Consistency evaluation of the current distribution of short-circuit tests in AT feeding systems for high-speed railways

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Abstract To improve the accuracy of fault location system, several short-circuit tests need to be conducted before being brought into service in autotransformer (AT) feeding systems for high-speed railways in China. However, no systematic algorithm yet exists to evaluate the consistency of the current distribution of short-circuit tests. A methodology is proposed in this paper to address this problem. Based on Kirchhoff's current law and the generalized method of symmetrical components, the current deviations of the AT feeding systems are analysed and then normalized with the short-circuit current as they vary greatly with systems and short-circuit sites. It is also found that the short-circuit current varies with the calculation methods, and its unbiased standard deviation also reflects the consistency of the short-circuit test. The mean and maximum of the current deviations, as well as the unbiased standard deviation of the short-circuit current, show the consistency of the short-circuit test from different aspects, although the last two items are highly relevant. Therefore, a unified evaluation index is defined as the sum of the three items, and then applied in two case studies to test its performance. The results show that, the proposed index can clearly distinguish the consistency of the short-circuit tests and may be used to sort the short-circuit tests for fault location systems. Besides, some short-circuit tests may have very poor consistency indices, and thus are not

applicable to the tuning of fault location systems. In the authors' opinion, the determination of the threshold of the proposed index needs further investigation.

Keywords Autotransformer feeding system · Short-circuit test · Consistency evaluation · Kirchhoff's current law · Generalized method of symmetrical components

1 Introduction

Autotransformer (AT) feeding systems are generally used in high-speed railways. Accurate fault location detection helps to reduce the outage duration and thus improve the efficiency of railway transport systems after the occurrence of short-circuit faults. As a result, several short-circuit tests are usually conducted for the tuning of a fault locator before being brought into service. Clearly, the accuracy of fault location depends on both precise current distribution measurement and excellent fault location algorithm.

To the authors' best knowledge, studies on fault locations are mainly focused on fault location algorithms [1–4], and very little literature focuses on whether the data obtained from the field tests are good enough for the tuning of the fault locator. The quality of the data can be evaluated by its consistency. In Refs. [5, 6], the consistency of current distribution is confirmed if the short-circuit currents that are derived by 0-sequence and 1-sequence currents are approximately equal. However, the algorithm seems incomplete and may lead to misjudgement. Thus, this topic should be further investigated.

This paper is focused on the consistency evaluation of the current distributions obtained from short-circuit tests. It is clear that the consistency of current distribution can be evaluated by to what extent Kirchhoff's current law (KCL)

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is met in the AT feeding system. Therefore, the current deviations at each node are to be calculated. As the current deviations vary with the fault site and system short-circuit capacity, they are normalized by the short-circuit current. The maximum and mean of the normalized current deviations are selected as the evaluation indices. The short-circuit current is estimated with the generalized method of symmetrical components (GMSC). It is found that the short-circuit current varies greatly with the utilized equation, and its standard deviation may also be selected as an index. To obtain a unified index, the three indices are summed. The unified index is then used in two case studies, and the results show that it can clearly distinguish the consistency of the short-circuit test.

This paper is arranged as follows. Section 2 briefly describes the AT feeding systems and the short-circuit procedure in China. Section 3 introduces GMSC and its application in AT feeding systems. Section 4 presents the formulae for current deviations and short-circuit current and gives a unified index for the consistency estimation of current distributions. Section 5 presents two case studies and draws the conclusions.

2 System description and short-circuit test procedure

A typical AT feeding system is shown in Fig. 1. In China, single-phase main transformers of $220/27.5$ kV are usually applied in high-speed railways [7]. In each AT post or section post, two ATs are usually installed: one is in service and the other in reserve. For clarity, circuit breakers, reserve ATs, and current return circuits that consist of protective wires, rails and grounding, are all omitted in

Fig. 1. The traction power network between the centre-tapped main transformer and the adjacent AT or two consecutive ATs is normally named as a cell. Thus, the feeding section shown in Fig. 1 contains two cells. Here, K denotes the possible sites for short-circuit tests; “T” denotes the trolley wire, which is a reduced representation of messenger wire and contact wire; “F” denotes the feeding wire; subscript “U” stands for up-track and “D” the down-track. For efficiency, only 15 current transformers are installed in the system, as shown in Fig. 1.

The procedure of the short-circuit test is described as follows [8]. First, several sites are selected in the AT feeding system. At each site, the catenary is uncharged, and a vehicle-mounted circuit breaker is then connected between the catenary and the rails. Next, the circuit breaker is open-circuited, and the catenary is charged. The circuit breaker is then closed, and the catenary is short-circuited. A huge short-circuit current flows through the AT feeding system, and current transformers are used to obtain the current distribution. The measured currents are transformed into voltage signals, which are used to compare with a threshold to determine whether the catenary is short-circuited or not. If yes, the measured currents will be sent to the traction substation, and a fault locator will estimate the short-circuit site based on the current distribution. Since current transformers are installed at different sites and the measured currents vary greatly in magnitude, the times to reach the threshold may be different, and current measurements may be non-synchronous. The magnitude and direction of the currents can only be determined from the measurements.

Improper configuration of the current transformer ratio will lead to the dissatisfaction of KCL as well, but it is easy to be identified [5]. Therefore, this paper mainly focuses on the consistency due to non-synchronization.

Fig. 1 Short-circuit tests in AT feeding systems

3 GMSC and its application in AT feeding systems

3.1 Outline of GMSC

Suppose that a system named S contains m similar conductors, $S_1, S_2, \dots$ and S_m . The system can be analysed by the method of symmetrical components [9]:

$$\mathbf{F}_{S_1 S_2 \dots S_m} = \mathbf{A}_m \mathbf{F}'_{01 \dots (m-1)}, \quad (1)$$

where $\mathbf{F}_{S_1 S_2 \dots S_m} = [\dot{F}_{S_1}, \dot{F}_{S_2}, \dots, \dot{F}_{S_m}]^T$ is the column vector of voltages or currents of all the m conductors, the superscript “T” denotes transpose; $\mathbf{F}'_{01 \dots (m-1)} = [\dot{F}'_0, \dot{F}'_1, \dots, \dot{F}'_{m-1}]^T$ is the column vector of sequence voltages or currents; and

$$\mathbf{A}_m = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & a^{m-1} & \dots & a \\ \vdots & \vdots & \ddots & \vdots \\ 1 & a & \dots & a^{m-1} \end{bmatrix},$$

where $a = e^{j2\pi/m}$.

Equation (1) may be extended if $S_1, S_2, \dots, S_m$ are m identical subsystems:

$$\begin{cases} F_{S_1} = F'_0 + F'_1 + \dots + F'_{m-1}, \\ F_{S_2} = F'_0 + a^{m-1}F'_1 + \dots + aF'_{m-1}, \\ \vdots \\ F_{S_m} = F'_0 + aF'_1 + \dots + a^{m-1}F'_{m-1}. \end{cases} \quad (2)$$

Assuming that each subsystem contains n conductors, the method of symmetrical components can still be used for each vector of $\mathbf{F}'_0, \mathbf{F}'_1, \dots, \mathbf{F}'_{m-1}$ as follows:

$$\begin{cases} \mathbf{F}'_0 = \mathbf{A}_n \mathbf{F}_0, \\ \mathbf{F}'_1 = \mathbf{A}_n \mathbf{F}_1, \\ \vdots \\ \mathbf{F}'_{m-1} = \mathbf{A}_n \mathbf{F}_{m-1}. \end{cases} \quad (3)$$

Substituting Eq. (3) into Eq. (2) and rearranging it in matrix form, we get

$$\mathbf{F}_{S_1 S_2 \dots S_m} = \mathbf{A}_{mn} [\mathbf{F}_0^T, \mathbf{F}_1^T, \dots, \mathbf{F}_{m-1}^T]^T, \quad (4)$$

where $[\mathbf{F}_0^T, \mathbf{F}_1^T, \dots, \mathbf{F}_{m-1}^T]^T$ denotes the column vector of the sequence components, and

$$\mathbf{A}_{mn} = \begin{bmatrix} \mathbf{A}_n & \mathbf{A}_n & \dots & \mathbf{A}_n \\ \mathbf{A}_n & a^{m-1}\mathbf{A}_n & \dots & a\mathbf{A}_n \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_n & a\mathbf{A}_n & \dots & a^{m-1}\mathbf{A}_n \end{bmatrix}.$$

It can be easily seen that $\mathbf{A}_{mn}$ is actually the Kronecker product of $\mathbf{A}_m$ and $\mathbf{A}_n$.

3.2 GMSC application in AT feeding systems

A double-track AT feeding system contains four phase conductors as T_U, F_U, T_D , and F_D . According to GMSC, AT feeding systems can be regarded as two subsystems, and each subsystem contains both T and F. For simplicity, these four conductors may be renamed as A, B, C , and D , respectively. Thus, Eq. (4) is concretized as follows [10, 11]:

$$\mathbf{F}_{ABCD} = \mathbf{A}_4 \mathbf{F}_{0123}, \quad (5)$$

where

$$\mathbf{A}_4 = \mathbf{A}_2 \otimes \mathbf{A}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

and

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

The inverse form of Eq. (5) is

$$\mathbf{F}_{0123} = \mathbf{A}_4^{-1} \mathbf{F}_{ABCD}, \quad (6)$$

where

$$\mathbf{A}_4^{-1} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

Equation (6) can be used to derive sequence currents throughout the AT feeding system. Sequence networks can also be determined according to GMSC [10, 12], as shown in Fig. 2. Here, Z_{AT} denotes the AT leakage impedance, TC denotes the transverse connector, and Z_s is the equivalent impedance of the main transformer and the upstream power supply system.

4 Consistency evaluation

4.1 Current deviations

(1) Applying KCL for node 1

It is easy to see that all the currents flowing into node 1 should meet KCL:

$$I_G - I_T - I_F = 0,$$

where I_T , I_F , and I_G are the currents flowing through the terminal T, terminal F, and neutral terminal of the main transformer, respectively, and their directions are shown in Fig. 1.

From Fig. 1, the last two items can be estimated as follows:

Fig. 2 Sequence networks of the double-track AT feeding system: **a** 0-sequence network; **b** 1-sequence network; **c** 2-sequence network; **d** 3-sequence network

$$\begin{cases} I_T = I_{TU} + I_{TD}, \\ I_F = I_{FU} + I_{FD}. \end{cases}$$

Therefore, the current deviation, which is denoted by $I_{\varepsilon(1)}$ and used to measure to what extent KCL is met at this node, is given as follows:

$$I_{\varepsilon(1)} = I_G - I_{TU} - I_{FU} - I_{TD} - I_{FD}, \quad (7)$$

where the number in parentheses in subscript is the number of current deviation.

(2) Applying KCL for nodes 3 and 5

The following can be derived easily by using KCL for node 3:

$$\begin{cases} I'_T + I'_{TU} + I'_{TD} = 0, \\ I'_F + I'_{FU} + I'_{FD} = 0. \end{cases}$$

As I'_F and I'_T are not measured, they are estimated as follows. The turn numbers of two windings in AT are identical, so the currents flowing in both windings will be equal to one another according to magneto-motive force balance. Thus, we have

$$I'_F = I'_T = I'_G/2.$$

The current deviation equations of node 3 are as follows:

$$\begin{cases} I_{\varepsilon(2)} = I'_{T1} + I'_{T2} + I'_G/2, \\ I_{\varepsilon(3)} = I'_{F1} + I'_{F2} + I'_G/2, \end{cases} \quad (8)$$

where $I_{\varepsilon(2)}$ and $I_{\varepsilon(3)}$ are the current deviations of conductors T and F at node 3, respectively.

Similarly, using KCL for node 5, we get

$$\begin{cases} I_{\varepsilon(4)} = I''_{TU} + I''_{TD} + I''_G/2, \\ I_{\varepsilon(5)} = I''_{FU} + I''_{FD} + I''_G/2, \end{cases} \quad (9)$$

where $I_{\varepsilon(4)}$ and $I_{\varepsilon(5)}$ are the current deviations of conductors T and F at node 5, respectively.

The currents from Eqs. (7) to (9) are measured at the same time, so large values of $I_{\varepsilon(2)}$ and $I_{\varepsilon(3)}$ are mainly due to improper configuration of the current transformer ratio.

(3) Applying KCL for nodes 2 and 4

When T_U is not short-circuited, using KCL at node 2, we have

$$I_{\varepsilon(6)} = I_{TU} - I'_{TU} - I''_{TU}; \quad (10)$$

similarly, using KCL to F_U at node 2, we obtain

$$I_{\varepsilon(7)} = I_{FU} - I'_{FU} - I''_{FU}, \quad (11)$$

where $I_{\varepsilon(6)}$ and $I_{\varepsilon(7)}$ are the current deviations of conductors T and F at node 2, respectively.

For the down-track conductors, we get

$$I_{\varepsilon(8)} = I_{FD} - I'_{FD} - I''_{FD}, \quad (12)$$

and

$$I_{\varepsilon(9)} = I_{TD} - I'_{TD} - I''_{TD}, \quad (13)$$

where $I_{\varepsilon(8)}$ and $I_{\varepsilon(9)}$ are the current deviations of conductors T and F at node 4, respectively.

The currents in Eqs. (10) to (13) may not be measured synchronously due to their different locations, so large values of $I_{\varepsilon(6)}$ to $I_{\varepsilon(9)}$ are mainly due to the measurement of non-synchronization or improper configuration of the current transformer ratio.

It is easy to see that Eqs. (7) to (13) hold whenever a short-circuit occurs in the first or second cell, and Eqs. (10) to (13) are irrelevant to the fault type; instead, Eqs. (7) to (9) are relevant to the fault type.

(4) Applying GMSC for ATs

It can be seen from Fig. 2 that the 1-sequence current is only fed by the main transformer. Thus, the 1-sequence current flowing through any AT should be zero. We obtain the current deviations as follows:

$$\begin{cases} I_{e(10)} = I'_{TU} - I'_{FU} + I'_{TD} - I'_{FD}, \\ I_{e(11)} = I''_{TU} - I''_{FU} + I''_{TD} - I''_{FD}, \end{cases} \quad (14)$$

where $I_{e(10)}$ and $I_{e(11)}$ are the current deviations of AT at AT post and section post, respectively.

The magnitudes of current deviations vary greatly with the site and the fault type of the short-circuit test, and thus, it is appropriate to use the per unit values to measure the degree of the consistency. Therefore, the current deviations should be divided by the short-circuit current.

4.2 Short-circuit current estimation

When a short circuit occurs in AT feeding systems, Eq. (6) can be used to calculate the sequence currents throughout the system, and the short-circuit current can be estimated according to the relationship between the short-circuit current and the sequence currents.

(1) T-R or F-R type short-circuits

If conductor B (F_U) is only short-circuited to the rails (R), supposing that the short-circuit current is I_K and $\mathbf{I}_{ABCD} = [0, I_K, 0, 0]^T$, where K stands for the short-circuit site, we get the following sequence currents using Eq. (6):

$$I_0 = -I_1 = I_2 = -I_3 = I_K/4.$$

Thus, the following holds:

$$\begin{cases} I_K = 4I_i \quad (i = 0, 2), \\ I_K = -4I_i \quad (i = 1, 3). \end{cases} \quad (15)$$

If the voltage of T to R is defined as positive, then the voltage of F to R is negative. As the direction of I_K is defined from F to R in GMSC, I_K is negative for F-R short circuits.

The short-circuit current may be estimated similarly for other types of short-circuit. From Fig. 2, the i -sequence component of the short-circuit current can be obtained as follows:

$$I_{K(i+1)} = 4(I_{iMT} - I_{iAT} - I_{iSP}), \quad (16)$$

where the subscript i denotes the current sequence number ($i = 0, 1, 2, 3$), and the number in parentheses in subscript denotes the number of short-circuit current calculation method, MT stands for the main transformer, and SP the section post.

After obtaining sequence currents of the short-circuit site, the short-circuit current can be calculated using any equation of Eq. (15).

In Eq. (7), the sum of the last four items should be equal to the neutral current of the main transformer, I_G , which also holds for AT. According to GMSC, the 0-sequence current of the main transformer or AT may also be estimated by its neutral current, and the short-circuit current may be calculated as follows:

$$I_{K(5)} = I_G + I'_G + I''_G. \quad (17)$$

In addition, regardless of short-circuit type, the short-circuit current can also be estimated using KCL for the short-circuit conductor. If T_U or T_D is short-circuited, the short-circuit current can be estimated as

$$I_{K(6)} = I_{Tj} - I'_{Tj} - I''_{Tj}, \quad (18)$$

where j ($j = U, D$) stands for up-track or down-track, respectively. Similarly, if conductor F_U or F_D is short-circuited, the following holds:

$$I_{K(7)} = I_{Fj} - I'_{Fj} - I''_{Fj}. \quad (19)$$

It can be seen that at least six equations are available to calculate the short-circuit current of T-R or F-R short-circuit.

(2) T-F short-circuit

Assuming that T_U - F_U is short-circuited and the short-circuit current is I_K , then $\mathbf{I}_{ABCD} = [I_K, -I_K, 0, 0]^T$, and we get the following by Eq. (6):

$$\begin{cases} I_0 = I_2 = 0, \\ I_1 = I_3 = I_K/2. \end{cases}$$

Thus, the following holds:

$$I_{K(7,8)} = 2I_i \quad (i = 1, 3). \quad (20)$$

After obtaining the 1-sequence and 3-sequence components, the short-circuit current can be estimated by Eq. (20).

Equation (18) or Eq. (19) can be used to calculate the short-circuit currents as well. Thus, four equations are available to estimate the short-circuit current for this kind of fault.

For current deviations, Eqs. (9) to (14) can be used. $I_0 = 0$, so the following two equations hold:

$$I_{0MT} - I_{0AT} - I_{0SP} = 0,$$

$$I_G + I'_G + I''_G = 0.$$

Thus, we get current deviations:

$$I_{e(12)} = I_{0MT} - I_{0AT} - I_{0SP}, \quad (21)$$

Table 1 Summaries of formulae for calculating current deviations and short-circuit currents for each type of short circuit

Contents	T-R		F-R		T-F	
	Up-track	Down-track	Up-track	Down-track	Up-track	Down-track
Current deviations	(7–9), (11–14)	(7–11), (13), (14)	(7–10), (12–14)	(7–12), (14)	(7–9), (12–14), (21–23)	(7–11), (14), (21–23)
Short-circuit currents	(15), (17), (18)	(15), (17), (18)	(15), (17), (19)	(15), (17), (19)	(18), (19), (20)	(18), (19), (24)

$$I_{\varepsilon(13)} = I_G + I'_G + I''_G, \quad (22)$$

where $I_{\varepsilon(12)}$ and $I_{\varepsilon(13)}$ are both the current deviations derived from the current distribution of the 0-sequence component.

Further, as $I_2 = 0$, the following holds:

$$I_{\varepsilon(14)} = I_{2MT} - I_{2AT} - I_{2SP}, \quad (23)$$

where $I_{\varepsilon(14)}$ is the current deviation derived from the current distribution of the 2-sequence component.

If T_D-F_D is short-circuited, then the following holds:

$$\begin{cases} I_{K(9)} = 2I_1, \\ I_{K(10)} = -2I_3. \end{cases} \quad (24)$$

Current deviations can be calculated similarly.

4.3 Consistency evaluation of the short-circuit test

After obtaining short-circuit currents by different formulae for each test, it is easy to see that big differences exist in short-circuit current magnitudes. The true value of the short-circuit current is unknown and may only be estimated by its mean value.

In each short-circuit test, when the current deviations are obtained, the mean and maximum values of the deviation currents can be calculated, respectively, as follows:

$$I_{\varepsilon\text{mean}} = \sum_{i=1}^{n_k} |I_{\varepsilon(i)}| / (\bar{I}_K \cdot n_k), \quad (25)$$

$$I_{\varepsilon\text{max}} = \max_i (|I_{\varepsilon(i)}| / (\bar{I}_K)), \quad (26)$$

where $\bar{I}_K$ denotes the mean value of short-circuit currents for each test and n_k is the number of current deviations. $I_{\varepsilon\text{mean}}$ emphasizes the overall level of current deviations. However, this index cannot differentiate the consistency as clearly as expected. $I_{\varepsilon\text{max}}$ is to highlight the extreme current deviations. If $I_{\varepsilon\text{max}}$ is only selected as the evaluation index, it is unable to distinguish two current distributions with similar $I_{\varepsilon\text{max}}$. Thus, both indices need be reserved.

It is obvious that the unbiased standard deviation of short-circuit currents also represents the consistency of the measurement. The per unit value of the unbiased standard deviation of the short-circuit currents is as follows:

$$I_{K\sigma} = \sigma(I_K) / \bar{I}_K, \quad (27)$$

where $\sigma(I_K)$ is the unbiased standard deviation of the short-circuit currents.

Although these three indices are relevant to each other, they reflect the consistency in different aspects and should all be reserved. To obtain a unified and strong indicator of the consistency, these indices can be summarized as follows:

$$I_p = I_{\varepsilon\text{mean}} + I_{\varepsilon\text{max}} + I_{K\sigma}, \quad (28)$$

where I_p is the proposed index.

4.4 Summaries of all types of short-circuits in AT feeding systems

The formulae for calculating current deviations and short-circuit currents are summarized in Table 1.

5 Case studies

All cases in this section are based on short-circuit tests conducted on a passenger-dedicated line in October 2014. The configuration of the AT feeding systems is as follows:

Overhead catenary: JTMH120 + CTS150

Positive feeder: LBGLJ-240/30

Protection wire: LBGLJ-120/20

Integrated grounding wire: TJ70

Contact line height: 5300 mm

System height: 1600 mm

The spatial arrangement of the traction network is shown in Ref. [7].

5.1 Case 1

An AT feeding system (System A) was short-circuited four times before being brought into service, and the current distributions are shown in Table 2. The current distribution of the 2nd short-circuit test is also shown in Fig. 3(a) for clarity. Using the proposed algorithm, the short-circuit current is calculated and listed in Table 3. It is estimated to be -7.837 kA by Eq. (13), and -7.813 kA by Eq. (21). If the

Table 2 Current distribution of short-circuit tests in System A (A)

Test	Fault type	I_{TU}	I_{FU}	I_{TD}	I_{FD}	I_G	I'_{TU}	I'_{FU}	I'_{TD}	I'_{FD}	I'_G	I''_{TU}	I''_{FU}	I''_{TD}	I''_{FD}	I''_G
1	T _D -R	1124	-966	5391	-957	4593	1212	-844	-2908	-853	3393	-95	-127	-140	-107	470
2	F _U -R	1092	-4745	1071	-905	-3488	968	2933	939	-1026	-3814	-127	-152	395	420	-534
3	T _D -R	1938	-1419	1946	-1419	1045	514	-397	-1421	-511	1815	1310	-911	-3132	-911	3644
4	T _D -F _D	2140	-1704	2137	-1717	857	675	-540	-729	481	118	1472	-1159	-1514	1117	89

evaluation algorithm in Refs. [6, 7] is used, the short-circuit currents are ideally consistent.

According to the proposed algorithm, the indices are shown in Table 4. It can be seen that I_p is 10.824%, and thus, the test is not particularly perfect. In fact, it can be seen from Fig. 3(a) that the current deviation of conductor T at node 4 is very large: up to -299 A. Thus, the algorithm in [6, 7] cannot be treated as a standard; otherwise, it may lead to a misjudgement.

The proposed algorithm is also applied to the 4th short-circuit test. The current distribution is shown in Fig. 3(b). It may be verified that the current deviations calculated by Eqs. (8), (9), (11), (12), and (21) are all very small. If the consistency was only evaluated with these current deviations, the consistency would be confirmed. However, it is obvious that the neutral currents of the main transformer and both ATs are in-phase, and the current deviation calculated by Eq. (22) is very large, up to 1.064 kA. Thus, the consistency of this test is very poor.

To illustrate the inconsistency of this short-circuit test, short-circuit currents are calculated as shown in Table 4. It is clear that the short-circuit currents vary greatly with the formulae; the difference between the maximum and the minimum is up to 1.064 kA, and the unbiased standard deviation is 434.8 A. Thus, the inconsistency of this test can be confirmed. In fact, I_p of this test is up to 42.164%. To sum up, the proposed index, I_p , is able to comprehensively show the consistency level of the short-circuit measurement and can be used for consistency evaluation.

5.2 Case 2

In this section, the aforementioned evaluation method is also used for other AT feeding systems (Systems B and C), and the current distributions are listed in Tables 5 and 6.

Using the proposed algorithm, the indices are calculated and shown in Tables 7 and 8, from which we obtained the following results:

1. The index I_{mean} is generally less than 3% and cannot significantly distinguish the consistency.
2. The huge difference between I_{mean} and I_{max} in the test means that only a few current deviations approach or reach I_{max} .
3. The correlation between $I_{K\sigma}$ and I_{max} is very strong, and calculations show that the correlation coefficient is up to 0.987 and 0.9985 for the data shown in Tables 5 and 6, respectively. However, taking into account the obvious difference between the short-circuit currents as shown in Table 3, it is recommended to regard I_p as the unified consistency index.

As for the 19 tests in Tables 2, 5, and 6, the relationship between the threshold and the percentage of the tests

Fig. 3 Current distributions of the short-circuit tests shown in Table 2. **a** The 2nd test. **b** The 4th test

Table 3 Short-circuit currents of the 2nd and 4th tests in System A

The 2nd test		The 4th test	
Formula	Value (kA)	Formula	Value (kA)
(16), 0-seq	- 7.837	(18)	4.380
(16), 1-seq	- 7.813	(19)	3.315
(16), 2-seq	- 6.713	(20), 1-seq	3.846
(16), 3-seq	- 7.741	(20), 3-seq	3.849
(17)	- 7.836		
(19)	- 7.526		

Table 4 Consistency evaluation of the short-circuit currents in System A (%)

Test	I_{mean}	I_{max}	$I_{k\sigma}$	I_p
1	0.024	0.083	0.127	0.233
2	1.077	3.946	5.802	10.824
3	0.355	1.754	2.190	4.300
4	3.216	27.650	11.299	42.164

Table 5 Current distribution of the short-circuit tests in System B (A)

Test	Fault type	I_{TU}	I_{FU}	I_{TD}	I_{FD}	I_G
1	T_D-R	1325	- 985	1198	- 979	559
2	F_U-R	1075	- 1083	1074	- 1003	96
3	T_D-R	2062	- 1117	1375	- 1074	1246
4	T_D-F_D	1255	- 1731	1213	- 1162	- 442

Table 6 Current distribution of the short-circuit tests in System C (A)

Test	Fault type	I_{TU}	I_{FU}	I_{TD}	I_{FD}	I_G	I'_{TU}	I'_{FU}	I'_{TD}	I'_{FD}	I'_G	I''_{TU}	I''_{FU}	I''_{TD}	I''_{FD}	I''_G
1	T _D -R	945	909	962	871	131	209	569	186	177	787	714	2188	787	688	2998
2	F _U -R	936	902	950	863	132	198	558	181	179	757	713	2161	774	674	2974
3	T _D -R	1222	1022	1177	1009	368	225	241	228	237	12	753	971	937	788	368
4	T _D -F _D	1233	915	1847	972	1192	1205	809	2301	533	2296	34	114	234	86	402
5	T _D -R	1166	912	1774	970	1059	1140	806	2575	686	2899	36	115	234	84	398
6	T _D -R	1231	918	1850	974	1188	1210	812	2468	446	2515	33	115	233	86	402
7	T _D -R	1229	912	1848	967	1198	1203	803	2657	651	2908	33	113	229	84	395
8	T _D -R	1230	913	1848	972	1192	1211	811	2369	551	2406	35	115	232	84	398
9	F _U -R	1029	1503	1025	1079	537	947	2964	908	1108	3712	96	210	126	12	446
10	T _D -R	1054	851	1180	876	506	212	195	645	239	866	902	625	2164	640	2531
11	T _D -R	1017	847	1139	875	434	197	196	526	156	666	833	650	2115	632	2564

Table 7 Consistency evaluation of the short-circuit currents in System B (%)

Test	$I_{\varepsilon\text{mean}}$	$I_{\varepsilon\text{max}}$	$I_{k\sigma}$	I_p
1	0.365	1.498	2.167	4.030
2	0.901	4.170	6.126	11.198
3	0.754	1.819	2.437	5.011
4	1.731	7.266	8.606	17.602

Table 8 Consistency evaluation of the short-circuit currents in System C (%)

Test	$I_{\varepsilon\text{mean}}$	$I_{\varepsilon\text{max}}$	$I_{k\sigma}$	I_p
1	0.145	0.600	0.632	1.377
2	0.144	0.691	0.721	1.556
3	0.113	0.363	0.481	0.958
4	0.318	0.722	0.630	1.670
5	2.005	8.209	8.705	18.919
6	0.739	4.400	4.440	9.580
7	1.043	9.873	9.727	20.644
8	0.524	4.941	4.864	10.329
9	1.781	7.705	8.097	17.583
10	0.2617	1.510	1.759	3.530
11	0.391	2.313	2.442	5.146

Fig. 4 Threshold versus percentage of meeting the threshold for the 19 tests

meeting this threshold is shown in Fig. 4. As seen, if the threshold chosen is very small, the percentage of tests meeting the threshold will be also low, and vice versa. The authors believe that the determination of the threshold needs further investigation.

6 Conclusions

The consistency evaluation of short-circuit current distribution in AT feeding system is studied, an algorithm is proposed, and the following conclusions are made:

1. If only a few current deviations are selected as the criterion, misjudgement may be led to, and thus, the evaluation should be conducted systematically.
2. The differences of I_p between the short-circuit tests are significant, so the index is most likely to distinguish the consistency of the short-circuit test.
3. The consistency of some tests may be very poor and they are not applicable to the tuning of fault location systems.
4. If the chosen threshold is very small, less data will meet this threshold and vice versa.

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