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A coordinated consistency voltage stability control method of active distribution grid

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Abstract The presence of distributed generators (DGs) with high penetration poses new challenges in the management and operation of electrical grids. Due to the local character of DGs, they could in principle be used in emergency situations to prevent a voltage instability event of the grid. In this paper, a certain method is proposed to coordinate the operation of virtual power plant (VPP) and conventional voltage regulation device to improve the static voltage stability of distribution network with the multi-agent framework. The concept and the general framework of this coordinated control system is introduced, and the voltage instable nodes are determined based on the voltage instability indicator. The voltage coordinated control model of the distribution system is established according to the multi-agent consistency control theory and the coordinated controllers for agents are designed by solving a problem with bilinear matrix inequality constraints. The suggested method is implemented on an IEEE 33 nodes test system and the simulation results show its efficiency and validity.

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1 Introduction

Various policies and incentives like feed-in tariff schemes facilitate the installations of distributed generators DGs and renewable energy resources (RES). As a result, current distribution network is evolving from passive grid to active and smarter grids [1, 2], in order to cope with the increasing pressures imposed by DGs in an economical, reliable and environmental friendly way. One of the innovative infrastructures proposed for active distribution network(ADN) is the VPP.

Comparable to a conventional power plant, the VPP, either operated in centralized(direct) control [3], multi agent based hierarchical control [4] or fully distributed control scheme [5], is to integrate distributed energy resource (DER), energy storage units and controllable loads that geographically dispersed in grid into a special power plant, with the aims to: facilitate the participation of DERs in energy markets in the same manner as conventional generating unit, thus DERs can experience economies of scale in market participation and benefit from intelligence on market participation to maximize revenue opportunities [6–8]; provide ancillary services needed by distribution and transmission network operators for system security needs, such as frequency control, voltage control, congestion management, improvement of voltage quality, network restoration, islanded operation, and optimization of grid losses.

The planning, management and operation of aggregated DGs provide ancillary services in terms of voltage control which already have been subject of the research in different works with varying contexts. Reference [9] proposes a hierarchical multi agent system to determine the coordinated control action in the case of voltage instability, and in [10] the control strategies for DGs is used as actuators to improve system voltage stability. In [11], the detailed control of the load tap changer(LTC) is given. A control system based on multi-agent technique that coordinates different discrete and continuous control devices during the post-disturbance period to prevent voltage collapse of the whole system is presented in [12]. Reference [13] describes the coordination of reactive power supply devices based on optimization of the system voltages.

Static voltage instability, which usually has to be faced by transmission network operators, is now a nuisance also for the operators of ADNs due to the connections of DGs [14]. Significant research on the influence of DGs on static voltage stability of distribution network, the static voltage stability index and the methods to improve the static voltage stability of ADNs are being conducted as shown in [15–23]. The static voltage stability based on the load flow equation theory of distribution system is proposed in [15, 16], and the further improvement of voltage stability index (VSI) is discussed in [17]. A new VSI is developed in [18] for identifying the most sensitive bus to the voltage collapse in distribution network by using the catastrophe theory; In [19], a method that using the load apparent power to characterize the VSI has been discussed; whereas [20, 21] analyze the impact of distributed generations access on voltage stability index in distribution network; Furthermore, the optimal voltage stability-based distributed generation placement method have been proposed in [22, 23] to improve the voltage stability of the whole system.

In this paper,a coordinated consistency control method in multi-agent framework that coordinates VPP and conventional voltage regulation means including e.g. static synchronous compensator STATCOM to improve the static voltage stability of ADNs is proposed. In Section II, the framework of multi-agent-based coordinated control for ADNs is presented, and the principle of the coordinated consistency control to improve the static voltage stability is given. Section III presents the index and method for the static voltage stability analysis of ADNs. Section V presents the main contribution of this paper, namely a coordinated consistency control method for improving the static voltage stability of ADNs. Case studies performed on an IEEE-33 nodes test system to demonstrate the validity and effectiveness of the coordinated consistency control is given in Section VI. Finally, in Section VII, the main findings of the paper are summarized.

2 Principle and voltage coordination control framework of AND with VPP

The general structure of active distribution network(AND) with VPP is shown in Fig. 1.

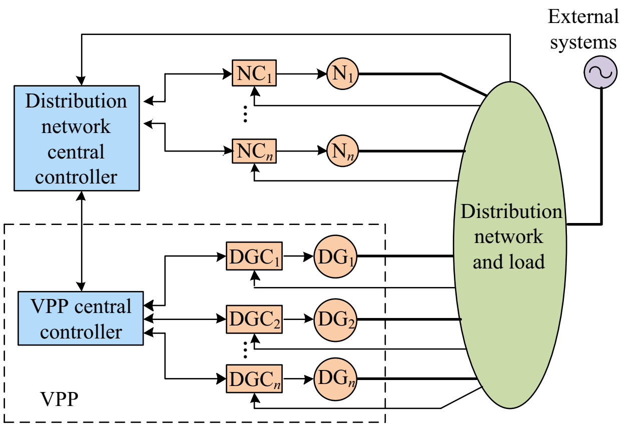


Fig. 1 General structure of distribution network with VPP

The network may consist of VPP (illustrated in the dashed box shown in Fig. 1, distribution grid, load, external systems and conventional voltage regulation means and devices, such as reactive compensation capacitor, on–load tap-changer, SVC and STATCOM. The conventional regulation means and their associated controllers are represented by N_1, N_2,…, N_n and NC_1, NC_2,…, NC_n in Fig. 1 respectively. DGs in VPP and their corresponding controllers are represented by DG_1, DG_2,…, DG_n and DGC_1, DGC_2,…, DGC_n in Fig. 1 respectively. Thick solid lines in Fig. 1 represent power connections between distribution network and external source, conventional voltage control apparatus, the DGs. The thin solid lines with arrows in Fig. 1 represent signal flows and their directions.

The distribution grid and VPP are equipped with central controller respectively, the controllers of conventional voltage control apparatus and DGs not only get local state quantities to control the corresponding object, but also exchange control and status information with the VPP central controller to realize coordination control. The central controller of the distribution grid obtains the distribution network status information and exchanges control and status information with the central controller of the VPP.

The proposed voltage coordination control system of distribution network with VPP based on multi-agent system is shown in Fig. 2. The article treats distribution network, VPP, conventional voltage control equipment (including STATCOM, reactive compensation capacitor, on–load tap-changer, etc), DGs and load as intelligent agents.

The voltage coordinated control system is divided into three layers.

1) The first layer is the intelligent layer of the distribution network, which organizes and manages the entire control system. Central controller is responsible for the judgment of the state of the distribution network and the operation mode switching (such as the flow calculation,

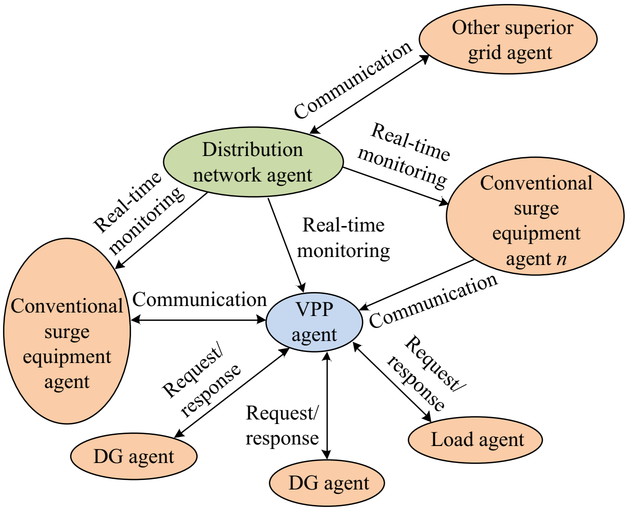


Fig. 2 Voltage coordinated control framework of distribution network

stability analysis, and emergency regulation, etc.), whereas controls the lower level intelligent agents coordinately.

2) The second layer includes VPP intelligent agent and the conventional voltage control intelligent agents. The VPP intelligent agent accomplishes the task announced by the upper intelligent agent through the coordination of the DG intelligent agent and load intelligent agent inside the VPP. It also completes the control strategies and parameters calculation for the lower level intelligent agent and uploads the states of the lower level intelligent agents to the upper intelligent agent. The conventional voltage control intelligent agent, including reactive compensation capacitor intelligent agent, on–load tap-changer intelligent agent and STATCOM intelligent agent and so on, is mainly to complete the adjustment of the local voltage in the distribution network.

3) The third layer includes DG intelligent agent as well as the load intelligent agent. The DG intelligent agent monitors the operation status of DG and adjusts their active and reactive power according to the requirement of the VPP intelligent agent. In some appropriate circumstances, it can prevent DG from operating in islanding mode to achieve the voltage regulator target. The load intelligent agent can perform the voltage adjustment task together with DG by shedding load to regulate voltage when the output of MG reaches their limits.

Each agent of multi-agent system has the characteristic of autonomy and sociality. The multi-agent systems replace the traditional centralized control work, and decompose the voltage cooperative control problem of the whole network into several sub-problems which will be assigned to specific agents, in order to achieve rapidly voltage regulation under weak communication conditions.

There is a certain voltage coordination control system within each voltage regulation interval, in which the agents can have data collection and assessment, and communicate with other agents in order to obtain more comprehensive voltage information. When the voltage stability problem occurs, the system will enter to a new regulation cycle. The coordinate agent will generate a regulation priority sequence contains neighboring agent equipment based on the received regulation requests and knowledge base information. The coordinate agent will send the regulation request to the agent with the highest priority device, then the highest priority agent can accept or reject the application considering its own circumstance. As for the regulation priority sequence, the regulation devices within the agent will certainly have the highest priority, and followed by the neighboring agent devices at the same level. If the voltage stability problem hasn't been eliminated at this time, then the agent regenerates a new regulation priority sequence, which will include the regulation device from the upper agent layer. It'll continue to send a regulation application to the agent with currently highest priority. The regulation cycle ends with the disappearance of abnormal voltage node.

3 Static voltage stability analysis method

In this paper, the static voltage stability of ADNs is analyzed according to the voltage probabilistic eigenvalue analysis method.

3.1 Probabilistic eigenvalue analysis

The parallel matrix is used to describe the connection of each variable in view of the active power distribution grid containing VPP:

$$\begin{bmatrix} \dfrac{\mathrm{d}\Delta\boldsymbol{x}}{\mathrm{d}t} \\ 0 \end{bmatrix} = \begin{bmatrix} \overline{\boldsymbol{A}} & \overline{\boldsymbol{B}} \\ \overline{\boldsymbol{C}} & \overline{\boldsymbol{D}} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{x} \\ \Delta\boldsymbol{V} \end{bmatrix} \tag{1}$$

where $\mathrm{d}\Delta\boldsymbol{x}/\mathrm{d}t$ is the input vector; $\Delta\boldsymbol{x}$ is the state variable vector; $\Delta\boldsymbol{V}$ is the non-state variable vector; $\overline{\boldsymbol{A}}$, $\overline{\boldsymbol{B}}$, $\overline{\boldsymbol{C}}$, $\overline{\boldsymbol{D}}$ are the submatrix.

The state space equation can be described as $\Delta\boldsymbol{x}' = \boldsymbol{E}\Delta\boldsymbol{x}$, and the coefficient matrix $\boldsymbol{E}$ can be expressed as:

$$\boldsymbol{E} = \boldsymbol{S}'(\boldsymbol{K}_a\boldsymbol{H} - \boldsymbol{K}_b) \tag{2}$$

where $\boldsymbol{S}$ is the node injection power vector and $\boldsymbol{S}' = (\boldsymbol{I} - \boldsymbol{K}_t\boldsymbol{H})^{-1}$, the variable $\boldsymbol{H}$ can be expressed as $\boldsymbol{H} = \overline{\boldsymbol{A}} + \overline{\boldsymbol{B}}(-\overline{\boldsymbol{D}}^{-1})\overline{\boldsymbol{C}}$, the detailed description of $\boldsymbol{K}_a$, $\boldsymbol{K}_b$ and $\boldsymbol{K}_t$ can be found in [24].

The mean vector of eigenvalue $\overline{\boldsymbol{\lambda}}$ can be obtained by the coefficient matrix $\boldsymbol{E}$ of the voltage mean value. The node voltage formula can be expressed as $\boldsymbol{\lambda} = \boldsymbol{G}(\boldsymbol{V})$, and $\boldsymbol{J}_{\lambda} = \partial\lambda/\partial V$, where $\boldsymbol{J}_{\lambda}$ is the Jacobian matrix of $\boldsymbol{\lambda}$. The linearization formula is:

$$\Delta\boldsymbol{\lambda} = \boldsymbol{J}_{\lambda}\Delta\boldsymbol{V} \tag{3}$$

The covariance matrix of eigenvalue can be expressed as:

$$\boldsymbol{C}_{\lambda} = \boldsymbol{J}_{\lambda}\boldsymbol{C}_{V}\boldsymbol{J}_{\lambda}^{\mathrm{T}} \tag{4}$$

The system can be judged as stable when the eigenvalue lies in the left half plane of the complex plane.

3.2 voltage stability analysis

The state variable vector $\Delta\boldsymbol{V}$ can be expressed according to (1) as:

$$\Delta\boldsymbol{V} = -\overline{\boldsymbol{D}}^{-1}\overline{\boldsymbol{C}}\Delta\boldsymbol{x} = \boldsymbol{E}'\Delta\boldsymbol{x} \tag{5}$$

where $\boldsymbol{E}^{'}$ is the matrix which contains the row of node voltage component from the $(-\overline{\boldsymbol{D}}^{-1}\overline{\boldsymbol{C}})$ matrix. The relationship between the node voltage amplitude deviation column vector and the state variable vector is:

$$\Delta\boldsymbol{V}_t = \begin{bmatrix} \dfrac{\boldsymbol{V}_R}{\boldsymbol{V}_t} & \dfrac{\boldsymbol{V}_J}{\boldsymbol{V}_t} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{V}_R \\ \Delta\boldsymbol{V}_J \end{bmatrix} = \begin{bmatrix} \dfrac{\boldsymbol{V}_R}{\boldsymbol{V}_t} & \dfrac{\boldsymbol{V}_J}{\boldsymbol{V}_t} \end{bmatrix} \boldsymbol{E}'\Delta\boldsymbol{x} = \boldsymbol{E}''\Delta\boldsymbol{x} \tag{6}$$

where $\boldsymbol{V}_t = [V_{t1}, V_{t2}, \ldots, V_{tn}]^{\mathrm{T}}$ is the node voltage amplitude vector; $\boldsymbol{V} = [\boldsymbol{V}_R, \boldsymbol{V}_J]^{\mathrm{T}}$ is the node voltage in rectangular coordinates, subscript R and J identify the real and imaginary part respectively.

The relationship between the node voltage amplitude deviation vector and the characteristic values can be obtained as:

$$\begin{aligned} \Delta\boldsymbol{V}_t &= \boldsymbol{E}^{''}\boldsymbol{U}\Delta\boldsymbol{Z} = \boldsymbol{W}\Delta\boldsymbol{Z} \\ &= z_{10}w_1e^{\lambda_1 t} + z_{20}w_2e^{\lambda_2 t} + \cdots + z_{n0}w_ne^{\lambda_n t} \end{aligned} \tag{7}$$

where $\boldsymbol{U}$ is the right eigenvector of the matrix $\boldsymbol{A}$. Node voltage instability coefficient relates to characteristic root is:

$$\boldsymbol{W} = \boldsymbol{E}''\boldsymbol{U} = [w_1, w_2, \ldots, w_n] \tag{8}$$

where $\boldsymbol{w_i} = [w_{1i}, \ldots, w_{ji}, \ldots, w_{ni}]$, and w_{ji} can reflect the impact of the characteristics value λ_i to the node voltage deviations ΔV_{tj}. If the w_{ji} is larger than one, the corresponding node is static voltage instable, and the larger the value of w_{ji}, the higher the degree of voltage instability. This paper will use these indicators to determine the static voltage instability of the distribution network.

4 Multi-agent-based coordinated control method for improving the static voltage stability

The problem to guarantee the coherence of several actuators in ADNs (e.g. VPP and conventional regulation means) to achieve a common goal (e.g. improving the static voltage stability) can be solved by using multi-agent-based coordinated consistency control method.

Graph theory is an indispensable tool for the analysis of the consistency problem. For a multi-agent-based system framework, a diagram is usually adopted to describe the exchange of information between individuals. The following is a brief introduction of graph theory knowledge.

4.1 Basic concepts about graph theory

For a node connection $\boldsymbol{V} = \{v_1, v_2, \ldots, v_n\}$ and a set of edges $\boldsymbol{E} = \{e_1, e_2, \ldots, e_m\}$, if any edge e_k in $\boldsymbol{E}$ has a correspond node pair (v_i, v_j) in $\boldsymbol{V}$, thus a graph can be constituted by $\boldsymbol{V}$ and $\boldsymbol{E}$ denoted as $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$, which is shown in Fig. 3.

In graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$, node set $\boldsymbol{V} = \{v_1, v_2, \ldots, v_n\}$ is a finite non-empty set which has n elements, and n is the order of graph $\boldsymbol{G}$. Each edge can be represent by node pair (v_i, v_j), where v_i is the starting point and v_j is the end point. In edge set $\boldsymbol{E} \subseteq v \times v$, graph edge $e_{ij} = (v_i, v_j) \subseteq \boldsymbol{E}$ represents that node j can transmit information to node i.

4.2 Matrix theory

Algebraic graph theory and linear algebra are closely linked, the edges and nodes may be corresponding to a matrix, then the analyses of the matrix can directly reflect

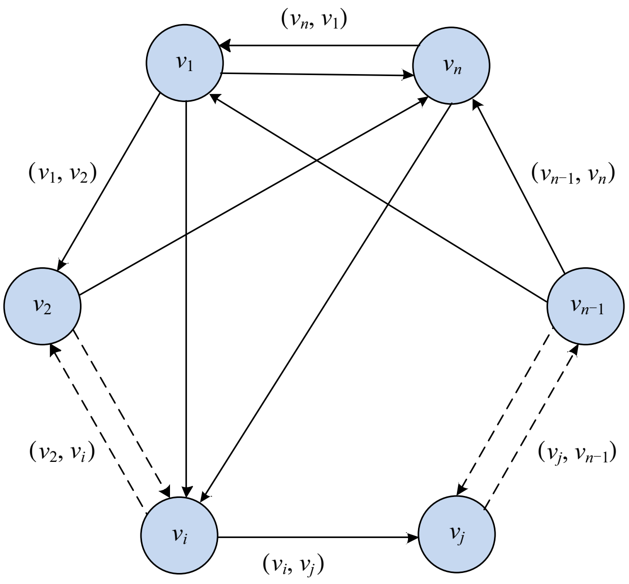


Fig. 3 Topology of a network with n agents

part characteristics of the graph. Following is an introduction of the relevant matrix theory.

Adjacency matrix $\boldsymbol{A} = [a_{ij}]$ of the graph $\boldsymbol{G}$ is used to describe the relationship between the nodes and edges, where $\boldsymbol{A}$ is defined as:

$$a_{ij} = \begin{cases} 1 & \left(v_i, v_j\right) \in \boldsymbol{E} \\ 0 & \text{the others} \end{cases} \tag{9}$$

For each agent, the communication direction is limited, and its state will change under the circumstance that the states of the neighbor agents which can communicate with it change. Agent neighborhood set is set as $\boldsymbol{N}$, and the neighborhood set of agent i is defined as:

$$\boldsymbol{N_i} = \left\{j \in \boldsymbol{V} : a_{ij} \neq 0\right\} = \{j \in \boldsymbol{V} : (i,j) \in \boldsymbol{E}\} \tag{10}$$

If agent j is a neighbor of agent i and $a_{ij} \neq 0$, this means i can accept information from j.

4.3 Voltage consistency control protocol

The information state of a single agent i in a multi-agent framework can be expressed as x_i and it represents the information that needed to be transferred to achieve consistency between agents. The information state can include the location, speed, voltage, etc. Assuming each agent as a node of a directed graph $\boldsymbol{G}$, each edge $(v_i,v_j) \in \boldsymbol{E}$ is corresponding to the reliable information transmission between agent i and j. At the same time, each agent can only change its state according to the information of itself and its neighbor agents. When utilizing VPP and conventional voltage regulation equipment to regulate voltage simultaneously, these objects can be seen as a single agent respectively. Thus, the information state of each agent is:

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}_i \boldsymbol{x}_i(t) + \boldsymbol{B}_i \boldsymbol{u}_i(t) \quad i = 1, 2, \cdots, n \tag{11}$$

where $\boldsymbol{x}_i(t) \in \boldsymbol{R}^m$, $i = 1,2,\ldots,n$ is information state value of agent i; $\boldsymbol{u}_i(t) \in \boldsymbol{R}^l$ is the control input (or protocol) at time t; $\boldsymbol{A}_i \in \boldsymbol{R}^{m\times m}$ and $\boldsymbol{B}_i \in \boldsymbol{R}^{m\times l}$ are known system matrix.

According to consistent control theory, if and only if all agent states meet: $\lim\limits_{t\to\infty} \left\|x_j(t) - x_i(t)\right\| = 0, \forall i,j \in n$, then the protocol u_i can solve the consistent problem gradually.

In the paper, voltage consistency control protocol is:

$$\boldsymbol{u}_i(t) = \sum_{j=1, j\neq i}^{n} a_{ij} \boldsymbol{K}_{ij} \left[\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)\right] \quad i = 1, 2, \cdots, n \tag{12}$$

where constant a_{ij} is the information flow from agent j to i, and $\boldsymbol{K}_{ij} \in \boldsymbol{R}^{l\times m}$ is the feedback gain matrix.

4.4 Solution of consistency voltage control

The designed consistency protocol (12) can make each agent to achieve consistent and the entire grid voltage global stable. It requires that all the agents involved to exchange information, as well as to complete the coordination control task. The closed-loop system defined by equation (11) and (12) is described as:

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}_d + \boldsymbol{B}_d \boldsymbol{K}) \boldsymbol{x}(t) \tag{13}$$

where:

$$\boldsymbol{A}_d = \mathrm{diag}(\boldsymbol{A}_1, \boldsymbol{A}_2, \cdots, \boldsymbol{A}_n) \tag{14}$$

$$\boldsymbol{B}_d = \mathrm{diag}(\boldsymbol{B}_1, \boldsymbol{B}_2, \cdots, \boldsymbol{B}_n) \tag{15}$$

$$\boldsymbol{x} = [x_1, x_2, \cdots x_n]^{\mathrm{T}} \tag{16}$$

$$\boldsymbol{K} = \begin{bmatrix} -\sum\limits_{j=1,j\neq 1}^{n} a_{1j}\boldsymbol{K}_{1j} & a_{12}\boldsymbol{K}_{12} & \cdots & a_{1n}\boldsymbol{K}_{1n} \\ a_{21}\boldsymbol{K}_{21} & -\sum\limits_{j=1,j\neq 2}^{n} a_{2j}\boldsymbol{K}_{2j} & \cdots & a_{2n}\boldsymbol{K}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}\boldsymbol{K}_{n1} & a_{n2}\boldsymbol{K}_{n2} & \cdots & -\sum\limits_{j=1,j\neq n}^{n} a_{nj}\boldsymbol{K}_{nj} \end{bmatrix} \tag{17}$$

Therefore, the problem of the coherence of the multi-agent system can be transformed into the solution of the asymptotic stability of closed-loop system (13) when converges to the equilibrium point.

The condition of multi-agent systems (11) with fixed topology $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ to be consistently stable and achieve consistence is that existing a positive definite matrices $\boldsymbol{P}_{ii}$, i=1,2,…,n, and matrices $\boldsymbol{P}_{ij}$, $i \neq j$, $i < j$, $\boldsymbol{K}_{ij}$, i,j = 1,2,…,n, to hold the following matrix inequality as:

$$\boldsymbol{L}\left(\boldsymbol{P}_{ij}\right) \triangleq \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} & \cdots & \boldsymbol{P}_{1n} \\ \boldsymbol{P}_{12}^{\mathrm{T}} & \boldsymbol{P}_{22} & \cdots & \boldsymbol{P}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{P}_{1n}^{\mathrm{T}} & \boldsymbol{P}_{2n}^{\mathrm{T}} & \cdots & \boldsymbol{P}_{nn} \end{bmatrix} > 0 \tag{18}$$

$$\boldsymbol{B}\left(\boldsymbol{P}_{ij}, \boldsymbol{K}_{ij}\right) \triangleq \begin{bmatrix} \boldsymbol{\Pi}_{11} & \boldsymbol{\Pi}_{12} & \cdots & \boldsymbol{\Pi}_{1n} \\ \boldsymbol{\Pi}_{12}^{\mathrm{T}} & \boldsymbol{\Pi}_{22} & \cdots & \boldsymbol{\Pi}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Pi}_{1n}^{\mathrm{T}} & \boldsymbol{\Pi}_{2n}^{\mathrm{T}} & \cdots & \boldsymbol{\Pi}_{nn} \end{bmatrix} < 0 \tag{19}$$

where:

$$\boldsymbol{\Pi}_{ij} = \begin{cases} \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_{ii} + \boldsymbol{P}_{ii}\boldsymbol{A}_i - \left(\sum\limits_{j=1,j\neq i}^{n} a_{ij}\boldsymbol{K}_{ij}\right)^{\mathrm{T}} \boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}_{ii} \\ \quad - \boldsymbol{P}_{ii}\boldsymbol{B}_i\left(\sum\limits_{j=1,j\neq i}^{n} a_{ij}\boldsymbol{K}_{ij}\right) \\ \quad + \sum\limits_{k=1}^{i-1}\left[(a_{ki}\boldsymbol{K}_{ki})^{\mathrm{T}}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{P}_{ki} + \boldsymbol{P}_{ki}^{\mathrm{T}}\boldsymbol{B}_k(a_{ki}\boldsymbol{K}_{ki})\right] \\ \quad + \sum\limits_{k=i+1}^{n}\left[(a_{ki}\boldsymbol{K}_{ki})^{\mathrm{T}}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{P}_{ki} + \boldsymbol{P}_{ki}^{\mathrm{T}}\boldsymbol{B}_k(a_{ki}\boldsymbol{K}_{ki})\right] \quad i = j \\ \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_{ij} + \boldsymbol{P}_{ij}\boldsymbol{A}_j - \left(\sum\limits_{j=1,j\neq i}^{n} a_{ij}\boldsymbol{K}_{ij}\right)^{\mathrm{T}} \boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}_{ij} \\ \quad - \boldsymbol{P}_{ij}\boldsymbol{B}_j\left(\sum\limits_{k=1,k\neq j}^{n} a_{jk}\boldsymbol{K}_{jk}\right) + \boldsymbol{P}_{ii}\boldsymbol{B}_i a_{ij}\boldsymbol{K}_{ij} \\ \quad + \left(a_{ji}\boldsymbol{K}_{ji}\right)^{\mathrm{T}}\boldsymbol{B}_j^{\mathrm{T}}\boldsymbol{P}_{jj} \\ \quad + \sum\limits_{k=1}^{i-1}\left[a_{ki}\boldsymbol{K}_{ki}^{\mathrm{T}}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{P}_{kj} + \boldsymbol{P}_{ki}^{\mathrm{T}}\boldsymbol{B}_k a_{kj}\boldsymbol{K}_{kj}\right] \\ \quad + \sum\limits_{k=i+1}^{j-1}\left[a_{ki}\boldsymbol{K}_{ki}^{\mathrm{T}}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{P}_{kj} + \boldsymbol{P}_{ik}^{\mathrm{T}}\boldsymbol{B}_k a_{kj}\boldsymbol{K}_{kj}\right] \\ \quad + \sum\limits_{k=j+1}^{n}\left[a_{ki}\boldsymbol{K}_{ki}^{\mathrm{T}}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{P}_{jk}^{\mathrm{T}} + \boldsymbol{P}_{ik}\boldsymbol{B}_k a_{kj}\boldsymbol{K}_{kj}\right] \quad i < j \\ \boldsymbol{\Pi}_{ji}^{\mathrm{T}} \quad i > j \end{cases}$$

From the above analysis, the consistency problem can be converted to solving the feasibility problem with BMI (bilinear matrix inequality) constraints as:

$$\begin{cases} \min t \\ \text{s.t.} - \boldsymbol{L}(\boldsymbol{P}_{ij}) < t\boldsymbol{I} \\ \boldsymbol{B}\left(\boldsymbol{P}_{ij}, \boldsymbol{K}_{ij}\right) < t\boldsymbol{I} \end{cases} \tag{20}$$

Formula (20) is a BMI about $\boldsymbol{P}_{ij}$ and $\boldsymbol{K}_{ij}$. The feedback gain matrix $\boldsymbol{K}_{ij}$ can be solved according to the following steps.

5 Case studies

5.1 System model and parameters

Extensive case studies are conducted on the IEEE-33 nodes test distribution system shown in Fig. 5 to illustrate the improvement on static voltage stability brought by the coordinated control method presented in above section for ADNs. The rated voltage of the network is 12.66 kV, and per-unit voltage of the root node 0 is 1.05 p.u. The loads of network nodes are modeled as motors with pure resistors [25]. Other detailed test system information can be found in [26].

Three DGs, rated at 500 kW+j100 kvar, 200 kW+j30 kvar and 500 kW+j100 kvar is added to node 17, 24 and 32 respectively, and two STATCOMs, both rated at 500 kvar is to be added to node 11 and 29 respectively.

5.2 Voltage stability coefficient of the grid when DGs are controlled independently

In this case, three independently controlled DGs are integrated to the network and their reduced-order models can be found in [27], the two STATCOMs are omitted in this case. Table 1 shows the voltage distribution of each nodes of the test network obtained from power flow calculation.

The state variables chosen for DGs are ΔP, ΔQ, ΔV_{od}, ΔV_{oq}. Through small signal stability analysis, the entire system has 12 characteristic roots, among which unstable characteristic roots is $\lambda_{7,8} = 1.137 \pm \mathrm{j}0.915$, $\lambda_{11,12} = 2.087 \pm \mathrm{j}1.286$. Table 2 lists the voltage instability coefficients of the nodes corresponding to the unstable characteristic roots.

As it can be seen from above table that node 11 and 29 have the largest voltage instability coefficients, so STATCOM is to be commissioned at each of these two nodes.

Table 1 Voltage distribution for independently controlled DGs

Node	Voltage	Node	Voltage	Node	Voltage
0	1.050	11	0.9330	22	1.011
1	1.044	12	0.9337	23	0.9984
2	1.018	13	0.9386	24	0.9920
3	1.005	14	0.9435	25	0.9587
4	0.9917	15	0.9496	26	0.9547
5	0.9619	16	0.9769	27	0.9403
6	0.9576	17	0.9880	28	0.9320
7	0.9492	18	1.044	29	0.9283
8	0.9413	19	1.038	30	0.9328
9	0.9358	20	1.038	31	0.9364
10	0.9348	21	1.041	32	0.9443

Table 2 Node voltage instability coefficients

Characteristic roots	Nodes					
	11	12	27	28	29	30
7	2.638	1.125	0.117	0.479	0.884	0.257
8	0.783	1.632	0.152	0.292	0.787	0.349
11	0.479	0.293	1.265	2.425	3.056	1.973
12	0.592	0.125	0.708	1.183	1.308	0.879

Table 3 Voltage distribution for independently controlled DGs and STATCOMs

Node	Voltage	Node	Voltage	Node	Voltage
0	1.050	11	0.9963	22	1.011
1	1.044	12	0.9924	23	0.9979
2	1.027	13	0.9955	24	0.9915
3	1.015	14	0.9950	25	0.9781
4	1.003	15	1.005	26	0.9743
5	0.9978	16	1.031	27	0.9705
6	0.9916	17	1.043	28	0.9651
7	0.9842	18	1.043	29	0.9613
8	0.9891	19	1.038	30	0.9579
9	0.9967	20	1.037	31	0.9588
10	0.9964	21	1.038	32	0.9632

Table 4 Voltage distributions for VPP and STATCOMS controlled coordinately

Node	Voltage	Node	Voltage	Node	Voltage
0	1.050	11	1.010	22	1.015
1	1.045	12	1.008	23	1.002
2	1.024	13	1.012	24	0.9952
3	1.014	14	1.017	25	0.9842
4	1.000	15	1.022	26	0.9803
5	0.9871	16	1.037	27	0.9718
6	0.9971	17	1.048	28	0.9729
7	0.9999	18	1.044	29	0.9709
8	1.002	19	1.039	30	0.9697
9	1.009	20	1.038	31	0.9722
10	1.009	21	1.041	32	0.9802

5.3 Voltage stability analysis when DGs and STATCOMs are controlled independently

The model in [28] is chosen for the STATCOM and the state variables are ΔI_d, ΔI_q, ΔI_0 and ΔU_{dj}. The voltage distribution of each nodes of the test network with the access of STATCOMs are listed in Table 3.

Figures 6 and 7 show the state variables trajectory of each of the DGs and STATCOMs respectively.

Based on the four state variables mentioned before, it was found that the real part of 20 eigenvalues (3 DGS and 2 STATCOMS) are all negative, which shows that it meets the voltage stability conditions. But form the state variables trajectories of each DG and STATCOM in Figs. 6 and 7, it can be found that only the state variables trajectories of DG2 finally converge to zero, while that other agents as DG1, DG3 and two STATCOMs are ultimately divergent, that is, the system is instable in terms of static voltage when DGs and STATCOMs are controlled independently, which need further coordinated control.

5.4 Voltage stability analysis for multi-agent coordinated control method

Table 4 shows the voltage distributions with DGs and STATCOMs are controlled coordinated using the method presented in this paper.

Considering the DGs and STATCOMs connecting with node 11, 17, 21, 28, 32 as the agent i (i = 1,2,3,4,5). In accordance with the multi-agent coordinated control algorithm, the gain matrix $\boldsymbol{K}_{ij}$ (i, j = 1,2,3,4,5, $i \neq j$) can be calculated based on the Solution flowchart shown in Fig. 4. And Fig. 8 shows all the system state trajectories when the VPP and STATCOMs are under the multi-agent-based coordinated control.

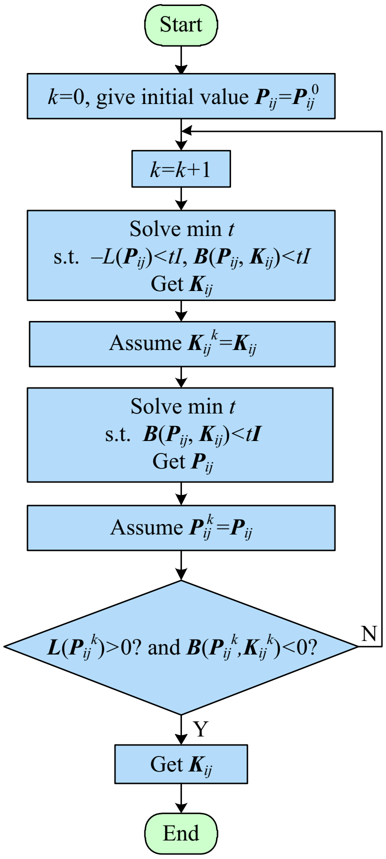


Fig. 4 Solution flowchart of feedback gain matrix

It can be seen from Fig. 8 that the system state trajectories converge to zero after using the multi-agent coordinated control strategy proposed in this paper. And the convergence time is less than one second which is much faster than the independent control situation, which shows

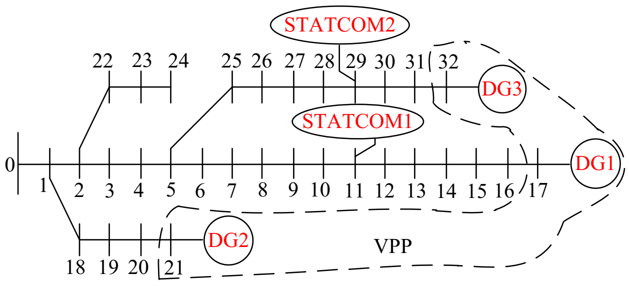
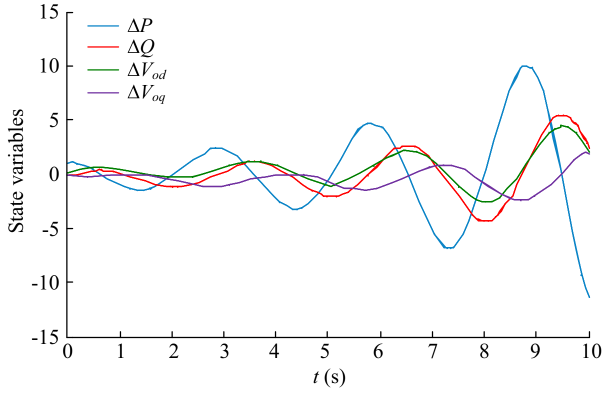
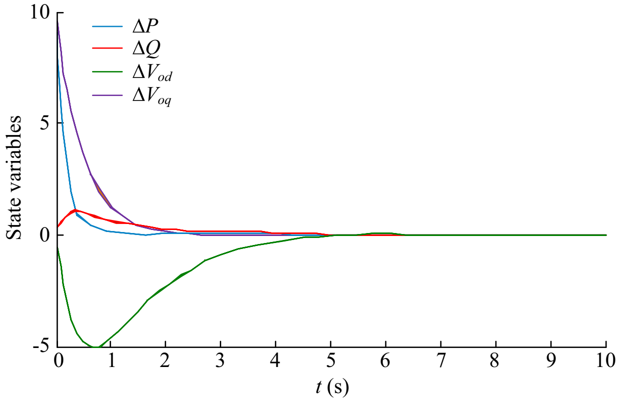


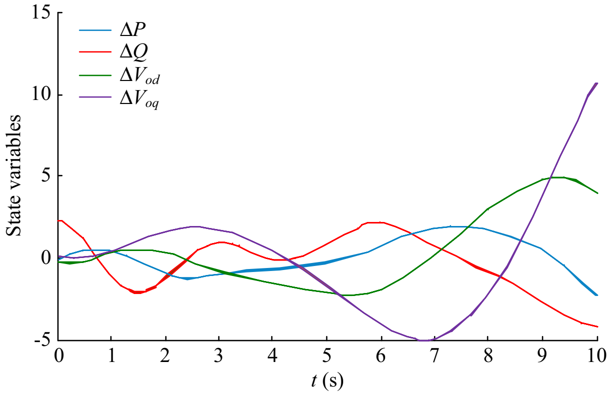
Fig. 5 IEEE 33 nodes test network



(a) DG1

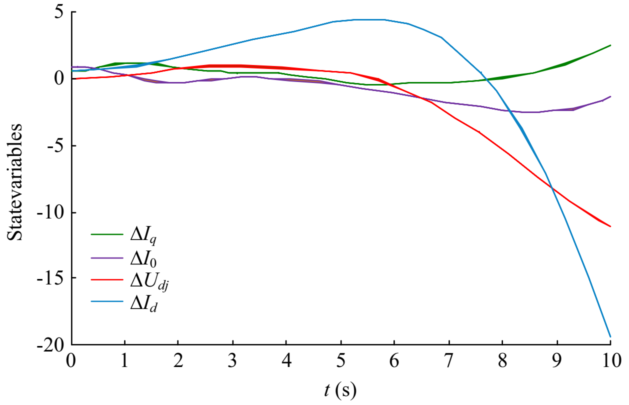


(b) DG2

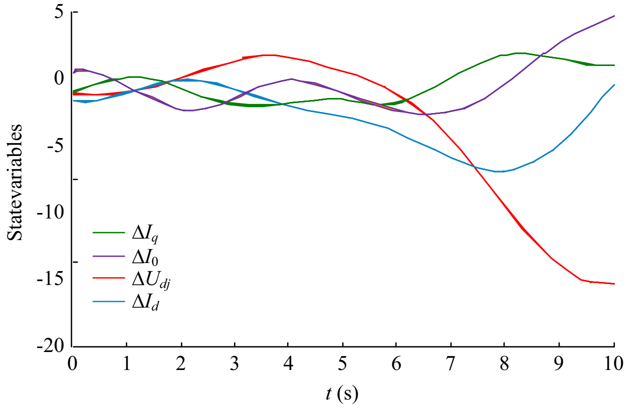


(c) DG3

Fig. 6 State variables trajectories of DGs



(a) STATCOM1



(b) STATCOM2

Fig. 7 State variables trajectories of STATCOMs

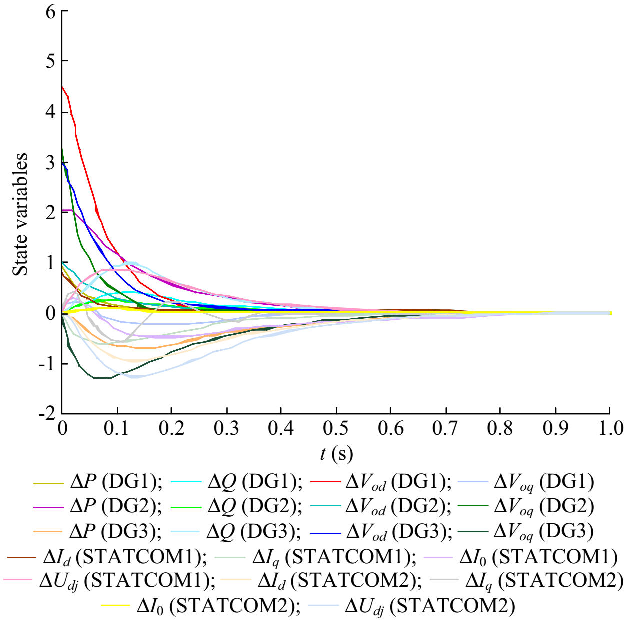


Fig. 8 State variables trajectory of multi-agent coordinated control algorithm

the multi-agent-based coordinated control method can be used for improving the static voltage stability of ADNs.

6 Conclusion

The paper analyzed the voltage stability of distribution network with DGs and found out the weak nodes in terms of static voltage stability of the system according to voltage instability coefficient of each node. The voltage coordinated control framework of active distribution network with DGs and the multi-agent consistency control theory have been applied to achieve the coordinated control of the VPP and conventional voltage control means, in order to improve the static voltage stability of distribution network. It also establishes a mathematical model of the consistency coordination control between the VPP and STATCOM agents, and solve this model by the bilinear matrix inequality (BMI) constraints feasible solution. Three cases of studies have been conducted with the characteristic are: independently controlled DGs; independently controlled DGs and conventional voltage control means; coordinately controlled VPP and STATCOMs. The result shows that in case of both the independently control schemes, the static voltage stability can't be guarantee for ADNs, whereas in coordinately control scheme, the static voltage stability of the whole system can be improved, which proves the validity and effectiveness of the proposed coordinately control method.

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