

On the relevance of analytical film thickness EHD equations for isothermal point contacts: Qualitative or quantitative predictions?

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Abstract: Thin film and elastohydrodynamic lubrication regimes are rather young domains of tribology and they are still facing unresolved issues. As they rely upon a full separation of the moving surfaces by a thin (or very thin) fluid film, the knowledge of its thickness is of paramount importance, as for instance to developing lubricated mechanisms with long lasting and efficient designs. As a consequence, a large collection of formulae for point contacts have been proposed in the last 40 years. However, their accuracy and validity have rarely been investigated. The purpose of this paper is to offer an evaluation of the most widespread analytical formulae and to define whether they can be used as qualitative or quantitative predictions. The methodology is based on comparisons with a numerical model for two configurations, circular and elliptical, considering both central and minimum film thicknesses.

Keywords: thin film lubrication; elastohydrodynamic lubrication (EHL); film thickness prediction; EHD analytical equation; central film thickness; minimum film thickness; circular contacts; elliptical contacts

1 Introduction

For almost four decades, semi-analytical expressions (simply named analytical in the following) were proposed to calculate film thickness in elastohydrodynamic lubrication (EHL) and especially for point contacts. They generally aimed to predict central and minimum film thicknesses (h_c and h_m) in elastohydrodynamic (EHD) circular contacts under pure rolling and isothermal conditions, and for lubricants considered as Newtonian fluids. Numerous formulas have been published, in particular during the last two decades during which progress in both experimental and computational techniques was substantial. They have been widely used by researchers to advance the knowledge in the fields of thin film lubrication and EHL, and by design and development engineers for estimating film thickness in mechanical devices, like gearboxes, rolling element bearings, cam-tappet assemblies, piston-ring-

liner systems, etc.

Surprisingly, the accuracy of the existing film thickness relationships has rarely been investigated in detail, and their application within the conditions for which they were originally established was not often verified nor respected. To the best of the authors' knowledge, very few—if not any—papers have dealt with these concerns. Except maybe those of van Leeuwen [1, 2] of whom it was not the primary objective: his aim was to derive the most accurate values of viscosity-pressure coefficients from, on one side, central film thickness measurements performed in circular contacts and, on the other side, a wide collection of EHD film thickness equations. Though indirectly, he showed that certain expressions were more relevant than others through their ability to provide correct values of viscosity-pressure coefficients. This is, however, a typical illustration of the classical approach of EHL, in which the author has chosen to derive the lubricants' properties from film thickness or friction measurements instead of relying on direct rheological measurements, obtained independently of tribological tests.

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Nomenclature			
a	contact length or dimension in the entrainment direction (m)	R_y	reduced radius of curvature perpendicular to the entrainment direction (m)
b	contact width or dimension perpendicular to the entrainment direction (m)	T_0	inlet temperature (K)
D	ratio of reduced radii of curvature, $D = R_x / R_y$	u_e	mean entrainment velocity (m/s) = $(u_1 + u_2) / 2$
E_1, E_2	Young moduli of solids 1 and 2 (Pa)	u_1, u_2	velocity in the x -direction of surfaces 1 and 2 (m/s)
E'	reduced modulus of elasticity (Pa) $2 / E' = (1 - \nu_1^2) / E_1 + (1 - \nu_2^2) / E_2$	U	dimensionless speed parameter (Hamrock & Dowson) = $\mu \cdot u_e / (E' \cdot R_x)$
G	dimensionless material parameter (Hamrock & Dowson) = $\alpha^* \cdot E'$	w	normal load (N)
h_c	central film thickness (m)	W	dimensionless load parameter (Hamrock & Dowson) = $w / (E' \cdot R_x^2)$
h_m	minimum film thickness (m)	α^*	reciprocal asymptotic isoviscous pressure, according to Blok [21] (Pa ⁻¹)
k	ellipticity ratio = $\frac{b}{a}$	μ	lubricant dynamic viscosity (Pa·s)
L	dimensionless material parameter (Moes) = $G \cdot (2U)^{0.25}$	μ_0	lubricant dynamic viscosity (Pa·s) at the inlet temperature T_0
M	dimensionless load parameter (Moes) for point contact = $W / (2U)^{0.75}$	ρ_0	lubricant density (kg·m ⁻³) at the inlet temperature T_0
p_H	Hertzian pressure (MPa)	σ	composite roughness of the mating surfaces (m)
R_x	reduced radius of curvature in the entrainment direction (m)		

Given the current trends towards more and more severe conditions applied to lubricated mechanisms due to technological, economic, and environmental constraints, and the unceasing film thickness decrease in lubricated contacts, the need to predict film thickness with high precision appears more than ever well founded. Specifically, new important questions have emerged and require clarification and verification, as for instance:

(1) a deviation of 10 or 20 nm, which seemed negligible 40 years ago, can nowadays have some dramatic consequences on the integrity of the mechanisms: this justifies the assessment of the analytical equations currently in use to make sure they are accurate enough;

(2) the relevance of the extrapolation to often much lower thicknesses as those used to design the analytical expressions should be checked to consider the latter appropriate for predicting very thin film thicknesses.

Furthermore, the related literature generally deals with central film thickness, h_c , whereas it is well known

that h_m , the minimum film thickness, is the crucial parameter for determining the lubrication regime through the h_m/σ ratio, σ being the composite roughness of the mating surfaces. Finally, in many applications the actual geometry of the contacting bodies leads to elliptical point contacts. These latter can be narrow (i.e., slender configuration) or wide, according to the orientation of the larger equivalent radius of curvature of the mating bodies with respect to the main rolling direction. Elliptical point contacts have received much less attention compared to circular ones and, as a consequence, a limited number of analytical expressions were published for the former.

Therefore, the aim of this work is to provide a new insight into the validity and accuracy of some among the most widely used analytical film thickness equations, established for circular and elliptical contacts. From a set of operating parameters leading to 5 reference cases, they will be confronted to a full EHD numerical model, taken here as a reference due to the numerous conditions considered for achieving its validation against

experimentation. Both the central and minimum film thicknesses will be studied in the case of a Newtonian lubricant operated under pure rolling and isothermal conditions. The purpose of this comparison is indeed not to rank the models against each other, but to evidence whether they can be considered sufficiently quantitative or just qualitative, in the domains investigated in this work.

2 Models and conditions

The choice of a reasonable number of EHD film thickness equations to be included in this work was dictated by different criteria (extensive use, circular and/or elliptical geometry). The widely-used expressions mentioned below were selected on the basis of (i) van Leeuwen studies [1, 2] and (ii) a previous experimental work [3] in which the capabilities of some of them were quantitatively compared with measurements performed over wide ranges of operating conditions and for numerous lubricants of different nature:

- (1) Hamrock & Dowson [4], for circular and elliptical (wide only) contacts;
- (2) Nijenbanning et al. [5] for h_c in circular and elliptical (wide only) contacts, combined with Chevalier h_c/h_m table [6] for calculating h_m (see Ref. [3]) in circular contacts;
- (3) Evans & Snidle [7], for circular contacts only;
- (4) Chittenden et al. [8], for circular and elliptical (slender and wide) contacts;
- (5) Masjedi & Khonsari [9], for circular and elliptical (wide only) contacts.

The analytical expressions and the numerical tables corresponding to these EHD film thickness equations and h_c/h_m ratios are given in Appendix.

Figure 1 provides a schematic description of the domains on which the analytical models above were established, as a function of M and L , the dimensionless load and material parameters as proposed originally by Moes [10] (M and L are defined in the Nomenclature). These ranges take into account the indirect (M, L) variations produced when considering elliptical contacts, except in the case of the Evans & Snidle equation which concerns circular cases only. Overall, the domains of validity of the analytical models, expressed in a (M, L) chart in Fig. 1, cover well the full range of EHL. However some of them were restricted

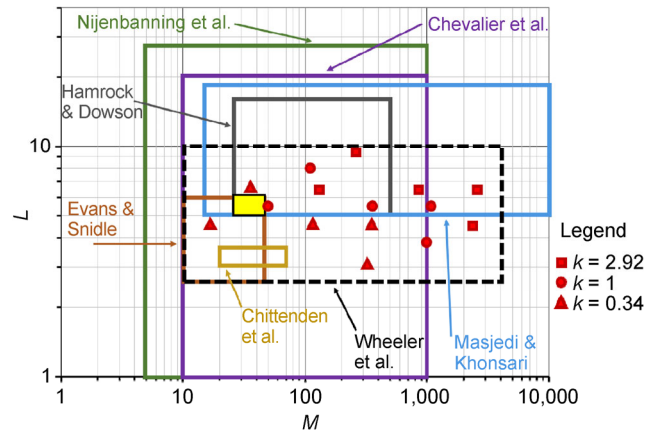


Fig. 1 Domains (expressed by empty rectangles) on which EHD film thickness equations for circular and elliptical contacts were established. The yellowed area represents the common area covered by all the analytical expressions considered here. The black bold dotted line indicates the region where the full numerical model was applied in Ref. [11]. The symbols show the domain corresponding to the 5 references cases of Table 1, in the circular ($k = 1$) and the elliptical configurations ($k = 2.92$ or 0.34).

to rather limited (M, L) areas and extrapolation could result in inaccurate results. There is a common area covered by all analytical models, given that Chittenden et al. [8] have also incorporated the results of Hamrock & Dowson [4] to derive their equations. This overlap extends to values of M and L between 25 and 45, and between 5 and 6, respectively, see the yellowed rectangle in Fig. 1. This area ultimately represents a very narrow domain compared to the full field of EHL.

The versatile EHD model used here as a reference has been already presented in Ref. [11] and will not be detailed further. It results from recent modeling developments performed at LaMCoS, after the works of Doki-Thonon in the case of spinning EHD contacts [12, 13] and those of Habchi who has laid the foundations inherent to this multiphysics model [14, 15]. The steady state problem concerns smooth surfaces, fully flooded, Newtonian and isothermal conditions. Based on the finite element method, the numerical model solves simultaneously the Reynolds, the solids deformation and the load balance equations. Typically, the Reynolds equation was solved using 2×10^4 degrees of freedom and the convergence was achieved when a relative deviation lower than 10^{-3} was obtained.

The physical behavior of the lubricant is taken into account through (i) a rheological equation which describes the viscosity changes with pressure and (ii)

a classical equation of state for the density variations: the Newtonian viscosity follows a modified Williams-Landel-Ferry (WLF) correlation [16] and the density varies according to the Murnaghan [17] equation. Both constitutive equations were fitted to independent characterizations carried out with high pressure devices, see Ref. [11] for more details.

Since its early developments, quantitative comparisons with experiment have proven the reliability and accuracy of this numerical model to predict film thickness in various configurations: for instance with conventional (mineral turbine oil [14]) and non-conventional lubricants (low viscosity working fluids or glycerol [18, 19]), or under complex kinematic conditions (spinning-skewing EHD contacts [13]). More recently, it was adapted for non-circular EHD contacts and successfully validated by quantitative confrontation with experiments [20].

A first reference case (called Case 3) was defined in the circular configuration, with $R_x = R_y = 80$ nm, an entrainment velocity $u_e = 2$ m/s, a normal load $w = 800$ N, a bearing steel material for the two solids ($E = 210$ GPa, $\nu = 0.3$), and an inlet lubricant temperature $T_0 = 313$ K, giving $\alpha^* = 20$ GPa⁻¹, $\mu_0 = 0.008$ Pa·s and $\rho_0 = 863$ kg·m⁻³. Then both the entrainment velocity, u_e , and the normal load, w , were varied in order to define 4 other reference cases to cover sufficient wide ranges of operating conditions, see Table 1 where they are also reported and expressed by the (M, L) dimensionless parameters. Apart from the central Case 3 already described, a low (120 N) and a high (2,500 N) normal load condition together with a low (0.5 m/s) and a high (10 m/s) entrainment speed condition are proposed. From these physical values, it is possible to compute the corresponding dimensionless

Table 1 Normal load, entrainment velocity (both in bold) and Hertzian contact pressure for the circular configuration (in italic) of the five reference cases. The (M, L) values are given under the Case number.

		u_e (m/s)		
		0.5	2	10
	120	—	<i>Case 1</i>	—
	<i>364</i>		<i>(51, 5.2)</i>	
w (N)	800	<i>Case 2</i>	<i>Case 3</i>	<i>Case 4</i>
	<i>686</i>	<i>(962, 3.7)</i>	<i>(340, 5.2)</i>	<i>(102, 7.8)</i>
P_h (MPa)	2500	—	<i>Case 5</i>	—
	<i>1002</i>		<i>(1,062, 5.2)</i>	

parameters M and L , and to compare them with those of Fig. 1 for the analytical models.

In the circular configuration, the 5 reference cases of Table 1 lead to a domain defined by $M \in [50, 1,062]$ and $L \in [3.7, 7.8]$, see the red dots in Fig. 1. Moreover in Ref. [11], the numerical experimentations covered a larger range delimited by $M \in [10, 4,000]$ and $L \in [2.5, 10]$, highlighted by the black dotted contour in Fig. 1 which shows a rather large overlap with the areas from which the analytical expressions were drawn. This enables to study and compare the dependence, for all the models considered here including the full EHD solution, of w and u_e on film thickness for both configurations, circular firstly, and then elliptical.

The last important point to consider for conducting an objective analysis concerns the integration of the lubricant properties. Indeed, the numerical model used in this work includes two physical laws that quantitatively describe the actual response of the lubricant subjected to contact conditions, while the analytical EHD models are based on empirical expressions, like the Barus, Roelands or Dowson-Higginson equations. Concerning the viscosity-pressure dependence, it should be reminded that Hamrock & Dowson [4] were aware of the weakness of the Barus law. In their expressions they preferred to consider α^* , the reciprocal asymptotic isoviscous pressure as proposed by Blok [21], instead of the classical secant pressure-viscosity coefficient based on an exponential dependence, i.e., on the so-called Barus law. Interestingly, the use of α^* some decades later has confirmed [22, 3], by comparison with experiments, that this parameter was really relevant to predict film thickness. Following this agreement, the reciprocal asymptotic isoviscous pressure was used to calculate film thickness from the analytical expressions.

3 Results and discussion on circular contacts

Results are expressed as the relative film thickness deviation given by each analytical equation to our numerical model (noted h_{ref} thereafter), a positive value meaning an overestimation:

$$\Delta h/h_{\text{ref}} = (h_{\text{mod}} - h_{\text{ref}})/h_{\text{ref}} \quad (1)$$

where h can be either h_c or h_{mv} and h_{mod} refers to a

prediction by an analytical model, i.e., Masjedi & Khonsari [9], Chittenden et al. [8], Evans & Snidle [7], Hamrock & Dowson [4], Nijenbanning et al. [5] or Chevalier [6] expressions. Throughout the rest of the paper, results are graphically reported according the following order: Case 1, Case 3, Case 5, Case 2, and Case 4. This enables first to assess and compare the influence of an increasing normal load (120, 800, and 2,500 N, respectively Case 1, Case 3 and Case 5), the remaining parameters being kept constant, and then to pursue the analysis to the entrainment speed influence (from 0.5 to 10 m/s), respectively for Case 2 and Case 4, w being constant and equal to 800 N.

Figure 2 presents a comparison of the central film thickness results, expressed by $\Delta h_c/h_{c,ref}$, given by the analytical film thickness equations mentioned before which are suitable for circular contacts. The 5 models are, in average, rather accurate and capable to estimate h_c with an acceptable precision (represented by a bold dotted line in Fig. 2) of 9% in average with a standard deviation of 6%: the interval of confidence (defined by \pm the standard deviation to the mean gap) is delimited by two thin dotted lines in Fig. 2. The results are not uniform across the models: those computed from the equation of Chittenden et al. [8] are in excellent agreement (within 3%) with the numerical solutions for the 5 reference cases, whereas the models of Evans & Snidle [7] (in particular at high load and/or low velocity) and of Nijenbanning et al. [5] (in a rather uniform manner) deviate more

significantly. A general and clear trend is however revealed, for the conditions simulated in this study: all the analytical EHD equations overestimate the central film thickness, on average by 9% which can be considered nevertheless as a moderate discrepancy.

The relative deviations on minimum film thickness predictions are reported in Fig. 3, expressed in the same way as in Fig. 2. For h_m , the discrepancy is much larger than for h_c and reaches an average value of 37% for the 5 reference cases of Table 1, with a standard deviation of 34%. Nevertheless, the combined Nijenbanning & Chevalier model [3, 5] provides a rather fair prediction of h_m with a mean overestimation of +11%, while the use of Evans & Snidle expression results in the unique, but very low, underestimation for Case 4, of -1.2%. The three other analytical equations predict strongly optimistic minimum film thicknesses: they overestimate h_m by nearly +50% with several occurrences exceeding +80%, especially at high load and/or low velocity conditions. These deviations are certainly too large—if not unacceptable—to insure safe working conditions of lubricated mechanisms, given the current technical and environmental demands that lead to lubricate with thinner and thinner lubricating films. Moreover, they could lead to erroneous lubrication regime estimation, the actual minimum film thickness being half-value of the analytically calculated ones.

In summary, the analytical EHD equations generally overestimate film thickness in circular contacts, to a

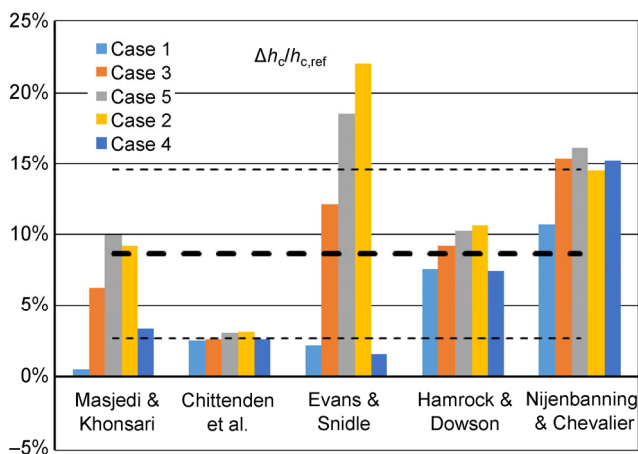


Fig. 2 Relative deviations given by analytical EHD film thickness expressions on h_c , the central film thickness, for the 5 circular reference cases defined in Table 1.

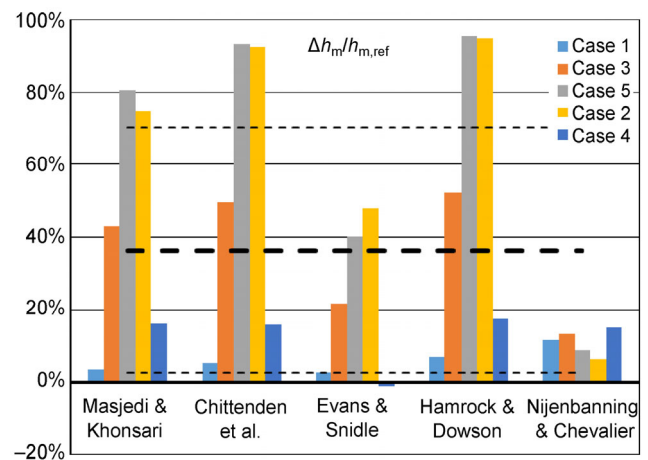


Fig. 3 Relative deviations given by analytical EHD film thickness expressions on h_m , the minimum film thickness, for the 5 circular reference cases defined in Table 1.

much larger extent for h_m , the minimum value, while the prediction appears acceptable, for an engineering point of view, for h_c , the central film thickness. This global discrepancy cannot be, to first order, attributed to the different manners of taking into account the physical properties of the lubricant in the full numerical model. If such were the cases, the agreement on h_m may have been more satisfying, given that minimum film thickness takes place where the pressure approaches its ambient value and therefore where the density and viscosity become closer to the ambient values. However, it is the opposite trend that is observed.

Clearly, the results of this comparison between analytical and numerical methodologies are in line with some previous findings. Concerning central film thickness, van Leeuwen [1, 2] concluded that for both moderately-loaded and highly-loaded EHD contacts, Chittenden et al. [8] formula was the more accurate and that its validity transcended the area where it was originally designed for. As for minimum film thickness, the use of Chevalier ratios [6] combined with the Nijebanning et al. [5] formula has shown, from experimental confrontation, to be the more consistent over very wide ranges of the (M, L) parameters [3]. But perhaps the crucial point to emphasize here lies in the fact that the results of the previous works were based on experimental measurements and are now fully confirmed by the current study which relies on a purely numerical and modeling approach.

4 Results and discussion on elliptical contacts

The consideration of elliptical contacts excludes the Evans & Snidle equations and the Chevalier table, all designed for the circular geometry. Moreover, there are two options to represent ellipticity, the first one based on $k = \frac{b}{a}$ where a is the contact length and b its width, the second relies on D , which expresses the ratio of the reduced radii of curvature at the contact center. k was selected here, given that most of the works on elliptical contacts have used this parameter to represent ellipticity.

Figures 4(a) and 4(b) compare, in $M-k/D$ and $k/D-L$ charts, the domains on which the Hamrock & Dowson

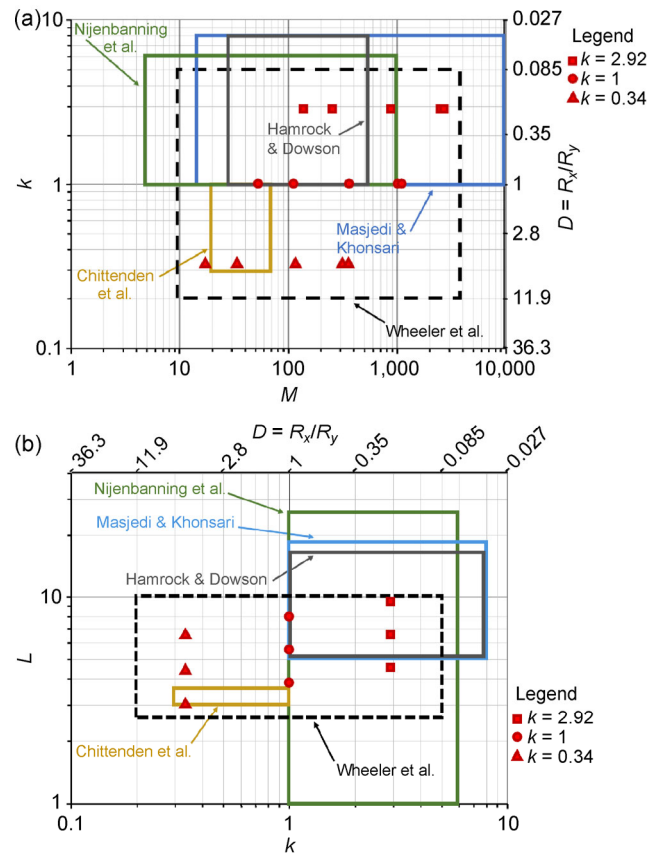


Fig. 4 Domains on which EHD film thickness equations for elliptical contacts were established. The black bold dotted line indicates the region where the full numerical model was applied in Ref. [11]. The square and triangle show the ranges on which the comparison was conducted for two elliptical configurations defined by $k = 2.92$ and 0.34 (or $D = 0.2$ and 5), respectively: (a) expressed in $M-k/D$ chart, and (b) expressed in $k/D-L$ chart.

[4], Nijebanning et al. [5], Chittenden et al. [8] and Masjedi & Khonsari [9] were derived for elliptical contacts. Note that Chittenden and his co-authors were the only ones to explore the case $k < 1$ (narrow or slender configuration), and that they included the Hamrock & Dowson results to establish their analytical models, thus valid for both slender and wide configurations. This is also the case in the paper by Wheeler et al. [11] who explored k values ranging from 0.2 to 5 delimited by the bold black dotted lines in Figs. 4(a) and 4(b).

In the following, the five reference cases of Table 1 will be combined with two configurations representative of wide and slender elliptical contacts, characterized by $k = 2.92$ and 0.34 (or $D = 0.2$ and 5), respectively. In the numerical model, the initial values of the radiuses

of curvature along the main axes were varied, keeping all the remaining parameters constant. As a consequence, the ranges of dimensionless parameters have been much extended compared to the circular cases and cover $M \in [131, 2,736]$, $L \in [4.5, 9.6]$ and $M \in [17,358]$, $L \in [3, 6.4]$, respectively. This is also visible in Figs. 4(a) and 4(b) where the values corresponding to the circular configuration (red dots) are exceeded for the elliptical cases in M and L , towards lower and higher extrema.

4.1 Wide elliptical contacts: $k = 2.92$

First note that Chittenden et al. [8], Hamrock & Dowson [4], and Nijenbanning et al. [5] formulae were extrapolated from the domains in M they were established for reference Case 2 (low speed, medium load, $M = 2,476$) and Case 5 (medium speed, high load, $M = 2,736$), see Fig. 4(a). However the results for these particular cases do not show significant differences with those calculated under regular conditions, i.e., without extrapolation, see Fig. 5 for the central film thickness for instance. In some ways, these results demonstrate the relative robustness of the 3 models mentioned just above.

The results are expressed as before for circular contacts, using Eq. (1). Overall, the confrontation between analytical film thickness expressions and the full EHD model for wide elliptical contacts results in similar trends as for the circular case: firstly h_c is systematically overestimated (see Fig. 5) and secondly,

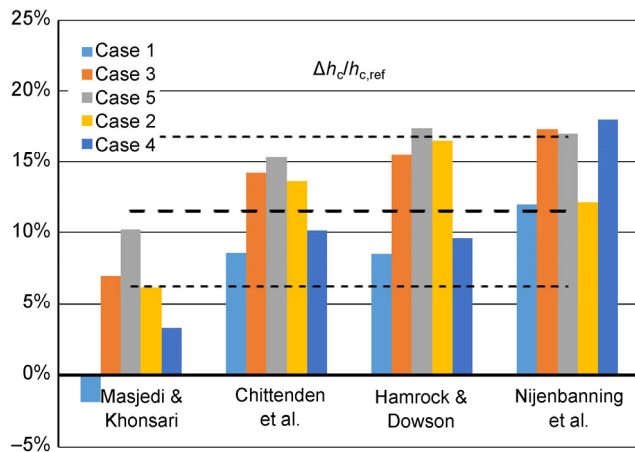


Fig. 5 Relative deviations given by analytical EHD film thickness expressions on h_c , the central film thickness, for the 5 wide elliptical reference cases defined by $k = 2.92$.

a mean discrepancy of +12% is found, with a standard deviation of 5%. In a logical way, the Masjedi & Khonsari model [9] proves to be the most accurate in predicting central thickness (within 5%) in wide elliptical contacts ($k = 2.92$ or $D = 0.2$). This is the most recent model (published in 2015), thus one can reasonably expect a fairer prediction compared with earlier models. Moreover, it has been established over the widest area in M [5, 10,000] and for k values ranging from 1 up to 8: it was thus applied within its domain of validity and any extrapolation was introduced which might have resulted in some further deviation.

When it comes to h_{mv} the same remark as for the central film thickness applies on the domains of validity of the analytical expressions, but here it was not possible to extrapolate the Nijenbanning et al. model because the tabulated ratio h_c/h_m at $M > 1,000$ has not been quantified in Ref. [5]. Thus results for Case 2 and Case 5 are missing for this expression. The minimum film thickness results for $k = 2.92$ are plotted in Fig. 6.

The minimum film thickness predictions by the analytical equations always lead to overestimation, and appear of a satisfying precision: the mean discrepancy is equal to +6%, and the standard deviation of the same value. The Nijenbanning et al. [5] table gives the more accurate estimate of h_{mv} but is limited up to $M = 1,000$, thus to Cases 1, 3 and 4 only in this study. Compared to the circular case, the fact that the

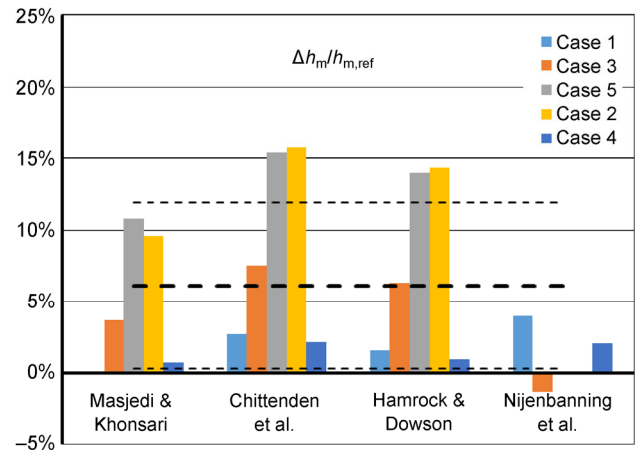


Fig. 6 Relative deviations given by analytical EHD film thickness expressions on h_m , the minimum film thickness, for 5 wide elliptical reference cases defined by $k = 2.92$.

analytical models appear more accurate, or less false in the perspective of a quantitative approach, in the elliptical configuration may seem surprising. However, in this section film thicknesses have been computed for $k = 2.92$ which still denotes a rather marked wide elliptical configuration. Under these circumstances, hydrodynamic effects are largely dominated by Poiseuille flows in the entrainment speed direction, the contact approaches the infinitely wide case and the places where the minimum film thickness occurs deviate from the lateral lobes towards the contact exit area [11]. For instance, with $k = 2.92$ and $W = 800$ N (the medium load case) one reaches the equality between the classical minimum thicknesses found on the lobes and the film thickness at the center of the exit zone of the contact [11], where the minima would occur if k was increased further, as in the case of line contacts. Here, for $k = 2.92$ the average h_c/h_m ratio obtained from all models (analytical and numerical) is equal to 1.28 (+28%), against 2.5 (+150%) in the circular configuration. The h_c and h_m values becoming closer, there is no reason why their prediction would give very different trends, in terms of accuracy.

4.2 Slender elliptical contacts: $k = 0.34$

In this configuration, the only available analytical model is that of Chittenden et al. [8]. However, even if the authors have specifically explored $k \in [0.3, 1]$, its range of application in terms of (M, L) domain was defined for $M \in [20, 70]$ and $L \in [3, 3.5]$ (see Fig. 1),

which is rather limited compared with the domain explored here ($M \in [17, 358]$, $L \in [3, 6.4]$), see Figs. 4(a) and 4(b). This model was thus significantly extrapolated for most cases to obtain the results reported in Fig. 7.

For the first time in this work, central film thickness is underestimated (see Fig. 7 left) with a mean relative gap of -20% and a standard deviation of 9% : these values are rather similar to those reported concerning h_c prediction in the circular and wide elliptical cases. In contrast, the minimum thickness is dramatically overestimated with an average relative difference of the order of 140% . This tendency clearly shows that the Chittenden et al. [8] model has no capability to properly capture the underlying mechanisms occurring in slender elliptical contacts when extrapolated to rather high M values. Two main phenomena intervene in such conditions. The hydrodynamics effects are dominated by the lateral Poiseuille flow rates along the directions transverse to the entrainment velocity [11]. In the meantime a relatively larger radius of curvature in the u_c direction reduces the wedge effect and thus the film building ability. The two effects are cumulative to generate a dramatic film thickness reduction especially on h_m , which leads to unusual h_c/h_m ratios. For the 5 reference cases considered here, this ratio is equal to 6.2 in average, which is a much larger value than in the circular or wide configurations. Furthermore, it can take values close to or higher than 9, as for the reference Case 2 and Case 5 where the minimum film thickness calculated from the full

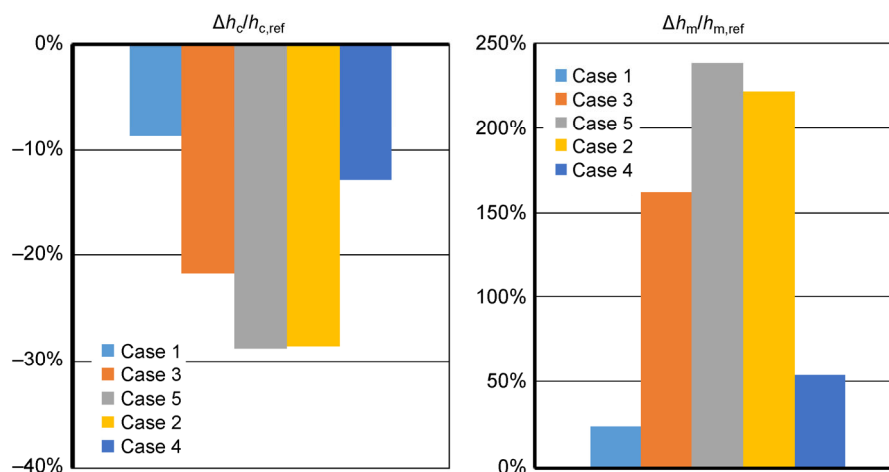


Fig. 7 Relative deviations given by the Chittenden et al. [8] analytical formulae on h_c (left) and h_m (right) for the 5 elliptical reference cases in a slender configuration defined by $k = 0.34$.

EHD model drops down to 13 and 28 nm, respectively. This underlines the impossibility, for the slender configuration, to extrapolate the Chittenden et al. [8] model to M values outside the range the expression was designed for.

5 Conclusions

Thin film lubrication and EHL are rather young domains of tribology and of science and technology in general, which really emerged about 70 years ago. One could have think, with the tremendous development of experimental techniques and computational tools, that they could become mature and well understood after this period. That was somehow one of the very first objectives of this work to ensure that one is able to predict analytically the lubricant film thickness in point contacts operated under the simplest conditions (Newtonian fluid, smooth surfaces, and no thermal effect).

Based upon a selection of well-known and widely used semi-analytical expressions, the first step consisted to present and compare their domains of validity, expressed through the M , L and k (or D) parameters. The differences among the models and the ranges not covered—or covered by only some of the formulae—have been identified and highlighted, especially when extrapolations were required to be carry out.

The comparison between the analytical predictions and the results from a full EHD solver has been then examined for circular contacts. In spite of being the most studied configuration from the earlier stages of development of thin film lubrication and EHL, this first assessment showed that film thickness was systematically overevaluated: the central film thickness was rather accurately predicted whereas a much larger discrepancy was obtained on the minimum film thickness.

The extension to elliptical cases, both slender and wide, was conducted with a more limited number of analytical models. The comparison was found to be more favorable in the case of wide elliptical contacts: film thick thickness was still over estimated but in a lower extent, and especially for the minimum film thickness where the best agreement between analytical

and numerical predictions was obtained. In contrast, the worst situation was pointed out in the case of slender contacts, for which only one analytical model was studied and showed its quasi inability for extrapolation to larger M values than those it was derived for.

Whatever the geometrical configuration, circular or elliptical, it is clear that the reference Case 5 and Case 2, namely the highly loaded and low velocity cases, gathered the largest discrepancies with the analytical models. This is certainly a major weakness because such cases correspond in fact to conditions more and more frequently found nowadays in lubricated systems: very thin lubricating films in line with the unceasing drop of film thickness with time, and heavily loaded contacts as those found for instance in rolling element bearings or in gears.

From the results of the current work, it is clear that analytical models can, at best, provide a qualitative estimate of film thickness. In such an approach, it could be recommended to use the Chittenden et al. [8] equation for estimating h_c and the Nijenbanning et al. expression [5] combined with the Chevalier table [6, 3] for predicting h_m in circular contacts, the Masjedi & Khonsari models [9] for h_c and h_m in wide elliptical contacts. The question of the slender elliptical contacts remains open, pending a suitable analytical model. A great care should be taken for establishing the lubrication regime: all the analytical models investigated in this work over predict minimum film thickness, which may lead to estimate erroneous frontiers between full film and mixed lubrication regimes.

Given the findings of this work and the conclusions and recommendations reported above, the most reliable approach to predict film thickness in EHD point contacts should, in authors' opinion, rely over a full numerical model. It is quantitative by nature and can include the actual lubricant behavior—obtained independently from tribological tests—and various other features not accounted for here.

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Appendix: Analytical expressions

Hamrock & Dowson [4]:

$$h_c / R_x = 2.69 U^{0.67} G^{0.53} W^{-0.067} (1 - 0.61e^{-0.75(R_y/R_x)^{0.64}})$$

$$h_m / R_x = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.70(R_y/R_x)^{0.64}})$$

Evans & Snidle [7]:

$$h_c / (R_x (2U)^{0.5}) = 1.7 M^{-0.026} L^{0.40}$$

$$h_m / (R_x (2U)^{0.5}) = 1.9 M^{-0.17} L^{0.34}$$

Chittenden et al. [8]:

$$h_c / R_x = 4.31 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-1.23(R_y/R_x)^{2/3}})$$

$$h_m / R_x = 3.68 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.67(R_y/R_x)^{2/3}})$$

Nijenbanning et al. [5]:

$$h_c / (R_x (2U)^{0.5}) = ((H_{RI}^{3/2} + (H_{EI}^{-4} + H_{00}^{-4})^{-3/8})^{2s/3} + (H_{RP}^{-8} + H_{EP}^{-8})^{-s/8})^{1/s}$$

with:

$$D = R_x / R_y$$

$$s = 1.5(1 + e^{-1.2/H_{EI}/H_{RI}})$$

$$H_{00} = 1.8 D^{-1}$$

$$H_{RI} = 145(1 + 0.796 D^{14/15})^{-15/7} D^{-1} M^{-2}$$

$$H_{EI} = 3.18(1 + 0.006 \ln(D) + 0.63 D^{4/7})^{-14/15} D^{-1/15} M^{-2/15}$$

$$H_{RP} = 1.29(1 + 0.691 D)^{-2/3} L^{2/3}$$

$$H_{EP} = 1.48(1 + 0.006 \ln(D) + 0.63 D^{4/7})^{-7/20} D^{-1/24} M^{-1/12} L^{3/4}$$

h_m is obtained from the h_c/h_m ratios reported in the following tables:

$D = 1$ from Chevalier [3, 6]	M							
	1	3	10	30	100	300	1000	
L	0	1.2645	1.2635	1.26	1.25	1.33	1.48	1.93
	2	1.2915	1.3045	1.35	1.48	1.8	2.23	3.28
	5	1.251	1.273	1.35	1.57	1.92	2.42	3.43
	10	1.2645	1.2835	1.35	1.54	1.87	2.33	3.2
	20	1.2425	1.2575	1.31	1.46	1.72	2.08	2.79
	40	1.1985	1.2055	1.23	1.3	1.42	1.58	1.97
	60	1.1545	1.1535	1.15	1.14	1.12	1.08	1.15

$D = 0.4$	M								
	5	10	20	50	100	200	500	1000	
L	0	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
	1	1.3	1.3	1.3	1.3	1.4	1.4	1.5	1.6
	2.5	1.3	1.3	1.3	1.4	1.5	1.6	1.7	1.8
	5	1.3	1.3	1.4	1.5	1.5	1.6	1.8	2
	10	1.3	1.4	1.4	1.5	1.5	1.6	1.8	2
	25	—	1.4	1.3	1.5	1.6	1.6	1.7	1.9

$D = 0.2$	M								
	5	10	20	50	100	200	500	1000	
L	0	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.3
	1	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.4
	2.5	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.4
	5	1.2	1.3	1.3	1.3	1.3	1.4	1.4	1.5
	10	1.2	1.3	1.3	1.4	1.4	1.4	1.5	1.5
	25	—	1.3	1.3	1.4	1.4	1.4	1.5	1.5

Masjedi & Khonsari [9]:

$$h_c / R_x = 3.672 U^{0.663} k^{0.025} G^{0.502} k^{0.064} W^{-0.045} k^{0.18} (1 - 0.573e^{-0.74k})$$

$$h_m / R_x = 1.637 U^{0.711} k^{-0.023} G^{0.650} k^{-0.045} W^{-0.09} k^{-0.15} (1 - 0.974e^{-0.676k})$$

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