Multiphysics Theoretical Evaluation of Thermal Stresses in Laser Machined Structural Alumina

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Abstract Lasers are being explored as innovative machining tools to efficiently fabricate complex shapes and contours in hard and brittle structural ceramics. Although, the concentrated nature of laser beam is capable of providing suitable conditions for efficient machining of structural ceramics, it often generates steep temperature gradient that leads to generation of the detrimental thermal stresses. These thermal stresses are likely to be sufficient to nucleate the micro-cracks and subsequent crack propagation or growth can lead to failure. In light of this, a theoretical approach was employed in the present efforts to evaluate the thermal stresses during laser machining of structural alumina ceramic. A multistep multiphysics computational model was designed and developed to estimate thermal stresses during multidimensional laser machining of structural alumina. The environmental scanning electron microscopy (ESEM) observations were carried out to detect the micro-cracks generated due to the thermal stresses on the surface of laser machined alumina under various machining conditions.

Keywords Nd:YAG pulsed laser · Laser machining · Alumina · Computational model · Thermal stresses

Abbreviations

Al_2O_3	Alumina or aluminum oxide
ZrO_2	Zirconia or zirconium oxide
MgO	Magnesia or magnesium oxide
SiC	Silicon carbide
Si_3N_4	Silicon nitride

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Nd:YAG	Neodymium-doped yttrium aluminum garnet
ρ	Density (kg/m^3)
C_p	Specific heat at constant pressure (J/kg.K)
K	Thermal conductivity (W/m.K)
T	Temperature (K)
t	Time (s)
X	x-coordinate (m)
Y	y-coordinate (m)
Z	z-coordinate (m)
$C_p l$	Modified specific heat at constant pressure (J/kg.K)
δ_m	Normalization function around the melting temperature (K^{-1})
δ_v	Normalization function around the vaporization temperature (K^{-1})
H'	Smooth heaviside function
L_m	Latent heat of melting (J/g)
T_m	Melting temperature (K)
ΔT	Half-width of the curve (K)
L_v	Latent heat of evaporation (J/g)
T_v	Vaporization temperature (K)
B	Fraction of a particular phase (solid, liquid or vapor)
T_x	Transition temperature between two phases, solid–liquid or vapor–liquid (K)
φ	Laser on/off function
P_g	Average laser power density (W/m^2)
H	Heat transfer coefficient ($\text{W/m}^2.\text{K}$)
T_a	Ambient temperature (K)
T_i	Initial temperature (K)
εl	Emissivity
σl	Stefan-Boltzmann constant ($\text{W/m}^2.\text{K}^4$)
A	Absorptivity of laser beam
t_1	Time just below melting temperature before solidification
t_2	Time just below melting temperature during solidification
f	Pulse rate or pulses per second
P_w	Pulse width (s)
D	Diameter of laser beam (m)
E_p	Energy per pulse (J)
E_d	Average laser energy density per pulse (J/m^2)
E_{eff}	Effective laser energy density per pulse (J/m^2)
tg	Time gap between two pulses (s)
V	Scanning speed (mm/s)
x_r	Reference point to represents the center of the laser beam along x-axis (m)
y_r	Reference point to represents the center of the laser beam along y-axis (m)
ϕ	Standard deviation of the Gaussian laser beam (m)
u	Velocity in the x-direction (m/s)
v	Velocity in the y-direction (m/s)
w	Velocity in the z-direction (m/s)
$u2$	Displacement in the x-direction (m/s)
$v2$	Displacement in the y-direction (m/s)
$w2$	Displacement in the z-direction (m/s)

E	Elastic modulus
ν	Poisson's ratio
ε	Total strain tensor
σ	Stress tensor
C	4th order elasticity tensor
$:$	Double-dot tensor product (or double contraction)
s_0	Initial stress (Pa)
ε_0	Initial strain
ε_{th}	Thermal strain
α	Thermal expansion coefficient (K^{-1})
T_{ref}	Reference temperature (K)
p	Pressure (N/m^2)
g	Gravitational constant (m/s^2)
P_r	Recoil pressure (N/m^2)
T_s	Instantaneous surface temperature (K)
M_v	Mass of vapor molecule (kg/atm)
γ	Surface tension ($N/m.K$)
μ	Dynamic viscosity ($Pa.s$)
R	Universal gas constant ($J/mol.K$)
SiO_2	Silicon dioxide
Fe_2O_3	Iron oxide
R_2O	Base or fluxing oxides
D_L	Lateral distance between two laser pulses (mm)
O_L	Lateral overlap (%)
D_T	Transverse distance between two laser tracks (mm)
O_T	Transverse overlap (%)

Introduction

Laser machining techniques have appeared as a promising technology to efficiently and effectively machine hard and brittle structural ceramics (alumina, magnesia, zirconia, silicon carbide, and silicon nitride) into complex shapes and geometries [1–29]. On the other hand, during pulsed mode laser machining of structural ceramics (due to higher melting temperatures and lower thermal conductivity), the localized nature of heating (small laser beam and laser-material interaction zone) creates the larger temperature gradient due to sudden temperature rise and fall [4, 6, 8, 10–17] that in turn likely to induce the thermal stresses and subsequent crack nucleation in the small laser-material interaction zone and surrounding heat affected zone (HAZ) [30–33]. Thus, generation of thermal stresses poses a serious limitation for this innovative laser machining technology to effectively fabricate structural ceramics.

Previously, a considerable amount of experimental and computational work [20, 22–26, 29–45] was carried out to investigate the role of thermal stresses during laser machining of structural ceramics and its effects on the machining quality. In these studies, researchers proposed various methodologies and approaches to effectively machine ceramics, such as by active stressing techniques to delay fracture during laser cutting of alumina [30, 34], by designing appropriate operating conditions [35], by controlled fracture techniques during laser cutting of alumina [36], by investigating

deformation mechanisms [45], by laser control melting during cutting of straight, rectangular, and triangular geometry into alumina tile [22–25], by using two combined laser sources during surface treatment of alumina-based ceramic [38], and by effective stress management [31–33, 37–44]. The laser processing parameters used by most of these works revealed that the large thermal gradient caused by the localized laser beam generated the thermal stresses, which were generally higher than the fracture and/or yield strength of structural ceramics and are sufficient enough to nucleate micro-cracks that grew during subsequent heating/cooling cycles. Therefore, to improve the quality of laser machining, it is very important to evaluate the magnitude and location of thermal stresses during laser machining and establish tight control on laser machining parameters. However, due to localized nature of laser beam and very short duration of laser-material interaction it is extremely challenging to evaluate thermal stresses via in-situ experimental analysis during laser machining of structural ceramics. Furthermore, difficulties associated with preparation of surfaces due to extremely brittle and hard nature of these ceramics it is very difficult, if not impossible, to conduct post-process stress analysis by experimental techniques. In view of this, continuum level (finite-element) computational modeling approach can be considered as an effective solution.

Previously, the present authors have employed an integrated experimental and computational methodology to investigate the influence of stationary (single [11] or multiple laser pulses [13]) and moving (multiple laser pulses [15]) laser beam during one-, two-, and three-dimensional (1D, 2D, and 3D) laser machining of structural alumina (Fig. 1) (to specifically replicate generic drilling, cutting, and milling operations, respectively) [10, 11, 13–17] and their effects on developing surface roughness. This multistep multiphysics computational model incorporated the temperature-dependent material properties and heat transfer and fluid flow boundary conditions to simulate the influence of various physical phenomena (Marangoni convection, recoil pressure, cooling rates, and surface tension) involved during laser machining and their resultant effects on evolving surface roughness, which was also successfully validated for ± 3 to 5 % error with the experimental observations.

Fig. 1 Classification of pulsed mode (PM) multidimensional laser machining: (a) one-dimensional (single and multiple laser pulses), (b) two-dimensional (multipulse), and (c) three-dimensional (multipulse)

In extension of these works and as part of the ongoing incremental efforts, an advanced multistep multidimensional multiphysics computational model was developed on commercially available COMSOLTM multiphysics finite-element platform. Further, this computational model combined heat transfer, fluid flow, and structural mechanics and incorporated the thermo-mechanic coupling to predict and/or evaluate not only temperature field, fluid flow, and resultant surface topography but also the corresponding thermal stresses during 1-, 2-, and 3D laser machining of structural alumina, which is schematically presented in Fig. 2. Although, the authors are aware that experimentally measured thermal stresses can also be helpful to validate the theoretical prediction, due to several limitations mentioned earlier such as but not limited to localized nature of laser beam-material interaction and difficulties associated with preparation of surface of these brittle and hard structural ceramics; no attempts were made to explore any experimental techniques.

Materials and Methods

Sample Preparation and Laser Machining

In the present efforts, an alumina slab (porosity <0.5 %) with an average equiaxed grain size of 20 μm obtained from Advalue Technology Inc., Tucson, AZ was used. The alumina had purity of 99.6 wt. % (<0.1 % SiO_2 , <0.05 % Fe_2O_3 , and <0.1 % R_2O all in wt.%). JK 701 Lumonics pulsed Nd:YAG (1064 nm wavelength) laser system was

Fig. 2 Schematic of multiphysics multistep computational model adopted in evaluation of thermal stresses

Table 1 Laser machining parameters employed for laser machining of structural alumina

Parameter	Value (units)
P_w	0.5 (ms)
D	0.6 (mm)
A	0.25
One-dimensional laser machining: single laser pulse	
E_p	1, 1.5, 2, 2.5, 3 (J)
f	1 pulse
$E_d = E_p / (\frac{\pi}{4} D^2)$	3.5, 5.3, 7.1, 8.8, 10.6 ($\times 10^6$ J/m ²) respectively
One-dimensional laser machining: multiple laser pulses	
E_p	1 (J)
$E_d = E_p / (\frac{\pi}{4} D^2)$	3.5×10^6 J/m ²
f	10, 20, 30, 40, 50 pulses
$E_{eff} = [E_p / (\frac{\pi}{4} D^2)] \times f$	3.5, 7.1, 10.6, 14.1, 17.7 ($\times 10^7$ J/m ²) respectively
Two-dimensional laser machining: multiple laser pulses	
E_p	1 (J)
$E_d = E_p / (\frac{\pi}{4} D^2)$	3.5×10^6 J/m ²
f	17, 20, 25, 34, 50, 100 pulses
$tg = 1/f$	0.06, 0.05, 0.04, 0.03, 0.02, 0.01 (s)
V	10 (mm/s)
$D_L = V/f$	0.6, 0.5, 0.4, 0.3, 0.2, 0.1 (mm) respectively
$O_L = (\frac{D-D_L}{D}) \times 100$	0, 17, 33, 50, 67, 83 (%) respectively
$E_{eff} = [E_p / (\frac{\pi}{4} D^2)] \times f$	6.02, 7.08, 8.85, 12.0, 17.7, 35.4 ($\times 10^7$ J/m ²) respectively
Three-dimensional laser machining: multiple laser pulses	
D_T	0.6, 0.5, 0.4, 0.3, 0.2, 0.1 (mm) respectively
$O_{T=} (\frac{D-D_T}{D}) \times 100$	0, 17, 33, 50, 67, 83 (%) respectively

employed for laser machining of structural alumina. The laser machining parameters used for 1D (single and multiple laser pulses), 2D and 3D laser machining are enumerated in Table 1 and corresponding schematics for multidimensional laser-machined alumina are presented in Fig. 1.

Multistep Multidimensional Multiphysics Computational Model

During multidimensional laser machining, material experiences various physical phenomena such as phase transition from solid-to-liquid-to-vaporization, material loss during evaporation, and solidification from liquid-to-solid. In addition, material also experiences the transition dependent effects such as thermal expansion during heating, recoil pressure during vaporization, and Marangoni convection and surface tension during solidification. In order to simulate such complex laser machining mechanism, the present model was designed and developed using multistep multiphysics computational modeling approach for multidimensional (1-, 2-, and 3D) laser machining

process on COMSOLTM multiphysics finite-element platform. The present computational model based on the multiphysics approach combined heat transfer, fluid flow, and structural mechanics for thermo-mechanical coupling (temperature and thermal expansion coefficient) to investigate the combinatorial effects of these physical phenomena on evolution of surface topography and thermal stresses as schematically presented in Fig. 2. The selective governing equations and boundary conditions for multiphysics computational model are presented in Table 2 [46–48]. Furthermore, the present computational model incorporated the heat transfer, fluid flow, and structural mechanics boundary conditions (Table 2) corresponding to various physical phenomena (Marangoni convection, surface tension, recoil pressure, and structural deformation) and temperature-dependent material properties (Table 3) (thermal conductivity, heat capacity, elastic modulus, coefficient of thermal expansion, and dynamic viscosity) in order to assess the temperature (T) profile, associated cooling rate, subsequent surface topography, and resultant thermal stresses during 1-, 2-, and 3D laser machining [10–17].

The present model used the thermal stress interface of structural mechanics module to couple solid mechanics interface with a heat transfer interface in such a way that the temperature from the heat transfer interface acts as a thermal load for the solid mechanics interface [49]. To define the linear elastic material properties, the linear elastic material node from the structural mechanics module was specifically assigned to

Table 2 Selective governing equations and boundary conditions for multiphysics computational model

Boundary condition	Equation	#	References
Heat flux, natural convection cooling and radiation	$\rho c_p [\partial T / \partial t] = (\nabla(k(T) \nabla T)) + \varphi P_g - k(T) \nabla T = -\varphi P_g + h[T - T_a] + \varepsilon 1 \sigma 1 [T^4 - T_a^4]$ <p>where, $\varphi = \text{rect1} \left(\text{mod} \left(t, \frac{1}{f} \right) \right)$ Here, $h = 10 \text{ W/m}^2 \cdot \text{K}$, $\varepsilon = 0.7$, and $T_a = 298.15 \text{ K}$</p>	(1) and (2)	[11–17]
Average laser power density in Gaussian distribution (W/m^2)	$P_g = A \left[\frac{E_p}{P_w \left(\frac{\pi}{4} D^2 \right)} \right] \cdot \exp \left[-\left(\frac{(x-x_r)^2}{2\sigma^2} \right) \right] \cdot \exp \left[-\left(\frac{(y-y_r)^2}{2\sigma^2} \right) \right]$	(3)	[11–17]
Level set method	$B = 1$ for $(T > T_x + \Delta T)$	(4)	[52, 53]
	$B = \frac{(T - T_x + \Delta T)}{2\Delta T}$ for $(T_x - \Delta T) \leq T \leq (T_x + \Delta T)$ Here $\Delta T = 10 \text{ K}$	(5)	
	$B = 0$ for $(T < T_x - \Delta T)$	(6)	
Volume force	$\rho g \alpha (T - T_m)$ given by Boussinesq approximation	(7)	[52, 53]
Recoil pressure	$P_r = (P_g) \frac{1.69}{\sqrt{L_v}} \left[\frac{\sqrt{\left(\frac{k T_s}{M_v L_v} \right)}}{1 + 2.2 \left(\frac{k T_s}{M_v L_v} \right)^2} \right]$ <p>only when $T \geq T_v$</p>	(8)	[48]
Surface tension	$\text{test}(u) * \left[\frac{\partial \gamma}{\partial T} \frac{\partial T}{\partial x} \right], \text{test}(v) * \left[\frac{\partial \gamma}{\partial T} \frac{\partial T}{\partial y} \right], \text{test}(w) * \left[\frac{\partial \gamma}{\partial T} \frac{\partial T}{\partial z} \right]$	(9), (10), and (11)	[11–17]
Total Strain tensor in terms of displacement gradient	$\nabla u_2 = \frac{\partial u_2}{\partial x}, \quad \nabla u_2 = \nabla u_2 \approx 1,$ $\therefore \varepsilon = \frac{1}{2} [\nabla u_2 + (\nabla u_2)^T]$	(12)	[22, 49, 51]
Stress tensor	$\sigma = s$ $s = s_0 + C : (\varepsilon - \varepsilon_0 - \varepsilon_{th})$ (Hooke's law)	(13)	[22, 49, 51]
Thermal strain	$\varepsilon_{th} = \alpha (T - T_{ref})$ Here, $T_{ref} = 298.15 \text{ K}$	(14)	[22, 49, 51]

Table 3 Material properties of alumina

Property	Value (units)	References
ρ (T)	3800 (kg/m ³)	[47]
k (T)	$58.868 - 0.22972T + 2.607 \times 10^{-4}T^2 - 1.3607 \times 10^{-7}T^3 + 2.7092 \times 10^{-11}T^4$ (W/m.K)	[25]
CpI (T)	$Cp = 880 + \delta_m(L_m) + \left(\frac{L_m}{T_m}\right) \cdot H'((T-T_m), \Delta T) + \delta_v(L_v)$ $+ \left(\frac{L_v}{T_v}\right) \cdot H'((T-T_v), \Delta T)$ (J/kg.K)	[11–17]
δ_m and δ_v	$\frac{\exp\left[-\left(\frac{(T-T_m)^2}{\Delta T^2}\right)\right]}{\Delta T \sqrt{\pi}}$, and $\frac{\exp\left[-\left(\frac{(T-T_v)^2}{\Delta T^2}\right)\right]}{\Delta T \sqrt{\pi}}$	
T_m	2324 (K)	[47]
T_v	3273.15 (K)	[47]
L_m	1067.43 (J/g)	[47, 48]
L_v	1066.5 (J/g)	[47, 48]
M_v	1.693e-25 (kg/atm)	[47, 48]
α (T)	$(-0.23036 + 7.0045 \times 10^{-4}T + 5.681 \times 10^{-8}T^2) (\times 10^6 \text{ 1/K})$ valid for $298 > T < 2000$ K	[25]
$\partial\gamma/\partial T$	-8e-5 (N/(m.K))	[11–17]
γ (T)	$0.64 - 8.2 \times 10^{-5}(T - T_m)$ valid for $2190 > T < 2500$ K	[11–17]
μ (T)	$3.2 \cdot \left[\exp\left(\frac{43.2 \times 10^3}{RT}\right)\right]$ valid for $2190 > T < 2500$ K	[11–17]
σI	5.67×10^{-8} (W/m ² .K ⁴)	[46]
R	8.314 (J/mol.K)	[46]
g	9.81 (m/s ²)	[46]
E (T)	$407.10 - 7.3407 \times 10^2 T$ (GPa) valid for $298 > T < 2000$ K	[25]
ν (T)	$0.2644 + 2 \times 10^{-5}T$ valid for $298 > T < 1300$ K	[25]

the linear elastic solid as well as its interface (Table 3). The temperature-dependent elastic modulus and coefficient of thermal expansion were, therefore, assigned to the model to effectively incorporate elastic behavior law (Table 3). In addition, the multiphysics node was added to the present model to incorporate the multiphysics coupling features: (i) thermal expansion and (ii) temperature coupling [49]. Particularly, due the localized nature of laser beam, material experienced higher temperature gradient and associated thermal expansion and strain/deformation during heating and cooling/solidification process. Previously many researchers [22, 50, 51] have shown by their solidification models that the thermal strains which dominate the thermo-mechanical behavior involved very small strain/deformation during solidification and hence suggested that the strain due to the thermal expansion must be subtracted from the total strain. In the present efforts, the thermal expansion coupling feature introduced by the multiphysics node automatically subtracted thermal expansion induced strain/deformation from the total strain [49, 51]. The governing equations related to thermal stress are listed in Table 2 by Eqs. (12–14). The temperature history evaluated from the present multiphysics model was used as input to the thermal stress analysis, which in turn was modeled as von Mises elastic–plastic material with isotropic hardening and a yield stress that changed with temperature [22].

The present multiphysics computational model utilized two-step modeling approach to simulate multidimensional (1-, 2-, and 3D) laser machining process for structural alumina. The Step-1 of the computational model (coupled with heat transfer, structural mechanics, and phase change kinetics) was specifically designed and developed to predict the geometry of a crater produced via evaporation losses of the material (Tables 2 and 3). In Step-1, the level-set method [52, 53] (Eqs. (4–6) in Table 2) was employed to trace the interfaces between liquid–solid and vapor–liquid phases so that the elements whose temperature reached above the vaporization temperature were excluded from the geometry. The dimensions of crater (depth and width) play a significant role in predicting the final surface profile and associated thermal stresses. Therefore, in Step-2 of the model the crater geometry predicted from Step-1 was considered as a starting surface profile. The main objective of Step-2 of the computational model (coupled with the heat transfer, fluid flow, structural mechanics, and phase change kinetics) was to predict the combinatorial effects of phase change and associated physical phenomena on the evolving surface topography and thermal stresses (Fig. 2, Table 2, and Table 3).

Nonetheless, the main efforts of the present article being to study the thermal stresses during multidimensional laser machining, the discussion on evolving surface topography is not presented here and can be found in the previous publications [10–17]. It was anticipated that the minimal thermal stress in the vicinity or equal to the fractural strength of the alumina (379 GPa) [54] is sufficient enough to induce/initiate (nucleate) micro-cracking during laser machining. Thermal stress higher than the fractural stress, although at present the magnitude of such high stress is unknown, is likely to not only induce/initiate (nucleate) crack but also contribute to its growth/expansion. Also, not only the magnitude of stress but number of pulses and overlap (lateral and transverse) are likely to influence the crack growth and expansion. Furthermore, during multi pulse and the multi-track overlap laser treatment, depending upon the level of thermal stress generated within the region of material treated by the previous laser pulse and previous track, an existing crack is likely to experience growth/expansion in subsequent laser pulses and tracks. Due to the intense stress concentration at the tip of such already existing micro crack it is highly likely to grow/expand at the level of thermal stress (depending upon the magnitude of the stress concentration at the crack tip) less than the theoretical flexural strength of alumina (379 GPa) in the subsequent laser pulses and overlaps. It should be noted that the present analysis only considered the thermal stress of solid phase (just below melting temperature before melting and just below melting temperature after solidification) with the assumptions that the thermal stress field is zero for liquid phase, even though the thermal stresses of all the phases (solid, liquid, and gas) can be numerically predicted [22, 23, 25], the micro cracks are be physically generated and retained only in a solid phase.

The geometry for computational model is presented in the Fig. 3. In order to effectively achieve the savings in computation time and cost, a two-dimensional geometry based on X-Y symmetric cutting plane (Fig. 3a) was designed for one-dimensional computational modeling, whereas, a three-dimensional geometry (Fig. 3b) was designed and used for the 2D and 3D computational models. For 1D single pulse laser machining, the dimensions of A , B , C , and D are 1200, 500, 200, and 600 μm , respectively. However, for 1D multipulse laser machining, the dimension of B is 2500 μm , while dimensions for A , C , and D are same as used for 1D single pulse laser machining. Similarly, for 2D and 3D multipulse laser machining, the dimensions of A , B , C , and D are 1600, 1200, 200, and 600 μm , respectively (Fig. 3).

Fig. 3 Geometry for computational model of (a) one-dimensional (single and multiple laser pulse), and (b) two- and three-dimensional multipulse laser machining

Topographical Observations

An environmental scanning electron microscope (ESEM, FEI Quanta 200) was employed to conduct topographical observations of laser machined alumina. The main objective of using ESEM in the present work was to observe and confirm the presence or absence of micro-cracks on the surface of laser machined alumina under various laser machining conditions.

However, in the present efforts, an advanced multiphysics multistep computational model was designed and developed to evaluate thermal stresses during multidimensional (1-, 2-, and 3D) laser machining of structural alumina. The present efforts mainly focused on establishing a link between the computationally predicted thermal stresses and their locations as function of laser processing parameters (energy density, number of pulses, and transverse and lateral overlap) and its correlation with the experimentally observed micro-cracks. Although the discussion/explanation behind the mechanism of

crack nucleation and growth/expansion during laser machining is important and interest to the authors, it is not a part of the current work. Separate efforts on this aspect are ongoing and will be presented in future publications.

Results and Discussions

One-Dimensional Laser Machining - Single Laser Pulse

1D single pulse laser machining was carried out (to replicate generic drilling process, Fig. 1a) by delivering only single laser pulse with stationary laser beam and workpiece to produce shallow hole or develop physical surface texture of shallow depth. Based on the multistep computational model developed for various laser processing conditions during one-dimensional (1D) single pulse laser machining of alumina [11], the transient temperature (T) (Fig. 4a) and thermal stresses (Fig. 4b) as function of time (t) were predicted from the domain probe for various laser energy densities ($3.5, 5.3, 7.1, 8.8, 10.6 \times 10^6 \text{ J/m}^2$). As described earlier, the present analysis only considered the computationally evaluated thermal stresses of solid phase (before melting and after melting/solidification) [22, 23, 25], even though the thermal stresses of all the phases (solid, liquid, and gas) can be numerically predicted (Fig. 4b). It should be noted that

Fig. 4 Computational plots (a) transient temperature and (b) thermal stresses as function of time, (c) temperature and (d) thermal stresses in alumina just below melting temperature during solidification and corresponding time= t_2 ms along boundaries 3, 6, and 8 during one-dimensional (single pulse) laser machining of alumina

the times t_1 and t_2 (Fig. 4a and b for all laser processing conditions) are times (in ms) corresponding to the times just before melting and below melting during solidification, respectively, and during this time period (between t_1 and t_2) the temperature (T) is higher than the melting temperature ($T_m=2324$ K) of alumina. In addition, it was also observed that the peak temperature (Fig. 4a) as well as corresponding time period between t_1 and t_2 , where $T>T_m$ increased with an increase in laser energy density.

The spatial temperature (Fig. 4c) and thermal stress (Fig. 4d) distribution just below melting during solidification (corresponding to time t_2) were also predicted from the boundary probe located on the top surface of the geometry (boundary #s 3, 6, and 8, Fig. 3a). Moreover, due to Gaussian distribution of the laser beam, the center of the laser beam supplied higher laser energy density and gradually decreased towards the edges that in turn developed higher temperature at center compared to edges of the laser beam (Fig. 4c). As a result, material experienced preferential heat conduction as well as higher temperature gradient thereby inducing the higher thermal stresses of tensile (+ve) in nature at the center than the edges of the laser beam (Fig. 4d). Furthermore, for 5.3 to 10.6×10^6 J/m² laser energy input (Fig. 4d), a substantial increase in thermal stress around the outer edge region of the laser beam was observed. This is primarily due to the evolution of specific surface topography/profile associated with the piling of solidified material around the lip of machined crater (cavity) during laser machining (Fig. 5) as well as due to the heat conduction (or dissipation) and associated large thermal gradient during single pulse laser machining. However, this effect is negligible for the 3.5×10^6 J/m² laser processing conditions (Fig. 4d) mainly due to the lower energy density (3.5×10^6 J/m²) that barely allowed temperature to reach above the vaporization temperature. This resulted in reduced material loss due to evaporation (Fig. 4a), reduced resultant temperature gradient and reduced pile up around the laser interaction zone thereby generating higher magnitude of thermal stresses mainly confined to the center region of the laser beam (Fig. 4d). Nonetheless, it is important to note that the predicted thermal stresses for all laser energy densities (3.5, 5.3, 7.1, 8.8, 10.6×10^6 J/m²) are less than the flexural strength of alumina (379 MPa).

The contour plot illustrating the spatial distribution of thermal stresses just below melting during solidification at time t_2 and at the room temperature for various laser energy densities (3.5, 5.3, 7.1, 8.8, 10.6×10^6 J/m²) are presented in Fig. 5a and b respectively. Based on the predicted temporally and spatially distributed temperature and thermal stress data (Figs. 4 and 5) corresponding to all presently employed 1D single pulse laser machining conditions (3.5, 5.3, 7.1, 8.8, 10.6×10^6 J/m²), it was observed that although the thermal stresses are lower they are close to the fracture strength of alumina (379 MPa). In such situation, it was anticipated that even though the micro-cracks nucleated they did not grow further due to very short duration of laser-material interaction (0.5 ms) during machining [11]. In view of this, few isolated micro cracks and network of micro-cracks located in center region of the crater of the laser machined alumina were detected during ESEM observations (Fig. 6).

Fig. 5 Contour plots of thermal stresses during one-dimensional (single pulse) laser machining of alumina (a) just below melting temperature during solidification at time= t_2 and (b) at room temperature

Fig. 6 ESEM photographs of top surface of one-dimensionally (single pulse) laser machined alumina

One-Dimensional Laser Machining - Multiple Laser Pulses

Unlike single pulse 1D laser machining, multipulse 1D laser machining was carried out (to replicate generic drilling process, Fig. 1a) by delivering multiple laser pulses at the same location (with stationary laser beam and workpiece) to create deeper holes at much higher material removal rates. Based on the computational model developed for 1D multipulse laser machining, the thermal stresses were estimated similar to 1D single pulse laser machining for various laser energy densities 3.5 , 7.1 , 10.6 , 14.1 , and $17.7 \times 10^7 \text{ J/m}^2$ with 10 , 20 , 30 , 40 , and 50 laser pulses/second, respectively. It should be noted that the energy per pulse for all 1D multipulse laser machining was 1 J , which was same as the first laser processing condition for 1D single pulse laser machining ($3.5 \times 10^6 \text{ J/m}^2$). However, due to subsequent variation in pulse rates (10 , 20 , 30 , 40 , and 50 pulses/second) the effective laser energy density varied (3.5 , 7.1 , 10.6 , 14.1 , and $17.7 \times 10^7 \text{ J/m}^2$) thereby influencing the associated temperature distribution and resultant thermal stresses. Figure 7 presents the contour plot illustrating the spatial distribution of thermal stresses just below melting temperature during solidification after application of the last laser pulse at time t_2 (Fig. 7a) and at the room temperature (Fig. 7b). The thermal stresses are of tensile (+ve) in nature at both time steps (t_2 and room temperature). The thermal stresses at time t_2 reached near or slightly above the fracture strength of alumina (379 MPa) within the center region of machined crater due to Gaussian beam profile of the laser beam (Fig. 7a). On the contrary, the estimated thermal stresses at room

Fig. 7 Contour plots of thermal stresses during one-dimensional (multiple laser pulses) laser machining of alumina (a) just below melting temperature during solidification corresponding to last laser pulse at time $= t_2$ and (b) at room temperature

temperature (Fig. 7b) mainly remained substantially below the fracture strength of alumina. This combination of states of stresses at time= t_2 and room temperature led to nucleation of few isolated cracks without their recognizable growth and expansion (Fig. 8).

Two-Dimensional Laser Machining- Multiple Laser Pulses

During multipulse 2D laser machining either the laser beam or the workpiece is travelled in relation to the other and simultaneously multiple laser pulses of short pulse width (0.5 ms) were delivered at defined pulse rate (f) to simulate/replicate generic cutting process (Fig. 1b). As the laser beam was continuously moved along the Z-axis (Fig. 1b), the lateral distance between two laser pulses (D_L = scanning speed/pulse rate= V/f) and corresponding lateral overlap (O_L), were considerably important for the present process (Fig. 1b). Since, the pulse rate (f) establishes the time duration between two consecutive laser pulses (t_g , Table 1), the temperature history and its subsequent effect on the thermal stresses varied. Thus, the appropriate selection of laser processing conditions (scanning speed (V) and pulse rate (f)) during 2D laser machining are extremely important to reduce the magnitude of the evolving thermal stresses below the fracture strength of alumina (379 MPa) to possibly produce crack-free components.

It should be noted that the 2D multipulse laser machining was carried out using 1 J energy per pulse, which was similar to the first laser processing condition for 1D single pulse laser machining (3.5×10^6 J/m²). However, the combination of scanning speed ($V=10$ mm/s) and pulse rates ($f=17, 20, 25, 34, 50$, and 100 laser pulse/second) provided variation in the lateral distance between two laser pulses ($D_L=0.6, 0.5, 0.4, 0.3, 0.2, 0.1$ mm) and lateral overlap ($O_L=0, 17, 33, 50, 67, 83$ %) and therefore corresponding variation in the effective laser energy density ($E_{eff}=6.02, 7.08, 8.85, 12.0, 17.7, 35.4 \times 10^7$ J/m², Table 1) thereby subsequently

Fig. 8 ESEM photograph of top surface of one-dimensionally (multiple laser pulses) laser machined alumina

influencing the associated temperature distribution and resultant thermal stresses. Based on the multiphysics computational model developed for 2D laser machining, the temperature history (Fig. 9a) and consequential thermal stresses (Fig. 9b) were predicted for various laser machining conditions ($E_{eff}=6.02, 7.08, 8.85, 12.0, 17.7, 35.4 \times 10^7 \text{ J/m}^2$, Table 1).

Within the range of laser processing conditions employed in the present work, the predicted temperature history did not indicate any significant temperature build up during the subsequent laser pulse delivery [15] (Fig. 9a). The counter plot of the spatial distribution of thermal stresses (Fig. 9b) just below melting temperature during solidification (at time= t_2) after second laser pulse during 2D laser machining of alumina indicated that the thermal stresses are tensile in nature (Fig. 9b). The stresses are highest in the center region of the crater and reached nearly the flexural strength of alumina (379 MPa) under all the processing conditions (Fig. 9b). Although, it was anticipated that the micro-cracks may nucleate under such conditions, their further growth/expansion and evolution into a network depended upon the pulse rate (f) and scanning speed (V) that together controlled the lateral overlap (O_L) between two pulses. As the two consecutive laser pulses were distinctly separated from each other for lateral overlap (O_L) of 0 % for $6.02 \times 10^7 \text{ J/m}^2$ no network of micro-cracks was observed (Fig. 9b). On the contrary, when the lateral overlap gradually increased (O_L from 17 to 83 %) for the lateral distances between two consecutive laser pulses (D_L) from 0.5 to 0.1 mm, the extent of micro-crack network also increased thereby becoming extensive for lateral overlap (O_L) of 67 and 83 % ($D_L=0.2$ and 0.1 mm) as observed in Fig. 10a and b, respectively.

Fig. 9 Computationally predicted (a) temperature history and (b) thermal stresses in alumina just below melting temperature during solidification (time= t_2) after second laser pulse during two-dimensional (moving) laser, lateral overlap) laser machining of alumina

Fig. 10 ESEM photographs of top surface of two-dimensionally laser machined alumina

Three-Dimensional Laser Machining- Multiple Laser Pulses

3D laser machining was carried out to machine large area and complex geometries (Fig. 1c). Due to smaller beam diameter (~ 0.6 mm), 3D laser machining involved laying of successive laser tracks with the combination of lateral overlap (overlap between two consecutive laser pulses, O_L) and transverse overlap (overlap between two laser tracks, O_T) to cover larger area or volume (Fig. 1c). Thus, unlike 2D laser machining, the 3D laser machining was carried out by using both lateral (O_L) and transverse (O_T) overlaps in various scanning configurations [16]. In view of this, appropriate multistep multiphysics computational model (moving laser beam, multiple laser pulses) was designed and developed to predict the temperature history and corresponding thermal stresses for various laser machining conditions ($E_{eff}=6.02, 7.08, 8.85, 12.0, 17.7, 35.4 \times 10^7$ J/m², corresponding to $D_L-D_T=0.6-0.6, 0.5-0.5, 0.4-0.4, 0.3-0.3, 0.2-0.2, 0.1-0.1$ mm and $O_L-O_T=0-0 \%, 17-17 \%, 33-33 \%, 50-50 \%, 67-67 \%, 83-83 \%$, respectively, Table 1). Similar to 2D laser machining, every pulse in all sets of the processing conditions of 3D laser machining employed 1 J laser energy per pulse (3.5×10^6 J/m², Table 1) and no temperature build up was experienced (Fig. 9a) [16]. Thus, although the thermal history of the individual pulse and individual laser track remained same, the combination of delivery of lateral (O_L) and transverse (O_T) laser tracks may affect the thermal stresses and their effect on the pre-existing micro-cracks and nucleation and growth of new micro-cracks.

Based on the present multistep multiphysics computational model for 3D laser machining, the spatial distribution of thermal stresses were computed for 3.5×10^6 J/m² laser energy density along the top surface (Fig. 11a) in X-Z cutting plane (Fig. 3b) and around the center axis of laser beam (Fig. 11b) in X-Y cutting plane (Fig. 3b) just below melting temperature during solidification (time= t_2) at the end of the last laser pulse during 3D laser machining of alumina (Fig. 11). As a consequence, just below melting temperature during solidification (time= t_2) the corresponding thermal stresses were tensile in nature, (+ve) in both spatial and depth distributions (Fig. 11). Due to the Gaussian laser beam profile the stresses were highest in the center region and dropped towards outer region (Fig. 11a).

Fig. 11 Predicted thermal stresses (a) on the top surface and (b) just below melting temperature during solidification (time= t_2) at the end of the last laser pulse during three-dimensional laser machining of alumina

Furthermore, the depth profile (Fig. 11b) indicated that the stresses were high on the surface and dropped off substantially through the depth of $\sim 200\ \mu\text{m}$. Thus, both surface and depth profiles (Fig. 11) indicated that the stresses on the surface evolved closed to the flexural strength of alumina (379 MPa) with the possibilities for nucleation and generation of micro-cracks. Accordingly, ESEM observations revealed the presence of few isolated cracks on the surfaces of samples processed under all six laser processing conditions employed during 3D laser machining in the present efforts (Fig. 12).

Although, laser processing conditions (laser energy densities) within the individual laser track of all six processing conditions of both 2D and 3D machining were same (Table 1), unlike 2D machining (Fig. 10), ESEM observations indicated the absence of network of micro-cracks on the surfaces of all 3D machined samples (Fig. 12). Such pronounced difference was expected due to various overlapped laser tracks (O_T) produced by transverse shifts (D_T) in subsequent laser tracks required during 3D machining. The various overlaps produced under six different laser processing conditions (O_L – O_T =0–0 %, 17–17 %, 33–33 %, 50–50 %, 67–67 %, 83–83 %, respectively, Table 1) employed in the present work likely to have melted/sealed the (network of) micro-cracks produced during previous laser tracks thereby generating the machined surfaces with few isolated micro-cracks (Fig. 12).

Theoretically the micro-cracks can be nucleated when the thermal stress reaches near or equal to the flexural strength limit of structural alumina (379 MPa). However, based on the observed results it was found that the number of laser pulses in 1D and 2D laser machining, the presence of lateral overlap (O_L) during 2D and 3D laser machining, and also transverse overlap (O_T) during 3D laser machining significantly influenced the

Fig. 12 ESEM photographs of top surface of three-dimensionally laser machined alumina

amount and nature of micro-cracks due to the thermal expansion difference between the laser material interaction region and surrounding substrate material. Depending upon the level of thermal stress generated within the region of material treated by the previous laser pulse (during multi pulse treatment) and the laser track overlap (lateral and transverse), it is likely to experience initiation (nucleation) and/or growth/expansion of micro crack(s). Furthermore, due to the intense stress concentration at the tip of such already existing micro crack is highly likely to grow/expand at the level of thermal stress (depending upon the magnitude of the stress concentration at the crack tip) less than the theoretical flexural strength of alumina (379 GPa) in the subsequent laser pulses and overlaps. The extent of micro crack and generation of micro crack network depends upon the magnitude of laser parameters (pulse frequency, pulse energy, and magnitude of overlap).

Conclusion

An advanced multiphysics multistep computational model was designed and developed to evaluate thermal stresses during multidimensional (1-, 2-, and 3D) laser machining of structural alumina. The present multistep model incorporated heat transfer, fluid flow, structural mechanics boundary conditions, multiphysics (thermo-mechanical) coupling, and temperature-dependent materials properties to effectively simulate multidimensional (1-, 2-, 3D) laser machining conditions. The results of this investigation indicated that the theoretically evaluated thermal stresses (in spatial and depth distribution) for multidimensional (1-, 2-, and 3D) laser machining were tensile (+ve) in nature and predominately high on the surface and in the center region of laser beam reaching nearly the flexural strength of structural alumina (379 MPa). Such state of stresses led to nucleation of few isolated micro-cracks during multidimensional (1-, 2-, and 3D) laser machining. However, the number of laser pulses in 1D and 2D laser

machining, the presence of lateral overlap (O_L) during 2D and 3D laser machining, and also transverse overlap (O_T) during 3D laser machining significantly influenced the amount and nature of micro-cracks due to the thermal expansion difference between the laser interaction region and surrounding substrate material. Furthermore, the present research approach/methodology indicated tremendous promise in adopting laser based machining to efficiently and effectively produce defect-free components at high material removal rates and desired surface roughness from difficult-to-machine materials.

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