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Spatial dispersion of wind speeds and its influence on the forecasting error of wind power in a wind farm

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Abstract Big wind farms must be integrated to power system. Wind power from big wind farms, with randomness, volatility and intermittent, will bring adverse impacts on the connected power system. High precision wind power forecasting is helpful to reduce above adverse impacts. There are two kinds of wind power forecasting. One is to forecast wind power only based on its time series data. The other is to forecast wind power based on wind speeds from weather forecast. For a big wind farm, due to its spatial scale and dynamics of wind, wind speeds at different wind turbines are obviously different, that is called wind speed spatial dispersion. Spatial dispersion of wind speeds and its influence on the wind power forecasting errors have been studied in this paper. An error evaluation framework has been established to account for the errors caused by wind speed spatial dispersion. A case study of several wind farms has demonstrated that even if the forecasting average wind speed is accurate, the error caused by wind speed spatial dispersion cannot be ignored for the wind power forecasting of a wind farm.

Keywords Wind farm, Wind speed, Spatial dispersion, Wind power, Forecasting error

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1 Introduction

Wind energy is one of the renewable energy options with the most potential for large-scale commercial development and use [1]. Grid integration of wind power is not only the way to develop large scale wind energy, but also the choice of the Sustainable Energy Development Strategy of China. According to the statistics of the Chinese Wind Energy Association, the cumulative installed wind power capacity has exceeded 90 GW by the end of 2013. Wind generation has been the third largest one after coal and hydro generation. The "Wind Power Development Roadmap" proposed jointly by the National Development and Reform Committee (NDRC) of China and the International Energy Agency (IEA) also set forth wind installation targets for China to reach 200, 400, and 1000 GW by 2020, 2030, and 2050, respectively [2].

Wind turbines are driven by near-ground wind, transforming wind energy into electrical energy. Compared with conventional power generations, wind power is fluctuating, intermittent, and of low energy density. For the grid integrating with large-scale wind farms, wind power fluctuations will lead to upset the equilibrium of power supply and demand, and even endanger the safety of the grid. The curtailment of wind power is implemented to insure power system safety, which decreases the utilization rate of wind power [3]. If wind power can be forecasted accurately, the adverse impacts of wind power fluctuations can be reduced by prearranging a scheduling plan. Therefore, it is important to improve the precision of wind power forecasting.

Although many researchers engaged in wind power forecasting, the forecasting precision is still not satisfied. Reference [4] reported statistics for the domestic Chinese wind power forecasting error. The root mean square errors (RMSEs) are about 10% ~ 20% for a day-ahead (24 hours) forecasting; the RMSEs are about 10% ~ 18% for a real time(4 hours) wind power forecasting for individual wind farm. For regional wind power forecasting, RMSEs are about 10% ~ 25%, as shown in Table 1.

There are two kinds of wind power forecasting. One is to forecast wind power only based on its time series data. The other is to forecast wind power based on wind speeds from weather forecast. Many studies have been focused on improving the forecasting precision of the wind speed. Reference [5], based on CFD pre-calculated flow fields (CPFF), established a wind speed database for each wind turbine under various inflow conditions and used the database to calculate the speed of each wind turbine under the mesoscale numerical weather prediction (NWP) input data conditions. Reference [6] proposed a method to forecast the turbine speed that combined the NWP input speed data with the turbine speed data using the fan standard power curve to calculate the power. Reference [7] proposed a f-ARIMA model that can forecast the wind speed up to 24 and 48 hours in advance. Reference [8] proposed a modified EMD-based artificial neural network model to forecast wind speed, improving the wind speed forecasting accuracy. Reference [9] forecasted the wind speed based on the hybrid forecasting model. Using a Kalman filter to revise the results forecasted by the artificial neural network, this method produced a good outcome. Reference [10] used the discrete Hilbert transform as minimum phase type filter to forecast wind speed and identify the characterization of wind speed. Reference [11] transformed wind into a two-dimensional input space, establishing a wind power forecasting model based on k-nearest neighbor (k-NN). A hybrid model (SAM–ESM–RBFN) was proposed for wind speed prediction in [12].

Table 1 Wind power forecasting errors [4]

Region	RMSE (%)	Mean absolute error (MAE) (%)
Eastern Inner Mongolia	13.9	12.32
Liaoning	11.73	9.99
Jilin	12.2	10.3
Heilongjiang	10.79	9.2
Gansu	16.23	14.15
Ningxia	20.21	17.7
Xinjiang	15.93	13.45

This model is based on the seasonal adjustment method (SAM), exponential smoothing method (ESM), and radial basis function neural network (RBFN). An effective results are achieved. Reference [13] applied a pseudo ensemble approach to integrate NWP forecasting and used the adjacent datasets of time and space in the forecasting point to obtain the probabilistic wind speed forecasting. The above conferences mainly concentrated on the study of wind speed forecasting. The goal of wind speed forecast is wind power forecast. The forecasting accuracy enhancement of wind speed is helpful to the wind power forecasting. However, it does not totally contribute to forecast accuracy of wind power.

Usually, for the wind power forecasting of a big wind farm, a representative wind speed (such as a spatial average wind speed) should be forecasted whether based on the time series method or NWP method.

A big wind farm normally contains more than hundreds of wind turbines and covers the area up to tens or hundreds of square kilometers. In such big area, due to its spatial scale and dynamics of wind, it is almost impossible that each turbine would be supplied the same wind speed at each moment, that is called the spatial dispersion of wind speeds.

Newly developed big wind turbines have been equipped with real-time data acquisition systems, recording mechanical and electrical operating data of wind turbines. This provided a foundation for analyzing the spatial dispersion of wind speeds and its influence on calculating the total wind power of a wind farm. In this paper, the spatial dispersion of wind speeds in wind farm has been studied. The framework for analyzing forecasting errors caused by neglecting spatial dispersion has been established. The results show that even if the spatial average wind speed forecasting is accurate, the error of the total wind power in a wind farm, caused by spatial dispersion of wind speeds, is still evident.

2 Spatial dispersion of wind speeds for each turbine in a wind farm

The wind speed $v_i(t)$ of a wind turbine i can be expressed as the sum of the average wind speed $\bar{v}(t)$ for a farm and the deviation $\Delta v_i(t)$ of the wind speed at moment t:

$$v_i(t) = \bar{v}(t) + \Delta v_i(t) \qquad (i = 1, 2, \cdots, n) \tag{1}$$

where $\bar{v}(t)$ is the average wind speed for a wind farm, and n is the number of the wind turbines in the farm,

$$\bar{v}(t) = \frac{1}{n}\sum_{i=1}^{n} v_i(t) \tag{2}$$

The deviation $\Delta v_i(t)$ of the wind speed is defined as the spatial dispersion of a turbine wind speed:

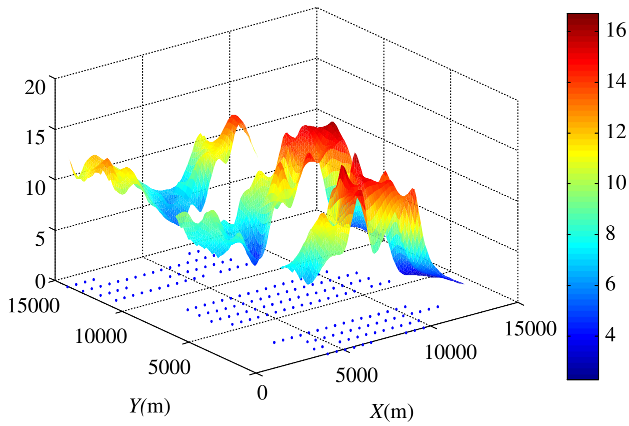

Fig. 1 Wind speeds distribution of 177 turbines at the moment of 6:20 am, on August 10, 2012, in the Jixi Wind Farm

$$\Delta v_i(t) = v_i(t) - \bar{v}(t) \qquad (i = 1, 2, \cdots, n) \tag{3}$$

It is evident that the summation of total deviations of the wind speeds equal to zero, i.e.:

$$\sum_{i=1}^{n} \Delta v_i(t) = 0 \tag{4}$$

The average speed $\bar{v}(t)$ is an unbiased estimate of the overall speed strength of all the turbines at time t.

As a real case, Fig. 1 shows the spatial dispersion of wind speeds at the Jixi Wind Farm, which contains 177 turbines of 1.5 MW. At the moment of 6:20 am, on August 10, 2012, all wind speeds distributed in the range from 3.00 to 17.29 m/s. The average wind speed was 10.07 m/s, and the standard deviation of wind speeds was 3.24 m/s.

Figure 2 shows the sequence diagram of the maximum wind speed $v_{\max}(t)$ and minimum wind speed $v_{\min}(t)$ at the Jixi wind farm over one day (August 27, 2012). The sampling interval is 10 min. In this case of one day, the maximum difference between $v_{\max}(t)$ and $v_{\min}(t)$ is 10.02 m/s, the minimum difference between them is 4.05 m/s, and the standard deviation of the difference between them is 1.38 m/s. The envelope curves of $v_{\max}(t)$, $v_{\min}(t)$ and $v_{av}(t)$ are shown in Fig. 2, which describes the strength of spatial dispersion of wind speeds.

Because at any time, $v_{\max}(t) \neq v_{\min}(t)$, the spatial dispersion of wind speeds is always existed.

The strength of the wind speed spatial dispersion can therefore be measured by the standard deviation of all deviations of wind speeds at moment t.

$$S(t) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \left(v_i(t) - \bar{v}(t)\right)^2} \tag{5}$$

where $S(t)$ is the standard deviation of spatial dispersion of all wind speeds at moment t.

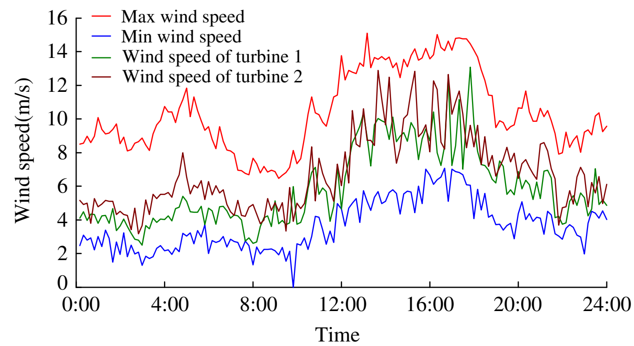

Fig. 2 The maximal and minimal wind speeds varied over one day (August 27, 2012) in the Jixi Wind Farm

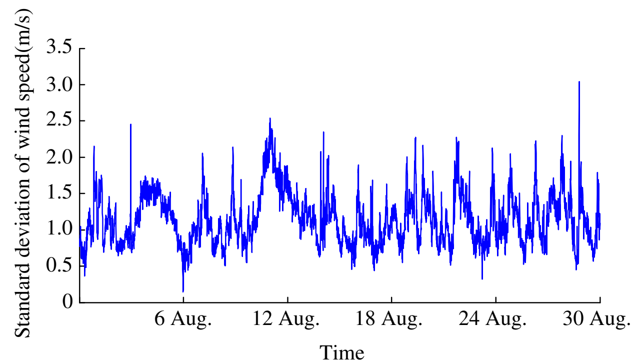

Fig. 3 Standard deviation of wind speeds varied over one month (August, 2012) in the Jixi Wind Farm

The standard deviation of the wind speed spatial dispersion at the Jixi Wind Farm over a month (August, 2012) is shown in Fig. 3. The average standard deviation is 1.15 m/s. That means the spatial dispersion of wind speeds is significant.

3 Wind power transformation and calculation of the wind power for a wind farm

The function of wind turbines is to transform the wind energy (input) exerted on the impeller into electric power (output). Describing the relationship between wind turbines input and output is the wind power transformation (power characteristics).

3.1 Standard wind power transformation

The standard wind power transformation of the wind turbine is provided by the turbine manufacturers based on the standards of the wind turbine power characteristics (GBT18451.2-2012). Standards stipulate that under the condition of the standard air density (1.225 kg/m^3) wind speed is the average speed on the fan during 10 min of

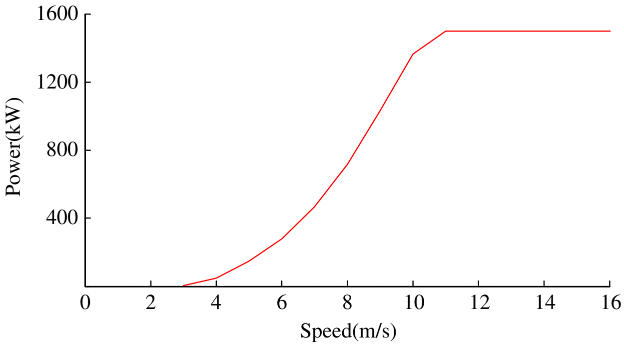

Fig. 4 The standard wind power transformation of a 1.5 MW wind turbine

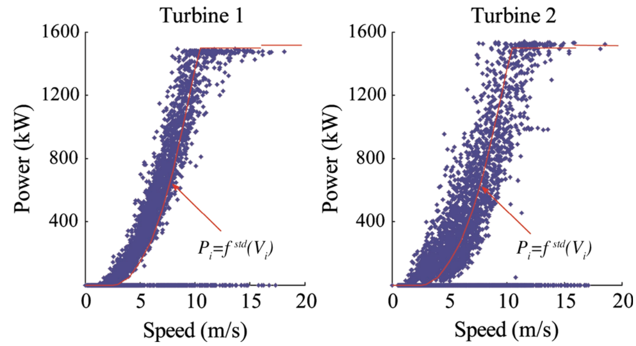

Fig. 5 Actual wind power transformation of two wind turbines

operation measured from the center height of the wind turbine wheel, and wind power is the average input power of the wind turbine during 10 min. The interval method is used to determine the power curve, which uses 0.5 m/s as an area to calculate the average value of the speed and the power within the area.

The standard wind power transformation of a wind turbine of 1.5 MW is shown in Fig. 4, which represents:

$$P_i(t) = f^{std}(v_i(t)) \tag{6}$$

where $P_i(t)$ is the wind power of wind turbine i at moment t calculated by standard wind power transformation, and $f^{std}(\cdot)$ is the common wind power transformation function.

After this process of averaging, the calculation of the wind power displays a single-value mapping relationship. It means that the wind power transformation is a pure static relationship, not related to the historical variation of the wind speed.

3.2 Actual wind power transformation of a wind turbine

Due to the volatility of wind speed, the operation state (v, P) rarely stay at steady state. If wind speed changes fast, the operation state will go out of the red line of Fig. 4.

Under real operating condition, the relationship between the speed and power is a multi-valued mapping, presenting as a scatterplot (see in Fig. 5), that is distributed in a wide range around the standard wind power transformation characteristic curve. That means the actual wind power relationship is dynamical and is related to the historical value.

Figure 5 gives wind-power transformations of two wind turbines in the Jixi Wind Farm based on the operational data. The red curve is the standard wind power transformation.

As shown in Fig. 5, different from the single-valued mapping of standard transformation curve of red line, the actual wind power transformation is a multi-valued mapping, shown as the blue scatterplot.

Even for the same type of wind turbines, the actual wind power transformations may be different.

The actual wind power transformation of wind turbine i cannot be described as an analytical form, be denoted as $f_i^*(\cdot)$. So the wind power of turbine i is described as:

$$P_i^{real}(t) = f_i^*(v_i(t)) \tag{7}$$

where $P_i^{real}(t)$ is the real wind power of wind turbine i at moment t.

3.3 The relationship between wind speeds and the total wind power of a farm

The total wind power $P_\Sigma^{real}(t)$ of the whole farm is described as:

$$P_\Sigma^{real}(t) = \sum_{i=1}^{n} f_i^*(v_i(t)) \tag{8}$$

In the situation of forecasting wind power based on wind speed, even if the average wind speed is accurately forecasted, due to the spatial dispersion of wind speeds and different wind power transformations, wind power forecasting errors would be existed.

3.3.1 Forecasting error evaluation of neglecting the difference of wind power transformations

The operating characteristics of all wind turbines can be represented by a common standard wind power transformation $f^{std}(\cdot)$ based on the assumption of neglecting the difference of wind power transformations in different turbines. The total wind power calculation therefore becomes:

$$P_\Sigma^{cal,std}(t) = \sum_{i=1}^{n} f^{std}(v_i(t)) \tag{9}$$

where $P_\Sigma^{cal,std}(t)$ is the total wind power calculated by standard wind power transformation.

The error of $\varepsilon_i^{*std}(v_i(t))$ caused by using $f^{std}(\cdot)$ to replace $f_i^*(\cdot)$, can be described as:

$$\varepsilon_i^{*std}(v_i(t)) = f_i^*(v_i(t)) - f^{std}(v_i(t)) \tag{10}$$

This error reflects the extent to which the scatterplot of $f_i^*(\cdot)$ deviates from the standard wind power transformation curve of $f^{std}(\cdot)$, in other words, the extent to which the actual wind power dynamic relationship deviates from the wind power static relationship. The error is related to the changing process of the wind speed (historical value), and it can only be used to observe the statistical properties, rather than be written in an analytic formula.

By substituting the wind speed of each turbine into the standard wind power transformation, the total wind power of the farm can be calculated.

$$P_\Sigma^{cal,std}(t) = \sum_{i=1}^{n} f^{std}(\bar{v}(t) + \Delta v_i(t)) \tag{11}$$

The relationship between the real total wind power and the calculated one based on the standard wind power transformation can be described as:

$$P_\Sigma^{real}(t) = P_\Sigma^{cal,std}(t) + \sum_{i=1}^{n} \varepsilon_i^{*std}(v_i(t)) \tag{12}$$

Considering constraint condition of $\sum_{i=1}^{n} \Delta v_i(t) = 0$, if $f^{std}(\cdot)$ is a linear function, then $f^{std}(\cdot) = f^{std,linear}(\cdot)$. Eq. (13) can easily be proved according to the additive property of linear function.

$$\begin{aligned} P_\Sigma^{cal,linear}(t) = P_\Sigma^{cal,std}(t) &= \sum_{i=1}^{n} f^{std}(\bar{v}(t) + \Delta v_i(t)) \\ &= \sum_{i=1}^{n} f^{std,linear}(\bar{v}(t) + \Delta v_i(t)) \\ &= \sum_{i=1}^{n} f^{std,linear}(\bar{v}(t)) + \sum_{i=1}^{n} f^{std,linear}(\Delta v_i(t)) \\ &= n f^{std,linear}(\bar{v}(t)) \end{aligned} \tag{13}$$

Eq. (13) indicates that if the function of $f^{std}(\cdot)$ is linear, the total wind power of $P_\Sigma^{cal,std}(t)$ can be accurately calculated by the average speed without the influence of the spatial dispersion of wind speeds.

In fact, due to wind speed changes within a large range, $f^{std}(\cdot)$ is indeed a nonlinear function as shown in Fig. 4. Nonlinear function does not have additive properties.

Therefore, taking into account the nonlinear properties of $f^{std}(\cdot)$, (13) should be rewritten as:

$$P_\Sigma^{cal,std}(t) = \sum_{i=1}^{n} f^{std}(\bar{v}(t) + \Delta v_i(t)) \neq n f^{std}(\bar{v}(t)) \tag{14}$$

It is obvious that the wind power calculated by the average wind speed and the standard wind power transformation does not exactly equal to the total wind power of $P_\Sigma^{cal,std}(t)$. It means that using the average wind speed or neglecting spatial dispersion of wind speeds would produce calculating error of total wind power in a wind farm.

If $f^{std}(\cdot)$ is a linear function, the power deviation $\Delta P_i(t)$ of turbine i and $\Delta P_j(t)$ of turbine j, caused by the speed deviations of $\Delta v_i(t) = -\Delta v_j(t)$, can be exactly offset, i.e.: $\Delta P_i(t) = -\Delta P_j(t)$.

If $f^{std}(\cdot)$ is a nonlinear function, the power deviation caused by speed deviations cannot be offset, thus neglecting spatial dispersion of the wind speeds will lead to the calculation error of total wind power for a wind farm.

3.3.2 *Forecasting error evaluation of considering individual wind power transformation*

Assumed that the wind power transformation of turbine i can be fitted to a single-valued mapping function, which is described as:

$$P_i^{curve}(t) = f_i^{curve}(v_i(t)) \tag{15}$$

where $f_i^{curve}(\cdot)$ is a fitted wind power transformation, and $P_i^{curve}(t)$ is the wind power of turbine i at moment t calculated by the fitted wind power transformation.

The error of $\varepsilon_i^{*curve}(v_i(t))$ caused by using the fitted wind power transformation $f_i^{curve}(\cdot)$ to replace $f_i^*(\cdot)$, can be described as:

$$\varepsilon_i^{*curve}(v_i(t)) = f_i^*(v_i(t)) - f_i^{curve}(v_i(t)) \tag{16}$$

This error reflects the extent to which the scattered dots of $f_i^*(\cdot)$ deviate from the fitted wind power transformation of $f_i^{curve}(\cdot)$. The error of $\varepsilon_i^{*curve}(v_i(t))$ cannot be written in an analytical formula, which is similar to the error of $\varepsilon_i^{*std}(v_i(t))$.

Hence, the actual wind power of turbine i can be written as:

$$P_i^{real}(t) = f_i^{curve}(v_i(t)) + \varepsilon_i^{*curve}(v_i(t)) \tag{17}$$

Only $f_i^{curve}(\cdot)$ can be used to calculate wind power based on the individual wind speed and individual wind power transformation.

Therefore, with well-known wind speeds, the total wind power of the wind farm can be calculated using wind speeds and considering individual wind power transformation.

$$P_\Sigma^{cal,curve}(t) = \sum_{i=1}^{n} f_i^{curve}(\bar{v}(t) + \Delta v_i(t)) \tag{18}$$

where $P_\Sigma^{cal,curve}(t)$ is the total wind power calculated by the fitted wind power transformation and individual wind speed.

Considering the nonlinear property of $f_i^{curve}(\cdot)$, assume at least for one turbine *i,* the wind power transformation of $f_i^{curve}(\cdot)$ is not equal to the common wind power transformation $f^{std}(\cdot)$, then, (19) can be derived.

$$P_{\Sigma}^{cal,curve}(t) = \sum_{i=1}^{n} f_i^{curve}(\bar{v}(t) + \Delta v_i(t)) \neq \sum_{i=1}^{n} f_i^{curve}(\bar{v}(t)) \neq nf^{std}(\bar{v}(t)) \tag{19}$$

That means neglecting spatial dispersion of wind speeds could raise errors of calculated total wind power (left hand side) whether individual wind power transformation or common wind power transformation is considered.

Defining the errors between $f_i^{curve}(\cdot)$ and $f^{std}(\cdot)$ as $\varepsilon_i^{std-curve}(v_i(t))$, which is expressed as:

$$\varepsilon_i^{std-curve}(v_i(t)) = f_i^{curve}(v_i(t)) - f^{std}(v_i(t)) \tag{20}$$

Given a group wind speeds of a specific moment, if $f_i^{curve}(\cdot)$ and $f^{std}(\cdot)$ are known, the errors caused by neglecting spatial dispersion under different wind power transformations can be calculated.

The relationship of the two methods for calculating total wind power, one is to include and the other is to exclude the individual wind power transformation, can be written as:

$$P_{\Sigma}^{cal,curve}(t) = P_{\Sigma}^{cal,std}(t) + \sum_{i=1}^{n} \varepsilon_i^{std-curve}(v_i(t)) \tag{21}$$

where the term of $\sum_{i=1}^{n} \varepsilon_i^{std-curve}(v_i(t))$ is the error caused by neglecting individual wind power transformation.

Furthermore, the relationship between the calculated total wind power and the actual total wind power can be written as:

$$\begin{aligned} P_{\Sigma}^{real}(t) &= P_{\Sigma}^{cal,curve}(t) + \sum_{i=1}^{n} \varepsilon_i^{*curve}(v_i(t)) \\ &= P_{\Sigma}^{cal,std}(t) + \sum_{i=1}^{n} \varepsilon_i^{std-curve}(v_i(t)) \\ &\quad + \sum_{i=1}^{n} \varepsilon_i^{*curve}|v_i(t)| \end{aligned} \tag{22}$$

Applying (12) again, (23) can be obtained.

$$\sum_{i=1}^{n} \varepsilon_i^{*std}(v_i(t)) = \sum_{i=1}^{n} \varepsilon_i^{std-curve}(v_i(t)) + \sum_{i=1}^{n} \varepsilon_i^{*curve}(v_i(t)) \tag{23}$$

The term of $\sum_{i=1}^{n} \varepsilon_i^{*std}(v_i(t))$ (left hand side) is the overall error of calculating the total wind power of a wind farm. In the right hand side of (23), both the two terms related to the spatial dispersion of wind speeds and the wind power transformation.

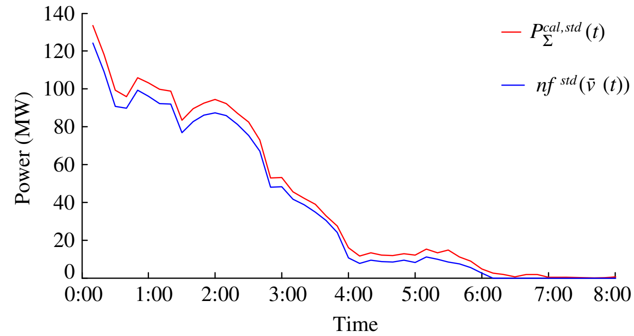

Fig. 6 Errors caused by neglecting the spatial dispersion of wind speeds in the Jixi wind farm (0:00–8:00 August 26, 2012)

A real case of the Jixi Wind Farm has been calculated, the error of $\sum_{i=1}^{n} \varepsilon_i^{std-curve}(v_i(t))$ varied in 8 hours is shown in Fig. 6, it is the difference of curves $nf^{std}(\bar{v}(t))$ and $P_{\Sigma}^{cal,std}(t)$. Due to the function of $f^{std}(\cdot)$ is a nonlinear one, there are clear errors when excluding the spatial dispersion of wind speeds.

4 Relationships between the spatial dispersion of wind speeds and errors of wind power forecasting

Obviously, a bad wind speed forecasting would lead to big error of total wind power forecasting. Consequently, much research has focused on improving the precision of wind speed forecasting.

On the other hand, the accuracy improvement of wind speed forecasting does not necessarily lead to an infinite increase in wind power forecasting accuracy. The errors produced in the process of recording the speed to calculate power will be unchanged because of the improvement of the accuracy of the wind speed forecasting. In other words, even if the forecasting of wind is completely accurate, a completely accurate forecasting of wind power still cannot be achieved.

4.1 Relationship among actual speed, forecasted wind speed and the calculated total wind power of a farm

Assuming that the forecasted wind speed of wind turbine *i* is $v_i^{forecast}(t)$ at time *t*, the actual wind speed is defined as:

$$v_i(t) = v_i^{forecast}(t) - \varepsilon_i^{forecast}(t) \tag{24}$$

where $\varepsilon_i^{forecast}(t)$ is the wind speed forecasting error of turbine *i*.

Therefore, the total forecasting wind power $P_{\Sigma}^{forecast}(t)$ can be calculated as:

$$P_{\Sigma}^{forecast}(t) = \sum_{i=1}^{n} f_i^* \left(v_i^{forecast}(t) - \varepsilon_i^{forecast}(t) \right) \tag{25}$$

The real wind power transformation for each wind turbine is considered.

If the differences of wind power transformations are ignored, this means that the input and output relationships of all the wind turbines in a farm can be expressed by a common standard wind power transformation, the total wind power can be defined as:

$$P_{\Sigma}^{forecast,std}(t) = \sum_{i=1}^{n} f^{std} \left(v_i^{forecast}(t) - \varepsilon_i^{forecast}(t) \right) \tag{26}$$

where $P_{\Sigma}^{forecast,std}(t)$ is the forecasting power of the whole farm calculated by standard wind power transformation.

The forecasting wind speed of turbine i, similar to (1), can be described as the sum of the forecasted average speed $\bar{v}^{forecast}(t)$ and the spatial dispersion of the forecasted wind speed $\Delta v_i^{forecast}(t)$ at moment t, which is given as:

$$v_i^{forecast}(t) = \bar{v}^{forecast}(t) + \Delta v_i^{forecast}(t) \quad (i = 1,\ 2, \cdots,\ n) \tag{27}$$

Substituting into (26), the forecasted total wind power based on a common wind power transformation can be obtained as:

$$\begin{aligned} P_{\Sigma}^{forecast,std}(t) &= \sum_{i=1}^{n} f^{std} \left(v_i^{forecast}(t) - \varepsilon_i^{forecast}(t) \right) \\ &= \sum_{i=1}^{n} f^{std} \left(\bar{v}^{forecast}(t) + \Delta v_i^{forecast}(t) - \varepsilon_i^{forecast}(t) \right) \end{aligned} \tag{28}$$

where the forecasted wind speed can be described as (29) at time t.

$$\bar{v}^{forecast}(t) = \frac{1}{n} \sum_{i=1}^{n} v_i^{forecast}(t) \tag{29}$$

Substituting into (24), the forecasted average wind speed can be obtained.

$$\bar{v}^{forecast}(t) = \bar{v}(t) + \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i^{forecast}(t) \tag{30}$$

This is the relationship between the forecasted average wind speed and the actual average wind speed. Normally, the two average speeds are different.

4.2 Relationship between the spatial dispersion of wind speeds and the calculating error of the total wind power

For investigating the relationship between the spatial dispersion of wind speeds and the calculating error of the total wind power of the wind farm, the forecasting error of wind speeds could be temporarily ignored. This means that the wind speed of each turbine is accurately forecasted. Therefore, the actual average wind speed can be taken as the forecasted average wind speed, which is defined as:

$$\bar{v}^{forecast}(t) = \bar{v}(t) \tag{31}$$

According to (30), the only condition to derive (31) is to satisfy $\sum_{i=1}^{n} \varepsilon_i^{forecast}(t) = 0$, rather than each $\varepsilon_i^{forecast}(t) = 0$. This condition does not mean that the spatial dispersion of wind speeds has disappeared.

$\Delta v_i^{forecast}(t)$ is the spatial dispersion of the forecasted wind speeds, and $\varepsilon_i^{forecast}(t)$ is the forecasting errors of wind speeds. They are completely different, and therefore no systematic cancellation will be resulted in.

If the actual average wind speed is taken as the forecasted average wind speed, the calculating error of total wind power would be the minimum forecasting error.

The error caused by neglecting the spatial dispersion of wind speeds can be described as:

$$\varepsilon^{SpatialDisp}(t) = P_{\Sigma}^{cal,std}(t) - P_{\Sigma}^{cal,linear}(t) \tag{32}$$

The total calculating error $\varepsilon^{Total}(t)$ is defined as:

$$\varepsilon^{Total}(t) = P_{\Sigma}^{real}(t) - P_{\Sigma}^{cal,linear}(t) \tag{33}$$

The total calculating error $\varepsilon^{Total}(t)$ includes the influence of spatial dispersion of wind speeds, the wind power transformation, and the difference between $\sum_{i=1}^{n} \varepsilon_i^{*std}(v_i(t))$ and $\sum_{i=1}^{n} \varepsilon_i^{*curve}(v_i(t))$.

$\varepsilon^{Total}(t)$ is the total wind power forecasting error of the wind farm when the forecasted average wind speed is completely accurate.

The standard deviation of different errors can be measured by $\underset{t\in[0,T]}{std}(\varepsilon^{SpatialDisp}(t))$ and $\underset{t\in[0,T]}{std}(\varepsilon^{Total}(t))$ within a specific period of time [0, T].

$\eta_{\mathrm{SpatialDisp/Total}}$ is defined to represent the proportion of error caused by the spatial dispersion in the total errors.

$$\eta_{SpatialDisp/Total} = \frac{\underset{t\in[0,T]}{std}(\varepsilon^{SpatialDisp})}{\underset{t\in[0,T]}{std}(\varepsilon^{Total})} \tag{34}$$

It is obvious that greater $\eta_{SpatialDisp/Total}$ means the greater proportion of the error caused by neglecting the spatial dispersion of the wind speeds. That is, the greater error of the forecasted total wind power will be occurred.

4.3 Case study

In order to examine the existence of the spatial dispersion of wind speeds and its influence on the forecasting error of total wind power in a wind farm, operating data including wind speeds and wind powers of 3 wind farms have been collected and used in the case studies. The relationship between the spatial dispersion of wind speeds and calculated error of wind power is analyzed.

The Jixi Wind Farm of 265 MW being connected to Jilin grid is taken as an example. The farm contains 177 wind turbines of 1.5 MW. All coverage of the farm is 100.3 km^2. The recorded operating data of wind turbine and generator included wind speed and wind power of each turbine. The data sampling interval of 10 min is adapted. And the time span of 30 days is enough for checking an ultra-short term (or real time) wind power forecasting.

Taking one day operation data for example,first, the spatial average wind speed of every moment is calculated by using the wind speeds of all turbines to calculating the spatial average wind speed.

If neglecting the spatial dispersion of wind speeds, and the common standard wind power transformation was adapted, the time series of wind power of each turbine is $f^{std}(\bar{v}(t))$, and the time series of the total wind power of the wind farm is $nf^{std}(\bar{v}(t))$.

Based on different calculation models, the time series of total wind power $P_{\Sigma}^{real}(t)$ and $P_{\Sigma}^{cal,std}(t)$ can also be calculated. All 3 time series of wind power are shown in Fig. 7 and Fig. 8.

The maximal error of neglecting the spatial dispersion of wind speeds reached 10.96 MW, the average value was

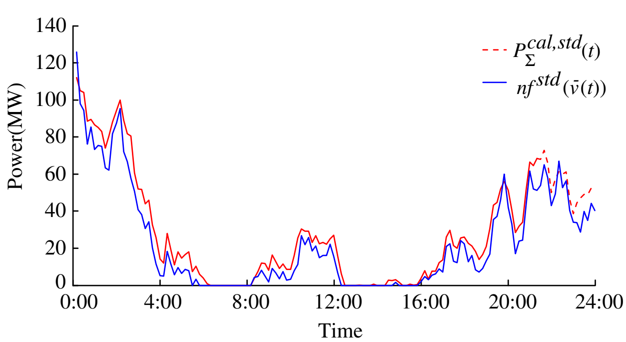

Fig. 7 Comparison of calculated total wind power using the average wind speed and real speed in the Jixi Wind Farm (August 26, 2012)

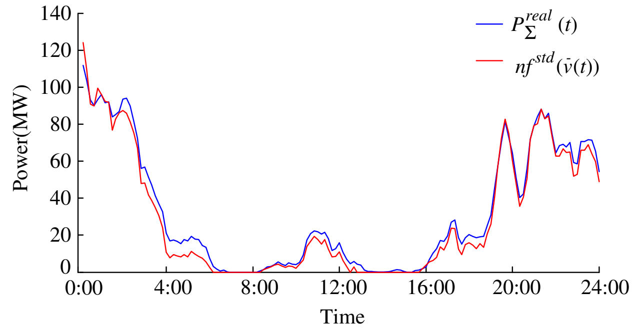

Fig. 8 Comparison of actual total wind power and the total wind power calculated using the average wind speed at the Ji xi Wind Farm (August 26, 2012)

Table 2 Error of neglecting spatial dispersion and the total error in some other days

Date	$\frac{\max(\varepsilon^{SpatialDisp})}{\max(\varepsilon^{Total})}$ (%)	$\frac{\text{mean}(\varepsilon^{SpatialDisp})}{\text{mean}(\varepsilon^{Total})}$ (%)	$\eta_{SpatialDisp/Total}$ (%)
Aug. 16	43.40	55.77	33.68
Aug. 17	51.02	32.71	64.42
Aug. 18	32.12	48.39	23.72
Aug. 19	39.40	66.88	28.59
Aug. 20	54.2	63.45	38.80

4.46 MW, and the standard deviation of the errors was 2.41 MW.

As shown in Fig. 8, neglecting the spatial dispersion of the wind speed and the deviation of wind power transformation, the greatest error during the day reaches 12.27 MW, the average value is 3.53 MW, and the standard deviation of the errors is 2.93 MW, 4.63%, 1.33% and 1.11% of the totally installed capacity of the farm respectively.

According to (34), $\eta_{SpatialDisp/Total} = 82.25\%$, therefore, the proportion of the standard deviation of the wind power calculation errors caused by neglecting the spatial dispersion of wind speeds in the standard deviation of the total errors was 82.25% in the day of 26 August.

Relationships between the errors caused by neglecting the spatial dispersion and the total errors in some other days are shown in Table 2.

In order to examine the effect of neglecting spatial dispersion of wind speeds on the forecasting error of total wind power in a wind farm for a long-term, the 30 days data in three different wind farms are summarized.

Tables 3, 4 and 5 represent the results of the wind power calculation errors in three different wind farms caused by neglecting the spatial dispersion of the wind speeds and the dispersion of the wind power transformations.

Table 3 Comparison of error caused by neglecting spatial dispersion and the total error in the Jixi Wind Farm

Error	Error of spatial dispersion $\varepsilon^{SpatialDisp}$ (MW)	Total Error ε^{Total} (MW)	$\varepsilon^{SpatialDisp}/\varepsilon^{Total}$ (%)	Installed capacity (%)
Max error	56.14	241.74	23.22	21.14
Average error	5.83	11.62	50.19	2.20
Standard deviation of error	8.19	25.30	32.37	3.08

Table 4 Comparison of error caused by neglecting spatial dispersion and the total error in the Liaotai Wind Farm

Error	Error of spatial dispersion $\varepsilon^{SpatialDisp}$(MW)	Total Error ε^{Total} (MW)	$\varepsilon^{SpatialDisp}/\varepsilon^{Total}$ (%)	Installed capacity (%)
Max error	6.83	35.50	19.23	18.97
Average error	1.05	2.44	43.06	2.92
Standard deviation of error	1.10	4.85	22.57	3.06

Table 5 Comparison of error caused by neglecting spatial dispersion and the total error in the Menggui Wind Farm

Error	Error of spatial dispersion $\varepsilon^{SpatialDisp}$ (MW)	Total Error ε^{Total} (MW)	$\varepsilon^{SpatialDisp}/\varepsilon^{Total}$ (%)	Installed capacity (%)
Max error	32.70	72.97	44.82	67.49
Average error	1.90	12.79	14.90	3.92
Standard deviation of error	3.10	14.72	21.07	6.40

As shown in Table 3, the calculating errors of total wind power reached 2.2% (average) and 3.08% (standard deviation). The proportion of errors caused by neglecting the spatial dispersion of wind speeds in the total errors was 50.19% (average) and 32.37% (standard deviation). The similar results for the Liaotai Wind Farm and the Menggui Wind Farm are shown in Table 4 and Table 5.

If actual forecasted average wind speed is used to calculate the wind power, the bigger errors will be emerged than the above errors.

Therefore, the error caused by neglecting spatial dispersion of wind speeds, shown in Tables 3, 4 and 5, could not be eliminated by increasing the forecasting precision of the average wind speed.

5 Conclusion

For a big wind farm, due to its spatial scale and dynamics of wind, wind speeds at different wind turbines are obviously different. The deviation of $v_i(t) - \bar{v}(t)$ is defined as spatial dispersion of wind speeds for turbine *i*.

For forecasting the total wind power of a farm, where the wind speed forecasting is based on. The forecasting errors of the total wind power are related to the spatial dispersion of wind speeds and wind power transformations. Neglecting spatial dispersion of wind speeds will lead to obvious forecasting errors.

The relationships between the spatial dispersion of wind speeds and wind power forecasting errors have been investigated. Both theoretical analysis and case study show that even if the average wind speed is accurately forecasted, neglecting spatial dispersion of wind speeds, the proportion of forecasting errors will be more than 20%. Therefore, the forecasting accuracy enhancement of wind speed could not entirely contribute to forecasting precision of the total wind power in a farm.

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