Electroadhesion between a flat touchscreen and the human finger with randomly self-affine fractal surface

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Abstract: In this study, the effects of finger roughness on the electrostatic potential, electrostatic field, and average effective squeezing pressure between a human finger and a touchscreen are calculated by the perturbation method. This theory is an extension of an earlier work by Persson. It is found that an additional potential $<\phi^{(2)}>$ will appear between the solids when the roughness effect is considered in calculating the perturbation potential. This additional potential is still proportional to the distance $\bar{u}$ from the bottom surface. Therefore, the effect of the roughness increases the effective potential $<\phi>$ between the two solids. As a result, the average electrostatic field and average effective squeezing pressure increase. Using the increased effective squeezing pressure, we obtain the contact area, average surface separation, and friction between a human finger and the surface of a touchscreen. The effect of the roughness of the finger skin on the increased average effective squeezing pressure (electroadhesion) increases the contact area and reduces the average surface separation between the finger skin and touchscreen. Therefore, the finger-touchscreen friction increases. The surface topography for the forefinger skin is also measured by atomic force microscopy to obtain more realistic results. The auto spectral density function for the forefinger skin surface is calculated as well.

Keywords: friction; touchscreen; perturbation method

1 Introduction

The first touchscreen was designed in 1960, and touchscreens are now ubiquitous in daily life. The use of touchscreens is not limited to smartphones and tablets; they are used in many portable computers, smart watches, game consoles, and ATMs, among others. Capacitive touchscreens are currently the most popular type of touchscreens due to their multi-touch technology.

A capacitive touchscreen panel consists of a glass substrate coated with a conductive layer such as indium tin oxide (ITO), and an insulating layer such as silicon dioxide (SiO₂). When an alternating voltage is applied to the conductive layer of a capacitive touchscreen, touching its surface generates an attractive

force between the finger (as the human body is an electrical conductor) and the touchscreen surface. This force is caused by the induction of charges with opposite signs on the insulating layer of the glass substrate and the finger. This electrostatic attraction force is called “electroadhesion” and increases the finger-touchscreen contact area and the sliding friction force [1–4]. Jonhnsen and Rahbek observed the attraction force between human skin and a charged surface [5], while Mallinckrodt et al. accidentally discovered that dragging a dry finger over a conductive surface covered with a thin insulating layer and in the presence of the alternating voltage can increase the friction during touch [6], and explained this phenomenon using a parallel-plate capacitor based model. Later, this phenomenon was termed “electrically induced

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vibrations” by Grimnes [7] who examined the effect of the roughness and moisture of the finger on the electrically induced vibrations. Recently, Persson derived a mathematical relation between the adhesion (and friction) force and the applied voltage for the capacitive touchscreen [8–10]. Based on the small slope approximation ($|\nabla u| \ll 1$), he only considered the influence of the surface roughness of the stratum corneum of the skin on the interfacial separation $u(x, y)$, while the roughness effect should also be considered for the potential and the electric field [11, 12]. Persson used $\nabla^2 \phi \approx \partial^2 \phi(x, y, z) / \partial z^2 \approx 0$ to calculate the effect of the applied voltage on the electroadhesion attraction force [8]. In the present work, the effect of the finger skin surface roughness on the electric potential, electric force, electroadhesion attraction force, and friction force between the skin of the finger and touchscreen will be calculated by the perturbation theory.

The rest of this paper is organized as follows: in Section 2, we introduce the model for considering the effects of the finger skin roughness. In Section 3, the integrals are solved numerically and the results are presented. Section 4 presents our concluding remarks.

2 Theory: Perturbation theory for electro-adhesion between a flat touchscreen and a rough finger

Figure 1 schematically shows a finger above a flat touchscreen. The ITO layer of the touchscreen and the finger are conductive, and insulating layers with the thicknesses d_1 (SiO_2 layer of the touchscreen) and d_2 (stratum corneum of the skin) and dielectric constants ε_1 and ε_2 , respectively, are present. When we press the finger against the touchscreen with a normal force $F_0 = P_0 A_0$, the finger makes partial contact with the touchscreen surface on the atomic scale. Therefore, there are many non-contact areas between the finger and the touchscreen surface that are filled with air with $\varepsilon_{\text{air}} = 1.00059 \approx 1$. The air gap between the two solids has a uniform thickness $u(x, y)$, and $u(x, y)$ is the interfacial separation between the solids [13–15]. In this study, we consider the simplest possible approximation for $u(x, y)$ and assume that the air gap between the two solids has a uniform

thickness $u(x, y) \approx \bar{u}$, where $\bar{u}$ is the average interfacial separation. As observed from Fig. 1, an electric potential $\phi = V$ is applied between the two electrically conducting layers of the finger and the flat touchscreen. The electric potential increases the electric field in the gap between the solids and therefore, an attractive force between the solids appears. $h_1(x, y)$ and $h_2(x, y)$ are the height fluctuations of the inner and outer surfaces of the stratum corneum (SC) layer after contact, respectively. As stated in Section 1, we now consider the roughness effects in the electric potential and the electric field, and then calculate the attractive force between the solids.

First, we must solve the Laplace equation to determine the electric potential and the electric field in the three areas shown in Fig. 1.

$$\nabla^2 \phi = 0 \quad (1)$$

which obeys the boundary conditions:

$$\phi(x, y, z = f_1(x, y)) = 0 \quad (2)$$

and

$$\phi(x, y, z = 0) = V \quad (3)$$

Here, ϕ is the electrostatic potential and $z = f_1(x, y) = d_1 + d_2 + \bar{u} + \gamma h_1(x, y)$ is the inner rough surface of SC, while at the other boundary, $h_2(x, y)$, $z = d_1 + \bar{u} + \gamma h_2(x, y)$. If we assume that $\gamma \ll 1$, we can apply a

Fig. 1 A schematic of a finger above a flat touchscreen.

perturbation method for the potential on the rough boundaries to solve the Laplace equation. Thus, the boundary condition (2) and the potential in the boundary $z = f_2(x, y) = d_1 + \bar{u} + \gamma h_2(x, y)$ can be expanded as the following Taylor series:

$$\phi_1(x, y, d_1 + d_2 + \bar{u}) + \gamma h_1(x, y) \phi_{1z}(x, y, d_1 + d_2 + \bar{u}) + \frac{1}{2} \gamma^2 h_1^2(x, y) \phi_{1zz}(x, y, d_1 + d_2 + \bar{u}) + \dots = 0 \quad (4)$$

and

$$\phi(x, y, d_1 + \bar{u} + h_2(x, y)) = \phi(x, y, d_1) + \gamma h_2(x, y) \phi_z(x, y, d_1) + \frac{1}{2} \gamma^2 h_2^2(x, y) \phi_{zz}(x, y, d_1) + \dots \quad (5)$$

where

$$\phi_z(x, y, z) = \frac{\partial \phi(x, y, z)}{\partial z}, \quad \phi_{zz}(x, y, z) = \frac{\partial^2 \phi(x, y, z)}{\partial z^2}$$

The perturbed potential can be written as:

$$\phi(x, y, z) = \phi^{(0)}(x, y, z) + \gamma \phi^{(1)}(x, y, z) + \gamma^2 \phi^{(2)}(x, y, z) + \dots \quad (6)$$

Substituting Eq. (6) into the Laplace Eq. (1), we find that for all orders of perturbation, $\phi^{(n)}(x, y, z)$ satisfies the Laplace equation

$$\nabla^2 \phi^{(n)}(x, y, z) = 0 \quad (7)$$

The electric potential and the normal component of the electric field must be continuous at $z = d_1$ and $z = d_1 + \bar{u}$. Therefore, we obtain:

$$\phi_2(d_1 + 0^+) = \phi_3(d_1 - 0^+) \Big|_{z=d_1} \quad (8)$$

$$\phi_1(d_1 + \bar{u} + 0^+) = \phi_2(d_1 + \bar{u} - 0^+) \Big|_{z=d_1 + \bar{u}} \quad (9)$$

$$\varepsilon_0 E_{2\hat{n}_1}(d_1 + 0^+) = \varepsilon_1 E_{3\hat{n}_1}(d_1 - 0^+) \Big|_{z=d_1} \quad (10)$$

$$\varepsilon_2 E_{1\hat{n}_2}(d_1 + \bar{u} + 0^+) = \varepsilon_0 E_{2\hat{n}_2}(d_1 + \bar{u} - 0^+) \Big|_{z=d_1 + \bar{u}} \quad (11)$$

where

$$\hat{n}_1 = \hat{z}, \quad \hat{n}_2 = \frac{\hat{z} - \gamma \nabla h}{\sqrt{1 + \gamma^2 (\nabla h)^2}}$$

and

$$E_{\hat{n}} = -\hat{n} \cdot \nabla \phi$$

Substituting Eq. (6) into Eq. (4), and also substituting Eq. (5) into Eqs. (7)–(10), and then considering the perturbed expansion of the potential (6), various orders of perturbation potential boundary conditions will be obtained. The zeroth-order of boundary conditions are given by:

$$\phi_3^{(0)}(x, y, z) = V \Big|_{z=0} \quad (12)$$

$$\phi_1^{(0)}(x, y, z) = 0 \Big|_{z=d_1 + d_2 + \bar{u}} \quad (13)$$

$$\phi_3^{(0)}(x, y, z) = \phi_2^{(0)}(x, y, z) \Big|_{z=d_1} \quad (14)$$

$$\phi_2^{(0)}(x, y, z) = \phi_1^{(0)}(x, y, z) \Big|_{z=d_1 + \bar{u}} \quad (15)$$

$$\varepsilon_1 \phi_{3z}^{(0)}(x, y, z) = \varepsilon_0 \phi_{2z}^{(0)}(x, y, z) \Big|_{z=0} \quad (16)$$

$$\varepsilon_0 \phi_{2z}^{(0)}(x, y, z) = \varepsilon_2 \phi_{1z}^{(0)}(x, y, z) \Big|_{z=d_1 + \bar{u}} \quad (17)$$

By calculating the boundary conditions at zeroth order, the zeroth-order potential is obtained as:

$$\begin{aligned} \phi_1^{(0)} &= \frac{-V(z - \bar{u} - d_1 - d_2)}{\varepsilon_2 \left(\bar{u} + \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)}, & d_1 + \bar{u} < z < d_1 + d_2 + \bar{u}; \\ \phi_2^{(0)} &= V \varepsilon_1 \left(\bar{u} + \frac{d_2}{\varepsilon_2} \right) - \frac{V(z - d_1)}{\left(\bar{u} + \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)}, & d_1 < z < d_1 + \bar{u}; \\ \phi_3^{(0)} &= V - \frac{Vz}{\varepsilon_1 \left(\bar{u} + \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)}, & 0 < z < d_1 \end{aligned} \quad (18)$$

where h and ϕ are used instead of $h(x, y)$ and $\phi(x, y, z)$ in order to simplify the notation.

The first-order boundary conditions are given by:

$$\phi_3^{(1)} = 0 \Big|_{z=0} \quad (19)$$

$$\phi_1^{(1)} = -h_1 \phi_{1z}^{(0)} \Big|_{z=d_1 + d_2 + \bar{u}} \quad (20)$$

$$\phi_3^{(1)} = \phi_2^{(1)} \Big|_{z=d_1} \quad (21)$$

$$\varepsilon_1 \phi_{3z}^{(1)} = \varepsilon_0 \phi_{2z}^{(1)} \Big|_{z=d_1} \quad (22)$$

$$\phi_1^{(1)} + h_2 \phi_{1z}^{(0)} = \phi_2^{(1)} + h_2 \phi_{2z}^{(0)} \Big|_{z=d_1 + \bar{u}} \quad (23)$$

$$\begin{aligned} \varepsilon_2 \phi_{1z}^{(1)} + \varepsilon_2 h_2 \phi_{1zz}^{(0)} - \varepsilon_2 \nabla h_2 \cdot \nabla \phi_1^{(0)} &= \varepsilon_0 \phi_{2z}^{(1)} + \\ \varepsilon_0 h_2 \phi_{2zz}^{(0)} - \varepsilon_0 \nabla h_2 \cdot \nabla \phi_2^{(0)} &\Big|_{z=d_1 + \bar{u}} \end{aligned} \quad (24)$$

The second-orders boundary conditions are:

$$\phi_3^{(2)} = 0 \Big|_{z=0} \quad (25)$$

$$\phi_1^{(2)} = -h_1 \phi_{1z}^{(1)} - \frac{h_1^2}{2} \phi_{1zz}^{(0)} \Big|_{z=d_1+d_2+\bar{u}} \quad (26)$$

$$\phi_3^{(2)} = \phi_2^{(2)} \Big|_{z=d_1} \quad (27)$$

$$\varepsilon_1 \phi_{3z}^{(2)} = \varepsilon_0 \phi_{2z}^{(2)} = 0 \Big|_{z=d_1} \quad (28)$$

$$\phi_1^{(2)} + h_2 \phi_{1z}^{(1)} + \frac{1}{2} h_2^2 \phi_{1zz}^{(0)} = \phi_2^{(2)} + h_2 \phi_{2z}^{(1)} + \frac{1}{2} h_2^2 \phi_{2zz}^{(0)} \Big|_{z=d_1+\bar{u}} \quad (29)$$

$$\begin{aligned} & \varepsilon_2 \phi_{1z}^{(2)} + \varepsilon_2 h_2 \phi_{1zz}^{(1)} + \frac{1}{2} \varepsilon_2 h_2^2 \phi_{1zzz}^{(0)} - \varepsilon_2 \nabla h_2 \cdot \nabla \phi_1^{(1)} - \\ & \varepsilon_2 (\nabla h_2 \cdot \nabla h_2) \phi_{1z}^{(0)} - \varepsilon_2 h_2 (\nabla h_2 \cdot \nabla \phi_{1z}^{(0)}) - \\ & \frac{1}{2} \varepsilon_2 \nabla^2 h_2 \phi_{1z}^{(0)} = \varepsilon_0 \phi_{2z}^{(2)} + \varepsilon_0 h_2 \phi_{2zz}^{(1)} + \frac{1}{2} \varepsilon_0 h_2^2 \phi_{2zzz}^{(0)} - \\ & \varepsilon_0 \nabla h_2 \cdot \nabla \phi_2^{(1)} - \varepsilon_0 (\nabla h_2 \cdot \nabla h_2) \phi_{2z}^{(0)} - \\ & \varepsilon_0 h_2 (\nabla h_2 \cdot \nabla \phi_{2z}^{(0)}) - \frac{1}{2} \varepsilon_0 \nabla^2 h_2 \phi_{2z}^{(0)} \Big|_{z=d_1+\bar{u}} \end{aligned} \quad (30)$$

We use the Fourier transformation technique to solve the higher-order Laplace perturbation equation.

$$\nabla^2 \phi^{(n)}(x, y, z) = \frac{\partial^2}{\partial z^2} \phi^{(n)}(x, y, z) + \nabla_\rho^2 \phi^{(n)}(x, y, z) = 0 \quad (31)$$

where $\rho = (x, y)$ is the position vector in the $(x-y)$ plane. We obtain the Laplacian equation of the Fourier transform in the $(x-y)$ plane as

$$\frac{\partial^2}{\partial z^2} \tilde{\phi}^{(n)}(q, z) - q^2 \tilde{\phi}^{(n)}(q, z) = 0 \quad (32)$$

where

$$\tilde{\phi}^{(n)}(q, z) = \frac{1}{(2\pi)^2} \int d^2 \rho e^{iq \cdot \rho} \phi^{(n)}(\rho, z) \quad (33)$$

$$\phi^{(n)}(\rho, z) = \int d^2 q e^{-iq \cdot \rho} \tilde{\phi}^{(n)}(q, z) \quad (34)$$

The general solutions of Eq. (32) are in the form of

$$\begin{aligned} \tilde{\phi}_1^{(n)}(q, z) &= A^{(n)}(q) e^{q(z-d_1-d_2-\bar{u})} + \\ & B^{(n)}(q) e^{-q(z-d_1-d_2-\bar{u})}, \quad d_1 + \bar{u} < z < d_1 + d_2 + \bar{u}; \\ \tilde{\phi}_2^{(n)}(q, z) &= C^{(n)}(q) e^{q(z-d_1)} + \\ & D^{(n)}(q) e^{-q(z-d_1)}, \quad d_1 < z < d_1 + \bar{u}; \\ \tilde{\phi}_3^{(n)}(q, z) &= E^{(n)}(q) e^{qz} + \\ & F^{(n)}(q) e^{-qz}, \quad 0 < z < d_1 \end{aligned} \quad (35)$$

Applying the boundary conditions at $z=0$, $z=d_1$, and $z=d_1+d_2+\bar{u}$, we respectively obtain:

$$\begin{aligned} E^{(n)}(q) &= -F^{(n)}(q), \\ \tilde{\phi}_3^{(n)}(q, z) &= 2 E^{(n)}(q) \sinh(qz) \end{aligned} \quad (36)$$

$$E^{(n)}(q) = \frac{C^{(n)}(q) + D^{(n)}(q)}{2 \sinh(qd_1)},$$

$$D^{(n)}(q) = \frac{(\tanh(qd_1) - \varepsilon_1)}{(\tanh(qd_1) + \varepsilon_1)} C^{(n)}(q),$$

$$\tilde{\phi}_2^{(n)}(q, z) = C^{(n)}(q) \left(e^{q(z-d_1)} + \frac{(\tanh(qd_1) - \varepsilon_1)}{(\tanh(qd_1) + \varepsilon_1)} e^{-q(z-d_1)} \right) \quad (37)$$

$$B^{(1)}(q) = \frac{V \tilde{h}_1(q)}{\varepsilon_2 \left(\bar{u} + \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)} - A^{(1)}(q),$$

$$\begin{aligned} B^{(2)}(q) &= \int d^2 q' q' \tilde{h}_1(q-q') \left(\frac{V \tilde{h}_1(q')}{\varepsilon_2 \left(\bar{u} + \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)} - 2 A^{(1)}(q') \right) - \\ & A^{(2)}(q) \end{aligned} \quad (38)$$

where

$$\tilde{h}_{1,2}(q) = \frac{1}{(2\pi)^2} \int d^2 \rho e^{iq \cdot \rho} h_{1,2}(\rho) \quad (39)$$

The $A^{(n)}(q)$ and $C^{(n)}(q)$ can be determined by the boundary conditions at $z=d_1+\bar{u}$. The attractive force in the air gap between the solids can be obtained from the zz component of the Maxwell stress tensor as:

$$\sigma_{zz} = \frac{\varepsilon_0}{2} (E_z^2 - E_{\parallel}^2) \quad (40)$$

where ε_0 is the dielectric constant of vacuum. The perturbed form of the zz component of the Maxwell stress tensor can be written as:

$$\sigma_{zz} \approx \sigma_{zz}^{(0)} + \sigma_{zz}^{(1)} + \sigma_{zz}^{(2)} + \dots \quad (41)$$

then, the averaged normal stress over the surface roughness can be calculated as:

$$\langle \sigma_{zz}^{(0)} \rangle = \frac{\varepsilon_0}{2} \frac{V^2}{\left(\bar{u} + \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)^2} \quad (42)$$

The first-order averaged normal stress is zero, because its expression contains $\langle h(x; y) \rangle = 0$, as shown below.

$$\langle \sigma_{zz}^{(1)} \rangle = 0 \quad (43)$$

$$\begin{aligned} \langle \sigma_{zz}^{(2)} \rangle = & \frac{\epsilon_0 V^2}{2\pi\epsilon_2 \left(\bar{u} + \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)^3} \int d^2 q' q' C_1(q') \times \\ & \left[G(q') e^{q'd_2} (1 - \tanh(q'd_2)) \left(e^{q'\bar{u}} - \frac{(\tanh(q'd_1) - \epsilon_1)}{(\tanh(q'd_1) + \epsilon_1)} e^{-q'\bar{u}} \right) + \right. \\ & \left. \frac{e^{q'd_2}}{\cosh(q'd_2)} - 1 \right] + \\ & \frac{\epsilon_0 (\epsilon_2 - 1)^2 V^2}{2\pi\epsilon_2 \left(\bar{u} + \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)^3} \int d^2 q' q' C_2(q') G(q') \cosh(q'd_2) \times \\ & \left(e^{q'\bar{u}} - \frac{(\tanh(q'd_1) - \epsilon_1)}{(\tanh(q'd_1) + \epsilon_1)} e^{-q'\bar{u}} \right) - \\ & \frac{\epsilon_0 V^2}{\pi \left(\bar{u} + \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)^2} \int d^2 q' q'^2 G(q')^2 \frac{(\tanh(q'd_1) - \epsilon_1)}{(\tanh(q'd_1) + \epsilon_1)} \times \\ & [C_1(q') e^{2q'd_2} (\cosh(q'd_2) - \sinh(q'd_2))^2 + \\ & C_2(q') (\epsilon_2 - 1)^2 \cosh^2(q'd_2)] \end{aligned} \quad (44)$$

where

$$\begin{aligned} G(q) = & \frac{e^{-q\bar{u}} (\tanh(qd_1) + \epsilon_1)}{[(\epsilon_2 \cosh(qd_2) + \sinh(qd_2))(\epsilon_1 + \tanh(qd_1)) + \\ & e^{-2q\bar{u}} (\epsilon_2 \cosh(qd_2) - \sinh(qd_2))(\tanh(qd_1) - \epsilon_1)]} \end{aligned} \quad (45)$$

$C_{1,2}(q)$ are the auto spectral density functions of the finger surface and are obtained from the following equation [16]:

$$\langle \tilde{h}(q) \tilde{h}(q') \rangle = \frac{1}{2\pi} \delta(q + q') C(q) \quad (46)$$

For homogeneous and isotropic surfaces, $C(q)$ is a real function that depends only on $q = |q|$, so that we obtain:

$$\langle \tilde{h}(q) \tilde{h}(-q) \rangle = \frac{A_0}{(2\pi)^3} C(q) \quad (47)$$

where A_0 is the nominal area of the rough surface. When a finger is squeezed against the touchscreen (with a force $F_0 = p_0 A_0$) in the presence of an electric potential between the two surfaces, the effective squeezing pressure will be given by:

$$p = p_0 + p_a, \quad p_a = \langle \sigma_{zz} \rangle \quad (48)$$

To consider the simplest possible approximation for solving p_a , we assume that first, the air gap between the two solids has a uniform thickness $\bar{u}$, and second, the dielectric functions of the two solids are real [17, 18]. Therefore, we obtain

$$\begin{aligned} p = p_0 + & \frac{\epsilon_0 V^2}{2 \left(\bar{u} + \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)^2} + \frac{\epsilon_0 V^2}{2\pi\epsilon_2 \left(\bar{u} + \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)^3} \int d^2 q' q' C_1(q') \times \\ & \left[G(q') e^{q'd_2} (1 - \tanh(q'd_2)) \left(e^{q'\bar{u}} - \frac{(\tanh(q'd_1) - \epsilon_1)}{(\tanh(q'd_1) + \epsilon_1)} e^{-q'\bar{u}} \right) + \right. \\ & \left. \frac{e^{q'd_2}}{\cosh(q'd_2)} - 1 \right] + \\ & \frac{\epsilon_0 (\epsilon_2 - 1)^2 V^2}{2\pi\epsilon_2 \left(\bar{u} + \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)^3} \int d^2 q' q' C_2(q') G(q') \cosh(q'd_2) \times \\ & \left(e^{q'\bar{u}} - \frac{(\tanh(q'd_1) - \epsilon_1)}{(\tanh(q'd_1) + \epsilon_1)} e^{-q'\bar{u}} \right) - \\ & \frac{\epsilon_0 V^2}{\pi \left(\bar{u} + \frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}\right)^2} \int d^2 q' q'^2 G(q')^2 \frac{(\tanh(q'd_1) - \epsilon_1)}{(\tanh(q'd_1) + \epsilon_1)} \times \\ & [C_1(q') e^{2q'd_2} (\cosh(q'd_2) - \sinh(q'd_2))^2 + \\ & C_2(q') (\epsilon_2 - 1)^2 \cosh^2(q'd_2)] \end{aligned} \quad (49)$$

For small applied pressures, the relationship between the average interfacial separation $\bar{u}$ and the small applied normal squeezing pressure p [13–15] is described by

$$p \approx \beta E^* \exp\left(-\frac{\bar{u}}{u_0}\right) \quad (50)$$

$$\bar{u} \approx u_0 \log\left(\frac{\beta E^*}{p}\right) \quad (51)$$

where $u_0 = 0.5 h_{\text{rms}}$, and h_{rms} is the root-mean-square (rms) roughness of the SC layer of the finger surface. Equation (49) shows that the surface roughness gives rise to an additional potential $\langle \phi^{(2)} \rangle$ between the solids, and this additional potential is still proportional to the distance $\bar{u}$ from the bottom surface. Therefore, the effect of the roughness increases the effective potential $\langle \phi \rangle$ between the two solids. As a result, the average electrostatic field and the average effective squeezing pressure increase.

Denoting the actual (microscopic) and the nominal (macroscopic) contact areas by A and A_0 , the relative contact area for a homogeneous elastic solid is given by [20]

$$\frac{A}{A_0} \approx \frac{\pi}{2} \operatorname{erf} \left(\frac{2p}{h' E^*} \right) \quad (52)$$

where $E^* = E/(1-\nu^2)$, and E and ν are the elastic modulus and the Poisson's ratio of the elastic solid, respectively, and h' is the root mean square (rms) slope of the rough surface (SC layer of finger). After calculating the relative contact area, we can obtain the friction force between the finger and the touchscreen.

Several studies have shown that the friction force in the solid-solid nanocontacts below the wear threshold is proportional to the real contact area, i.e., the number of interfacial atoms [21–25]. This is demonstrated by the experimental measurements of both the friction and the nano-scale contact area. In other words, the friction force F_f is given by:

$$F_f = A \tau_f \quad (53)$$

where A is the interfacial contact area and τ_f is the frictional shear stress. The frictional shear stress for human skin is $\tau_f = 5-13$ MPa [26, 27].

3 Numerical results

As stated in Section 2, we consider the thickness of the insulating layer of the touchscreen (made of SiO_2) and its dielectric function to be $d_1 = 1 \mu\text{m}$ and $\epsilon_1 \approx 3.9$ [17, 18], respectively. The electrostatic potential between the two solids is constant for the static state (the frequency $f = 0$). The dielectric function for the SC layer is $\epsilon_2 \approx 10^4$ [9] with the thickness $d_2 = 100 \mu\text{m}$. We also assume that the skin of the finger only undergoes elastic deformation and no plastic deformation occurs during the contact with the touchscreen (wet finger) [9]. Therefore, the Young's modulus of the skin of the finger is $E^* = 1,10$ MPa and $\nu = 0.5$ [9]. After the contact, there are many non-contact areas between the finger and the touchscreen surface, and the applied squeezing pressure is small ($p_0 = 10$ kPa). Therefore, we assume that the height fluctuations of the SC layer after the contact are similar to the height

fluctuations prior to the contact. The auto spectral density functions of the dry and wet human skin can be considered as self-affine fractal surfaces [9, 10]. To simplify, we consider two rough surfaces of the SC layer that have same parameters $H_1 = H_2 = 0.8$, $h_{1\text{rms}} = h_{2\text{rms}} = 22 \mu\text{m}$ and the rms slope $h'_1 = h'_2 = 0.91$.

In the figures present below, the roughness effects of the SC layer are considered for the electrostatic potential, electrostatic field, and average effective squeezing pressure between the surfaces in the solid curve, while the roughness effects on the average effective squeezing pressure are not included in the dotted curve. In Eq. (49), the integrals are numerically solved.

3.1 Auto spectral density function for the human wrist skin

For human wrist skin, $q_{01} = q_{02} = 4 \times 10^5 \text{ m}^{-1}$ and $q_1 = q_2 = 3 \times 10^6 \text{ m}^{-1}$ are used, as obtained from Refs. [9, 10]. Figures 2 and 3 show (a) the normalized contact area (A/A_0) and (b) the average surface separation $\bar{u}$ as a function of the applied voltage, for the materials with the effective Young's modulus $E^* = 1$ MPa and $E^* = 10$ MPa, respectively. When the effect of the skin roughness is included in the calculations of the average effective squeezing pressure (perturbation theory), the normalized contact area increases. The roughness leads to an increase in the area of the rough surface (the skin of finger) and in the magnitude of the positive and negative charges on the surfaces [11, 12, 28], effectively giving rise to a larger charge induction. Therefore, the electrostatic attraction (electroadhesion) between the skin of the finger and the touchscreen increases due to the increased magnitude of the surface charges. The normalized contact area then increases while the average surface separation $\bar{u}$ decreases. Comparing Figs. 2 and 3, we conclude that the increase in the elastic modulus reduces the normalized contact area and increases the average surface separation [8–10].

We show the results for the friction force F_f as a function of the normal force $F_N = 0.5-2$ N, with $E^* = 10$ MPa, $p_0 = 10$ kPa, and the applied voltages 500 and 800 V in Fig. 4. The upper and lower solid curves show the results for the case where the effect of the surface roughness is included in the calculation

Fig. 2 (a) Normalized contact area A/A_0 and (b) average surface separation $\bar{u}$ as a function of the applied voltage for the wrist skin, with the effective Young's modulus $E^* = 1$ MPa and $p_0 = 10$ kPa. The roughness effects of the SC layer are considered for the data shown by the solid curve, while the roughness effects are not included for the data shown by the dotted curve.

of the average effective squeezing pressure with voltages 800 and 500 V, respectively. The asterisk and dotted curves correspond to the cases where the effect of the surface roughness is not included in the calculations on the average effective squeezing pressure with voltages 800 and 500 V, respectively. Here, the frictional shear stress $\tau_f = 5$ MPa which is suitable for the wet skin is used. It is clear from Fig. 4 that the friction force increases with increasing normal force and applied voltage [9, 10]. As observed from Figs. 2 and 3, the effect of the roughness of the SC layer on the average effective squeezing pressure increases the normalized contact area, so that the friction force also increases.

Fig. 3 (a) Normalized contact area A/A_0 and (b) average surface separation $\bar{u}$ as a function of the applied voltage for the wrist skin, with the effective Young's modulus $E^* = 10$ MPa and $p_0 = 10$ kPa. The roughness effects of the SC layer are considered for the data shown by the solid curve, while the roughness effects are not included for the data shown by the dotted curve.

3.2 Auto spectral density function for the human finger skin

In the above figures, the results for the auto spectral density function of the wrist skin are presented. Due to the small roughness of wrist skin $q_{01} = q_{02} = 4 \times 10^5 \text{ m}^{-1}$, the results obtained using the perturbation model in this study are not significantly different from the results obtained by the non-perturbed model [9, 10]. Therefore, the surface roughness of the top of the forefinger used in this study is measured by atomic force microscopy (AFM), and the auto spectral density function of the surface height profile is calculated for the finger skin (Fig. 5). Figure 5 shows that the surface

Fig. 4 Friction force F_f as a function of the normal force $F_N = 0.5\text{--}2$ N for the wrist skin, with $E^* = 10$ MPa and $p_0 = 10$ kPa. The upper and lower solid curves are for the case where the surface roughness effect is included in the calculation of the average effective squeezing pressure with the voltages of 800 and 500 V, respectively. The asterisk and dotted curves correspond to the cases where the surface roughness effect is not included in the calculation of the average effective squeezing pressure with the voltages of 800 and 500 V, respectively.

Fig. 5 Auto spectral density function as a function of the wave vector q for the finger skin. The straight line has the slope $-2(1+H) = -3.6$ corresponding to the Hurst exponent $H = 0.8$ and fractal dimension $D_f = 3 - H = 2.2$.

is a self-affine fractal because the logarithm of the correlation function is linear in the logarithm of the wave number. The slope of the straight line is $-2(1+H)$, corresponding to the fractal dimension $(3-H)$, where H is the Hurst exponent. The slope is -3.6 , corresponding to the Hurst exponent $H = 0.8$, and fractal dimension 2.2. The largest value of the roughness of the finger skin is observed to be approximately 0.5 mm [29], corresponding to $q_{01} = q_{02} =$

$2 \times 10^3 \text{ m}^{-1}$. Then, we use the obtained roughness of the finger skin in the above equations.

Figure 6 shows the normalized contact area as a function of the applied voltage for the finger skin, with the effective Young's modulus $E^* = 1$ MPa and $p_0 = 10$ kPa. The roughness effects of the SC layer are considered for the results presented in the solid curve, while the roughness effects are not included for the results presented in the dotted curve. The friction force F_f as a function of the normal force $F_N = 0.5\text{--}2$ N for the finger skin, with $E^* = 1$ MPa, $p_0 = 10$ kPa and the applied voltages (a) 200 V, (b) 300 V, and (c) 500 V are shown in Fig. 7. Comparing Figs. 2 and 6 and Figs. 4 and 7, it is observed that when the wrist skin roughness is replaced by the finger skin roughness in the equations, the perturbed model shows significant difference from the non-perturbed model. For example in Fig. 6, the increase in the friction forces for the voltages of 200, 300 and 500 V at the normal force of 0.5 N are greater than 6.5%, 17%, and 75%, respectively.

4 Concluding remarks

In this study, in addition to the effect of the roughness of the finger skin on the interfacial separation (Persson study), the effects of the roughness of the skin (SC layer) on the electrostatic attraction between the human

Fig. 6 Normalized contact area A/A_0 as a function of the applied voltage for the finger skin, with the effective Young's modulus $E^* = 1$ MPa and $p_0 = 10$ kPa. The roughness effects of the SC layer are considered for the data shown by the solid curve, while the roughness effects are not included for the data shown by the dotted curve.

Fig. 7 Friction force F_f as a function of the normal force $F_N = 0.5\text{--}2\text{ N}$ for the finger skin, with $E^* = 1\text{ MPa}$ and $p_0 = 10\text{ kPa}$. The solid curves are for the case where the effect of surface roughness is included in the calculation of the average effective squeezing pressure. The asterisk curves correspond to the case where the effect of surface roughness is not included in the calculations of the average effective squeezing pressure.

finger and the touchscreen were investigated by perturbation theory. It was found that the inclusion of the effect of the surface roughness in perturbation

theory gives rise to an additional potential $\langle \phi^{(2)} \rangle$ between the solids, and this additional potential is still proportional to the distance $\bar{u}$ from the bottom surface. Therefore, the effect of the roughness included by using perturbation theory increases the effective potential $\langle \phi \rangle$ between the two solids. As a result, the average electrostatic field and the average effective squeezing pressure increase. In other words, in the presence of the applied voltage, the roughness of the finger leads to an increase in the area of the finger surface, increasing the charges induced on the SC layer and the touchscreen. This increase in the surface charges increases the electrostatic attraction between the finger and the touchscreen. Due to this phenomenon, the normalized contact area between the two surfaces increases while their average surface separation decreases. Since the friction force is proportional to the real contact area between the two surfaces, the friction force also increases with increased actual contact area.

The surface topography for the forefinger skin was also measured by AFM to obtain more realistic results. Therefore, the auto spectral density function for the forefinger skin surface was calculated. It was found that when the auto spectral density function of the finger skin is used in the equations, the perturbed model shows a significant difference from the non-perturbed model.

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