A geomechanics classification for the rating of railroad subgrade performance

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Abstract The type of subgrade of a railroad foundation is vital to the overall performance of the track structure. With the train speed and tonnage increase, as well as environmental changes, the evaluation and influence of subgrade are even more paramount in the railroad track structure performance. A geomechanics classification for subgrade is proposed coupling the stiffness (resilient modulus) and permanent deformation behaviour evaluated by means of repeated triaxial loading tests. This classification covers from fine- to coarse-grained soils, grouped by UIC and ASTM. For this achievement, we first summarize the main models for estimating resilient modulus and permanent deformation, including the evaluation of their robustness and their sensitivity to mechanical and environmental parameters. This is followed by the procedure required to arrive at the geomechanical classification and rating, as well as a discussion of the influence of environmental factors. This work is the first attempt to obtain a new geomechanical classification that can be a useful tool in the evaluation and modelling of the foundation of railway structures.

Keywords Subgrade · Resilient modulus · Permanent deformation · Geomechanical classification

Nomenclature

PI Plasticity index

γ	Dry unit weight
S	Degree of saturation
a	Initial tangent modulus
q_u	Unconfined compressive strength
% finer #200	Percent passing #200 sieve
LL	Liquid limit
% clay	Percent finer than 0.002 mm
W_L	Liquid limit
W_P	Plastic limit
IP	Index plasticity
W	Moisture content
W	Optimum moisture content
C_u	Coefficient of uniformity
C_c	Coefficient of gradation
D_n	Diameter corresponding to $n\%$ of passed material in a grain size distribution curve
CBR	California bearing ratio

1 Introduction

The understanding and knowledge about the deformation and failure mechanisms of the geomaterials under repeated loadings are extremely important for the proper design and maintenance planning in the railway structures [1–4]. Typically, the subgrade layer of railway track is composed of geomaterials (mainly coarse-grained soils due to its better performance when compared to the fine-grained soils) and presents two types of deformation when submitted to cyclic loads: the recoverable (quasi-elastic, resilient or reversible) and permanent (or plastic)

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deformations, which have significant importance in the long-term performance of the substructure [5–7]. Regarding the resilient deformation, the concepts were introduced by AASHTO in 1986 [8]. This property is often used to characterize the materials in several railway layers [9–11]. About the permanent deformation, an accurate estimation of the amount of cumulative settlement will benefit railway structures to avoid a mediocre performance and consequently higher annual maintenance costs [12–14]. Overall, resilient and permanent deformations are both important parameters for the design of railway structures [15–18]. However, the complexity and time-consuming of laboratory tests, mainly for permanent deformation evaluation, conduct to the development of predictive models based on the available data from the existing laboratory and/or field results.

In this line, one of the main objectives of this work is to review the available resilient and permanent deformation models in the scope of the geomaterials published in the specialized bibliography considering the information about the conditions for its development (properties of the materials tested, compaction tests, degree of compaction, moisture content, etc.) and the main variables/factors that can influence the response of the material. This means that this analysis also includes external factors as the physical state of the soil, which is often difficult to control because it depends on other environmental aspects. A first attempt was already performed for permanent deformation models in the work developed by [19]. Consequently, this work intends to complement it with the resilient deformation models, including comparisons among some of the selected models that are based on different materials with different classifications (UIC and ASTM), properties, granulometry and physical states. By coupling both resilient- and permanent-based deformations, we rate the geomaterial classifications according to the predicted resilient and permanent deformation performances, which is believed to be a helpful tool in the design of the railway structures. It is also noted that this rating should be interpreted under normalized conditions because it depends on several properties and soil state conditions. Based on these results, a novel geomechanical classification for track subgrade/formation that couples the resilient modulus and the permanent deformation is purposed using a similar approach developed by [20].

2 Influence of subgrade in design and performance of railroad track structures

The quality and the support given by the subgrade are important aspects in the performance of the railroad track structure. The subgrade is an integrant component of the

track structures (ballasted and ballastless tracks), and its properties are important in the performance of track and track quality. Indeed, this is why subgrade performance indicators are important and should be implemented into the track designs and assessments. Thus, there are numerous means of measuring and predicting subgrade performance, which include the laboratory and in situ tests. The in situ tests can be advantageous since the results are representative of the real conditions and some can be obtained quickly and more cost-effective. However, laboratory tests could be also very interesting and necessary to carry out parametric studies considering variations occurring during the service life of the structure. Indeed, recently, new methods have been developed to determine the resilient modulus, such as the performance of the cyclic lightweight deflectometer test [21].

Thus, despite the importance of the subgrade and its significant impact on the track quality and required maintenance, the evaluation of the subgrade is not straightforward since there is an extensive list of factors that can affect its short- and long-term performance: the moisture content and/ or, suction, and/ or degree of saturation, temperature (freezing and thawing), shear strength, stiffness parameters, consolidation, etc. Indeed, recent studies show the influence of soil suction on the deformation characteristics of railway formation materials [22]. Furthermore, other variables can affect the performance of the subgrade related to the train type (passenger or freight), axle load, train speed, train configuration, drainage, rail condition, tie spacing, and wheel condition [23–25]. Thus, these factors added to the environmental factors can affect the performance of the subgrade and also the global performance of the track structure. Indeed, the poor performance of the subgrade can lead to some issues in the railroad track structure: ballast fouling, ballast pockets, pumping of soil fines through the ballast, and slope stability failure. Thus, there are two main types of subgrade failure (mostly related to the fine subgrade soils): subgrade progressive shear failure and excessive subgrade plastic deformation which are well documented [15, 17, 26]. More recent works in Refs. [27] and [28] emphasize that low- and medium-plasticity soils can also be subjected to different types of failure such as fluidization and mud pumping due to excessive increase in axle loads of freight trains in recent years. Nevertheless, these phenomena and many others like liquefaction (large displacement caused by repeated loading, saturated silt, and fine sand), massive shear failure (slope stability—this failure is associated with the weight of the train, inadequate soil strain and increase of water content), consolidation settlement (the static soil stress increased due to the embankment weight and saturated fine-grained soils), frost action (the track become rough caused by periodic freezing), swelling/shrinkage,

slope erosion and soil collapse caused by ground settlement are out of scope of this paper.

The design of the subgrade (design period corresponds to 50–100 years) is defined according to the type of traffic, bearing capacity of the subgrade, configuration of the track, climatic and hydro-geological conditions [15, 29, 30]. Therefore, the subgrade should be designed to show a good short- and long-term performance, resisting to failure and excessive deformation, respectively, induced by repeated loads. The short performance is influenced directly by the resilient modulus (that has influence on the stress levels on the subgrade) and by the strength parameters, among other factors. The stress paths and the strength parameters (such as cohesion and friction angle) have a significant effect on the development of the permanent deformation. (This topic will be analysed in more detail further in this work.) Usually, the design of the ballasted track involves the determination of the deformation and stresses at critical locations in the track (sleeper-ballast or ballast-sub-ballast contacts). Posteriorly, the magnitude of the settlements and stresses (induced by the passage of the trains) is compared with allowable values in an iterative process where the dimensions of the sleepers and/or thickness of the granular layers are adjusted. These methodologies are applied by several authors and standards:

- Reference [15] developed a methodology that prevents progressive shear failure and also excessive plastic deformation where the plastic strain (ε_p) and the cumulative deformation should be limited to 2% and 25 mm, respectively.
- Reference [31] presented a set of references for the design and maintenance of track substructure. According to [31], there are four soil quality classes: QS0 (unsuitable soils), QS1 (poor soils), QS2 (average soils) and QS3 (good soils). This concept will be explored further in this work. From the soils' classification and thickness of the granular layers, the track is evaluated in three categories: P1, P2 and P3 (mediocre, average and good, respectively).
- British Rail Method and Network Rail Code defined recommendations related to the thickness of the trackbed layers [17].
- West Japan Railways defined a limit value for bearing capacity (288 kPa). Below this value, the improvement of the foundation is required. Thus, the bearing capacity of 288 kPa equates to a compressive strength σ_s of approximately 112 kPa assuming a cohesion model plastic solution to a simple strip footing where $\sigma_b = 2.57\sigma_s$.

These methods allow preventing the development of an overstressed subgrade that can present cumulative permanent deformation (formation of ballast pocket) or

progressive shear failure (subgrade squeezing) under repeated loads. Indeed, these are common failure modes of typical subgrades composed of fine-grained cohesive soils [15, 26, 30, 32, 33]. These types of problems can lead to an increase in dynamic loads and accelerate the deterioration of the track.

Regarding the ballastless track, the design theories of the ballastless track vary across the world according to the experience and background of each country [34].

Thus, two important parameters should be included in the design of railway tracks related to the short- and long-term performance: the resilient modulus (M_r) and the permanent deformation (ε_p). These two parameters should be analysed together in an integrated approach since, from a simplistic point of view, the stress levels at the rail track subgrade vary with the value of its M_r , which have impact on the permanent deformation. These parameters will be analysed together with the UIC classification in this work to purpose a novel geomechanical classification.

3 Resilient modulus

The resilient modulus is used to characterize the recoverable, reversible or quasi-elastic behaviour. In the traditional theory of elasticity, the elastic properties (which are material parameters) are defined, mostly, by the Poisson's ratio (ν) and Young's modulus (E). In the analysis of the response of the material when submitted to cyclic loads, the modulus of the elasticity is replaced by the resilient modulus to consider the nonlinearity (i.e. dependence on stress level) on the performance of the material, as well as the inelastic properties of the materials, i.e. the loading and unloading of the stress–strain curve are not totally overlapped due to the dissipation of the energy, as depicted in Fig. 1.

Fig. 1 Definition of the recoverable and permanent deformation

Indeed, in the case of repeated load tests with constant confining pressure, the resilient modulus and the Poisson's ratio are defined, respectively, by the following expressions:

$$M_r = \frac{\sigma_d}{\varepsilon_{1;r}}, \nu = -\frac{\varepsilon_{3;r}}{\varepsilon_{1;r}}, \quad (1)$$

where σ_d is the deviatoric stress, $\varepsilon_{1;r}$ is the recoverable axial deformation and $\varepsilon_{3;r}$ is the recoverable horizontal deformation.

The test method used to determine the experimental value of the resilient modulus is described in [8] and is based on the application of deviatoric stress under constant confining stress. The response of the materials varies according to its own nature. The cohesive materials are more sensitive to the deviator stress than the confining pressure since the experimental results show that there is a decrease of the resilient modulus with the increase in the deviatoric stress. On the other hand, the granular materials show a different tendency since the resilient modulus increases with confining stress. The more recent models are dependent on important factors regarding environmental aspects such as moisture content, suction and degree of saturation [9, 35–40]. Additionally, recent work by [41] also proposes a methodology to obtain the damping from these type of tests using the standard AASHTOT-307–93 [42].

The computational modelling of resilient behaviour is a very difficult task, due to the necessity of decoding complex behaviours into simple mathematical expressions and procedures for routine analysis.

3.1 Mechanistic-empirical modelling approach

The empirical approaches can be used in the design procedures of railway structures. This means that a relationship is established between the design inputs (materials, loads, environment, geometry) and, for example, rail track failures through experience, experimentation or a combination of both. The more complex empirical approaches are based on empirical equations derived from experimentations. The relationship between rail track failure and the physical phenomena is described by empirical equations.

The mechanistic-empirical modelling approaches used to determine the resilient modulus are divided according to the type of material tested (granular or cohesive). Furthermore, the models more complex depending on the physical state of the material are also presented. Moreover, in appendix, the state of the art of resilient models developed based on laboratory tests (mostly cyclic triaxial tests) considering all types of materials (such as clays, silts, sands and gravels) is summarized.

3.1.1 Cohesive materials

In the cohesive materials, the fines content and its mineralogy are important factors in the analysis, which means that the resilient modulus should be dependent on the soil physical state. Indeed, in cohesive materials, certain characteristics may have more importance in the value of the resilient modulus, besides the stress state, such as the moisture content, suction and saturation degree. In the work developed by [43], the authors state that the new generation models not only provide a better fit than the older models, but they also provide a reasonable fit to the data that can capture the effects of stress state, soil type, soil structure and the soil physical state quite effectively. Furthermore, according to [44], there are prediction models that combine the effect of the stress state and matric suction on the resilient modulus. These models not only reflect the relationship between the resilient modulus of subgrade soils and the stresses but also the effects of seasonal variation of moisture content on the resilient modulus. The authors also conclude that the degrees of stress and moisture content have a significant impact on the resilient modulus of compacted cohesive soils. Moreover, the results show that the resilient modulus of the tested cohesive soils increases with the increase in effective confining pressure, matric suction, and degree of compaction and decreases with the increase in deviator stress and moisture content. Therefore, lower moisture content concomitantly with a higher degree of compaction is beneficial to the stiffness of the subgrade, mainly in those not sensitive to water content changes.

Considering the extensive bibliography about this topic, several models were developed to describe the resilient modulus and are summarized in Table 1.

Furthermore, it is also important to take into account the type of approach in terms of total and/or effective stresses. In the recent work developed by [52], the developed model can describe the phenomena of modulus development during the cyclic undrained condition and takes into account the actual values of effective stress (p'), excess pore water pressure, the loading characteristic and the position of the effective stress path to the failure line.

3.1.2 Granular materials

The modelling of the resilient behaviour in the case of granular materials can be performed through two different approaches: simulation of the resilient modulus considering a constant value of the *Poisson's* ratio and by separation of the deformation response of the material into the volumetric and shear components, which is more complex [53].

Table 1 Resilient moduli for cohesive materials (adapted from [45])

Models	Expressions	Parameters	Authors
Bilinear	$M_r = K_1 + K_2\sigma_d$ when $\sigma_d < \sigma_{di}$ $M_r = K_3 + K_4\sigma_d$ when $\sigma_d > \sigma_{di}$	K_1, K_2, K_3 and K_4	Thompson and Robnett [46]
Power	$M_r = k\sigma_d^n$	k and n are parameters dependent on the type of soil and physical state; the parameter n usually has a negative value	Moossazadeh and Witczak [47]
Power (with the influence of the confining stress for overconsolidated saturated soils)	$M_r = k\left(\frac{\sigma_d}{\sigma'_3}\right)^n$	σ'_3 is the confining stress	Brown, Lashine [48]
Semi-log	$M_r = 10^{(k-n\sigma_d)}$	k and n are empirical parameters	Fredlund, Bergan [49]
Hyperbolic	$M_r = \frac{k + n\sigma_d}{\sigma_d}$	M_r in ksi and σ_d in psi	Drumm and Pierce [50]
Octahedral	$M_r = k \frac{\sigma_{oct}^n}{\tau_{oct}^m}$	σ_{oct} and τ_{oct} are the shear and normal octahedral stresses	Shackel [51]

The models expressed by the resilient modulus and constant Poisson's ratio are very easy to understand, very simple and easy to implement numerically. In fact, initially, in terms of variables, the models were only dependent on the confining stress and material parameters and/or the sum of the principal stresses [54, 55]. Due to the necessity to include the influence of the deviator stress, [56] modified the well-known k - θ model, introducing the influence of the mean and deviatoric stress (p and q , respectively):

$$E = k_1\theta^{k_2}q^{k_3}, \quad (2)$$

where θ is the sum of principal stresses, and k_1 , k_2 and k_3 are parameters dependent on the state of the soil and its characteristics.

Posteriorly, some models were improved considering, for example, the porosity of the material and variable confining stress [57, 58].

However, some of the previous models show drawbacks concerning the assumption of a constant value of the Poisson's ratio: despite the good results regarding the axial deformations, it was difficult to simulate correctly the volumetric and radial deformations. Indeed, some authors also suggested that the use of these models could lead to Poisson's ratios superiors to 0.5. However, some of these models are still used and were also improved because of their simplicity [59].

Nevertheless, in order to solve Poisson's ratio problem, other models were developed, expressing the resilient behaviour through the shear modulus (G) and bulk modulus (K). This formulation (in terms of G and K) is more comprehensive than the models only characterized by the resilient modulus and Poisson's ratio since it is dependent

on more complex laboratory tests and with a more generic interpretation than a cyclic axial loading.

Indeed, Ref. [48] cited by [60] identifies three important conditions in the application and formulation of these models:

- In each increment of calculus, a linear elastic behaviour is adopted.
- The shear and volumetric components of stress and strain are analysed independently.
- Better adjust to the experimental results, mostly when the solicitation presents a 3D character.

Reference [61] developed a well-known model dependent on bulk modulus and shear modulus:

$$K = \frac{p}{\varepsilon_v}, \quad (3)$$

$$G = 3 \frac{q}{\varepsilon_q}, \quad (4)$$

where ε_v and ε_q are respectively the volumetric and shear resilient strains:

$$\varepsilon_v = \frac{1}{K_1} p^n \left[1 - \beta \left(\frac{q}{p} \right)^2 \right], \quad (5)$$

$$\varepsilon_q = \frac{1}{3G_1} p^n \left(\frac{q}{p} \right), \quad (6)$$

K_1 , β and G_1 are parameters of the model, with $\beta = (1 - n)K_1/6G_1$.

In this model, there is a clear division between the volumetric and shear deformation and the values of K and G are stress-dependent (p and q , respectively). Posteriorly, this model was updated by other authors [53, 62–64] in

order to consider the anisotropy of the material, which is an important characteristic regarding the deposition and compaction conditions. Indeed, [63] and [53] purpose and apply an orthotropic version of Boyce's model introducing an anisotropic coefficient (γ).

3.2 Models dependent on the physical state—suction and degree of saturation

Recently, some studies were developed in order to include the physical state into the resilient models [9, 35, 36, 65].

In this section, a brief reference is made regarding the models dependent on the suction and degree of saturation. These models were developed based on experimental results and include suction as one of the main parameters. In the case of non-saturated soils, three important variables influence recoverable/reversible behaviour [66]:

- net confining stress: $\sigma_3 - u_a$;
- deviatoric stress: $\sigma_1 - \sigma_3$;
- suction matrix: $\psi_m = u_a - u_w$,

where σ_3 is the confining stress, σ_1 is the axial stress, u_a is the air pressure and u_w is the water pressure.

Based on Uzan's model, Ref. [67] developed a model that includes the suction in the determination of the resilient modulus:

$$M_r = k_1 p_a \left(\frac{\theta_{\text{net}} - 3\Delta u_{w-\text{sat}}}{p_a} \right)^{k_2} \left(\frac{\tau_{\text{oct}}}{p_a} + 1 \right)^{k_3} \left(\frac{\psi_m - \Delta\psi_m}{p_a} + 1 \right)^{k_4}, \quad (7)$$

where p_a is atmospheric pressure; τ_{oct} is octahedral shear stress; θ_{net} is net bulk stress, $\theta_{\text{net}} = \theta - 3u_a$, and θ is bulk stress; $\Delta\psi_m$ is the variation of the suction matrix regarding the initial suction ($\Delta u_{w-\text{sat}} = 0$); and $\Delta u_{w-\text{sat}}$ represents the increase of the pore pressure in saturated conditions ($\psi_m = 0$).

Despite the simplicity of the model formulation (which is based on the work developed by [56]), the characterization of the suction increases the complexity and application of these types of models since this is a parameter that is difficult to characterize.

Recent studies show the influence of the water content (with special attention on the role of the suction in the interpretation of the results) and consider the possibility to apply an effective stress approach to characterize the changes in normalized resilient modulus as a function of a single parameter that includes the total stresses and suction [37]. Moreover, some studies are focused on the importance of the degree of saturation (and the controlling in the soil compaction) and its relationship with the soil structure design. In fact, the degree of saturation and also the CBR (California bearing ratio) are parameters that are easy to obtain and easy to use when compared to the suction. The authors conclude that CBR (soaked and unsoaked) and also

the elastic shear modulus (representative of the stress-strain behaviour at small strains of the subgrade), unconfined compression strength, cyclic undrained shear strength, among other parameters, of unsaturated soil are controlled by ρ_d and S_r at the end of compaction [39]. More specific studies about compaction conditions and percentage of fines can be found in [68] and [69].

3.3 Main differences of the resilient modulus formulation—analysis and comparison

As previously described, the resilient behaviour is dependent on several factors such as the stress level, fines contents, particle shape, grading, density, maximum grain size, aggregate type, moisture content, stress history and number of load cycles. From all of them, two important factors stand out: the applied stress level and moisture content. The stress (in terms of confining stress, sum of principal stresses and deviator stress) is the most important factor in the analysis of the resilient behaviour since its impact on the response of the material is very significant. Indeed, most of the geomaterials (especially the granular materials) show a stress dependency under repeated loads. To replicate this nonlinearity and time dependency, the resilient response of the materials is not solved by the traditional elasticity theory. However, as mentioned previously, there are other formulations to characterize the response of the material such as the replacement of the resilient modulus and Poisson's ratio by the shear and bulk modulus.

This formulation is simple and based on curve-fitting. Indeed, this approach deals better with the nonlinear response of the geomaterials (mostly in the case of granular materials) from a theoretical point of view, and it is more realistic regarding the physical meaning in a 3D stress regime. However, as mentioned previously, these models have a more complex formulation and the parameterization is also difficult to obtain through the available test data [70].

Another relevant aspect is related to the fact that some of the resilient models only fit the laboratory data used for their own development, which means that the mathematical formulation can only be used in a particular situation.

The models that include the influence of the moisture content and/or degree of saturation describe better the material conditions. In the case of the granular materials, the granulometry and the fines content (in a compacted well-graded material) have a significant influence on the response of the material since the water cannot drain freely since it is kept in the pores of the materials [71]. Indeed, when the saturation is close to the maximum, the resilient behaviour is affected [72]. Nevertheless, these models are more complex since it is difficult to characterize these parameters related to the suction and degree of saturation.

More recently, some resilient models have been developed using machine learning and statistical techniques and optimization tools as described in the work developed by [73] and [74].

Despite the several advantages of the empirical models, the main limitation is related to the confidence in the extrapolation of the analysis beyond the conditions in which they were defined. The tables presented in appendix (Tables 17–21) contain information regarding the type of soils (UIC and ASTM classification), source (authors), the mathematical models and respective variables and empirical constants. This information is quite significant in the modelling of the subgrade of the railway tracks since supports information about the materials used to define the model, as well as its physical conditions such as moisture content and dry density.

4 Permanent deformation

The permanent deformation is induced by repeated traffic loading and may occur in railway structures. The development of permanent deformation is the result of the accumulation of deformation throughout millions of cycles, which is a complex process and depends on several factors such as the number of load cycles (N), stress levels, loading history, the effect of the principal stress rotation, moisture content, density, aggregate type and particle size distribution. This development of permanent deformation during N loading cycles can stabilize or, in the worst situation, may lead to the ultimate collapse of the structure (excessive rutting). Regarding the moisture content, despite the extensive research about the influence of this parameter on the reversible deformation, a recent study was developed to study the effects of the compaction moisture content on the plastic behaviour of fine soils [75].

To predict permanent deformation, several approaches were proposed: numerical simulations using elastoplastic models, shakedown theory or mechanistic-empirical permanent deformation models based on laboratory tests such as cyclic triaxial tests or the hollow cylinder tests [19]. Throughout the analyses presented by these authors, more emphasis is given to the mechanistic-empirical permanent deformation models from the laboratory testing until the modelling. In fact, the work done in this paper for the resilient modulus is a mirror of the one for permanent deformation.

5 Geomechanics classification and the rating

5.1 Resilient modulus: parametric study

Considering the extended information presented in Sect. 3 and also the tables presented in appendix, a first attempt was done to compare the resilient modulus of different materials under a certain stress level. This analysis is important to comprehend how the models differ from each other in terms of formulation and the importance of certain variables.

The models were calibrated considering the same reference stress path for all materials during the calibration process. The stress path described by [76] was used. This stress path is characterized by a cyclic deviator stress of 24 kPa and a constant confining stress of 60 kPa. During the cyclic tests, the stress ratio (σ_d/σ_c) was kept constant at 0.4, which is a representative ratio in the subgrade of a rail track full-scale model test. It is noted that other confining stresses from 60 to 210 kPa were also tested.

In this analysis, four models were applied in order to compare the resilient modulus for the coarse-grained soils, as depicted in Table 2. In order to facilitate the interpretation of Table 2, C and m are material constants and σ'_v is the effective vertical stress (more details in Tables 19 and 20)).

The first part of this work is based on the experimental results presented in [80]. The authors studied six granular materials representative of the base and sub-base materials used on flexible pavements. [80] performed rapid shear tests and repeated load tests in order to determine the shear strength parameters (cohesion and friction angle), resilient modulus, rutting potential and moisture susceptibility. The properties of the materials are presented in Table 3.

The constants of each model were determined by regression analyses. The values of these constants reflect the material properties as well as the loading cycle applied in each test that may vary from author to author. For example, in the work developed by [80], the specimens were conditioned for 1000 load repetitions while in the

Table 2 Models considered in the analysis of the coarse-grained soils

Refs.	Models
[55, 77, 78]	$M_r = k_1(\theta)^{k_2}$
[56]	$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{q}{p_0} \right)^{k_3}$
[79]	$E_v = C \sigma_v'^m$
[59]	$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{\tau_{oct}}{p_0} + 1 \right)^{k_3}$

Table 3 Properties of the materials (coarse-grained soils)

No.	UIC and ASTM classification	Materials	Properties	Observations
1	QS3 GW	Well-graded gravel	$C_u = 27$; $C_c = 2$ ($PI_{\min} = NP$; $PI_{\max} = 6$) $W_{L;\max} = 25$ $W = 6.3\%$ $W_{OMC} = 6.8\%$	Granular material used as based and sub-base in the flexible pavement sections;
2	QS2/QS3 SW/SM-ML	Sand	$C_u = 20$; $C_c = 0.8-1.1$ ($PI_{\min} = NP$; $PI_{\max} = 6$) $W_{L;\max} = 25$ $W = 7.7\%$ $W_{OMC} = 7.7\%$	The target moisture content and densities were selected based on AASHTO T99 test results and field-measured values [80]

Table 4 Summary of the values of the regression analysis based on experimental results

Models	Material classification	Parameters	Regression analysis R^2
$M_r = k_1 \theta^{k_2}$	Well-graded gravel	$k_1 = 5176$	0.997
	QS3-GW	$k_2 = 0.64$	
	Sand	$k_1 = 36,942$	0.933
	QS2/QS3-SW/SM-ML	$k_2 = 0.32$	
$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{q}{p_0} \right)^{k_3}$	Well-graded gravel	$k_1 = 5306$	0.998
	QS3-GW	$k_2 = 0.59$ $k_3 = 0.05$	
	Sand	$k_1 = 34,336$	0.942
	QS2/QS3-SW/SM-ML	$k_2 = 0.45$ $k_3 = -0.15$	
$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{\tau_{oct}}{p_0} + 1 \right)^{k_3}$	Well-graded gravel	$k_1 = 1.02$	0.997
	QS3-GW	$k_2 = 0.62$ $k_3 = -0.01$	
$E_v = C \sigma_v'^m$	Well-graded gravel	$C = 6768$	0.988
	QS3-GW	$m = 0.65$	

work developed by [9], the authors follow the procedures described in AASHTO T307-99. However, for practical purposes, the conditioning adopted for resilient modulus determination following the standard procedures (AASHTO T307-99, [40]) intends to assure that the number of cycles during resilient modulus determination will not affect the results. The results of the four models and the two materials are presented in Table 4. It is important to refer that for material 2, due to the lack of data, it was not possible to present the regression results for the model developed by [79] and MEPDG model. Analysing Table 4, in general, the models present similar r -squared values. Therefore, considering the obtained constants and the stress path selected, the models were compared in terms of the

resilient modulus, as depicted in Table 5. It can be observed that the models present similar results. Material 2 presents an inferior resilient modulus when compared to the well-graded gravel, as expected. Regarding the material classified as QS3-GW, it is evident the close values between the Uzan's model, $k-\theta$ and MEPDG model.

After the analysis of the coarse-grained soils, the fine-grained soils were evaluated. In the case of the fine-grained soils, four models (Table 6) and two materials (Table 7) were compared. The materials were tested through the triaxial test and both materials were compacted at optimum water content and 2% above the optimum. The measured resilient modulus is based on the work developed by [9].

Table 5 Determination of the resilient modulus for the coarse-grained materials

Models	Material classification	Resilient modulus (MPa)
$M_r = k_1 \theta^{k_2}$	Well-graded gravel QS3-GW Sand QS2/QS3-SW/SM-ML	156 109
$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{q}{p_0} \right)^{k_3}$	Well-graded gravel QS3-GW Sand QS2/QS3-SW/SM-ML	143 131
$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{\tau_{\text{oct}}}{p_0} + 1 \right)^{k_3}$	Well-graded gravel QS3-GW	159
$E_v = C \sigma_v'^m$	Well-graded gravel QS3-GW	118

Table 6 Models selected for the fine-grained soils

Refs.	Models
[59]	$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{\tau_{\text{oct}}}{p_0} + 1 \right)^{k_3}$
[35]	$M_r = k_1 (\sigma_d + \chi \psi_m)^{k_2}$
[9]	$M_r = k_1 p_a \left(\frac{\theta + \chi \psi_m}{p_a} \right)^{k_2} \left(\frac{\tau_{\text{oct}}}{p_a} + 1 \right)^{k_3}$
[81]	$E_r = A + B p'_0 + C q_r$

Considering the results obtained by [9] regarding the measured resilient modulus, the constants of the parameters were determined through regression analyses (Table 8). However, it is important to refer that the MEPDG model is defined in terms of total stresses, which means that the regression analysis is performed for each material on both moisture contents (opt and opt + 2%). The remaining models are dependent on effective stresses, which means that the regression analysis is performed considering the whole set of moisture content (opt and opt + 2%). This

type of information allows analysing the sensibility of the resilient modulus regarding the moisture content. The models developed by [9, 35] and [81] are dependent on the ψ_m and χ_m parameters, which means that the results are presented considering M_r^{opt} and $M_r^{\text{opt}+2\%}$, although the regression is performed with the whole set (opt and opt + 2%).

The model developed by [81] is simpler than the remaining models, and the influence of the moisture content is indirectly included in the suction matrix in the initial mean normal stress (p'_0). From multiple linear regression analyses, the empirical constants were determined, and the values are presented in Table 8.

From the constant parameters and the selected stress path, the resilient moduli of the materials were found, as depicted in Table 9.

Considering the obtained results for the fine- and coarse-grained materials, the MEPDG model [59] will be adopted in the following analysis, despite its complexity when compared to the remaining models. The main reason is related to its universal character since the model can be

Table 7 Properties of the fine-grained materials

No.	UIC and ASTM classification	Materials	Properties	Observations
1	QS1-CL Low plasticity	Lean clay	$W_L = 30.8\%$ $W_P = 18.4\%$ $IP = 12.3\%$	The effect of the matric suction will be included in the effective stress
2	QS1-CL-ML Low plasticity	Silt-lean clay	$W_L = 27.8\%$ $W_P = 19.8\%$ $IP = 8\%$	The triaxial tests following the AASHTO T307-99 procedure were conducted on both materials

Table 8 Regression results for the fine-grained materials

Models	Material classification	Parameters	Regression analysis R^2
$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{\tau_{oct}}{p_0} + 1 \right)^{k_3}$	QS1-CL	$k_1 = 0.55$	0.98
	Low plasticity	$k_2 = 0.15$	
	opt	$k_3 = -1.82$	
	QS1-CL	$k_1 = 0.35$	0.90
	Low plasticity	$k_2 = 0.14$	
	opt + 2%	$k_3 = -1.61$	
	QS1-CL-ML	$k_1 = 0.97$	0.96
	Low plasticity	$k_2 = -0.17$	
	Opt	$k_3 = -1.97$	
	QS1-CL-ML	$k_1 = 0.68$	0.87
	Low plasticity	$k_2 = -0.10$	
	opt + 2%	$k_3 = -1.13$	
$M_r = k_1 (\sigma_d + \chi \psi_m)^{k_2}$	QS1-CL	$k_1 = 5.45$	Difficult adjustment
	Low plasticity	$k_2 = 0.36$	
	QS1-CL-ML	$k_1 = 72.11$	Difficult adjustment
	Low plasticity	$k_2 = -0.04$	
$M_r = k_1 p_a \left(\frac{\theta + \chi \psi_m}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3}$	QS1-CL	$k_1 = 0.24$	0.62
	Low plasticity	$k_2 = 0.85$	
		$k_3 = -2.29$	
	QS1-CL-ML	$k_1 = 0.78$	0.61
	Low plasticity	$k_2 = 0.13$	
		$k_3 = -2.01$	
$E_r = A + B p'_0 - C q_r$	QS1-CL	$A = 3499.51$	0.91
	Low plasticity	$B = 261.25$	
		$C = -190.78$	
	QS1-CL-ML	$A = 60,443.76$	0.69
	Low plasticity	$B = -167.96$	
		$C = -474.39$	

Table 9 Resilient modulus of the fine-grained materials

Models	Material classification	Resilient modulus (MPa)
$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{\tau_{oct}}{p_0} + 1 \right)^{k_3}$	QS1-CL	50 (opt)
	Low plasticity	33 (opt + 2%)
	QS1-CL-ML	70 (opt)
	Low plasticity	56 (opt + 2%)
$M_r = k_1 (\sigma_d + \chi \psi_m)^{k_2}$	QS1-CL	Difficult adjustment
	Low plasticity	
	QS1-CL-ML	Difficult adjustment
$M_r = k_1 p_a \left(\frac{\theta + \chi \psi_m}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3}$	QS1-CL	53 (opt)
	Low plasticity	47 (opt + 2%)
	QS1-CL-ML	73 (opt)
	Low plasticity	72 (opt + 2%)
$E_r = A + B p'_0 - C q_r$	QS1-CL	83 (opt)
	Low plasticity	76 (opt + 2%)
	QS1-CL-ML	80 (opt)
	Low plasticity	71 (opt + 2%)

Fig. 2 Resilient modulus predicted by MEPDG for the different types of soil

applied in the prediction of the resilient modulus of all types of geomaterials.

This analysis shows a possible way to rank materials according to the resilient modulus, as depicted in Fig. 2. In the case of the material classified as QS2/QS3-SW/SM-ML, instead of the MEPDG model, the Uzan's model was used due to the lack of information. (It was not possible to perform a regression analysis.) However, as the previous analysis shows, the MEPDG and Uzan's models present similar results. All the materials present in Fig. 2 are at optimum moisture content, except the material classified as QS2/QS3 SW/SM-ML. In this case, the moisture content is close to the optimum conditions.

5.2 Permanent deformation: parametric study

The comparison regarding the permanent deformation models was already presented in the work developed by [19]. As in the previous exercise, in this parametric study, the authors attempted to compare different permanent deformation models available for different types of soils considering the empirical permanent deformation model developed by [76]. Thus, the results showed that two soils can be integrated into the same classification (UIC or ASTM), but the laboratory conditions may differ greatly and therefore lead to different results in terms of permanent deformation. The calibration was performed to find the best-fit for the experimental data through the parameters $\varepsilon_{\pi,0}$, α and B (which correspond to the model constants

associated with the material properties). The models were calibrated considering the same stress paths (described by [76]). As in the resilient modulus analysis, the materials selected are representative of different types of materials.

In order to understand better the ranking, the materials selected for the preliminary analysis of the permanent deformation are depicted in Table 10. This table also includes information about the ASTM and UIC classifications. To complete the information about the materials, the physical properties, state conditions and strength properties of the materials are depicted in Tables 11, 12 and 13, which would be extremely important to understand the novel geomechanical classification purposed.

In the work developed by [19], the ranking process is described. The final ranking is depicted in Fig. 3.

6 Geomechanical classification

Considering the results presented in the previous sections, two models were selected to characterize the resilient modulus and permanent deformation of the geomaterials. Indeed, this is an attempt to relate the resilient modulus with the permanent deformation under a certain stress level. As in the previous analysis, the selected stress path is characterized by a cyclic deviator stress equal to 24 kPa and a confining constant stress equal to 60 kPa. The permanent deformation was determined considering a number of load cycles equal to 30,000. The selected materials are

Table 10 Materials selected for the preliminary analysis of the permanent deformation [19]

Refs	Soils	ASTM classification	UIC classification	Observations
[76]	Sand	SP	QS2	Compacted at field conditions and saturated
	Silt	CL-ML	QS1	Compacted at field conditions and saturated
[82] and [66]	Silty sand (42.2% fines)	SM	QS1	Compacted at optimum compaction—standard proctor
	Silty sand (27.4% fines)	SM	QS1	Compacted at optimum compaction —standard proctor
[83]	Well-graded Sand	SP	QS2	Compacted at optimum compaction—standard proctor
	Poor-graded Sand	SW	QS3	Compacted at optimum compaction—standard Proctor

Table 11 Physical properties of the materials—granulometry [19]

Material Properties	C. Bruynweg [83]	Crusher [83]	Silty clay (42.2% fines content) [82]	Silty clay (27.4% fines content) [66]	Silt [76]	Coarse sand [76]
C_u	2.100	10.50	28.0	33.0	2.51	4.68
C_c	1.050	1.250	0.54	0.75	1.32	0.62
D_{10}	0.148	0.217	—	—	—	—
D_{30}	0.219	0.784	—	—	—	—
D_{50}	0.280	1.722	—	—	—	—
D_{60}	0.310	2.280	—	—	—	—
CBR (%)	22.00	15.70	—	—	—	—

Table 12 State conditions of the materials [19]

Material properties	C. Bruynweg [83]	Crusher [83]	Silty clay (42.2% fines content) [82]	Silty clay (27.4% fines content) [66]	Silt [76]	Coarse sand [76]
$\gamma_{\text{dry,max}}$ (kg/m ³)	1723	1755	1998	2070	1620	2110
W_{opt} (%)	12.5	10.5	10.1	7.6	—	—
γ_{wet} (kg/m ³)	1942	1937	—	—	—	—
Liquid limit (%)	Non-plastic	Non-plastic	Non-plastic	Non-plastic	35	Non-plastic
Plastic limit	Non-plastic	Non-plastic	Non-plastic	Non-plastic	24	Non-plastic

Table 13 Strength properties of the materials [19]

Material properties	C. Bruynweg [83]	Crusher [83]	Silty clay (42.2% fines content) [82]	Silty clay (27.4% fines content) [66]	Silt [76]	Coarse sand [76]
ϕ (°)	48.2	50.20	36.18	45.66	11.7*	0.00*
c (kPa)	5.60	8.680	15.82	10.43	16.4*	33.0*
s (kPa)**	9.90	14.90	31.80	19.10	24.8	0.00
m **	1.98	2.070	1.470	1.880	0.62	1.33

* In this case, the author described the saturated samples (silt and coarse sand) in ϕ' and c'

** s and m are the parameters used to define the failure envelope: $q = s + mp$

presented in Table 14. Some of the materials are the same used in Sects. 5.1 and 5.2, and others were added to increase the robustness of the analysis, the ranking and the impact of the final result of the geomechanical classification. These “auxiliary” data only have information about the resilient modulus or permanent deformation (they were used to define the “limits” of the classification). The work developed by [84] was also included but due to the significant information about both types of deformation (resilient and permanent) under different water content conditions. It is important to refer that, for these particular materials, are presented, whenever it is possible, the values of both deformations: resilient and permanent deformation.

This table also includes information about compaction conditions.

The resilient modulus and permanent deformation of the selected materials (Table 14) are presented in Table 15. The materials with both values of M_r and ε_p allow validating the mechanical classification.

From the results presented in Table 15, it is possible to define guiding/limit values associated with the UIC classification: QS1, QS2 and QS3 [31]. This work only presents a preliminary geomechanical classification. The lack of information about the models' parameters found in the bibliography (namely the models that include the suction and other complex model coefficients) and the necessity and difficulty to performed back analysis to find important

Fig. 3 Permanent deformation predicted by Chen et al. (2014)'s model for different soils ($N = 30,000$ cycles) under a stress ratio $\sigma_1/\sigma_{1,f} = 0.4$ [19]

parameters (as the cohesion and friction angle used to define the failure criterion) reduced the number of models and materials that could be used in this geomechanical classification. Besides, some of the models found in the bibliography tested materials classified as QS0, which implies its elimination in the geomechanical classification, since these materials require mechanical improvement.

The geomechanical classification is an attempt to define a novel helpful guide to be used as a support for the modelling and design of the substructure. Indeed, this novel geomechanical classification tries to rank soils in a very similar way developed by [20] that was inspired by [40]. This work allows understating which are the acceptable values of the permanent deformation and resilient modulus according to the type of material and its classification. In this particular case, [31] classification was used as depicted in Fig. 4.

Analysing Fig. 4, a significant range of values was obtained for each classification. This was expected due to the mineralogical nature and physical properties of each material, which can have an influence on several parameters used in the determination of the resilient modulus and permanent deformation. Indeed, in the case of the materials classified as QS1 (fine soils), the type of the material (CL, CH, ML, or MH) associated with its plasticity's characteristics and consistency index (as well as the percentage of fines) can have a significant impact in the permanent deformation and resilient behaviour, as depicted in Fig. 5. For the materials classified as QS2 and QS3, the type of materials (sand or gravel) as well as its granulometry (well- or poor-graded) and the percentage of fines (in the well-graded materials) can also influence the response of the material in terms of resilient and permanent deformations.

With this work, it was possible to define subsets associated with the properties of the soils, despite the need for more information. For example, as Fig. 5 shows, a silty soil

(classified as QS1-ML) presents a superior resilient modulus when compared to clayed soils (QS1-CL). Regarding the granular materials, the subsets can be defined by the granulometry of the materials (well- or poor-graded). However, it is important to refer that the percentage of fines can also influence the obtained results. Indeed, despite the importance of the granulometry and plasticity properties, the fines content and the moisture content have also significant importance, mostly in the fine soils. As mentioned previously, the work developed by [84] shows the influence of the moisture content in the soils. Thus, [84] selected and studied the behaviour of the clay considering three compaction moisture content and dry unit weight conditions representing dry of optimum moisture content level, optimum moisture content level (corresponds to 95% of maximum dry unit weight), and wet of optimum moisture content level. From the results obtained by [84], an estimation was performed (in terms of resilient modulus and permanent deformation), and the results are presented in Figs. 6 and 7. These results are based on the properties of the clay described in Table 16. Analysing Fig. 7, the moisture content can influence both deformations, which means that can affect severely the track performance. In a more detailed analysis, it is possible to identify a reduction of around 200% in the resilient modulus from dry to wet conditions. This difference is significant, which means that the moisture content is an important parameter since influences the performance of the subgrade and corroborates the findings of [20, 86] in the case of non-standard unbound granular materials for pavements.

Table 14 Properties of the materials

Information	UIC and ASTM classification	Materials	Properties	Observations
[50]	QS1-CL	–	$W_L = 28.5\%$; $W_P = 19.2\%$; $IP = 9.3\%$; $G_s = 2.73$; Clay content 20.0%; Passing #200 73.0%; Maximum dry density 17.36 kN/m ³	–
From the resilient modulus parametric study	QS1 Low plasticity CL	Lean clay	$W_L = 30.8\%$; $W_P = 18.4\%$; $IP = 12.3\%$	The effect of the matric suction will be included in the effective stress; the triaxial tests following the AASHTO T307-99 procedure were conducted on both materials
[84]	QS1 Lean clay (CL)	Silty clay	$W_L = 28.19\%$; $G_s = 2.63$; $IP = 12.55\%$ (low plasticity); Passing #200 80%; Maximum dry density 17.10 kN/m ³ ; $W_{opt} = 17.11\%$; $c = 60$ kPa; $\phi = 18^\circ$	Compacted at optimum compaction—standard proctor
From the permanent deformation parametric study	QS1 CL-ML	Silt	$W_L = 35\%$; $W_P = 24\%$; $IP = 11\%$ (low to medium plasticity); $G_s = 2.67$; $C_u = 2.51$; $C_c = 1.32$; Maximum dry density 15.89 kN/m ³ ; Hydraulic permeability 5.3×10^{-7} m/s;	Compacted at field conditions and saturated
From the resilient modulus parametric study	QS1 Low plasticity CL-ML	Silt-lean clay	$c' = 11.7$ kPa; $\phi' = 16.4^\circ$; $W_L = 27.8\%$; $W_P = 19.8\%$; $IP = 8\%$	The effect of the matric suction will be included in the effective stress; The triaxial tests following the AASHTO T307-99 procedure were conducted on both materials

Table 14 continued

Information	UIC and ASTM classification	Materials	Properties	Observations
From the permanent deformation parametric study	QS1 SM	Silty sand (42.2% fines)	$C_u \approx 28$; $C_c \approx 0.54$; $G_s = 2.68$; $W_{opt} = 10.1\%$; Fines content = 42.2%; Maximum dry density 19.6 kN/m ³	Compacted at optimum compaction—standard proctor
From the permanent deformation parametric study	QS1 SM	Silty sand (27.4% fines)	$C_u \approx 33$; $C_c \approx 0.75$; $G_s = 2.67$; $W_{opt} = 7.6\%$; Fines content = 27.4%; Maximum dry density 20.3 kN/m ³	Compacted at optimum compaction—standard proctor
[84]	QS1 Silt (SM)	Silty clayed sand	$W_L = 16.70\%$; $G_s = 2.70$; IP = 7.50% (low plasticity); Passing #200 38%; Maximum dry unit weight 16.9 kN/m ³ ; $W_{opt} = 19.3\%$; $c = 103$ kPa; $\phi = 35^\circ$	Compacted at optimum compaction—standard proctor
From the permanent deformation parametric study	QS2 SP	Well-graded sand	$C_u = 2.10$; $C_c = 1.05$; Maximum dry density 16.90 kN/m ³ ; $W_{opt} = 12.5\%$; $c = 5.60$ kPa; $\phi = 48.2^\circ$	Compacted at optimum compaction—standard proctor
[58]	QS2 SP	Poorly graded Sand	$G_s = 2.74$; $C_u \approx 2.0$; $C_c \approx 1.04$	—
[83]	QS2 SP	Poorly graded Sand	$C_u \approx 1.88$; $C_c \approx 1.04$; Maximum dry density 16.56 kN/m ³ ; $W_{opt} = 14.0\%$; $c = 6.34$ kPa; $\phi = 41.8^\circ$	Standard proctor
From the permanent deformation parametric study	QS2 SP	Sand	$G_s = 2.66$; $C_u = 4.8$; $C_c = 0.62$; Maximum dry density 20.69 kN/m ³ ; Minimum dry density 15.89 kN/m ³ ; Hydraulic permeability 3.2×10^{-5} m/s; $c' = 0$ kPa; $\phi' = 33^\circ$	Compacted at field conditions and saturated

Table 14 continued

Information	UIC and ASTM classification	Materials	Properties	Observations
[84]	QS2 SP	Poorly graded sand	$W_L = 26.40\%$; $G_s = 2.71\%$; Passing #200 0.70%; $C_u = 1.79$; $C_c = 0.89$; Maximum dry unit weight 15.70 kN/m^3 ; $W_{opt} = 13.70\%$; $c = 20 \text{ kPa}$; $\phi = 42^\circ$ $LA = 22$; $MED = 15$; Fines content = 10%	Compacted at optimum compaction—standard proctor
[85]	QS2-QS3	—	$C_u = 20$; $C_c = 0.8-1.1$; $(PI_{min} = NP; PI_{max} = 6)$ $W_L, max = 25$; $W = 7.7\%$; $W_{OMC} = 7.7\%$	Modified proctor
From the resilient modulus parametric study	QS2/QS3 SW/SM-ML	Sand	$C_u = 20$; $C_c = 0.8-1.1$; $(PI_{min} = NP; PI_{max} = 6)$ $W_L, max = 25$; $W = 7.7\%$; $W_{OMC} = 7.7\%$	Granular material used as base and sub-base in the flexible pavement sections; The target moisture content and densities were selected based on AASHTO T99 test results and field-measured values [80]
From the permanent deformation parametric study	QS3 SW	Poor-graded sand	$C_u = 10.5$; $C_c = 1.25$; Maximum dry density 17.21 kN/m^3 ; $W_{opt} = 10.5\%$; $c = 8.68 \text{ kPa}$; $\phi = 50.2^\circ$	Compacted at optimum compaction—standard proctor
From the resilient modulus parametric study	QS3 GW	Well-graded gravel	$C_u = 27$; $C_c = 2$; $(PI_{min} = NP; PI_{max} = 6)$ $W_L, max = 25$; $W = 6.3\%$; $W_{OMC} = 6.8\%$	Granular material used as base and sub-base in the flexible pavement sections; The target moisture content and densities were selected based on AASHTO T99 test results and field-measured values [80]
[58]	QS3 GW	Well-graded gravel	$G_s = 2.69$; $C_u \approx 61$; $C_c \approx 2.11$	—

Table 15 Resilient modulus and permanent deformation of the selected materials

Information	Materials classification	Resilient modulus (Mpa)	Permanent deformation ε_p (%)
[50]	QS1-CL	37	–
From the resilient modulus parametric study	QS1-CL	50	–
	Low plasticity		
[84]	QS1-CL	–	0.01864
From the permanent deformation parametric study	QS1-CL/ML	–	0.03126
	Silt		
From the resilient modulus parametric study	QS1-CL-ML	70	–
	Low plasticity		
From the permanent deformation parametric study	Silty sand (42.2% fines)	258	0.01385
	QS1-SM		
From the permanent deformation parametric study	Silty sand (27.4% fines)	294	0.01251
	QS1-SM		
[84]	QS1-SM	511	0.00658
From the permanent deformation parametric study	Well-graded sand	509	0.00003
	QS2-SP		
[58]	QS2-SP	673	–
[83]	QS2-SP	–	9.85×10^{-10}
	Sand	–	0.01170
[84]	QS2-SP	172	0.01873
	QS2-QS3	–	0.0002
[85]	Sand	131	–
	QS2/QS-SW/SM-ML		
From the permanent deformation parametric study	Poor-graded sand	178	0.00084
	QS3-SW		
From the resilient modulus parametric study	Well-graded gravel	159	0.00320
	QS3-GW		
[58]	QS3-GW	862	–

Fig. 4 Mechanical classification based on the resilient modulus and permanent deformation

Fig. 5 Variation of the type of materials with the classification QS1

Fig. 6 Variation of the moisture content with the permanent deformation (clay)

Fig. 7 Variation of the moisture content in the clay with the resilient modulus and permanent deformation

7 Conclusions

The capability to determine the resilient modulus and also the permanent deformation of geomaterials is imperative in the modelling, design and evaluation of the performance of railway structures. This work attempts to identify and summarize the main models to estimate the resilient modulus and also permanent deformation. At the beginning of the analysis, the most simplistic models are presented. In the case of the resilient modulus, the models should include the influence of the deviatoric and confining stresses and the constant parameters should reflect the influence of other factors as the moisture content and physical state of the material. The mechanistic-empirical models summarized in Annex were divided according to the type of material (clay, silt, sand and gravel) to better understand the model's formulation and the conditions of the material when tested in terms of its granulometry (C_u and C_c) and plasticity properties (W_L , W_P , and IP). Tables also include some observations regarding the laboratory tests and classification of the material (UIC and ASTM).

From the available mechanistic-empirical approach, it was important to evaluate the robustness of the models and their sensitivity. Thus, some models and materials were selected to perform a parametric study in terms of resilient modulus. A similar analysis was already performed for the permanent deformation in the work developed by [19]. The selection depended on the data available for each material and the variability of the geomaterials (type of soils) in terms of granulometry, percentage of fines, moisture content and plasticity properties. The analysis shows better results in the case of the coarse-grained soils. (The R -

Table 16 Properties of the clay [84]

Clay	W_{opt} (%)	ϕ (°)	c (kPa)	Liquid limit (%)	Plasticity index (%)	Maximum dry unit weight (kN/m ³)
Optimum	17.11	18	60	28.19	12.55	17.10
Dry of optimum	11.10	8	152	–	–	–
Wet of optimum	21.50	16	42	–	–	–

squared value is higher and the obtained values for resilient modulus are similar.) This fact can be explained by the complexity of the nature of fine-grained soils, namely the variation of its properties in terms of plasticity and fines content. At the end of this analysis, one model was selected to rating the geomechanical performance. The MEPDG model shows better results for both materials. Indeed, this choice is also justified by its universal character, which means that can be applied to all types of materials. As expected, the comparison of the results of the resilient modulus shows that the values increase with the UIC classification: higher resilient modulus is related to well-graded materials (QS2 and QS3).

The geomechanical classification is based on the parametric studies of the resilient modulus (applying the MEPDG model) and the permanent deformation (applying Chen's model). Moreover, other materials were added to this analysis because of the available information about the moisture content. The classification presents a wide range of results in terms of permanent deformation and resilient modulus. This range is a reflection of the variation of the properties of the materials with the same classification (QS1, QS2 and QS3) that influence both types of deformations. Indeed, this work is the first attempt to obtain a novel geomechanical classification that can be a helpful tool in the modelling of the foundation of the railway structures since gather information about the UIC classification (which is a reference in the railway works) and permanent and resilient deformations. Nevertheless, several limitations are recognized in this proposal, which were discussed throughout this paper. These limitations should be overcome in future work updating this proposal with a wide range of geomaterials properly characterized (physical and mechanical characteristics) and tested under different state conditions (deviation to the optimum

compaction conditions), which should include variations of water content /suction/degree of saturation and density.

Appendix

An extensive list of resilient models based on laboratory tests is presented in this appendix. Although these tables were constructed uniformly (especially regarding the nomenclature of the variables and the parameters and characteristics of the materials), the authors maintained the original symbols described in the original paper. For example, the symbol E_r is used to describe the resilient modulus, which is usually represented by the symbol M_r .

Some models were developed using certain SI units, and where possible, these units are described in the column "variables and empirical constants".

Regarding the resilient modulus tables, [87] defined w_c as optimum moisture content (which is equal to W_{opt}) and w_{cr} as the relation between the actual moisture content and the optimum moisture content. In this analysis, the fines are measured through the passing sieve #200 or through the percentage of fines. Furthermore, the maximum dry unit weight is also defined as maximum dry density. The constants such as A , B , C or k_i are defined as fitting, regression, material and model parameters according to the original formulation defined by the author. The symbols p_a and p_0 are defined as atmospheric stress/pressure and reference stress and it is different from p_u which means unit pressure.

Regarding the stress variables, the symbol θ is defined as bulk stress or the sum of principal stresses, and the deviator stress can be represented by the symbols q , q_r and σ_d . The symbol q is different from q_m which represents the mean value of the deviator stress (Tables 17–21).

Table 17 Models that describe resilient modulus of fine-grained soils—clays

UIC	ASTM classification	Ref.	Equation model	Variables and empirical constants	Parameters and characteristics	Observations
QS0*	CH (no information about the particle size curve) High plasticity	[88]	$M_r = k_1 p_a \left(\frac{\theta_{\text{net}}}{p_a} \right)^{k_2} \left(\frac{\tau_{\text{oct}}}{p_a} + 1 \right)^{k_3} \left(\frac{\psi_m}{\theta_{\text{net}}} + 1 \right)^{k_4}$	θ_{net} is the net bulk stress ($\theta_{\text{net}} = \theta - 3u_a$); τ_{oct} is the octahedral shear stress; ψ_m is the matric suction; p_a is the atmospheric stress; k_i are the material parameters	$W_L = 61\%$; $W_P = 25\%$; $IP = 36\%$; $G_s = 2.69$	<p>The model is similar to Cary and Zapata [67] model.</p> <p>The last term related to the suction parameter is normalized with the net bulk stress instead of the atmospheric pressure.</p> <p>The model does not consider the effect of the pore-water pressure under saturated conditions</p>
QS0	CL High plasticity	[36]	$M_r = k_1 p_a \left(\frac{\theta - 3k_4}{p_a} \right)^{k_2} \left(\frac{\tau_{\text{oct}}}{p_a} + k_5 \right)^{k_3}$ $MR_{\text{opt}} = \frac{M_{\text{so}}}{M_{\text{so, opt}}} = c + d \log \psi_m$	MR_{opt} is the modulus ratio; M_{so} is the modulus at certain suction (or moisture); $M_{\text{so, opt}}$ is the modulus at optimum moisture content; c and d are model parameters; p_a is the atmospheric pressure (101 kPa); k_i are fitting parameters	$W_L = 85\%$; $IP = 52\%$; $G_s = 2.75$; Sand content = 3.1%; Silt content = 21.2%; Clay content = 75.2%; Fines content = 96.4%; $W_{\text{opt}} = 27.5\%$; Maximum dry unit weight = 14.4 kN/m ³	<p>The model is developed based on four tested fine-grained soils.</p> <p>This model demands significant regression parameters and several combinations of parametric values to fit laboratory tests.</p> <p>The test specimens were initially saturated and then subjected to desorption until the matric suction was 154 or 350 kPa.</p> <p>All the test soils were prepared at OPT water content and at a dry density corresponding to 98% or 103% relative compaction for a standard proctor</p>

Table 17 continued

UIC	ASTM classification	Ref.	Equation model	Variables and empirical constants	Parameters and characteristics	Observations
QS0-QS1	CL Medium plasticity	[50]	$E_r = \frac{a' + b'\sigma_{dv}}{\sigma_{dv}}$ for $\sigma_{dv} > 0$	a' and b' are material parameters: $a' = 318.2 + 0.337(q_u) + 0.73(\% \text{clay}) + 2.26(\text{PI}) - 0.915(\gamma) - 2.19(S) - 0.304(\% \text{finer} \#200)$ $b' = 2.10 + 0.00039\left(\frac{1}{a'}\right) + 0.104(q_u) + 0.09(\text{LL}) - 0.10(\% \text{ finer} \#200)$	$W_L = 42.1\%$; $W_P = 22.0\%$; $IP = 20.1\%$; $G_s = 2.76$; Clay content = 36%; Passing sieve #200 = 93% Maximum dry density = 15.20 kN/m ³ $W_L = 30.5\%$; $W_P = 22.1\%$; $IP = 8.4\%$; $G_s = 2.66$; Clay content = 17.0%; Passing sieve #200 = 95.0%; Maximum dry density = 16.28 kN/m ³ $W_L = 38.8\%$; $W_P = 23.3\%$; $IP = 15.5\%$; $G_s = 2.65$; Clay content = 18.0%; Passing sieve #200 = 100.0%; Maximum dry density = 15.98 kN/m ³ $W_L = 29.5\%$; $W_P = 20.1\%$; $IP = 9.4\%$; $G_s = 2.67$; Clay content = 16.0%; Passing sieve #200 = 78.0%; Maximum dry density = 17.16 kN/m ³ $W_L = 28.5\%$; $W_P = 19.2\%$; $IP = 9.3\%$; $G_s = 2.73$; Clay content = 20.0%; Passing sieve #200 = 73.0%; Maximum dry density = 17.36 kN/m ³	The hyperbola predicts excessive values of E_r when σ_d is close to zero
QS1	CL Low plasticity					
QS1	CL Medium plasticity					
QS1	CL Low plasticity					
QS1	CL Low plasticity					

Table 17 continued

UIC	ASTM classification	Ref.	Equation model	Variables and empirical constants	Parameters and characteristics	Observations
QS1	CL Low plasticity	[36]	$M_r = k_1 p_a \left(\frac{\theta - 3k_4}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + k_5 \right)^{k_3}$ $MR_{opt} = \frac{M_{rs}}{M_{rs,opt}} = c + d \log \psi_m$	<p>MR_{opt} is the modulus ratio; M_{rs} is the modulus at certain suction (or moisture); $M_{rs,opt}$ is the modulus at optimum moisture content; c and d are model parameters; p_a is the atmospheric pressure (101 kPa); k_i are fitting parameters</p>	<p>$W_L = 26.0\%$; $IP = 9.0\%$; $G_s = 2.66$; Sand content = 36.3%; Silt content = 45.3%; Clay content = 14.5%; Per cent fines = 59.7%; $W_{opt} = 16.0\%$; Maximum dry unit weight = 17.70 kN/m³</p>	<p>Model developed based on four tested fine-grained soils. This model demands significant regression parameters and several combinations of parametric values to fit laboratory tests. The test specimens were initially saturated and then subjected to desorption until the matric suction was 154 or 350 kPa. All the test soils were prepared at OPT water content and at a dry density corresponding to 98% or 103% relative compaction for a standard proctor The model takes into account, explicitly, the effect of the confining and octahedral shear stress. It is used for a large range of unbound materials. The effect of the matric suction is included in the effective stress. The model is similar to the model developed by Uzan [56]</p>
QS1	CL Low plasticity	[9]	$M_r = k_1 p_a \left(\frac{\theta + \chi \psi_m}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3}$	<p>θ is the bulk stress; τ_{oct} is the octahedral shear stress; χ is the Bishop's effective stress parameter; ψ_m is the matric suction; k_i are the material parameters</p>	<p>$W_L = 30.8\%$; $W_P = 18.4\%$; $IP = 12.3\%$; Passing sieve #200 = 68.8%; $W_{opt} = 16.5\%$; Maximum dry density = 17.70 kN/m³</p>	

Table 17 continued

UIC	ASTM classification	Ref.	Equation model	Variables and empirical constants	Parameters and characteristics	Observations
≤ QS1 (based on AASHTO classification)	–	[87]	$M_r = k_1 p_a \left(1 + \frac{\sigma_d}{1 + \sigma_c} \right)^{k_2}$	<p>M_r is the resilient modulus (MPa);</p> <p>σ_d is the deviator stress (kPa);</p> <p>σ_c is the confining pressure (kPa);</p> <p>P_a is the atmospheric pressure (kPa);</p> <p>k_1 and k_2 are model parameters</p>	<p>$\gamma_d = 16.92 \text{ kN/m}^3$; $\gamma_{dr} = 0.97$; $w_{cr} = 1.48$; $w_c = 20.7\%$; Optimum $w_c = 14.0\%$; Maximum dry density = 17.4 kN/m^3; Passing sieve #200 = 89% $\gamma_d = 19.87 \text{ kN/m}^3$; $\gamma_{dr} = 1.10$; $w_{cr} = 0.82$; $w_c = 11.5\%$; Optimum $w_c = 14.0\%$; Maximum dry density = 18.4 kN/m^3; Passing sieve #200 = 44%;</p>	<p>k_1 is dependent on γ_{dr}, LL and w_c.</p> <p>k_2 is dependent on γ_{dr}, LL, w_{cr} and #200, where γ_{dr} and w_{cr} are the density and moisture ratio, respectively</p>
QS1	CL-ML Low plasticity	[9]	$M_r = k_1 p_a \left(\frac{\theta + \chi \psi_m}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3}$	<p>θ is the bulk stress;</p> <p>τ_{oct} is the octahedral shear stress;</p> <p>χ is the Bishop's effective stress parameter;</p> <p>ψ_m is the matric suction;</p> <p>k_i are the material parameters</p>	<p>$W_L = 27.8\%$; $W_P = 19.8\%$; IP = 8%; Passing sieve #200 = 56.3%; $W_{opt} = 14.2\%$; Maximum dry density = 17.75 kN/m^3</p>	<p>The model takes into account, explicitly, the effect of the confining and octahedral shear stress.</p> <p>It is used for a large range of unbound materials;</p> <p>The effect of the matric suction is included in the effective stress.</p> <p>The model is similar to the model developed by Uzan [56]</p>
QS1**	CL (no information about the particle size curve) Low to medium plasticity	[88]	$M_r = k_1 p_a \left(\frac{\theta_{net}}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3} \left(\frac{\psi_m}{\theta_{net}} + 1 \right)^{k_4}$	<p>θ_{net} is the net bulk stress ($\theta_{net} = \theta - 3u_a$);</p> <p>$\tau_{oct}$ is the octahedral shear stress;</p> <p>ψ_m is the matric suction;</p> <p>p_a is the atmospheric stress;</p> <p>k_i are the material parameters</p>	<p>$W_L = 37\%$; $W_P = 18\%$; IP = 19%; $G_s = 2.69$</p>	<p>The model is similar to Cary and Zapata [67] model.</p> <p>The last term related to the suction parameter is normalized with the net bulk stress instead of the atmospheric pressure.</p> <p>The model does not consider the effect of the pore-water pressure under saturated conditions</p>

*Classification based on the plasticity index and AASHTO classification (A-7-5: > 35% passing the 0.075 mm sieve)

** Classification based on the plasticity index and AASHTO classification (A-7-6: > 35% passing the 0.075 mm sieve)

Table 18 Models that describe resilient modulus of fine-grained soils—sils

UIC	ASTM classification	Ref.	Equation model	Variables and Empirical Constants	Parameters and characteristics	Observations
QS0	MH (no information about the particle size curve) High plasticity	[35]	$M_r = k_1 p_a (\sigma_d + \chi \psi_m)^{k_2}$	χ is the Bishop's effective stress parameters (it is function of the degree of saturation of the soil); ψ_m is the matric suction; k_i are the materials' parameters;	$W_L = 54.0\%$; $IP = 20\%$; $G_s = 2.71$; $W_{opt} = 18\%$; Maximum dry density = 17.26 kN/m ³	This model does not take into account the effect of confining stress. The model can be applied to a large range of moisture content. The model is dependent on only two material parameters
QS0	MH (no information about the particle size curve) High plasticity	[50]	$E_r = \frac{a' + b' \sigma_d}{\sigma_d}$ for $\sigma_d > 0$	a' and b' are material parameters: $a' = 318.2 + 0.337(q_u) + 0.73(\% \text{clay}) + 2.26(PI) - 0.915(\gamma) - 2.19(S) - 0.304(\% \text{finer} \#200)$ $b' = 2.10 + 0.00039 \left(\frac{1}{a} \right) + 0.104(q_u) + 0.09(LL) - 0.10(\% \text{finer} \#200)$	$W_L = 68.5\%$; $W_P = 39.2\%$; $IP = 29.3\%$; $G_s = 2.71$; Clay content = 28.7%; Passing sieve #200 = 80.0%; Maximum dry density = 12.55 kN/m ³ $W_L = 69.5\%$; $W_P = 42.6\%$; $IP = 26.9\%$; $G_s = 2.73$; Clay content = 55.0%; Passing sieve #200 = 80.0%; Maximum dry density = 13.83 kN/m ³	The hyperbola predicts excessive values of E_r when σ_d is close to zero
QS0-QS1	MH (no information about the particle size curve) Medium to high plasticity	[35]	$M_r = k_1 p_a (\sigma_d + \chi \psi_m)^{k_2}$	χ is the Bishop's effective stress parameters (it is function of the degree of saturation of the soil); ψ_m is the matric suction; k_i are the materials' parameters;	$G_s = 2.67$; $W_L = 50.0\%$; $IP = 23\%$; $W_{opt} = 17\%$; Maximum dry density = 17.65 kN/m ³	This model does not take into account the effect of confining stress. The model can be applied to a large range of moisture content. The model is dependent on only two material parameters

Table 18 continued

UIC	ASTM classification	Ref.	Equation model	Variables and Empirical Constants	Parameters and characteristics	Observations
QS1	ML Low plasticity	[36]	$M_r = k_1 p_a \left(\frac{\theta - 3k_4}{p_a} \right)^{k_2} \left(\frac{\tau_{oct} + k_5}{p_a} \right)^{k_3}$ $MR_{opt} = \frac{M_{rs}}{M_{rs,opt}} = c + d \log \psi_m$	MR_{opt} is the modulus ratio; M_{rs} is the modulus at certain suction (or moisture); $M_{rs,opt}$ is the modulus at optimum moisture content; c and d are model parameters; p_a is the atmospheric pressure (101 kPa); k_i are fitting parameters	$W_L = 28.0\%$; $IP = 11.0\%$; $G_s = 2.69$; Sand content = 11.9%; Silt content = 82.4%; Clay content = 5.7%; Fines content = 88.1%; $W_{opt} = 13.5\%$; Maximum dry unit weight = 17.9 kN/m ³ $W_L = 42.0\%$; $IP = 24.0\%$; $G_s = 2.69$; $W_{opt} = 22.0\%$; Sand content = 8.9%; Silt content = 63.8%; Clay content = 27.3%; Fines content = 91.1%; Maximum dry unit weight = 15.8 kN/m ³	The model is developed based on four tested fine-grained soils. This model demands significant regression parameters and several combinations of parametric values to fit laboratory tests. The test specimens were initially saturated and then subjected to desorption until the matric suction was 154 or 350 kPa. All the test soils were prepared at OPT water content and at a dry density corresponding to 98% or 103% relative compaction for a standard proctor
QS1	ML Medium plasticity	[88]	$M_r = k_1 p_a \left(\frac{\theta_{net}}{p_a} \right)^{k_2} \times \left(\frac{\tau_{oct} + 1}{p_a} \right)^{k_3} \left(\frac{\psi_m + 1}{\theta_{net}} \right)^{k_4}$	k_i are fitting parameters θ_{net} is the net bulk stress ($\theta_{net} = \theta - 3u_a$); τ_{oct} is the octahedral shear stress; ψ_m is the matric suction; p_a is the atmospheric stress; k_i are the material parameters	$W_L = 43\%$; $W_P = 29\%$; $IP = 14\%$; $G_s = 2.73$; Sand content = 24%; Silt content = 72%; Clay content = 4%; $C_u = 4.55$; $C_c = 0.61$; $W_{opt} = 16.3\%$; Maximum dry density = 17.26 kN/m ³	The model is similar to Cary and Zapata [67] model. The last term related to the suction parameter is normalized with the net bulk stress instead of the atmospheric pressure. The model does not consider the effect of the pore-water pressure under saturated conditions

Table 18 continued

UIC	ASTM classification	Ref.	Equation model	Variables and Empirical Constants	Parameters and characteristics	Observations
QS1	ML Low plasticity	[50]	$E_r = \frac{a' + b'\sigma_d}{\sigma_d} \text{ for } \sigma_d > 0$	<p>a' and b' are material parameters:</p> $a' = 318.2 + 0.337(q_u) + 0.73(\% \text{clay}) + 2.26(\text{PI}) - 0.915(\gamma) - 2.19(S) - 0.304(\% \text{finer} \#200)$ $b' = 2.10 + 0.00039\left(\frac{1}{a}\right) + 0.104(q_u) + 0.09(\text{LL}) - 0.10(\% \text{ finer } \#200)$	$W_L = 36.2\%$; $W_P = 34.1\%$; $IP = 2.1\%$; $G_s = 2.66$; Clay content = 18.0%; Passing sieve #200 = 80.0%; Maximum dry density = 13.14 kN/m ³ $W_L = 37.1\%$; $W_P = 27.0\%$; $IP = 10.0\%$; $G_s = 2.67$; Clay content = 35.0%; Passing sieve #200 = 68.0%; Maximum dry density = 14.91 kN/m ³	The hyperbola predicts excessive values of E_r when σ_d is close to zero
QS1	ML Medium plasticity	[89]	$M_r = k_1 p_a^{k_2} \sigma_d^{k_3} \psi_m^{k_3}$	σ_d is the deviator stress; ψ_m is the matric suction; k_i are the regression parameters	$W_L = 46.4\%$; $W_P = 28.8\%$; $IP = 17.6\%$; $G_s = 2.66$ Clay content = 38% Passing sieve #200 = 52%; $W_{opt} = 19.5\%$ Maximum dry density = 16.28 kN/m ³	<p>The model does not take into account the effect of confining stress.</p> <p>It is only valid for a limited range of moisture content.</p> <p>When the soils get saturated and matric suction is equal to zero (high moisture content) the resilient modulus is close to zero.</p> <p>Low moisture content values lead to high (unrealistic) resilient modulus.</p> <p>The suction was determined through the paper filter method</p>

Table 19 Models that describe resilient modulus of granular soils—sands

UIC	ASTM classification	Ref.	Equation model	Variables and empirical constants	Parameters and characteristics	Observations
QS1	SM-CL Low plasticity	[50]	$E_r = \frac{a' + b'\sigma_d}{\sigma_d} \text{ for } \sigma_d > 0$	a' and b' are material parameters: $a' = 318.2 + 0.337(q_u) + 0.73(\% \text{clay}) + 2.26(\text{PI}) - 0.915(\gamma) - 2.19(S) - 0.304(\% \text{finer} \#200)$ $b' = 2.10 + 0.00039\left(\frac{1}{a}\right) + 0.104(q_u) + 0.09(\text{LL}) - 0.10(\% \text{ finer } \#200)$	$W_L = 21.0\%$; $W_P = 14.1\%$; $IP = 6.9\%$; $G_s = 2.65$; Clay content = 16.0%; Passing sieve #200 = 36.0%; Maximum dry density = 18.04 kN/m ³ ; $W_L = 20.7\%$; $W_P = 19.0\%$; $IP = 1.7\%$; $G_s = 2.60$; Clay content = 17.0%; Passing sieve #200 = 20.0%; Maximum dry density = 17.55 kN/m ³	The hyperbola predicts excessive values of E_r when σ_d is close to zero
QS1	SM Low plasticity					
QS1	SC (the fines are classified as CL) Low plasticity	[67]	$M_r = k_1 p_a \left(\frac{\theta_{\text{net}} - 3\Delta u_{w-\text{sat}}}{p_a} \right)^{k_2} \left(\frac{\tau_{\text{oct}}}{p_a} + 1 \right)^{k_3} \left(\frac{\psi_m - \Delta\psi_m}{p_a} + 1 \right)^{k_4}$	θ_{net} is the net bulk stress ($\theta_{\text{net}} = \theta - 3u_a$); $\Delta u_{w-\text{sat}}$ is the pore-water pressure build-up in a saturated condition ($\psi_m = 0$); $\Delta\psi_m$ is the relative change in matric suction with respect to the initial matric suction in an unsaturated condition ($\Delta u_{w-\text{sat}} = 0$)	$W_L = 22\%$; $W_P = 18\%$; $IP = 5\%$; $G_s = 2.71$; Passing sieve #200 = 47% $W_{\text{opt}} = 12.1\%$ Maximum dry density = 18.73 kN/m ³	The model is based on the universal model with an additional term that includes the matric suction effects into the resilient modulus
QS1	SC (clayed sand) Low plasticity	[51]	$M_r = k \frac{\sigma_{\text{oct}}^m}{\tau_{\text{oct}}^m}$	σ_{oct} and τ_{oct} are the octahedral normal and shear stresses, respectively	$W_L = 26\%$; $W_P = 17.6\%$; $IP = 8.4\%$; 60% sand and 40% kaolinite by weight	This model is difficult to apply [86]

Table 19 continued

UIC	ASTM classification	Ref.	Equation model	Variables and empirical constants	Parameters and characteristics	Observations
QS2	SP	[58]	$M_r = A \left(\frac{p_m}{p_u} \right)^B \left(\frac{p_u}{p} \right)^C$	q_m is the mean value of deviator stress; p_m is the mean value of mean normal stress; p is the mean normal stress; p_u is unit pressure (1 kPa); A , B and C are constants	$G_s = 2.64$; $C_u \approx 2.5$; $C_c \approx 1.5$ $G_s = 2.64$; $C_u \approx 2.0$; $C_c \approx 1.04$ $G_s = 2.61$; $C_u \approx 28$; $C_c \approx 0.54$ Aggregate abrasion value = 1.9; $G_s = 2.24$; $C_u \approx 30$; $C_c \approx 0.5$ $W_{opt} = 8-12\%$; Permeability index = 0.0002 m/s	This model is based on the triaxial tests carried out in six aggregates. The model is based on laboratory triaxial testing (constant and variable confining pressure) [70]
QS3						
QS3	SW	[57]	$M_r = A(n_{max} - n)p_0 \left(\frac{\theta}{p_0} \right)^{0.5}$ $M_r = B(n_{max} - n)p_0 \left(\frac{\theta}{p_0} \right)^{0.7} \left(\frac{q}{p_0} \right)^{-0.2}$	θ is the sum of principal stresses; p_0 is the reference stress, 100 kPa; n is the porosity of the material; n_{max} is the theoretical maximum value of porosity when $M_r = 0$; A is a material parameter	$C_u \approx 5.9$; $C_c \approx 1.2$ $C_u \approx 1.6$; $C_c \approx 1.0$ $C_u \approx 50$; $C_c \approx 1.0$	This model is based on laboratory triaxial testing (constant confining pressure) [70]. The model is based on tests carried out in several types of coarse-grained granular materials typically used in unbound layers of road and railroad pavements. The density was incorporated in the model by the porosity parameter. Materials classified as natural gravels. The study also includes the crushed gravel and crushed rock
QS3	SW	[79]	$E_v = C\sigma_v'^m$	C is a parameter that depends on the mineralogy of the particles, shape of the grains and particle size curve (in the case of granular materials. In the case of the cohesive materials; it also depends on the percentage of fines and Atterberg limits	$D_{50} = 3.5$ mm; $G_s = 2.71$; $C_u = 28$; $C_c \approx 1.0$	The vertical Young's modulus is independent of the lateral stress

Table 20 Models that describe resilient modulus of granular soils—gravels

UIC	ASTM classification	Ref.	Equation model	Variables and empirical constants	Parameters and characteristics	Observations
QS2	GP-GM	[67]	$M_r = k_1 p_a \left(\frac{\theta_{\text{net}} - 3\Delta u_{w-\text{sat}}}{p_a} \right)^{k_2} \left(\frac{\tau_{\text{net}} + 1}{p_a} \right)^{k_3} \left(\frac{\psi_m - \Delta\psi_m}{p_a} + 1 \right)^{k_4}$	θ_{net} is the net bulk stress ($\theta_{\text{net}} = \theta - 3u_a$); $\Delta u_{w-\text{sat}}$ is the pore-water pressure build-up in a saturated condition ($\psi_m = 0$); $\Delta\psi_m$ is the relative change in matrix suction with respect to the initial matrix suction in an unsaturated condition $(\Delta u_{w-\text{sat}} = 0)$	IP = NP; $G_s = 2.71$; Passing sieve #200 = 7%; $W_{\text{opt}} = 7.2\%$ Maximum dry density = 22.16 kN/m ³	The model is based on the universal model with an additional term that includes the matrix suction effects into the resilient modulus
QS2	GP	[58]	$M_r = A \left(\frac{p_m}{p_u} \right)^B \left(\frac{p_u}{p} \right)^C$	q_m is the mean value of deviator stress; p_m is the mean value of mean normal stress; p is the mean normal stress; p_u is unit pressure (1 kPa); A , B and C are constants	$G_s = 2.69$; $C_u \approx 5.3$; $C_c \approx 4.7$	This model is based on the triaxial tests carried out in six aggregates.
QS3	Poor-graded gravel GW Well-graded gravel				$G_s = 2.69$; $C_u \approx 61$; $C_c \approx 2.11$; Aggregate abrasion value = 5.1;	The model is based on laboratory triaxial testing (constant and variable confining pressure) [70]
QS2-QS3	GP Poor-graded gravel	[79]	$E_v = C \sigma_v^m$	C is a parameter that depends on the mineralogy of the particles, shape of the grains, particle size curve (in the case of the granular materials)	$D_{50} = 8.5$ mm; $G_s = 2.71$; $C_u = 53$; $C_c \approx 4.5$	The vertical Young's modulus is independent of the lateral stress. The parameter C , in the case of the cohesive materials, also depends on the percentage of fines and Atterberg limits
$\geq$ QS2 (based on AASHTO classification)		[87]	$M_r = k_1 p_a \left(1 + \frac{\sigma_d}{1 + \sigma_c} \right)^{k_2}$	M_r is the resilient modulus (MPa); σ_d is the deviator stress (kPa); σ_c is the confining pressure (kPa); P_a is the atmospheric pressure (kPa); k_1 and k_2 are model parameters	$\gamma_d = 16.07$ kN/m ³ $\gamma_{\text{dr}} = 1.02$; $w_c = 16.3\%$; $w_{\text{cr}} = 1.05$; Optimum $w_c = 15.5\%$; Maximum dry density = 15.7 kN/m ³ ; Passing sieve #200 = 11% $\gamma_d = 18.04$ kN/m ³ $\gamma_{\text{dr}} = 1.10$; $w_{\text{cr}} = 1.15$; $w_c = 17.32\%$; Optimum $w_c = 15.0\%$; Maximum dry density = 16.7 kN/m ³ ; Passing sieve #200 = 34%	k_1 is dependent on γ_{dr} , w_{cr} , #200 and C_u , where γ_{dr} and w_{cr} are the density and moisture ratio, respectively. k_2 is dependent on γ_{dr} , w_{cr} , #200 and C_u

Table 21 Other models

Ref.	Equation model	Variables and empirical constants	Observations
[90]	$M_r = k_1 (\sigma_3)^{k_2}$	σ_3 is the confining pressure; k_1 and k_2 are material constants	This model is based on the resilient modulus and Poisson's ratio; Model based on laboratory triaxial testing (constant confining pressure) [70]
[77]	$M_r = k_1 (\theta)^{k_2}$	θ is the bulk stress (sum of the principal stresses σ_1 , σ_2 and σ_3); k_1 are the regression parameters	Commonly known as the k - θ model; hyperbolic relationship. This model assumes a constant Poisson's ratio. However, several studies show that this parameter varies with the applied stress. The effect of stress on the resilient modulus is only considered by the bulk stress (sum of the principal stresses). It is a simplified version of the universal model.
[91]			This model is based on laboratory triaxial testing (constant confining pressure)[70].
[78]			The model fits well in the case of tests with constant confining pressure and shows poor results in the case of variable confining pressure [53]
[92]	$M_r = K_1 + K_2 \sigma_d$, when $\sigma_d < \sigma_{di}$ $M_r = K_3 + K_4 \sigma_d$, when $\sigma_d > \sigma_{di}$	K_1 , K_2 , K_3 , and K_4 are model parameters dependent upon soil type and its physical state (normally, K_2 and K_4 are negative); σ_{di} is the deviator stress at which the slope of M_r versus σ_d changes	This is a bilinear model
[93]	$\log M_r = c_{ld} - m_{ld}(\sigma_1 - \sigma_3)$	c_{ld} and m_{ld} are functions of matrix suction; σ_1 and σ_3 are the major and minor principal stresses	The model was not fully validated with laboratory data. This model is characterized as a semi-log model
[61]	$\varepsilon_v = \frac{1}{K_1} p^n \left[1 - \beta \left(\frac{q}{p} \right)^2 \right]$ $\varepsilon_s = \frac{1}{3G_1} p^n \left(\frac{q}{p} \right)$ $K = \frac{p}{\varepsilon_v}$ $G = 3 \frac{q}{\varepsilon_q}$	p is the mean normal stress; q is the deviator stress; ε_v is the volumetric strain and ε_s is the shear strain; K is the bulk modulus; G is the shear modulus	This model uses shear-volumetric approach. This is a nonlinear elastic model that takes into account the effect of the stress path and it is expressed in terms of the bulk and shear modulus, that are stress-dependent. The model is elastic and follows the application of the theorem of reciprocity. The shear and volumetric components of stress and strain are treated separately. The granular materials show an inelastic response and this elastic model can lead to inaccurate predictions. The model is based on variable confining pressure. Sweere [94] used this model but keep the relationship between shear and volumetric strains with stresses independent from each other

Table 21 continued

Ref.	Equation model	Variables and empirical constants	Observations
[47]	$M_r = k_1 (\sigma_d)^{k_2}$	k_1 and k_2 are constants dependent on the material; σ_d is the deviator stress;	The model is characterized by a power model. A modified elastic layered computer program was made to determine equivalent subgrade modulus values. Multiple regression techniques were used to determine predictive equations for the equivalent subgrade modulus. This model was widely used by other authors and good results were obtained (in agreement with test results)
[56]	$M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{q}{p_0} \right)^{k_3}$ or $M_r = k_1 p_0 \left(\frac{\theta}{p_0} \right)^{k_2} \left(\frac{\tau_{oct}}{p_0} \right)^{k_3}$	θ is the bulk stress (sum of the principal stresses σ_1 , σ_2 and σ_3); τ_{oct} is the octahedral shear stress; p_0 is the atmospheric pressure; k_i are the material parameters	The model shows good results in the case of pre-failure stresses. However, in the case of stresses that go beyond the static failure, the predictions are poor. This model is suitable for a large range of materials: from fine-grained soils to coarse-granular materials. This model is based on laboratory triaxial testing (constant confining pressure) [70]
[95]	$\epsilon_v = \delta \left(\frac{p}{A} \right)^B \left[1 - C \left(\frac{q}{p} \right)^2 \right]$ $\epsilon_s = D \delta \left[\frac{q}{p+E} \right] \left[\frac{\sqrt{p^2 + q^2}}{p_m} \right]^F$	p is the mean normal stress; q is the deviator stress; A , B , C , D and E are material constants	This model uses shear-volumetric approach. Two materials were considered: well-graded crushed limestone and uniformly graded limestone. This model is also known as the contour model and takes into account effective mean and deviatoric stresses and stress path dependency. This model predicts the shear strain (considering the length of the stress path corresponding to the load cycle). The material parameters in the two parts of the contour model are independent of each other. In fact, independent material parameters mean that the model is inelastic (unlike the Boyce model)
[96]	$M_r = k_1 \left(\frac{J_2}{\tau_{oct}} \right)^{k_2}$	k_1 and k_2 are constants; J_2 is the second stress invariant; τ_{oct} is the octahedral shear stress	Model based on laboratory triaxial testing (constant confining pressure) [70]
[97]	$M_r = k_1 \left(\frac{p}{q} \right)^{k_2}$	k_1 is a material constant; p is the mean normal stress; q is the deviator stress;	Model based on laboratory triaxial testing (constant confining pressure) [70]
[98]	$\epsilon_v = A (\delta \ln p)^B (\delta p)^C - D \left[\delta \left(\ln \frac{q}{\sigma_3} \right)^2 \right]^E$ $\epsilon_s = E \left[\delta \left(\ln \frac{q}{\sigma_3} \right)^2 \right]^F \left(\delta \tau + \frac{1}{3} \delta S \right)$	σ_1 and σ_3 are the major and minor principal stresses T and S are 2D stress invariants	Model developed for prediction of the results of triaxial and hollow cylinder tests. The model separates the volumetric and shear components as the Boyce's and contour models. Karasahin [58] verified the model but found it very difficult to obtain the material parameters by nonlinear regression analysis

Table 21 continued

Ref.	Equation model	Variables and empirical constants	Observations
[99]	$M_r = k_1 \frac{\theta^{k_2}}{10^{k_1}}$	θ is the bulk stress (sum of the principal stresses σ_1 , σ_2 and σ_3); k_1 is a constant parameter; $A_1 = mR^3$, m is a regression coefficient determined from laboratory data and R is the stress/strength ratio (deviator stress divided by the failure deviator stress)	k - θ model that includes a failure term (the impact is insignificant until the failure is approached). The failure deviator stress is determined based on the confining pressure and the static triaxial shear stress. This model is based on laboratory triaxial testing (constant confining pressure) [70]
Developed by Mayhew [100] and re-written by Brown and Selig [101]	$\epsilon_v = \frac{1}{K_1} p^A \left[1 - C \left(\frac{q}{p} \right)^2 \right]$ $\epsilon_s = \frac{1}{3G_1} p^B \left(\frac{q}{p} \right)$	p is the mean normal stress; q is the deviator stress; $\epsilon_{v,r}$ is the recoverable volumetric strain and $\epsilon_{s,r}$ is the shear strain; K is the bulk modulus; G is the shear stress	This model is similar to the contour model. The author plots the shear and volumetric strains in the p - q space. The author concludes that the stress path length (included in the contour model) had little impact on the shear strain response. The model removes the stress path dependence and the equations of the contour model turn into the inelastic modified version of the Boyce model
[102]	$\epsilon_v = p_0^{1-B} p^B \left[\frac{1}{A} - \frac{(1-B)}{6C} \left(\frac{q}{p} \right)^2 - \frac{B}{D} \left(\frac{q}{p} \right) \right]$ $\epsilon_s = p_0^{1-B} p^B \left[\frac{1}{3C} \left(\frac{q}{p} \right) - \frac{1}{D} \right]$ $M_r = N_1 q^{N_2} (\sigma_3)^{N_3}$	–	Model based on Boyce's model; the author introduced the concept of anisotropy
[103]		σ_3 is the confining pressure; q is the deviator stress	Model based on laboratory triaxial testing (constant confining pressure) [70]
[80]		$R_{m1} = M_r/M_{r,opt}$ for the case of constant dry density; M_r is the resilient modulus at moisture content w (%) and the same dry density as $M_{r(opt)}$; $M_{r(opt)}$ is the resilient modulus at the maximum dry density and optimum moisture content w_{opt} (%) for any compaction effort	This model includes the soil's physical state by two quantities: moisture content and dry density that are related to the soil-compaction curve.
[86]	$R_{m1} = 0.98 - 0.28(w - w_{opt}) + 0.029(w - w_{opt})^2$		The results are based on repeated load triaxial test results on fine-grained soils from literature (Seed, Chan [104]; Sauer and Monismith [105]; Culley [106]; Robnett and Thompson [107]; Fredlund, Began [93]; Edil and Moran [108]; Kirwan, Farrell [109]; Elfino and Davidson [110]).
			The power-law model was selected for the representation of the relationship between resilient modulus and deviator stress

Table 21 continued

Ref.	Equation model	Variables and empirical constants	Observations
[86]	$R_{m2} = 0.96 - 0.18(w - w_{opt}) + 0.0067(w - w_{opt})^2$	$R_{m2} = M_r/M_{r,opt}$ for constant compaction effort; M_r is the resilient modulus at moisture (%) and the same compaction effort as $M_{r,opt}$	<p>This model includes the soil's physical state by two quantities: moisture content and dry density that are related to the soil-compaction curve.</p> <p>The results are based on repeated load triaxial test results on fine-grained soils from literature (Seed, Chan [104]; Kallas and Riley [111]; Shifley and Monismith [112]; Tanimoto and Nishi [113]; Edris and Lytton [114]; Fredlund, Began [93]; Kirwan, Farrell [109]; Pezo, Kim [115].</p> <p>The power-law model was selected for the representation of the relationship between resilient modulus and deviator stress</p> <p>This model is based on Boyce's model and is also known as the anisotropic Boyce model</p>
[116]	$e_v = \frac{p^{*A}}{p_0^{A-1}} \left[\frac{\gamma+2}{3K_1} + \frac{A-1}{18G_1} (\gamma+2) \left(\frac{q^*}{p^*} \right)^2 + \frac{\gamma-1}{3G_1} \left(\frac{q^*}{p^*} \right) \right]$ $e_s = \frac{2}{3} \frac{p^{*A}}{p_0^{A-1}} \left[\frac{\gamma-1}{3K_1} + \frac{A-1}{18G_1} (\gamma-1) \left(\frac{q^*}{p^*} \right)^2 + \frac{2\gamma+1}{6G_1} \left(\frac{q^*}{p^*} \right) \right]$	$p^* = \frac{\gamma G_1 + 2\sigma_1}{3} q^*$ γ is the coefficient of anisotropy; <p>The parameter γ means that the material is isotropic (when $\gamma = 1$) and the material behaves more anisotropically as γ deviates further from $\gamma = 1$; when $\gamma < 1$, the material is stiffer vertically than horizontally;</p>	
[117]	$E_v = \frac{\partial \sigma_v}{\partial e_v} = E_{v0} \cdot \left(\frac{\sigma_v}{\sigma_0} \right)^{nv} \cdot F(e)/F(e_0)$ $E_h = \frac{\partial \sigma_h}{\partial e_h} = E_{h0} \cdot \left(\frac{\sigma_h}{\sigma_0} \right)^{nv} \cdot F(e)/F(e_0)$ $\frac{E_v}{E_h} = \frac{E_{v0}}{(E_{h0})_0} \cdot R^n = aR^n$ (when powers nv and nh are the same and equal to n)	<p>The power n represents the stress-dependency of the quasi-elastic vertical Young's modulus E_v and E_h;</p> <p>a quantifies the inherent anisotropy;</p> <p>$R = \frac{e_h}{e_v}$ and quantifies the stress-induced anisotropy at the isotropic stress states;</p> <p>E_{v0} is the reference vertical stress</p> <p>$F(e)$ is the void ratio function;</p> <p>$A = a + \frac{b-a}{1+\exp(\beta+K_s(S-S_{opt}))}$</p> <p>$S$ is the degree of saturation and S_{opt} is the saturation at OPT conditions;</p> <p>θ is the bulk stress;</p> <p>τ_{oct} is the octahedral shear;</p> <p>p_a is atmospheric pressure;</p> <p>k_1, k_2, k_3, a, b and k_s are fitting parameters; $\beta = \ln(b/a)$</p>	<p>Cross-anisotropic-hypo-quasi-elasticity model</p> <p>This model depends on four material constants (E_{v0}, v_0, n and a).</p> <p>The model can predict nonlinear and anisotropic resilient stress-strain properties for general stress paths under general stress conditions</p>
[118]	$M_r = 10^4 k_1 p_a \left(\frac{\theta}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3}$		<p>The first term of the expression represents the environment related to the contribution of changes in moisture content</p>

Table 21 continued

Ref.	Equation model	Variables and empirical constants	Observations
[59]	$\log \frac{M_r}{M_{\text{ropt}}} = a + \frac{b - a}{1 + \exp[\ln(-b/a) + k_m(S - S_{\text{opt}})]}$	M_r/M_{ropt} is the resilient modulus ratio, where M_r is the resilient modulus at optimum moisture content; a is the minimum of $\log(M_r/M_{\text{ropt}})$; b is the maximum of $\log(M_r/M_{\text{ropt}})$; k_m is a regression parameter; $(S - S_{\text{opt}})$ is the variation in the degree of saturation	<p>This model can be used in the case of fine-grained and coarse-grained materials (the difference is expressed in terms of the parameters a, b and k_m).</p> <p>The constants of the model are determined based on RLT testing data.</p> <p>The model does not combine the effects of the stress state and the moisture content.</p> <p>The model estimated the change of the resilient modulus at a certain moisture content and a certain known stress state.</p> <p>Based on an intensive literature review study, it was developed (select) a model that analytically predicts changes in modulus due to changes in moisture</p>
[81]	$E_r = 49200 + 950p'_0 - 370q_r - 2400w_p$	p'_0 is the initial mean normal effective stress (in kPa); q_r is the repeated deviator stress (in kPa); w_p is the plastic limit in %	<p>This is an analysis based on repeated load triaxial testing carried out on recompacted soils</p>

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