{ik}(t) \xi_{ki} r_{ij}^k \tilde{v}_{jk}(t) \\
 &\leq \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \Phi_i \tilde{\mathbf{v}}_i(t) - \frac{\alpha}{2} \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{x}}_i(t) \\
 &\quad - \frac{\alpha}{2} \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) \tilde{\mathbf{v}}_i(t) \\
 &= -\frac{\alpha}{2} \sum_{i=1}^N \tilde{\mathbf{x}}_i^T(t) \tilde{\mathbf{x}}_i(t) + \sum_{i=1}^N \tilde{\mathbf{v}}_i^T(t) (\Phi_i - \frac{\alpha}{2} \mathbf{I}_n) \tilde{\mathbf{v}}_i(t)
 \end{aligned} \tag{15}$$

From the definitions of measurement errors, we found that $\mathbf{e}_{xi}(t)$ and $\mathbf{e}_{vi}(t)$ will be set to zeros at each event sampling instant t_k^i . The next sampling time instant t_{k+1}^i is the moment when the term $\|\mathbf{e}_{xi}(t)\|_1 + \|\mathbf{e}_{vi}(t)\|_1$ becomes the time-varying threshold

$$\begin{aligned}
 &\varphi_1 \sum_{j \in N_i} \left[\|\mathbf{R}_{ij}(\mathbf{x}_i(t_k^i) - \mathbf{x}_j(t_{k'}^j))\|_1 \right. \\
 &\quad \left. + \|\mathbf{R}_{ij}(\mathbf{v}_i(t_k^i) - \mathbf{v}_j(t_{k'}^j))\|_1 \right] + \varphi_2 e^{-\gamma t}
 \end{aligned}$$

Then, for all $t \geq 0$,

$$\begin{aligned}
 &\|\mathbf{e}_{xi}(t)\|_1 + \|\mathbf{e}_{vi}(t)\|_1 \\
 &\leq \varphi_1 \sum_{j \in N_i} \left[\|\mathbf{R}_{ij}(\mathbf{x}_i(t_k^i) - \mathbf{x}_j(t_{k'}^j))\|_1 \right] \\
 &\quad + \varphi_1 \sum_{j \in N_i} \left[\|\mathbf{R}_{ij}(\mathbf{v}_i(t_k^i) - \mathbf{v}_j(t_{k'}^j))\|_1 \right] + \varphi_2 e^{-\gamma t} \\
 &\leq \varphi_1 \sum_{j \in N_i} \left[\|\mathbf{R}_{ij}\|_1 \|\mathbf{x}_i(t_k^i) - \mathbf{x}_j(t_{k'}^j)\|_1 \right] \\
 &\quad + \varphi_1 \sum_{j \in N_i} \left[\|\mathbf{R}_{ij}\|_1 \|\mathbf{v}_i(t_k^i) - \mathbf{v}_j(t_{k'}^j)\|_1 \right] + \varphi_2 e^{-\gamma t} \\
 &\leq \varphi_1 N_M \bar{r} (\|\mathbf{e}_x(t)\|_1 + \|\mathbf{e}_v(t)\|_1) + \varphi_2 e^{-\gamma t} \\
 &\quad + \varphi_1 N_M \bar{r} (\|\tilde{\mathbf{x}}(t)\|_1 + \|\tilde{\mathbf{v}}(t)\|_1)
 \end{aligned} \tag{16}$$

where $\bar{r} = \max_{1 \leq i \leq N, j \in N_i, 1 \leq k \leq n} r_{ij}^k$, $N_M = \max_{1 \leq i \leq N} |N_i|$. By adding i from 1 to N , we can obtain from (16) that

$$\begin{aligned}
 &\|\mathbf{e}_x(t)\|_1 + \|\mathbf{e}_v(t)\|_1 \leq \varphi_1 N_M N \bar{r} (\|\mathbf{e}_x(t)\|_1 + \|\mathbf{e}_v(t)\|_1) \\
 &\quad + \varphi_1 N_M N \bar{r} (\|\tilde{\mathbf{x}}(t)\|_1 + \|\tilde{\mathbf{v}}(t)\|_1) + \varphi_2 N e^{-\gamma t}
 \end{aligned}$$

i.e.,

$$\begin{aligned}
 &\|\mathbf{e}_x(t)\|_1 + \|\mathbf{e}_v(t)\|_1 \\
 &\leq \frac{\varphi_1 N_M N \bar{r}}{1 - \varphi_1 N_M N \bar{r}} (\|\tilde{\mathbf{x}}(t)\|_1 + \|\tilde{\mathbf{v}}(t)\|_1) \\
 &\quad + \frac{\varphi_2 N}{1 - \varphi_1 N_M N \bar{r}} e^{-\gamma t} \\
 &= \phi_1 (\|\tilde{\mathbf{x}}(t)\|_1 + \|\tilde{\mathbf{v}}(t)\|_1) + \phi_2 e^{-\gamma t}
 \end{aligned}$$

Thus, we can obtain from I_3 that

$$\begin{aligned}
 I_3 &= \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Pi_{ij} (\mathbf{e}_{xj}(t) + \mathbf{e}_{vj}(t)) \\
 &\quad + \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Pi_{ij} (\mathbf{e}_{xj}(t) + \mathbf{e}_{vj}(t)) \\
 &\leq \left| \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{x}}_i^T(t) \Pi_{ij} (\mathbf{e}_{xj}(t) + \mathbf{e}_{vj}(t)) \right| \\
 &\quad + \left| \sum_{i=1}^N \sum_{j=1}^N \tilde{\mathbf{v}}_i^T(t) \Pi_{ij} (\mathbf{e}_{xj}(t) + \mathbf{e}_{vj}(t)) \right| \\
 &\leq M \|\tilde{\mathbf{x}}^T(t)\|_1 (\|\mathbf{e}_x(t)\|_1 + \|\mathbf{e}_v(t)\|_1) \\
 &\quad + M \|\tilde{\mathbf{v}}^T(t)\|_1 (\|\mathbf{e}_x(t)\|_1 + \|\mathbf{e}_v(t)\|_1) \\
 &= \left[2M \phi_1 N n + \frac{M^2 \phi_2^2 N n}{4\kappa^2} \right] \tilde{\mathbf{x}}^T(t) \tilde{\mathbf{x}}(t) + \eta e^{-2\gamma t} \\
 &\quad + \left[2M \phi_1 N n + \frac{M^2 \phi_2^2 N n}{4\sigma^2} \right] \tilde{\mathbf{v}}^T(t) \tilde{\mathbf{v}}(t)
 \end{aligned} \tag{17}$$

where

$$\begin{aligned}
 \Pi_{ij} &= \Phi_i R_{ij}; \kappa > 0; \sigma > 0; \eta = \kappa^2 + \sigma^2. \\
 I_4 &= \sum_{i=1}^N \tilde{x}_i^T(t) \Phi_i [f(x_i(t), v_i(t), t) - f(\bar{x}(t), \bar{v}(t), t)] \\
 &\quad + \sum_{i=1}^N \tilde{v}_i^T(t) \Phi_i [f(x_i(t), v_i(t), t) - f(\bar{x}(t), \bar{v}(t), t)] \\
 &\leq \frac{1}{2} \sum_{i=1}^N \tilde{x}_i^T(t) \Phi_i \tilde{x}_i(t) + \frac{1}{2} \sum_{i=1}^N \tilde{v}_i^T(t) \Phi_i \tilde{v}_i(t) \\
 &\quad + \sum_{i=1}^N [f(x_i(t), v_i(t), t) - f(\bar{x}(t), \bar{v}(t), t)]^T \Phi_i \\
 &\quad [f(x_i(t), v_i(t), t) - f(\bar{x}(t), \bar{v}(t), t)] \\
 &\leq (l_1^2 + l_1 l_2 + \frac{1}{2}) \sum_{i=1}^N \tilde{x}_i^T(t) \Phi_i \tilde{x}_i(t) \\
 &\quad + (l_2^2 + l_1 l_2 + \frac{1}{2}) \sum_{i=1}^N \tilde{v}_i^T(t) \Phi_i \tilde{v}_i(t)
 \end{aligned} \tag{18}$$

From (14), (15), (17) and (18), we can obtain the following:

$$\begin{aligned}
 D^\beta V(t) &\leq \sum_{i=1}^N \tilde{x}_i^T(t) \left[-\frac{\alpha}{2} + 2M\phi_1 Nn + \frac{M^2 \phi_2^2 Nn}{4\kappa^2} \right] \\
 &\quad \cdot \tilde{x}_i(t) + \sum_{i=1}^N \tilde{x}_i^T(t) \left[(l_1^2 + l_1 l_2 + \frac{1}{2}) \Phi_i \right] \tilde{x}_i(t) \\
 &\quad + \sum_{i=1}^N \tilde{v}_i^T(t) \left[-\frac{\alpha}{2} + 2M\phi_1 Nn + \frac{M^2 \phi_2^2 Nn}{4\sigma^2} \right] \tilde{v}_i(t) \\
 &\quad + \sum_{i=1}^N \tilde{x}_i^T(t) \left[(l_2^2 + l_1 l_2 + \frac{3}{2}) \Phi_i \right] \tilde{v}_i(t) \\
 &\quad + (\kappa^2 + \sigma^2) e^{-2\gamma t} \\
 &= - \sum_{i=1}^N (\tilde{x}_i^T(t), \tilde{v}_i^T(t)) \Theta \begin{bmatrix} \tilde{x}_i(t) \\ \tilde{v}_i(t) \end{bmatrix} + \eta e^{-2\gamma t}
 \end{aligned} \tag{19}$$

where

$$\Theta = \begin{bmatrix} \mu_1 I_n - (l_1^2 + l_1 l_2 + \frac{1}{2}) \Phi_i & \mathbf{0} \\ \mathbf{0} & \mu_2 I_n - (l_2^2 + l_1 l_2 + \frac{3}{2}) \Phi_i \end{bmatrix}.$$

From the condition of the theorem, we know that $\Theta > \mathbf{0}$. Choosing $0 < \theta < \frac{\lambda_{\min}(\Theta)}{\lambda_{\max}(P)}$, we have

$$D^\beta V(t) \leq -\theta V(t) + h(t) \tag{20}$$

where $h(t) = \eta e^{-2\gamma t}$; $P = \begin{bmatrix} \alpha I_n & \Phi_i \\ \Phi_i & \Phi_i \end{bmatrix}$. Therefore, there exists a non-negative function $q(t)$ such that $D^\beta V(t) +$

$q(t) = -\theta V(t) + h(t)$. We can obtain its solution by the Laplace transform method [44]. Then,

$$\begin{aligned}
 V(t) &= E_\beta(-\theta t^\beta) V(0) + h(t) * t^{\beta-1} E_{\beta,\beta}(-\theta t^\beta) \\
 &\quad - q(t) * t^{\beta-1} E_{\beta,\beta}(-\theta t^\beta) \\
 &= E_\beta(-\theta t^\beta) V(0) - q(t) * t^{\beta-1} E_{\beta,\beta}(-\theta t^\beta) \\
 &\quad + \int_0^t (t-\tau)^{\beta-1} E_{\beta,\beta}(-\theta(t-\tau)^\beta) h(\tau) d\tau
 \end{aligned}$$

where $*$ stands for the convolution between two functions,

$$\begin{aligned}
 E_\beta(-\theta t^\beta) &= \sum_{k=0}^{\infty} \frac{(-\theta t^\beta)^k}{\Gamma(k\beta + 1)}, \\
 E_{\beta,\beta}(-\theta(t-\tau)^\beta) &= \sum_{k=0}^{\infty} \frac{(-\theta(t-\tau)^\beta)^k}{\Gamma(k\beta + \beta)}.
 \end{aligned}$$

Because the non-negative function $q(t) \geq 0$, $t^{\beta-1} \geq 0$ and $E_{\beta,\beta}(-\theta t^\beta) \geq 0$, we have $q(t) * t^{\beta-1} E_{\beta,\beta}(-\theta t^\beta) \geq 0$. Let $\tau = t, \tau_0 = \theta^\beta$, therefore, $\tau/\tau_0 \rightarrow \infty$ when $\tau \rightarrow \infty$. Then, $E_\beta[-(\tau/\tau_0)^\beta] \sim (\tau/\tau_0)^{-\beta}/\Gamma(1-\beta)$ from Lemma 5. Thus, $\lim_{t \rightarrow \infty} E_\beta(-\theta t^\beta) V(0) = 0$. It is noteworthy that $h(t) = \eta e^{-2\gamma t}$. Therefore, we have

$$\begin{aligned}
 V(t) &= E_\beta(-\theta t^\beta) V(0) \\
 &\quad + \varrho(t) - q(t) * t^{\beta-1} E_{\beta,\beta}(-\theta t^\beta)
 \end{aligned}$$

where

$$\varrho(t) = \eta \int_0^t (t-\tau)^{\beta-1} E_{\beta,\beta}(-\theta(t-\tau)^\beta) e^{-2\gamma\tau} d\tau.$$

With variable substitution,

$$\varrho(t) = \eta e^{-2\gamma t} \int_0^t \tau^{\beta-1} E_{\beta,\beta}(-\theta\tau^\beta) e^{2\gamma\tau} d\tau.$$

We then denote $\varpi(t) = \tau^{\beta-1} E_{\beta,\beta}(-\theta\tau^\beta)$. It is obvious that $\lim_{t \rightarrow \infty} \varpi(t) = 0$. Similar to the analysis in [47], one can obtain that, $\lim_{t \rightarrow \infty} \varrho(t) = 0$. Subsequently, $\lim_{t \rightarrow \infty} V(t) = 0$. Thus, the consensus in the fractional-order MAS (6) is achieved asymptotically, implying that the position and speed of every agent will approach a time-varying common state.

Next, we will show that the Zeno behavior would be excluded. Therefore, there exists a strictly positive number such that for each $i, i = 1, 2, \dots, N$, we have $t_{k+1}^i - t_k^i > \tau$ for all $k \in N$. For $t \in [t_k^i, t_{k+1}^i)$, letting $u_i(t_k^i, t_{k'}^i) = \sum_{j=1, j \neq i}^N [R_{ij}(x_j(t_{k'}^i) - x_i(t_k^i)) + R_{ij}(v_j(t_{k'}^i) - v_i(t_k^i))]$,

$$\begin{aligned}
 D_+^\alpha (\|e_{xi}(t)\|_1 + \|e_{vi}(t)\|_1) &\leq \|\dot{e}_{xi}(t)\|_1 + \|\dot{e}_{vi}(t)\|_1 \\
 &= \|v_i(t)\|_1 + \|f(x_i(t), v_i(t), t) + u_i(t_k^i, t_{k'}^j(t))\|_1 \\
 &\leq \|e_{vi}(t) + v_i(t_k^i)\|_1 + l_1 \|x_i(t_k^i) + e_{xi}(t)\|_1 + l_2 \|e_{vi}(t) \\
 &\quad + v_i(t_k^i)\|_1 + \|u_i(t_k^i, t_{k'}^j(t))\|_1 \\
 &\leq \max\{l_1, 1 + l_2\} [\|e_{xi}(t)\|_1 + \|e_{vi}(t)\|_1] + l_1 \|x_i(t_k^i)\|_1 \\
 &\quad + (1 + l_2) \|v_i(t_k^i)\|_1 + \|u_i(t_k^i, t_{k'}^j(t))\|_1 \\
 &= l [\|e_{xi}(t)\|_1 + \|e_{vi}(t)\|_1] + l_1 \|x_i(t_k^i)\|_1 \\
 &\quad + (1 + l_2) \|v_i(t_k^i)\|_1 + \|u_i(t_k^i, t_{k'}^j(t))\|_1.
 \end{aligned}$$

Let $\chi(t_k^i, t_{k'}^j(t)) = l_1 \|x_i(t_k^i)\|_1 + (1 + l_2) \|v_i(t_k^i)\|_1 + \|u_i(t_k^i, t_{k'}^j(t))\|_1$, and integrating from t_k^i to t , we obtain $t \in [t_k^i, t_{k+1}^i)$,

$$\|e_{xi}(t)\|_1 + \|e_{vi}(t)\|_1 \leq \frac{\chi(t_k^i, t_{k'}^j(t))}{l} e^{l(t-t_k^i)} - \frac{\chi(t_k^i, t_{k'}^j(t))}{l},$$

Using $t = t_{k+1}^i$, we have

$$\begin{aligned}
 \|e_{xi}(t_{k+1}^i)\|_1 + \|e_{vi}(t_{k+1}^i)\|_1 \\
 \leq \frac{\chi(t_k^i, t_{k'}^j(t))}{l} e^{l(t_{k+1}^i - t_k^i)} - \frac{\chi(t_k^i, t_{k'}^j(t))}{l}.
 \end{aligned}$$

It is noteworthy that the next sampling time instant t_{k+1}^i will yield the following equality:

$$\begin{aligned}
 \|e_{xi}(t_{k+1}^i)\|_1 + \|e_{vi}(t_{k+1}^i)\|_1 \\
 > \varphi_1 \sum_{j \in N_i} [\|x_i(t_k^i) - x_j(t_{k'}^j(t))\|_1 + \|v_i(t_k^i) - v_j(t_{k'}^j(t))\|_1] \\
 &+ \varphi_2 e^{-\gamma t_{k+1}^i} = \varphi_1 \|u_i(t_k^i, t_{k'}^j(t))\|_1 + \varphi_2 e^{-\gamma t_{k+1}^i}
 \end{aligned}$$

Thus, we have

$$\begin{aligned}
 \varphi_1 \|u_i(t_k^i, t_{k'}^j(t))\|_1 + \varphi_2 e^{-\gamma t_{k+1}^i} \\
 < \frac{\chi(t_k^i, t_{k'}^j(t))}{l} e^{l(t_{k+1}^i - t_k^i)} - \frac{\chi(t_k^i, t_{k'}^j(t))}{l}
 \end{aligned}$$

i.e.,

$$\begin{aligned}
 \varphi_1 \|u_i(t_k^i, t_{k'}^j(t))\|_1 + \varphi_2 e^{-\gamma t_k^i} e^{-\gamma \tau} \\
 < \frac{\chi(t_k^i, t_{k'}^j(t))}{l} e^{l\tau} - \frac{\chi(t_k^i, t_{k'}^j(t))}{l}
 \end{aligned}$$

where $\tau = t_{k+1}^i - t_k^i$. Let

$$\begin{aligned}
 k_1 &= \varphi_1 \|u_i(t_k^i, t_{k'}^j(t))\|_1 + \frac{\chi(t_k^i, t_{k'}^j(t))}{l} \\
 k_2 &= \varphi_2 e^{-\gamma t_k^i}, \quad k_3 = \frac{\chi(t_k^i, t_{k'}^j(t))}{l}
 \end{aligned}$$

Then, we have

$$k_1 + k_2 e^{-\gamma \tau} < k_3 e^{l\tau}$$

Because $k_1 + k_2 > k_3$, a strictly positive τ must exist that satisfies the equality $k_1 + k_2 e^{-\gamma \tau} = k_3 e^{l\tau}$, as illustrated in Fig. 1. The Zeno behavior is excluded for all agents during the sampling process, i.e., $t_{k+1}^i - t_k^i > \tau$ for all $k \in N$ and $i = 1, 2, \dots, N$. The proof is completed.

When the information among the agents can be perfectly transmitted through the communication system, meaning the channel matrix is equal to an identity matrix I_n , from the definitions of R_{ij}, R_k, Φ_i in the former part, they can be redefined as $R_{ij} = a_{ij} I_n, R_k = -L, \Phi_i = \xi_i I_n$, and $\Xi_k = \Xi = \text{diag}\{\xi_1, \xi_2, \dots, \xi_N\}$, respectively, where L is the Laplacian matrix of the network. Subsequently, the fractional-order MAS by the event-trigger sampling scheme would be

$$\begin{cases} D^\beta x_i(t) = v_i(t) \\ D^\beta v_i(t) = u_i(t) \\ u_i(t) = f(x_i(t), v_i(t), t) \\ \quad - \sum_{j=1, j \neq i}^N [a_{ij}(x_i(t_k^i) - x_j(t_{k'}^j)) + a_{ij}(v_i(t_k^i) - v_j(t_{k'}^j))] \end{cases} \quad (21)$$

Corollary 1 Suppose Assumption 1 holds and $t_0^i = t_0 = 0$ under the event condition. R_k is irreducible. Therefore, the consensus in network (21) is achieved under the designed control input if there exists a scalar $\alpha > 0$ satisfying the following LMIs:

Fig. 1 Solution of the equality for τ

$$\begin{cases} Q^T(\Xi L + L^T \Xi - \alpha I_N)Q > 0 \\ \mu_1 > 0 \\ \mu_2 > 0 \\ \alpha - \xi_i > 0 \end{cases}$$

where

$$\mu_1 = \frac{\alpha}{2} - 2M\phi_1 Nn - \frac{M^2\phi_2^2 Nn}{4\kappa^2} - (l_1^2 + l_1 l_2 + \frac{1}{2})\xi_i,$$

$$\mu_2 = \frac{\alpha}{2} - 2M\phi_1 Nn - \frac{M^2\phi_2^2 Nn}{4\sigma^2} - (l_2^2 + l_1 l_2 + \frac{3}{2})\xi_i,$$

$$\bar{r} = \max_{1 \leq i \leq N, j \in N_i} a_{ij}, M = \max_{1 \leq i, j \leq N} \xi_i a_{ij}, \gamma > 0,$$

$$\phi_1 = \frac{\varphi_1 N_M N \bar{r}}{1 - \varphi_1 N_M N \bar{r}}, \phi_2 = \frac{\varphi_2 N}{1 - \varphi_1 N_M N \bar{r}}.$$

3 Simulation example

In this section, a numerical example is presented to demonstrate the effectiveness of the primary results.

Consider a 5-generator 14-bus cyber-physical power system, as shown in Fig. 2. The related data are obtained from IEEE 5-generator 14-bus standard test system. Each of the generators is assigned with an agent such that it can obtain the others information through the communication of the agents, which is shown in Fig. 3. Here, the double arrow solid line indicates the cyber network, and the single arrow solid line represents the physical network. Based on this distributed multi-agent mechanism, the power cyber networks and physical networks are effectively integrated together. Supposing that each agent (generator) contains two levels of information, i.e., $N = 5, n = 2$. The

Fig. 2 5-Generator 14-bus cyber physical power system

Fig. 3 Physical interconnection and cyber interconnection of CPPS

connection of each agent can be represented by

$$A = \begin{bmatrix} 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 6 & 7 & 0 \\ 0 & 5 & 0 & 2 & 0 \\ 0 & 0 & 5 & 0 & 6 \\ 7 & 4 & 0 & 5 & 0 \end{bmatrix}. \text{ The channel matrices are given}$$

as below:

$$\begin{aligned} H_{12} &= \text{diag}\{0, 1\}, H_{15} = \text{diag}\{1, 0\}, H_{21} = \text{diag}\{1, 0\}, \\ H_{23} &= \text{diag}\{1, 1\}, H_{24} = \text{diag}\{0, 1\}, H_{32} = \text{diag}\{1, 1\}, \\ H_{34} &= \text{diag}\{1, 1\}, H_{43} = \text{diag}\{1, 0\}, H_{45} = \text{diag}\{1, 1\}, \\ H_{51} &= \text{diag}\{0, 1\}, H_{52} = \text{diag}\{1, 1\}, H_{54} = \text{diag}\{1, 0\}. \end{aligned}$$

According to the definition of the channel Laplacian matrices, we can obtain them as follows:

$$\begin{aligned} R_1 &= \begin{bmatrix} -4 & 0 & 0 & 0 & 4 \\ 5 & -11 & 6 & 0 & 0 \\ 0 & 5 & -7 & 2 & 0 \\ 0 & 0 & 5 & -11 & 6 \\ 0 & 4 & 0 & 5 & -9 \end{bmatrix} \\ R_2 &= \begin{bmatrix} -5 & 5 & 0 & 0 & 0 \\ 0 & -13 & 6 & 7 & 0 \\ 0 & 5 & -7 & 2 & 0 \\ 0 & 0 & 0 & -6 & 6 \\ 7 & 4 & 0 & 0 & -11 \end{bmatrix} \end{aligned}$$

Let the nonlinear dynamics of the system be given by

$$f(x_i, v_i, t) = \begin{bmatrix} 6\sin(4t) + 0.01\sin(2x_{i1}) + 0.01\cos(4v_{i1}) \\ 6\sin(4t) + 0.01\sin(2x_{i2}) + 0.01\cos(4v_{i2}) \end{bmatrix}$$

Combing with Assumption 1, one can obtain $l_1 = 0.0008, l_2 = 0.0032$. Select $\beta = 0.8, \varphi_1 = 0.00009, \varphi_2 = 1.8, \gamma = 0.1, \kappa = \sigma = 500$ and $N_M = 3$. Using MATLAB LMI Toolbox, we can obtain a feasible solution $\alpha = 1.8031$ that satisfy the LMIs of Theorem 1. The cyber-physical power systems of the rotor angle and rotor speed are given by Figs. 4 and 5, respectively; one can find that all of the rotor angles and rotor speeds approach the common time-

Fig. 4 Rotor angle of CPPS

Fig. 5 Rotor speed of CPPS

Fig. 6 Sampling instants of five agents

varying value. Figure 6 shows the event-triggered sampling instants of all five agents of the system during the time period $[0, 1.5]$.

4 Conclusion

Herein, the consensus of the CPPS based on the fractional-order MAS with communication constraints was investigated, and a sufficient consensus condition was derived. Owing to practical limitations, only partial information could be transmitted among agents. For the energy resource conservation, the event-trigger sampling strategy was considered and the Zeno behavior of the triggered time sequence was excluded. Thus, the proposed consensus protocol of the CPPS based on the fractional-order MAS model with communication constraints was more realistic and general. Therefore, the related consensus problem was challenging. Finally, a numerical example was presented to demonstrate the validity of the theoretical results.

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