A finite element analysis (FEA) approach to simulate the coefficient of friction of a brake system starting from material friction characterization

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Abstract: The coefficient of friction (COF) is one of the most important parameters to evaluate the performance of a brake system. To design proper brake systems, it is important to know the COF when estimating the brake force and resulting torque. It is challenging to simulate the COF since friction in disc brakes is a complex phenomenon that depends on several parameters such as sliding velocity, contact pressure, materials, and temperatures, etc. There is a lack of studies found in the literature focusing on simulation of the COF for a full brake system based on tribometer material characterization. The aim of this work is therefore to investigate the possibility to use a finite element analysis (FEA) approach combined with a COF pv -map to compute the global COF of a disc brake system. The local COF is determined from a pv -map for each local sliding velocity and contact pressure determined by the FEA. Knowing the local COF, the braking force of the entire brake system and the global COF can be evaluated. Results obtained by the simulation are compared with dyno bench test of the same brake system to investigate the validity of the simulation approach. Results show that the simulation is perfectly in line with the experimental measurements in terms of in-stop COF development, but slightly higher with a positive offset for every braking.

Keywords: disc brakes; friction coefficient; simulation; brake performance; pin-on-disc

1 Introduction

The coefficient of friction (COF) is one of the main parameter determining a brake system performance. A brake system is generally composed by a calliper, one or more pistons, two pads, and a rotating disc. When the pressure is applied, the pistons are pushed by the fluid and they push the pads against the disc, generating the braking force. The braking force is strongly dependent of the COF, which is, in turn, strongly affected by the contact situation in terms of local contact pressure and sliding velocity at the pads-to-disc interface [1–7]. The local sliding velocity

is mainly given by the rotating disc rate, while the local pressure conditions are mainly given by how the pads are put in contact with the disc and it could be influenced by the entire calliper and the disc. Therefore, different brake systems can produce different COF with the same disc and pads materials. This difference in friction conditions brings to different performance which have to be evaluated before the brake system is produced. Since it is difficult, in general, studying the contact during braking [8, 9] and in particular impossible to evaluate it experimentally before having the components produced, a simulation tool able to predict it could have a key role in the brake industries.

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To simulate brake systems from a macroscopic point of view, different finite element analyses (FEAs) have been done by several authors. The contact pressure at the pads-to-disc contact interface has been studied and compared with tests by Abukabar and Ouyangb [10] focussing its influence on the wear and brake squeal. Han et al. [11] also considered temperature effects performing a thermo-mechanical analysis to study the influence of contact pressure on wear and they used the analysis inside an optimization loop to design the pad shape in order to have a more uniform contact pressure distribution. The influence of the contact pressure distribution on the wear has been also studied by Söderberg et al. [12] and Valota et al. [13]. They used a FEA to compute the pressure distribution on the pad and then, knowing the disc rotation rate, applied a generalization of the Archard's wear law [14] to compute the pad wear. Wahlström et al. [15] used a similar approach to investigate the wear particles emission from disc brakes, considering the friction coefficient independent by contact pressure and sliding velocity. Wahlström et al. [16, 17] worked on this approach to study friction, wear, and emission from the sliding contact between disc and pads. This work included the friction dependence by the contact pressure at the pad-to-disc interface. Riva et al. [18] extended the approach to include the local pressure and velocity dependency on wear and emissions. They used a FEA to compute the local contact pressure and developed a subroutine to take into account the local pressure and velocity conditions in the wear and emissions computation. In the previous mention studies on macroscopic size-scale, the COF is seen as independent on contact pressure and sliding velocity. There are some studies in which the COF is modelled as a function of pressure, velocity, and/or temperature [19]. In particular, Ostermeyer [20] developed a two-equations dynamic model in which contact pressure, sliding velocity, and temperature are taken into account. Into the model five constant parameter are taken into account to distinguish different chemical compositions of the friction materials and need to be set through experimental tests. Recently, Ricciardi et al. [21] developed a three-equations semi-empirical model based on the work of Ostermeyer. The model is parametrized and tested for three

different brake systems and the results show a good representation of the experiments in terms of COF. In this model every parameter has to be calibrated according to the experimental test. Nobody, known to the authors, considered a characterisation of the local COF dependency on the local contact pressure and sliding velocity typical of a specific couple pad–disc. This characterization results in a friction map of the COF as a function of contact pressure and sliding velocity. Knowing the material characterization, these data are used to simulate the braking force of the whole brake system, aiming to study the brake system performance during the very early design phase. Also, this kind of FEA approach could be used to simulate friction phenomena, such as in-stop increase of COF and in-stop semi-local temperature rise which can be responsible of some negative phenomena as fading of friction material.

The aim of this work, therefore, is to develop a FEA to compute the local contact pressure. Knowing the contact pressure and the sliding velocity from the disc rotation rate, it is possible to compute the COF and the resulting braking torque using a COF p - v -map based on pin-on-disc tribometer (POD) tests [5]. The simulated COF is compared with experimental results obtained by a dyno bench test [18] to validate the simulation approach.

2 Simulation methodology

An overview of the proposed simulation methodology is presented in Fig. 1. First (box-1), pre-process operations consist in choosing the brake system and driving cycle conditions. Thereafter (box-2), tribometer tests are run in order to get the local COF dependence on the nominal contact pressure and sliding velocity. The results of the tribometer tests are then used as input data to a FEA (box-3) to simulate the global COF of the brake system. Inertia dynamometer tests (box-4) are run with the chosen brake system and driving cycle. Box-4 can be performed in parallel with box-2 and box-3. Finally, the results from the FEA and dynamometer tests are compared in the post-processing operations (box-5). The activities of the methodology are explained more in detail in the following sub-sections.

Fig. 1 An overview of the simulation methodology.

2.1 Pin-on-disc tribometer

The materials considered in the simulation and used in the dyno bench test were tested by Wahlström et al. [5] using a POD at different sliding velocities (v) and nominal contact pressures (p) values to obtain a COF pv -map. This map is used by the FEA to have the local COFs. The pin-on-disc experiments are described below.

The components from a real brake system were used to manufacture the pins and discs. The pin samples were made from low-metallic pads and the disc samples were made from a grey cast iron disc [22]. The pin is cylindrical with a diameter of 10 mm while the disc has a diameter of 60 mm. The nominal contact pressures and sliding velocities tested are summarized in Table 1.

All the tests are conducted for two hours to ensure the steady condition has been reached. The COF value in Table 1 corresponds to the one measured when the steady condition is reached. The normal force (F_N) is applied using weights and the nominal contact pressure is then computed dividing this normal force by the pin area. The tangential force (F_T) is measured using a load cell (HBM® Z6FC3/10kg). The COF (μ_{POD}) is

Table 1 Sliding velocities and contact pressures tested with POD.

Velocity (m/s)	Pressure (MPa)	COF
2	0.59	0.48
2	1.14	0.50
1	1.14	0.50
2	0.31	0.52
1	0.59	0.57
4	0.59	0.54
1	0.31	0.58
3	0.31	0.43
3	0.59	0.46
3	0.86	0.50

obtained by dividing the tangential by the normal force

$$\mu_{\text{POD}} = \frac{F_T}{F_N} \quad (1)$$

Every test has been repeated three times and the mean value after running-in between the tests has been considered. The friction map obtained is shown in Fig. 2.

2.2 Disc brake system

The studied brake system is used in a typical C-segment car. The brake system considered is made by a grey cast iron rotor and a floating calliper. The floating calliper consists in two pads with steel backplates and friction material, a piston, a calliper

Fig. 2 Nominal contact pressure (p) and sliding velocity (v) map of the friction coefficient. The p and v used in the pin-on-disc tribometer are marked with circles. The pv -values are represented with dashed isolines.

body, a carrier, and two pins with two bushings where the calliper can slide. The entire brake system is represented in Fig. 3. The disc braking ring has an internal and external radius respectively of 79.5 and 138.7 mm. The pad area is 5,088 mm² and the piston diameter is 28.5 mm.

2.3 Finite element analysis

The FEA has been performed with the commercial software Abaqus [23]. A representation of the meshed components is shown in Fig. 4. A parabolic tetrahedral mesh has been used for the calliper, while linear hexahedral mesh has been used for all the other components. The average size of the mesh is 4 mm. A constraint of zero displacement has been set in correspondence of the fixing points of the carrier. The system pressure p_{sys} is applied to the back of the piston and on the wall of the calliper canalization. The rotation rate is applied to the disc.

Fig. 3 Disc brake system.

Fig. 4 Brake system mesh.

A quasi-static analysis is performed for every braking considered. At every time step of the braking, the FEA is able to compute a contact pressure and a slip rate distribution between pads and disc. The contact pressure distribution is determined by the way the pads are pushed against the disc and it is sensitive to the brake system geometry. The slip rate is proportional to the rotation rate of the disc, so it is higher for higher disc radii, and it decreases time step by time step during the same braking. The contact pressure and the slip rate are computed in every node in contact. Therefore, each node in contact will be characterized by a contact pressure p_{ci} and a slip rate v_i . The friction coefficient map presented in Fig. 2 can be written as a function of contact pressure and slip rate as follow:

$$\mu_{\text{POD}} = f(p, v) \quad (2)$$

where a linear interpolation is considered between the twelve conditioned tested, while the nearest value is considered if a condition outside the map is verified. Evaluating this function for every node of the pads in contact—characterized by p_{ci} and v_i —it is possible to obtain the local friction coefficient μ_i distribution. Knowing the local friction coefficient distribution, the system friction coefficient μ at every time step is obtained dividing the normal braking force by the tangential braking force:

$$\mu = \frac{1}{2p_{\text{sys}}A_p} \sum_{i=1}^N (\mu_i p_{ci} A_{ni}) \quad (3)$$

where p_{sys} is the brake system pressure, A_p is the piston area, μ_i is the local friction coefficient, p_{ci} is the local contact pressure, A_{ni} is the nodal area, and N is the number of pads nodes.

2.4 Braking case

An inertia brake dynamometer [6] has been used to run a Los Angeles City Traffic (LACT) [24] cycle. Ten brake events of the LACT cycle have been chosen to be simulated with the FEA procedure. The system pressure and car velocity during braking have been considered linear and are shown in Table 2. The velocity values reported in Table 2 represent the vehicle velocity, while the pressure values represent the fluid pressure inside the brake system.

Table 2 Brake events: Vehicle velocity and brake system pressure.

Braking number	Initial velocity (kph)	Final velocity (kph)	Initial pressure (bar)	Final pressure (bar)
1	25.87	10.68	8.10	6.10
2	24.46	6.09	7.30	6.10
3	21.63	6.44	11.50	10.00
4	26.40	6.18	10.70	8.80
5	25.52	8.39	8.10	6.90
6	22.78	6.00	8.20	7.40
7	24.11	5.38	6.80	6.10
8	27.02	7.59	6.80	5.60
9	16.16	5.74	7.00	6.30
10	21.46	7.06	11.30	10.00
11	24.73	6.53	8.50	6.80
12	26.93	6.00	9.90	7.70

The braking in Table 2 is chosen to have the nominal contact pressure and the sliding velocity at the effective radius inside the friction map in Fig. 5, with a tolerance of 5% for the final velocity. This is done to consider only the area where the friction coefficient value has been investigated in the POD tests. Considering these restrictions, the chosen braking all results in the lower-left part of the friction map, which means low pressure and velocity. Note that choosing low pressure and velocity braking is consistent also with the neglecting of temperature of this simulation approach. In fact, the lower the pressure

Fig. 5 Simulated braking checked to be inside the friction map with a tolerance of 5%. Every line in the figure represents a single brake event.

and the velocity are, the lower is the heat generation, and the lower is the temperature increase. The temperature topic will be treated more in detail in the discussion section.

3 Results

The contact pressure computed with FEA analysis is shown in Fig. 6 for braking #2 and braking #11 in Table 2, for the piston and finger side pads. The condition represented corresponds to the end of the braking. The contact pressure has a very similar map for both the braking. The pad on the piston side shows a higher pressure toward the disc inner side, which is also a higher for higher radii. The result is a transversal gradient for low radius-outer side to high radius-inner side. The finger pad has a contact pressure distribution with a gradient from low to high radii not much influenced by the disc sliding direction. The slip rate is shown in Fig. 7. As evident in the picture the slip rate is a mainly affected by the disc sliding velocity and is higher for higher radii in all the distributions analysed.

The COF generated by the piston and the finger side pads during all the chosen braking in Table 2 is represented in Fig. 8. The resulting COF is very similar for both the pad, which means that for the braking analysed the different contact pressure in Fig. 6 does not significantly influence the global friction coefficient generated. The finger side pad results with a slightly lower friction coefficient almost in every braking and during all the single braking time.

Fig. 6 Contact pressure of braking #2 and #11 in Table 2. Arrows show the disc sliding direction.

Fig. 7 Slip rate of braking #2 and #11 in Table 2. Arrows show the disc sliding direction.

Fig. 8 Internal and external side friction coefficient.

Simulated COF has been compared to the experimental one measured during dyno bench tests. The comparison is shown in Fig. 9. The simulated COF is always higher than the experimental one in terms of absolute value, with an offset varying braking by braking between 0.03 in braking #2 and 0.1 in braking #3, 4, 5, and 6. The simulated trend inside every braking, instead, seems to be very representative of the experiments. The comparison removing the offset has been done in Fig. 10.

Fig. 9 COF time history during the selected braking. Experimental COF in black, FEA COF in red.

Fig. 10 COF time history with offset removed from simulated curve. Experimental COF in black, FEA COF in red.

4 Discussion

The aim of the presented approach is to propose a simulation methodology which could be applied to investigate and predict a brake system friction performance for the considered friction material contact pair by using a p - v -map of the local COF. This can be useful during the design phase of a new brake system, when a prototype is not yet available and a specific friction material has to be chosen to better define the system requirements and friction performance. Moreover, this approach can be used to investigate phenomena which are difficult to study using experimental tests such as in-stop COF increase and semi-local contact temperature increase.

Twelve different stops coming from a city-traffic cycle (LACT) have been simulated and the results have been compared with the correspondent experimental measurements. For two brake events the contact pressure and the slip rate have been shown. It is possible to see in Fig. 6 that the contact pressure distribution is different on the piston side compared to the finger side for both brake events presented. This is explained by that the braking pressure of the analysed brake events is relatively low and due to the nature of the floating callipers. The piston pushes just one pad on the disc while the other is pushed by reaction of the floating body. By looking at the contact pressure distribution it seems that the brake torque generated by the rotating disc has an effect only on the piston-side pad that has a gradient in the tangential direction from the outer to the inner side. The slip rate (Fig. 7) is directly related to the radius. This seems to be obvious since the disc is in rotation, but it is

also a signal that there are no transversal movements of the pads which can usually be more typical at higher pressure brake events. The COF simulated for the piston and the finger side (Fig. 8) seem to be very similar. Moreover, the slip rate is the same for both the pads, the contact pressure, as discussed above, has a different distribution which does not seem to affect the global COF.

The correlation between experimental and simulated results (Figs. 9 and 10) shows that the absolute value of the COF simulated is always slightly higher and that the trend during every brake event is well represented by the simulation. This is important to be able to study phenomena such as the in-stop increase of the COF. It remains to better investigate the simulation offset compared to the experiments. A hypothesis is that the measurement on the POD, where the input COF map is created, is different compared to the measurement at the dyno bench, where the experimental results come from.

Inside every single brake event it is clear that a rise of the COF occurs. This is in line with what has been numerically simulated by Wahlström [17] where in the first part of the braking there is an increase of the COF, while it stays constant after a run in phase. In Ref. [17] the braking considered are more powerful compared to the brake events analysed in this present work and therefore the run in phase is shorter and they reach a situation similar to a steady state. Considering lower power braking, instead, the run in time is increased. Looking at the experimental results presented in Ref. [17], after the run in phase the COF is not constant, but it has some decreasing phases that differs between brake events. This can be explained by the effect of contact temperature which influences the material properties and decreases the performance of the friction material by causing some local fading effects. In this work the decreasing of the in-stop COF is not visible neither in the simulation nor in the experimental tests. Note that considering the friction map in Fig. 5 the twelve braking have been chosen in the lower-left side of the map, which means low pressure and low velocity, and, therefore, low braking power. Though, at low power and energy braking it seems reasonable to neglect the temperature. For higher power and energy braking, the number of experiments in the POD can be increased and a 3 dimensional (3D) map including

temperature as a variable could be implemented. It has to be considered that there are phenomena, especially in the noise-vibration-harshness (NVH) field, where the contact pressure and the sliding velocity are much lower. In these phenomena the temperature does not play a primary role, therefore it is also more consistent to neglect it and apply the approach directly as it is. Ostermeyer [19] considered this fading effects introducing a two-equations dynamic model where the second equation is focused on temperature effects. The tested case is a sort of constant velocity braking where a real variation of velocity is not considered which results in a COF as a function of contact pressure and temperature. The five constant parameters included in the model have been identified matching the experimental measurements with the simulation results. The difference between the two approaches is that the one presented is focused on evaluating the performance of a new system by only knowing the friction material. In Ref. [19], instead, a model to find a law for the friction coefficient has been formulated including some constant parameters which have to be set case by case.

At the beginning of a braking most of the contact is carried by the metal fibres [21] which are inside the friction material mixture. During braking pads and disc wear, and the worn material can fill the gaps in the disc and pad surface generating secondary plateaus [8]. This secondary plateaus contribute to increase the real contact area and increase the friction coefficient during a braking. Looking at the meso-scales, every portion of the pad is made by different materials, therefore different properties. These different properties, combined with the different local values of pressure and velocity, generate different contact conditions and in particular different friction coefficient values [5]. Higher friction coefficient brings to higher power and higher temperature. At the same time, lower thermal diffusivity contributes to lower dissipation of the heat generated and then higher temperature. These higher temperature areas could generate a strong decrease of friction performance and then fading phenomena. It would be interesting to study the possibility to use the proposed simulation methodology to investigate these phenomena if the *pv*-map is expanded for conditions when fading could occur.

In the last paragraph we mentioned the gaps in the

disc and pad surfaces which are responsible of the roughness of the material, and the wear which modifies this surface roughness. The roughness influences phenomena that act at the meso-scales, as investigated by Riva et al. [25]. The focus of this work is on the macro-scales where the overall brake system acts, therefore this is not explicitly developed in the simulation methodology. However, this is implicitly taken into account in the POD friction map. In fact, during the POD experiments, the pin and the disc tested have a specific roughness, are subjected to a specific wear, and have a specific real contact area, which is different compared with the nominal pin area used to compute the contact pressure reported on the y-axis of Fig. 5. During the FEA local conditions are considered for every node in contact. This local condition is characteristic of a nodal area that is on the order of the pin area used in the POD test. This means that we can use the nominal contact pressure computed in that specific node to query the friction map and all the roughness and contact area effects at the smaller scales are included in that specific condition tested. Instead, what it is explicitly computed in the simulation is the real macro-contact area determined by the contact pressure distributions in the FEA. Since the piston and the floating calliper push the pads in a non-uniform way, the pad-to-disc contact will produce a non-uniform pressure distribution which is computed in the analysis. Also, this pressure distribution in some cases can avoid to some nodes of the pad to be in contact with the disc, and reducing then the real contact area. The macroscopic wear of the pad can also affect the contact pressure distribution and therefore the local friction coefficient. However, the macroscopic wear of pads and disc acts at different time scales compared to the one studied here. In brake disc applications wear can be usually considered important after a series of high-energy braking or even after some braking cycles. In this study only twelve braking have been investigated and the wear as first approximation can be neglected to reduce the computational time.

To summarize, the results from the present study seem promising. The simulation methodology developed can be useful in the early design phase of a brake system when there are no components to

test, but the friction material is already known. This gives the possibility to avoid several iterations after prototyping. Feature developments are needed to test/simulate other brake systems using the same friction material and new friction materials. In fact, other POD friction maps with other friction materials can be developed to build a material database. Moreover, it could be interesting to develop a 3D map of the COF including local temperature effects and implement the temperature variable into the simulation, being able to investigate higher power brake events and possible fading effects. This approach also gives the possibility to study how design changes of the calliper impact the global COF. That is, how should the calliper be designed in order to have a pressure distribution that results in a COF that one wants.

5 Conclusions

A simulation methodology based on a FEA has been developed. The methodology consists of computing the global COF of an entire brake system starting from the local COF dependence of contact pressure and sliding velocity. By analysing brake events from a city traffic cycle and comparing the simulation and experimental results, the following conclusions can be done:

- 1) For the studied brake system and brake events there is no significant difference between the COF generated by the piston and the finger side pads;
- 2) Experimental and simulation results are qualitatively in line especially in the in-stop behaviour, while the simulated results show a positive offset in the COF compared to the experimental ones;
- 3) No decreasing of COF has been seen neither in experimental nor in simulated results, possibly explained by the low power braking considered which allow to neglect the thermal effects.

Further studies with different brake systems have to be done to fully validate the model and other friction materials can be tested to generate a friction *p-v*-map database.

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