Stability analysis of long hydrodynamic journal bearings based on the journal center trajectory

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Abstract: In this study, we observe that there are two threshold speeds (stability threshold speed and second threshold speed) for the long journal bearing, which is different for the short bearing. When the rotating speed is below the stability threshold speed, the stability boundary nearly coincides with the clearance circle, and the journal center gradually returns to the equilibrium point after being released at an initial point. If the rotating speed is between the stability threshold speed and the second threshold speed, after being released at an initial point, the journal center converges to a contour containing the equilibrium point. In this situation, for a higher rotating speed, the corresponding contour is also larger. When the rotating speed exceeds the second threshold speed, the journal gradually moves towards the bearing surface after being released at an initial point.

Keywords: long journal bearings; stability threshold speed; stability boundary; journal center trajectory

1 Introduction

Hydrodynamic bearings are widely used in high-speed rotating machinery and equipment. During high-speed rotation, the oil whirl cannot be ignored, which restricts the rotating speed of the bearings. Over the last few decades, several studies have been conducted to discuss the nonlinear stability of oil film bearings considering the oil whirl.

Hollis and Taylor [1] discussed the critical speed stability boundary of short bearings by applying Hopf bifurcation and linear stability theory. Khonsari and Chang [2] studied the stability boundary of short hydrodynamic bearings lubricated with Newtonian fluids applying the fourth order Runge–Kutta method and the Routh–Hurwitz stability criterion. Bouaziz et al. [3]

discussed the dynamic performances of a misaligned rotor supported by hydrodynamic bearings. Jang and Yoon [4] analyzed the stability of hydrodynamic journal bearings with herringbone grooves. Smolík et al. [5] investigated the threshold speed and threshold curve of nonlinear hydrodynamic bearings by applying a short bearing approximation. Kushare and Sharma [6] displayed the nonlinear stability of hybrid journal bearings with valve restrictors lubricated using non-Newtonian fluids, and Li et al. [7] illustrated the nonlinear stability boundaries of hybrid journal bearings with orifice restrictors. Lin et al. investigated the effects of surface topography [8] and non-Newtonian lubricants [9] on the stability boundary of short journal bearings. Wang and Khonsari [10, 11] proposed a method to predict the stability boundary of short bearings by applying the Hopf bifurcation theory, and their

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results agreed well with those of a trial-and-error method. Miraskari et al. [12] developed a nonlinear coefficient method and successfully predicted the bifurcation types of the short bearing system as well as obtained journal center trajectories for rotating speeds below and above the threshold speed. Chasalevris [13] investigated the effects of the design parameters of the short bearing system on the type of Hopf bifurcations.

The above-mentioned studies have discussed the stability of short bearings using different methods. For long journal bearings, the research method has always been focused on the Hopf bifurcation theory. Myers [14] introduced the Hopf bifurcation theory into the discussion of long bearings. Amamou and Chouchane [15] discovered the nonlinear threshold speed of long journal bearings and observed nonlinear phenomena near the stability limits. Wang and Khonsari discussed the static performance [16] and nonlinear instability [17] of long-grooved journal bearings. Kumar and Sarangi [18] discussed the dynamic response of long bearings in an unbalanced state considering the nonlinear oil film forces.

In this study, the nonlinear stability of the long journal bearing was discussed by applying a trial-and-error method. First, the motion equations are established with the long bearing approximation. After introducing the state vector, the Jacobi matrix of the stiffness and damping coefficients in rectangular coordinates can be obtained. On applying the Routh–Hurwitz stability criterion to the characteristic equation of the Jacobi matrix, the stability threshold speed of the journal bearing can be determined. For a given rotating speed, the journal center trajectory can be confirmed using the fourth-order Runge–Kutta method. Subsequently, the stability of the bearing could be discussed with a known journal center trajectory.

2 Analysis

The journal bearing discussed herein is displayed in Fig. 1. The radius of the bearing is R , and the anticlockwise rotating angular speed is ω . The radial clearance is C , eccentricity is e , angular

Fig. 1 Schematic diagram of the journal bearing.

coordinate along the bearing surface is θ , and film thickness of the bearing can be expressed as $h = C + e \cos \theta$. The attitude angle is φ , and the fluid force components in the eccentric direction and the direction of the attitude angle are f_ε and f_φ , respectively. The external load on the journal is W .

We introduce the non-dimensional parameters as follows:

$$\begin{aligned} x &= R\theta, h = Ch^* = C(1 + \varepsilon \cos \theta), e = C\varepsilon, p = \mu\omega \left(\frac{R}{C}\right)^2 p^*, \\ \omega^* &= \sqrt{\frac{W}{mC}}\omega, \dot{\varepsilon} = \frac{1}{\omega} \frac{d\varepsilon}{dt}, \dot{\varphi} = \frac{1}{\omega} \frac{d\varphi}{dt}, S = \frac{\mu\omega R^3 L}{WC^2}, \\ f_\varepsilon &= \frac{\mu\omega R^3 L}{C^2} f_\varepsilon^*, f_\varphi = \frac{\mu\omega R^3 L}{C^2} f_\varphi^*, F_X = F_X^* W, F_Y = F_Y^* W \end{aligned} \quad (1)$$

where m denotes the mass of the journal, μ is the dynamic viscosity of the lubricant, ε is the eccentricity ratio, S is the Sommerfeld number, and F_X and F_Y are the oil film force in the x and y directions, respectively.

On applying the long journal bearing approximation, the non-dimensional Reynolds equation is

$$\frac{\partial}{\partial \theta} \left(h^{*3} \frac{\partial p^*}{\partial \theta} \right) = 6[\varepsilon(2\dot{\varphi} - 1) \sin \theta + 2\dot{\varepsilon} \cos \theta] \quad (2)$$

Using the half-Sommerfeld boundary conditions $p^*(\theta=0, \theta=\pi)=0$, the non-dimensional Reynolds equation can be solved. Subsequently, the non-dimensional film forces in the eccentric direction and the direction perpendicular are obtained [19] as follows:

$$f_\varepsilon^* = \frac{(16 - \pi^2(2 + \varepsilon^2))\dot{\varepsilon}}{\pi(1 - \varepsilon^2)^{\frac{3}{2}}(2 + \varepsilon^2)} + \frac{2\varepsilon^2(2\dot{\phi} - 1)}{(1 - \varepsilon^2)(2 + \varepsilon^2)} \quad (3)$$

$$f_\phi^* = \frac{4\varepsilon\dot{\varepsilon}}{(2 + \varepsilon^2)(1 - \varepsilon^2)} - \frac{\pi\varepsilon(2\dot{\phi} - 1)}{(2 + \varepsilon^2)(1 - \varepsilon^2)^{\frac{1}{2}}} \quad (4)$$

Assume that the journal is rigid and no tilt exists, and the non-dimensional resultant forces of the journal in Fig. 1, the X and Y directions are

$$F_X^*(X^*, Y^*, \dot{X}^*, \dot{Y}^*) = S(f_\varepsilon^* \cos \phi - f_\phi^* \sin \phi) + 1 \quad (5)$$

$$F_Y^*(X^*, Y^*, \dot{X}^*, \dot{Y}^*) = S(f_\varepsilon^* \sin \phi + f_\phi^* \cos \phi) \quad (6)$$

The non-dimensional motion equations are obtained

$$\ddot{X}^* = \frac{F_X^*}{\omega^{*2}} = \frac{S(f_\varepsilon^* \cos \phi - f_\phi^* \sin \phi) + 1}{\omega^{*2}} \quad (7)$$

$$\ddot{Y}^* = \frac{F_Y^*}{\omega^{*2}} = \frac{S(f_\varepsilon^* \sin \phi + f_\phi^* \cos \phi)}{\omega^{*2}} \quad (8)$$

where ω^* is the non-dimensional angular speed of the journal.

Using vector $\mathbf{c}$ to represent the state vector

$$\mathbf{c} = \begin{bmatrix} X^* \\ Y^* \\ \dot{X}^* \\ \dot{Y}^* \end{bmatrix} \quad (9)$$

and the state equation is

$$\dot{\mathbf{c}} = \begin{bmatrix} \dot{X}^* \\ \dot{Y}^* \\ F_X^* / \omega^{*2} \\ F_Y^* / \omega^{*2} \end{bmatrix} \quad (10)$$

At the equilibrium point, the Jacobi matrix of state Eq. (10) is

$$\mathbf{J}(\omega)_s = \frac{1}{\omega^{*2}} \begin{bmatrix} 0 & 0 & \omega^{*2} & 0 \\ 0 & 0 & 0 & \omega^{*2} \\ -K_{XX}^* & -K_{XY}^* & -C_{XX}^* & -C_{XY}^* \\ -K_{YX}^* & -K_{YY}^* & -C_{YX}^* & -C_{YY}^* \end{bmatrix} \quad (11)$$

The subscript s indicates the Jacobi matrix under the steady state, whereas K_{ij}^* and C_{ij}^* ($i, j = X, Y$) express the four stiffness coefficients and four damping coefficients in rectangular coordinates

defined as follows:

$$\begin{aligned} K_{XX}^* &= -\left(\frac{\partial F_X^*}{\partial X^*}\right)_s, & K_{XY}^* &= -\left(\frac{\partial F_X^*}{\partial Y^*}\right)_s, & K_{YX}^* &= -\left(\frac{\partial F_Y^*}{\partial X^*}\right)_s, \\ K_{YY}^* &= -\left(\frac{\partial F_Y^*}{\partial Y^*}\right)_s, & C_{XX}^* &= -\left(\frac{\partial F_X^*}{\partial \dot{X}^*}\right)_s, & C_{XY}^* &= -\left(\frac{\partial F_X^*}{\partial \dot{Y}^*}\right)_s, \\ C_{YX}^* &= -\left(\frac{\partial F_Y^*}{\partial \dot{X}^*}\right)_s, & C_{YY}^* &= -\left(\frac{\partial F_Y^*}{\partial \dot{Y}^*}\right)_s \end{aligned} \quad (12)$$

The non-dimensional stiffness and damping coefficients in rectangular coordinates can be obtained by applying the transformation [20]:

$$\begin{bmatrix} K_{XX}^* & K_{XY}^* \\ K_{YX}^* & K_{YY}^* \end{bmatrix}_s = -S \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}_s \begin{bmatrix} \frac{\partial f_\varepsilon^*}{\partial \varepsilon} & \frac{\partial f_\varepsilon^*}{\partial \phi} - \frac{f_\phi^*}{\varepsilon} \\ \frac{\partial f_\phi^*}{\partial \varepsilon} & \frac{\partial f_\phi^*}{\partial \phi} + \frac{f_\varepsilon^*}{\varepsilon} \end{bmatrix}_s \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}_s^T \quad (13)$$

$$\begin{bmatrix} C_{XX}^* & C_{XY}^* \\ C_{YX}^* & C_{YY}^* \end{bmatrix}_s = -S \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}_s \begin{bmatrix} \frac{\partial f_\varepsilon^*}{\partial \dot{\varepsilon}} & \frac{\partial f_\varepsilon^*}{\partial \dot{\phi}} \\ \frac{\partial f_\phi^*}{\partial \dot{\varepsilon}} & \frac{\partial f_\phi^*}{\partial \dot{\phi}} \end{bmatrix}_s \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}_s^T \quad (14)$$

The characteristic equation is $\det[\mathbf{J}(\omega)_s - \lambda \mathbf{I}] = 0$, where $\mathbf{I}$ is the unit matrix. Subsequently, on applying the Routh–Hurwitz stability criterion to the characteristic equation, the stability threshold speed of the journal bearing ω_s^* is obtained:

$$\omega_s^* = \sqrt{\frac{d_1 d_3 d_4}{d_4^2 - d_1 d_2 d_4 + d_1^2 d_5}} \quad (15)$$

$$d_1 = C_{XX}^* + C_{YY}^*$$

$$d_2 = K_{XX}^* + K_{YY}^*$$

$$\text{where } d_3 = C_{XX}^* C_{YY}^* - C_{XY}^* C_{YX}^* \quad (16)$$

$$d_4 = K_{XX}^* C_{YY}^* + K_{YY}^* C_{XX}^* - K_{XY}^* C_{YX}^* - K_{YX}^* C_{XY}^*$$

$$d_5 = K_{XX}^* K_{YY}^* - K_{XY}^* K_{YX}^*$$

When the stability threshold speed ω_s^* is observed, by choosing a rotating speed ω^* that is below the stability threshold speed ω_s^* , substituting the chosen speed ω^* into state Eq. (10), and applying the fourth-order Runge–Kutta method, the journal center trajectory of the given rotating speed ω^* can be confirmed.

For each given rotating speed ω^* ($\omega^* < \omega_s^*$), a

stability boundary exists. When the journal rotating speed ω^* is below the stability threshold speed ω_s^* , after the journal is released on an initial point in the stability boundary, the journal center will gradually return to the equilibrium point. The program flow chart in Fig. 2(a) is used to obtain the stability boundary. Further descriptions are

Fig. 2 Program flow chart: (a) program calculating the stability boundary; (b) illustration of Step (vi).

explained as follows.

- (i) Start the program.
- (ii) Preset an equilibrium point $A_0(\varepsilon_s, \varphi_s)$ that is a stable point.
- (iii) Calculate the stability threshold speed ω_s^* of the given equilibrium point.
- (iv) Input rotating speed $\omega^* < \omega_s^*$.
- (v) Divide the clearance circle into 36 parts ($0^\circ, 10^\circ, 20^\circ, 30^\circ, \dots, 350^\circ$). The eccentric ratio on the clearance circle is 1, and all 36 points are unstable.
- (vi) Choose the first point $A_1(0^\circ)$ on the clearance circle and obtain the midpoint of line A_0A_1 , which is indicated by $A_{1,1}$. Then determine if point $A_{1,1}$ is stable. If the point $A_{1,1}$ is stable, replace the previous stable point A_0 with $A_{1,1}$. The new pair of stable and unstable points is $A_{1,1}$ and A_1 . If the point $A_{1,1}$ is unstable, replace the previous unstable point A_1 with $A_{1,1}$. The new pair of stable and unstable points is A_0 and $A_{1,1}$. In the same manner, we discuss the stability of the midpoint of $A_{1,1}A_1$ (or $A_{1,1}A_0$). These steps are repeated until the distance between the stable point and the unstable point $\Delta < 0.001$. Subsequently, a point on the stability boundary is found. This step is illustrated in Fig. 2(b).
- (vii) Change the angle ($10^\circ, 20^\circ, 30^\circ, \dots, 350^\circ$) of the initial unstable point on the clearance circle and repeat Step (vi); the remaining 35 points on the stability boundary can then be obtained.
- (viii) Connect the 36 points; for a given rotating speed ω^* , the stability boundary is obtained.

3 Results and discussion

Based on the above analysis, the stability boundaries and journal center trajectories of the long bearings are presented in Figs. 3 and 4. Different from the results of the short bearings shown in Ref. [2], the stability boundary of the long bearings nearly coincides with the clearance circle. With an increase in the rotating speed (not above the stability threshold speed ω_s^*), the variation of the stability boundary is negligible. When the rotating speed of the journal is below the stability threshold speed, if the journal is released at an arbitrary point in the stability

boundary, it can gradually return to the equilibrium point. The stability of the long bearing is considerably good when the rotating speed of the journal is under the stability threshold speed. The journal center trajectories are also presented in Figs. 3 and 4. When the journal rotating speed is below the stability threshold speed, the journal would gradually return to the equilibrium point when it is in the stability boundary. For the initial location out of the stability boundary, the journal center moves toward the bearing surface, as shown in Fig. 4.

Once the rotating speed of the journal is above the stability threshold speed, for a short bearing,

Fig. 3 The trajectories of journal center and the stability boundaries on different rotating speeds: $\omega^* = 0.5, 0.7$, and 0.9 ($\omega_s^* = 0.98, \varepsilon_s = 0.2$).

Fig. 4 The trajectories of journal center and the stability boundaries with different rotating speeds: $\omega^* = 0.7, 0.9$, and 1.1 ($\omega_s^* = 1.21, \varepsilon_s = 0.5$).

after being released at an initial point, the journal center will gradually move away from the equilibrium point until arriving at the bearing surface, leading to the bearing be damaged. However, for the long bearings, there is a second threshold speed ω_2^* that is higher than the stability threshold speed ω_s^* . When the rotating speed of the journal is below the stability threshold speed ω_s^* , the journal center gradually returns back to the equilibrium point, which is the same as the situation of the short bearings. However, when the journal rotating speed is above the stability threshold speed, not exceeding the second threshold speed ω_2^* , the journal center gradually converges to a contour containing the equilibrium point. After the rotating speed exceeds the second threshold speed ω_2^* , the journal center of the long bearing would gradually get to the bearing surface and lose its stability. The relation between the second threshold speed ω_2^* and the eccentric ratio of the equilibrium point is presented in Fig. 5, which is a comparison of the relation between the stability threshold speed and the eccentric ratio of the equilibrium point. The results of the stability threshold speed agree very well with the results in Refs. [15] and [19]:

For a given rotating speed that is between the stability threshold speed ω_s^* and the second threshold speed ω_2^* , the trajectories of the journal center are shown in Figs. 6 and 7. The polar coordinates of the initial release points in Fig. 6 are $(0.3, \pi/4)$, $(0.9, 4\pi/3)$, and $(0.75, 5\pi/3)$. In Fig. 7, the initial release points are $(0.4, \pi/3)$, $(0.92, 3\pi/4)$, and $(0.8, 11\pi/6)$. When the rotating speed is fixed, regardless whether the journal is released in the

Fig. 5 Relation between the threshold speed and eccentricity ratio of the equilibrium point.

Fig. 6 Journal center trajectories on different initial release points with a given rotating speed $\omega^* = 1.2$ ($\omega_s^* = 0.98$, $\omega_2^* = 1.54$, $\varepsilon_s = 0.2$).

Fig. 7 Journal center trajectories on different initial release points with a given rotating speed $\omega^* = 1.6$ ($\omega_s^* = 1.21$, $\omega_2^* = 2.18$, $\varepsilon_s = 0.5$).

contour or outside the contour, the journal center gradually converges to the same contour containing the equilibrium point.

When the rotating speed of the journal exceeds the stability threshold speed ω_s^* , the trajectories of the journal center are presented in Figs. 8 and 9. The polar coordinates of the equilibrium point in Fig. 8 are $(0.2, 82.6^\circ)$, and the journal rotating speeds are: $\omega^* = 1.0, 1.2, 1.4$, and 1.8 . In Fig. 9, the polar coordinates of the equilibrium point are $(0.5, 69.8^\circ)$ and the journal rotating speeds are: $\omega^* = 1.4, 1.7, 2.0$, and 2.3 . When the rotating speed of the journal is between the stability threshold speed ω_s^* and the second threshold speed ω_2^* (i.e., $\omega^* = 1.0, 1.2$, and 1.4 in Fig. 8 and $\omega^* = 1.4, 1.7$, and 2.0 in Fig. 9), with an increase in the rotating speed, the contour would become increasingly larger. When

Fig. 8 Journal center trajectories of different rotating speeds ($\omega^* = 1.0, 1.2, 1.4$, and 1.8) between the stability threshold speed ($\omega_s^* = 0.98$) and the second threshold speed ($\omega_2^* = 1.54$).

Fig. 9 Journal center trajectories of different rotating speeds ($\omega^* = 1.4, 1.7, 2.0$, and 2.3) between the stability threshold speed ($\omega_s^* = 1.21$) and the second threshold speed ($\omega_2^* = 2.18$).

the rotating speed exceeds the second threshold speed ω_2^* (i.e., $\omega^* = 1.8$ in Fig. 8 and $\omega^* = 2.3$ in Fig. 9), the journal center would gradually reach the bearing surface.

4 Conclusions

Herein, a nonlinear stability study on a long journal bearing is presented. Through the above discussion, the stability boundary of the long journal bearing is observed. Moreover, the journal center trajectories of different operating situations

are provided. Based on the aforementioned analysis, the following conclusions can be obtained:

1) When the journal rotating speed is below the stability threshold speed, the nonlinear stability boundary of the long bearing is nearly the entire clearance circle, which is different from short bearings. This indicates that the stability of the long journal bearing is considerably good.

2) For a long journal bearing, there exists a second threshold speed that is higher than the stability threshold speed. When the journal rotating speed is between the stability threshold speed and the second threshold speed, after the journal center is released at an initial point that is not the equilibrium point, the journal center converges to a contour containing the equilibrium point. When the rotating speed is above the second threshold speed, the journal center gradually reaches the bearing surface.

3) When the journal rotating speed lies between the stability threshold speed and the second threshold speed, regardless of whether the journal is released in the clearance circle (except the equilibrium point), the journal center gradually converges to the same contour.

4) When the journal rotating speed is varied between the stability threshold speed and the second threshold speed, the convergence contour also varies. At a higher rotating speed, the corresponding contour is also larger.

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