Experimental study on permanent deformation characteristics of coarse-grained soil under repeated dynamic loading

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Received: 16 June 2020 / Revised: 20 January 2021 / Accepted: 22 January 2021 / Published online: 14 February 2021
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Abstract Practical assessment of subgrade settlement induced by train operation requires developing suitable models capable of describing permanent deformation characteristics of subgrade filling under repeated dynamic loading. In this paper, repeated load triaxial tests were performed on coarse-grained soil (CGS), and the axial permanent strain of CGS under different confining pressures and dynamic stress amplitudes was analysed. Permanent deformation behaviors of CGS were categorized based on the variation trend of permanent strain rate with accumulated permanent strain and the shakedown theory. A prediction model of permanent deformation considering stress state and number of load cycles was established, and the ranges of parameters for different types of dynamic behaviors were also divided. The results indicated that the variational trend of permanent strain rate with accumulated permanent strain can be used as a basis for classifying dynamic behaviors of CGS. The stress state (confining pressure and dynamic stress amplitude) has significant effects on the permanent strain rate. The accumulative characteristics of permanent deformation of CGS with the number of load cycles can be described by a power function, and the model parameters can reflect the influence of confining pressure and dynamic stress amplitude. The

study's results could help deepen understanding of the permanent deformation characteristics of CGS.

Keywords Repeated load triaxial tests · Coarse grained soil · Shakedown theory · Dynamic stress · Accumulated permanent strain · Railway subgrade

1 Introduction

Heavy haul railways have been internationally-recognized as the future direction of freight transportation because of their higher transport capacity and efficiency, lower energy consumption, and transport cost [1], and the development of heavy haul freight transportation has become a recent focus of China's railway construction. Because of the greater axle loads of heavy haul trains, the dynamic loading exerted on subgrades is much greater and more complex than that induced by ordinary trains, and consequent subgrade problems, particularly excessive settlement, are commonly observed. The Code for Design of Heavy Haul Railway [2] issued by the State Railway Administration of China in 2017 is China's first industry standard for heavy-haul railway.

In order to avoid common issues in subgrade of heavy haul railways such as mud pumping, the code has recommended the use of coarse-grained soil (CGS) due to several reasons, including reducing deformation, controlling horizontal and vertical movement of railway foundations, providing better drainage and structural support for subgrade bed in the construction of heavy haul railway subgrade in China [3–5]. Therefore, study of the permanent

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deformation characteristics of CGS is very important for heavy haul railway subgrade design and maintenance.

Many scholars have devoted significant effort to studying the permanent deformation characteristics of subgrade soils under train dynamic loads. Most studies were mainly focused on clay and other fine-grained soils [6–13]. Health, et al. [6] studied the permanent deformation of London clay when subjected to repeated triaxial loading. Monismith et al. [7] performed a series of repeated-load undrained triaxial compression tests on a silty clay subgrade soil, and introduced a power-law model for predicting the permanent strain induced by a number of stress applications. Indraratna et al. [11–14] investigated the subgrade fluidization (forming the slurry and mud pumping tracks) and the effect of cohesive fines on cyclic behavior under heavy haul load. Li et al. [8] improved existing methods in the literature for predicting cumulative plastic deformation for fine-grained subgrade soils. With the development of repeated load triaxial (RLT) apparatus, RLT tests were conducted on specimens of greater dimension to investigate permanent deformation characteristics of unbound granular materials [9, 10, 15–28] that are widely used in road surface, base, and subbase layers. Based on the shakedown concept, Werkmeister et al. [15, 16] defined three possible responses of crushed rock aggregates under repeated cyclic loading, and this method was subsequently widely used to classify permanent deformation behaviors of unbound granular materials [19, 22, 26–28].

Establishing a permanent deformation prediction model that comprehensively considers various factors (confining pressure, axial stress, load cycles, etc.) is the key to quantitatively analyze the long-term deformation characteristics of subgrade soil. Because of its simplicity and that the model parameters can be obtained by RLT tests, empirical formula methods are widely used to predict permanent deformation [8, 19, 22, 29–39]. Since empirical models take different forms and have specifically applicable conditions, only a few models can reflect the effects of stress state on the development of accumulated plastic strain [8, 22], so further study is necessary to establish a prediction model in which the main influencing factor (stress state) can be considered, and one capable of predicting different kinds of permanent deformation behaviours. Previous studies investigated the dynamic behavior and permanent deformation and degradation of coarse-grained soils under dynamic loads [40–45]. Lazorenko et al. [1] performed a general review on the effects of dynamic loads on soil materials used under heavy train loads and stated that there was a high dependency between shear deformations and number of cyclic loads. In addition, Touqan et al. [40] indicated that particle size of the subgrade soils and the rate of loading along with plasticity,

relative density and effective confining pressure are of important factors in permanent deformation of ballast under dynamic loads. However, it is important to study whether model parameters can be used to categorize different permanent deformation behaviors.

This paper focuses on the long-term permanent deformation of CGS subgrade filling. Large-scale RLT tests were conducted on CGS specimens, and variations of accumulated plastic strain with number of load cycles were analysed. Permanent deformation characteristics of CGS specimens were analysed based on the variation of accumulated strain rate with accumulated plastic strain. Moreover, a new model is proposed to predict the accumulation of plastic strain with the number of load cycles, and the ranges of model parameters for different types of dynamic behaviors are given. The study is significant to understanding the dynamic stability and deformation prediction of CGS subgrade under long-term train loading.

2 Repeated load triaxial tests

2.1 Tested materials and test equipment

The CGS tested in this study was collected from the subgrade bed of the Shuozhou–Huanghua heavy-haul railway located in Pingshan in Heibei province, China. The gravel (> 4.75 mm), sand (4.75 mm $>$ and > 0.075 mm), and fines (< 0.075 mm) contents of the CGS materials used in this study, respectively, are 43.2%, 44.3%, and 12.5% according to the ASTM D 422. Subgrade materials in this study could be considered as low to medium plasticity where the results of Atterberg limits test showed a plasticity index (PI) of 22, a liquid limit (LL) of 42 and a plastic limit (PL) of 20 for subgrade materials. Previous studies showed that there is a high probability of mud pumping, defined as migration of the subgrade fines toward ballast for subgrade materials with low to medium elasticities ($20 < LL < 50$ and $PI < 30$) [46, 47]. Figure 1 shows the gradation curve, and Table 1 presents the common physical properties of CGS.

The specimens were prepared in accordance with the Code for Soil Test of Railway Engineering [48]. Since heavy haul railways have high standards requiring that the degree of compaction of a subgrade bed should not be less than 97%, the target degree of compaction coefficient was set to 97%. The largest grain size was approximately 50 mm, as shown in Fig. 1, resulting in CGS specimen diameter and height of 300 mm and 600 mm, respectively [49]. A compaction method was used to make the cylindrical specimens. The specimens were compacted in six layers, and identical wet soil weight and compaction height of each layer were used to guarantee uniform density

Fig. 1 Gradation curve of the tested CGS

Table 1 Physical properties of tested CGS

Parameter	Value
Maximum dry density ρ_{dmax} ($\text{g}\cdot\text{cm}^{-3}$) ^a	2.21
Optimum moisture content ω_{opt} (%) ^a	6.0
Moisture content of saturated specimen ω_{sat} (%)	9.3
Void ratio	0.248
Coefficient of uniformity C_u	80
Coefficient of curvature C_c	1.25

^aASTM D698

conditions. All specimens were compacted at optimum moisture content. For saturated specimens, vacuum pumping was used to pre-saturate the specimens after compaction, and the specimens were then further saturated using a back-pressure method, with the specimens deemed to be completely saturated when a minimum B (pore-pressure coefficient) value of 0.95 was achieved.

A continuous dynamic test system for large-scale specimens was used to perform the test (Fig. 2). This system

Fig. 2 Large dynamic triaxial test system

was based on a YS30-3A static triaxial pressure chamber and an MTS dynamic loading system. A corresponding measurement system and rigid reaction frame were also configured, allowing both a stress control mode and a strain control mode to be used. This test system met the test requirements of a low confining pressure, a high number of load cycles, and of the ability to generate a variety of loading waveforms [49].

2.2 Testing program

Since the actual confining pressure of railway subgrade soil is typically relatively low, the effective confining pressures σ_3 were set at levels of 15, 30, and 60 kPa in the tests corresponding to lateral pressure environments at respective depths of 0.5, 1.5, and 3 m below the top surface of the subgrade. While a dynamic stress σ_d of 125 kPa or less was established when simulating a train load with axle load not exceeding 35 tons [50], a σ_d value of 150 kPa or greater was aimed at studying the dynamic characteristics of CGS used to improve the axle load of heavy haul trains. A detailed test program, considering effective confining pressure σ_3 and dynamic stress amplitude σ_d , is summarized in Table 2.

For saturated specimens, isotropic consolidation was performed until a negligible rate of volume change was observed, followed by an initial static deviatoric stress, σ_s , of 15 kPa applied on the specimens upon closing the drainage valve to simulate the stress transferred downward from the ballast, sleeper, and rail. Thereafter, a cyclic deviatoric stress with a sinusoidal wave was added. At the dynamic-stress wave trough, the axial deviator stress, σ_{1min} was 15 kPa, while at the dynamic-stress wave peak of the axial deviator stress, σ_{1max} was $\sigma_d + 15$ kPa, where σ_d is the dynamic stress amplitude at a 1 Hz cyclic loading frequency, as shown in Fig. 3. Previous studies showed that frequency has influence on the ballast performance. Low frequencies may not be representative of high train speeds. However, there are number of previous studies that used lower frequencies (< 5 Hz) to simulate low train speeds [51–60]. For failing specimens under cyclic loading, the stop criterion of the test was that the axial accumulated plastic strain had reached 15%. For specimens whose strain ceased increasing or were in a slowly increasing state, the test was terminated when the number of load cycles reached 50,000.

Table 2 Program of RLT tests

Specimen type	Moisture content w (%)	σ_3 (kPa)	σ_d (kPa)
Saturated specimen	9.3	15	50, 100, 125, 150, 200, 250
		30	50, 100, 125, 150, 200, 250
		60	50, 100, 125, 150, 200, 250
Unsaturated specimen	6.0	15	100, 150, 200, 250
		30	250, 275, 300
		60	200, 250, 300

Fig. 3 Schematic diagram of axial stress time history

3 Results

3.1 Classification of permanent deformation behavior of CGS specimens

Previous studies investigated the effects of loading condition to find critical deviator stress for subgrade under train loads. Cyclic axial strain changes with number of loading cycles were used to evaluate critical deviator stress under different loading conditions. Accordingly, a critical value was investigated for cyclic stress ratio that was defined as the ratio of the half of deviator stress to the effective confining pressure used in previous studies and was found to be between 0.3 and 0.4 for subgrade materials with different degree of compactions [13, 61–63].

The variation of accumulated plastic strain ε_p in saturated CGS specimens as a function of the number of load cycles N is shown in Fig. 4.

As can be seen in Fig. 4, under different stress states (confining pressure and dynamic stress amplitude), the accumulated plastic strain ε_p of CGS specimens develops differently over the number of load cycles. Different behaviors of accumulated plastic strains under repeated loads were grouped into behaviors: A, B, and C. When the applied dynamic stress amplitude σ_d is relatively small ($\sigma_d = 50$ kPa in Fig. 4), ε_p increases at initial stages of repeated loading and gradually levels off, after which the

ultimate response can be deemed purely elastic (A). However, when σ_d is relatively large (σ_d is greater than or equal to 150 kPa in Fig. 4), ε_p accumulates rapidly, with failure occurring in a relatively short time (C). When σ_d is at an intermediate level, ε_p accumulates at a high level during the initial period of repeated loading, and the growth rate then gradually decreases to a low, nearly constant level, and does not reach a stable value for up to 50,000 load cycles (B).

The variation of accumulated plastic strain ε_p with number of load cycles N for unsaturated CGS specimens is shown in Fig. 5. The development characteristics of accumulated plastic strain in Fig. 5 are similar with those in Fig. 4. However, it is worth mentioning that saturated specimens under all different loading conditions would reach failure much faster than unsaturated specimens as it is apparent in Figs. 4 and 5. In addition, the behavior under small deviator stress in saturated specimens (behavior “A”) was not observed for unsaturated specimens.

The relationship between the ε_p of CGS specimens and the number of load cycles can be used to describe CGS permanent deformation behavior. Werkmeister [15] pointed out that permanent strain behavior of unbound granular material specimens can be categorized into three types: plastic shakedown (Range A), where permanent strain progressively decreases leading to an asymptotic final value; plastic creep (Range B), where permanent strain decreases initially from high to low during the initial load cycles and then slowly decreases to a constant level; and incremental collapse (Range C), where permanent strain levels accumulates leading to failure.

Range A:

$$\varepsilon_{p,5000} - \varepsilon_{p,3000} < 4.5 \times 10^{-5}, \quad (1)$$

Range B:

$$4.5 \times 10^{-5} < \varepsilon_{p,5000} - \varepsilon_{p,3000} < 4.0 \times 10^{-4}, \quad (2)$$

Range C:

$$\varepsilon_{p,5000} - \varepsilon_{p,3000} > 4.0 \times 10^{-4}, \quad (3)$$

where $\varepsilon_{p,5000}$ and $\varepsilon_{p,3000}$ are the accumulated plastic strains at the 3000th and 5000th cycles of loading, respectively.

Fig. 4 Accumulated plastic strain ε_p versus number of load cycles N for saturated CGS specimens: **a** $\sigma_3 = 15$ kPa, **b** $\sigma_3 = 30$ kPa, **c** $\sigma_3 = 60$ kPa. The black solid and red dotted lines represent the measured and fitting results, respectively

In terms of variation in the permanent strain of CGS specimens with number of load cycles, and the classification method commonly found in the literatures [15],

Fig. 5 Accumulated plastic strain ε_p versus number of load cycles N for unsaturated CGS specimens: **a** $\sigma_3 = 15$ kPa, **b** $\sigma_3 = 30$ kPa, and **c** $\sigma_3 = 60$ kPa. The black solid and red dotted lines represent the measured and fitting results, respectively

permanent deformation behavior of CGS specimens in Figs. 4 and 5 can be divided into the same three categories:

Fig. 6 Photographs of specimens after testing: **a** plastic shakedown behavior, **b** plastic creep behavior, and **c** incremental collapse behavior

plastic shakedown (labeled A), plastic creep (labeled B) and incremental collapse (labeled C). Photographs of specimens after testing are shown in Fig. 6, where the specimen with plastic shakedown behavior happened to show less deformation compared to the specimens with plastic creep and incremental collapse behaviors. On the other hand, the specimen with incremental collapse behaviors had maximum deformation compared to the other two due to the higher loads applied on it.

It is worth noting that the classification of permanent deformation behavior of CGS specimens can also be analyzed from the perspective of permanent strain rate [15]. Permanent strain rate, usually a very small value, reflects the accumulated value of permanent strain per loading cycle. For study of long-term dynamic behavior of CGS specimens under repeated loading, the number of load cycles N is usually very large, so in this study, the percentage of accumulated plastic strain per 1,000 loading cycles is defined as the accumulated strain rate $\dot{\varepsilon}_p$. The method for determining the accumulated strain rate is as follows. The number of load cycles corresponding to the accumulated plastic strain ε_p is defined as N , and the percentage of plastic strain accumulated within the range $(N - 500, N + 500)$ is the $\dot{\varepsilon}_p$ value corresponding to ε_p .

As shown in Figs. 4 and 5, for plastic shakedown behavior, $\dot{\varepsilon}_p$ develops slightly at the initial stage of loading and attenuates rapidly to nearly zero. For plastic creep behavior, $\dot{\varepsilon}_p$ changes dynamically at the initial stage of loading ($N < 10,000$), then decreases gradually and tends to be stable, with permanent strain increasing slowly with the increase of load cycles. Uthus [64] applied up to 2 million load cycles and recognized that permanent deformation was still increasing even toward the end of the tests. In this case, when the number of load cycles is small, the permanent deformation of the specimen is usually small, but a larger permanent deformation will accumulate and

failure will occur when the number of load cycles becomes large enough. For incremental collapse behavior, $\dot{\varepsilon}_p$ of the specimens is always high and constantly fluctuating, with ε_p developing rapidly and specimens failing accordingly.

Figure 7 presents relationships between accumulated strain rate $\dot{\varepsilon}_p$ and accumulated plastic strain ε_p . Each range in Fig. 7 can be divided into three different ranges indicated by two dashed lines: Range A, Range B and Range C. This result is consistent with the division method in the literatures [15].

To allow clearer description of the trend of accumulated strain rate $\dot{\varepsilon}_p$ versus accumulated plastic strain ε_p , each range in Fig. 7 are magnified as shown in Figs. 8 and 9, respectively. Continuous red lines show the general trend of accumulated strain rate change versus accumulated plastic strain.

As shown in Figs. 8 and 9, in which the black curves represent plastic shakedown behavior, the accumulated strain rate $\dot{\varepsilon}_p$ rapidly decreases to zero when accumulated plastic strain ε_p is less than 1%. The blue curves representing plastic creep behavior show that the accumulated strain rate $\dot{\varepsilon}_p$ decreases rapidly and stabilizes gradually when accumulated plastic strain ε_p is less than 4%. The red curves representing incremental collapse behavior show that the accumulated strain rate $\dot{\varepsilon}_p$ remains unabated even if the accumulated plastic strain ε_p reaches 15%. Therefore, it could be important to investigate the behavior of accumulated strain rate under repeated loading relative to accumulated plastic strain changes.

3.2 Modelling of accumulated plastic strain

Further analysis shows that the relationship curves between accumulated strain rate $\dot{\varepsilon}_p$ and accumulated plastic strain ε_p are consistent with the power-function variation used to fit the experimental data in Fig. 7, in which it can be seen that

Fig. 7 Accumulated strain rate $\dot{\epsilon}_p$ versus accumulated plastic strain ϵ_p : **a** saturated specimens and **b** unsaturated specimens. The black, blue and red solid lines represent plastic shakedown, plastic creep, and incremental collapse behaviors in Figs. 4 and 5, respectively

the predicted curves are very close to the measured curves (in Figs. 8c and 9b, the red lines fluctuate markedly, and the power-function variation is not obvious). A power function proposed by Monismith [7] for predicting accumulated plastic strain, can thus be used to describe the change of accumulated strain rate $\dot{\epsilon}_p$ with accumulated plastic strain ϵ_p , as in Eq. (4).

$$\dot{\epsilon}_p = a \cdot \epsilon_p^b, \quad (4)$$

where a and b are determined regression coefficients ($a > 0$ and $b \neq 1$).

Power exponent b reflects variational characteristics of accumulated strain rate $\dot{\epsilon}_p$. When b is less than a certain value b_1 , the accumulated strain rate $\dot{\epsilon}_p$ rapidly decreases to nearly zero, representing plastic shakedown behavior. When b is larger than a certain value b_2 , the accumulated strain rate $\dot{\epsilon}_p$ either remains constant or increases gradually, the accumulated plastic strain ϵ_p develops rapidly, and the specimen failed quickly, representing incremental collapse behavior. When $b_1 < b < b_2$, the accumulated strain rate $\dot{\epsilon}_p$ decreases with an increase in the accumulated plastic strain ϵ_p . During the initial stage of accumulated plastic strain ϵ_p , $\dot{\epsilon}_p$ decreases dramatically, after which it decreases

Fig. 8 Accumulated strain rate $\dot{\epsilon}_p$ versus accumulated plastic strain ϵ_p for saturated specimens: **a** Range A, **b** Range B, and **c** Range C. The black, blue and red solid lines represent plastic shakedown, plastic creep, and incremental collapse behaviors, respectively

Fig. 9 Accumulated strain rate $\dot{\epsilon}_p$ versus accumulated plastic strain ϵ_p for unsaturated specimens: **a** Range B and **b** Range C. The blue and red solid lines represent plastic creep and incremental collapse behaviors, respectively

gradually. Therefore, the dynamic behavior of CGS specimens under repeated loading could be classified depending on the range of power exponent b and further analysis could provide better understanding of such behavior.

As described above, the accumulated strain ϵ_p is a function of cycle number N :

$$\epsilon_p(N) = f(N). \quad (5)$$

Substituting Eq. (5) into Eq. (4) produces Eq. (6):

$$\frac{d\epsilon_p}{dN} = a \cdot \epsilon_p^b. \quad (6)$$

Solving this differential equation produces Eq. (7):

$$\epsilon_p = [a(1-b)]^{\frac{1}{1-b}} \cdot N^{\frac{1}{1-b}} \quad (a > 0, b < 1). \quad (7)$$

Equation (7) is an expression describing the

Fig. 10 Variation of parameter a with dynamic stress: **a** saturated specimens and **b** unsaturated specimens

accumulated axial strain ε_p as a function of the number of load cycles N . According to Sect. 3.1, there are three possible types of permanent deformation behaviors for CGS specimens, and the accumulated plastic strain of three dynamic behaviors can be predicted in terms of different values of parameters a and b .

Equation (7) can be used to fit the measured data of different specimens to verify its accuracy, as represented by the red dotted lines in Figs. 4 and 5. It can be seen that the fitted curves have a satisfactory agreement with the measured curves except for some incremental collapse specimens; those measured data fluctuate sharply, resulting in the fitting curves deviating slightly from the measured curves.

3.2.1 Analysis of parameter a

By fitting the measured data by Eq. (7), values for parameters a and b under different loading conditions can

be obtained. From the above analysis, it can be seen that the confining pressure σ_3 and the dynamic stress amplitude σ_d have significant effects on the dynamic behavior of CGS specimens and consequently on parameters a and b . The relationship between parameter a and σ_d under different σ_3 values is shown in Fig. 10.

Figure 10 shows that parameter a increases with an increase in σ_d and decreases with an increase in σ_3 . The relationship between parameter a and σ_d can be described by a function of a power exponent as follows:

$$a = \alpha \cdot \sigma_d^\beta, \quad (8)$$

where α and β are constants.

Fitting the data in Fig. 10 with Eq. (8), corresponding values of α and β shown in Table 3 can be obtained.

Table 3 shows that the experimental data can be accurately fitted by Eq. (8). At the same CGS moisture content, parameter α in general tends to decrease rapidly with the increase of σ_3 , while parameter β conversely seems to increase slightly, so the following three laws describing the functional relationships between these parameters and σ_3 can be hypothesized:

$$\alpha = a_1 \sigma_3^{a_2} \quad (\text{power law}), \quad (9)$$

$$\alpha = a_1 e^{a_2 \sigma_3} \quad (\text{exponential law}), \quad (10)$$

$$\beta = a_3 + a_4 \sigma_3, \quad (11)$$

where a_1 , a_2 , a_3 and a_4 are regression coefficients that depend on the material tested.

By substituting the values of α and β in Table 3 into Eqs. (9)–(11), the corresponding parameters a_1 , a_2 , a_3 and a_4 shown in Table 4 can be obtained.

Table 4 shows that the power function can be used to describe the relationship between parameter α and σ_3 , so Eq. (9) can be expressed as follows:

$$a = a_1 \sigma_3^{a_2} \sigma_d^{a_3 + a_4 \sigma_3}. \quad (12)$$

Table 3 Summary of fitting results of parameters α and β

Specimen type	σ_3 (kPa)	α	β	R^2
Saturated specimens ($w = 9.3\%$)	15	1.25×10^{-14}	5.03	0.90
	30	9.75×10^{-26}	8.59	0.91
	60	1.37×10^{-34}	13.16	0.92
Unsaturated specimens ($w = 6.0\%$)	15	1.25×10^{-10}	3.16	0.91
	30	4.23×10^{-17}	6.10	0.99
	60	8.03×10^{-23}	7.51	0.99

R^2 is measure of goodness of fit

Table 4 Summary of fitting results of parameters a_1 , a_2 , a_3 and a_4

Specimen type	Equation	Parameters	Values	R^2
Saturated specimens ($w = 9.3\%$)	Equation (9)	a_1	7.27×10^{24}	1
		a_2	-32.96	
	Equation (10)	a_1	Cannot be fitted	
		a_2		
	Equation (11)	a_3	2.75	0.97
		a_4	0.18	
Unsaturated specimens ($w = 6.0\%$)	Equation (9)	a_1	7.20×10^{13}	1
		a_2	-20.20	
	Equation (10)	a_1	Cannot be fitted	
		a_2		
	Equation (11)	a_3	1.96	0.99
		a_4	0.09

Fig. 11 Variation of parameter b with dynamic stress: **a** saturated specimens and **b** unsaturated specimens

3.2.2 Analysis of parameter b

Similar to parameter a , σ_3 and σ_d significantly affect the development of parameter b . The variation of parameter b with σ_d for different σ_3 values is shown in Fig. 11.

As shown in Fig. 11, the variation of parameter b with σ_d can be described by a logarithmic function:

$$b = m \ln(\sigma_d) + n, \quad (13)$$

where m and n are constants.

Table 5 shows that parameter m tends to increase slightly with an increase of σ_3 , while parameter n conversely tends to decrease slightly with an increase of σ_3 . A simple linear equation can be used to describe the variation of m and n with σ_3 .

For saturated specimens:

$$m = 2.98 + 0.04\sigma_3, \quad (14)$$

$$n = -15.2 - 0.26\sigma_3. \quad (15)$$

For unsaturated specimens:

Table 5 Summary of fitting results of parameters m and n

Specimen type	σ_3 (kPa)	m	n	R^2
Saturated specimens ($w = 9.3\%$)	15	3.52	-18.54	0.95
	30	4.39	-23.91	0.98
	60	5.47	-30.59	0.96
Unsaturated specimens ($w = 6.0\%$)	15	1.89	-12.28	0.99
	30	1.92	-12.70	0.99
	60	2.08	-13.78	0.97

$$m = 1.81 + 0.004\sigma_3, \quad (16)$$

$$n = -11.74 - 0.03\sigma_3 \quad (17)$$

Therefore, the prediction model of accumulated plastic strain for CGS specimens can be written as follows:

$$\varepsilon_p = [a(1-b)]^{\frac{1}{1-b}} \cdot N^{\frac{1}{1-b}} \quad (a > 0, b < 1), \quad (18)$$

where

$$a = a_1 \sigma_3^{a_2} \sigma_d^{a_3 + a_4 \sigma_3}, \quad (19)$$

$$b = (m_1 + m_2 \sigma_3) \ln(\sigma_d) + n_1 + n_2 \sigma_3, \quad (20)$$

a_1 – a_4 , m_1 , m_2 , n_1 , and n_2 are regression parameters depending on the physical properties of tested materials, and in this paper, they mainly change with the variation of moisture content.

3.2.3 Different types of dynamic behavior according to a and b

The above analysis shows that when σ_3 is kept unchanged, the dynamic behavior of specimens changes from plastic shakedown to incremental collapse with an increase of dynamic stress amplitude σ_d , and parameters a and b increase accordingly, indicating that the dynamic behavior of CGS specimens can be judged in terms of the ranges of parameters a and b , as shown in Fig. 12. Because of the limited number of specimens used in this study, it is impossible to accurately determine the range of parameters a and b corresponding to different dynamic behavior, but some criticality zones can be described, and within these criticality zones, the types of dynamic behavior should be further analyzed. In addition, it should be noted that the three dynamic behaviors themselves cannot be accurately divided, and there are criticality zones for the

transformation of dynamic behavior, so the values of parameters a and b correspond to the division of dynamic behavior.

4 Discussion

Werkmeister's classification criterion [15] has been then adopted by European Standard [65]. However, Werkmeister's classification criterion is not suitable for Texas's basic materials based on RLT tests conducted by Gu et al. [66], who proposed a new criterion to redefine shakedown range boundaries for flexible base materials in Texas. It can be seen that the quantitative classification of dynamic behaviors are not completely applicable to different granular materials.

These two aforementioned scholars classified types of dynamic behaviors based on the accumulated plastic strain within the range of 3000th to 5000th cycles. In this study, the number of load cycles chosen to identify specimens without incremental collapse is 50,000, and the number of cycles applied on subgrade induced by traffic load can reach millions per year, so the long-term dynamic behavior of CGS specimens cannot be accurately reflected by considering only development of accumulated deformation between the 3000th and 5000th cycles.

In terms of dynamic triaxial test results of fine-grained soil, Li and Selig [8] performed parameter analysis using a power exponential model (Eq. 7) proposed by Monismith et al. [7], with results indicating that while parameter b is only related to the type of soil, parameter a is significantly affected by stress state and physical state. The results of this study show that parameter b directly reflects that the accumulated deformation either gradually stabilizes or fails with an increasing number of load cycles. Parameters a and b vary with the changes in dynamic stress and moisture content, as shown in Tables 3 and 4. Therefore, specific analysis is needed for different types of soils.

Because only CGS specimens with two moisture contents were studied in this RLT tests, so the influences of soil type and moisture content on the development of accumulated plastic strain cannot be analyzed in this paper, and the proposed permanent strain prediction model was not validated by other test results. This limitation needs to be overcome in future studies. Perhaps the values of parameters a and b in Fig. 12 in this paper do not apply to all soils for classifying dynamic behavior, however, the methods of establishing plastic strain model and classifying dynamic behavior types of soils under repeated loading may provide reference for studying the permanent deformation characteristics of soils.

Fig. 12 Classification of dynamic behavior types based on different values of parameters a and b : **a** parameter a and **b** parameter b

5 Conclusions

Based on shakedown theory, permanent deformation behavior of CGS specimens under repeated loading was systematically analyzed using RLT tests. The major findings of this paper can be summarized as follows:

- (1) The permanent deformation behavior of CGS specimens under repeated dynamic loads was significantly affected by stress state. The variational trend of accumulated strain rate with accumulated plastic strain can be used as a basis for classifying dynamic behavior of CGS specimens.
- (2) A critical value for deviator stress was found to be 150 kPa for unsaturated subgrade. In addition, saturated soils would reach failure faster than unsaturated soils under different loading conditions.
- (3) Based on the variation in accumulated strain rate with accumulated plastic strain, a prediction model of accumulated plastic strain with the number of load cycles was proposed. The model can accurately describe the accumulated characteristics of plastic strain for CGS specimens and reflect the effect of stress state on accumulated plastic strain.
- (4) The proposed permanent strain model is suitable for different dynamic behaviors, and the ranges of parameters for different types of dynamic behaviors were given. The conclusions of this study may provide a reference for revealing the dynamic behavior and permanent deformation characteristics of CGS specimens.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant Nos. 51878666 and 51678572). The authors are grateful to Shuozhou–Huanghua Railway Development Co., Ltd. for the support and resources provided to undertake this research.

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