Transportation Infrastructure Geotechnology
https://doi.org/10.1007/s40515-024-00390-x

TECHNICAL NOTE

Unique Relationship Between Optimum Compaction Properties of Fine-Grained Soils Across Rational Compactive Efforts: A Validation Study

Amin Soltani[1,2] · **Brendan C. O'Kelly**[3] · **Suksun Horpibulsuk**[4,5] · **Abbas Taheri**[6]

Accepted: 2 April 2024

Abstract

Among the many proposals for estimating the compaction characteristics of fine-grained soils for different compaction energy levels (CELs), energy conversion (EC) models are gaining increased attention. These models work on the premise of employing measured optimum moisture content (OMC) and maximum dry unit weight (MDUW) values obtained for a rational CEL (e.g., standard or reduced-standard Proctor (SP or RSP)) to predict the same for other CELs. This study revisits the most recently proposed EC-based compaction modeling framework, critically examining its asserted accuracy and hence identifying its true potentials. This was achieved by performing comprehensive statistical analyses on a newly compiled database of 206 compaction test results, entailing 70 different fine-grained soils (with liquid limits ranging 19–365%) and accounting for CELs of 202.0–2723.5 kJ/m^3. It was demonstrated that 99% and 96% of the differences between the SP-converted and measured values for OMC and MDUW, respectively, fall within the allowable margins of OMC and MDUW measurement errors permitted by the Australian AS 1289.5.1.1/AS 1289.5.2.1 standards (satisfying their recommended $\geq$95% requirement). Equally favorable results were also obtained for the RSP-based conversions. These findings reaffirmed that the optimum compaction parameters across rational CELs are somewhat uniquely related, and the effects of fine-grained soil attributes on soil compactability are adequately captured/explained by the measured OMC and MDUW values employed as the conversion inputs/predictors.

Keywords Fine-grained soil · Optimum moisture content · Maximum dry unit weight · Compactive effort · Energy conversion model · Statistical analysis

Extended author information available on the last page of the article

Abbreviations

AC	Aided [energy] conversion (model)
AS	Australian Standard
ASTM	American Society for Testing and Materials
BS	British Standard
CEL	Compaction energy level
DC	Direct [energy] conversion (model)
EC	Energy conversion (model)
MC	Gravimetric moisture content
MDUW	Maximum dry unit weight (for compaction)
MDUW_C	Converted/predicted maximum dry unit weight
MDUW_M	Measured maximum dry unit weight
MP	Modified Proctor (compaction test)
OMC	Optimum moisture content (for compaction)
OMC_C	Converted/predicted optimum moisture content
OMC_M	Measured optimum moisture content
RMP	Reduced modified Proctor (compaction test)
RSP	Reduced standard Proctor (compaction test)
SP	Standard Proctor (compaction test)
ZAV	Zero-air-voids (saturation line)

Notations

A	Soil activity (defined as $A = \text{PI}/C_\text{F}$)
C_F	Soil clay (< 2 μm sized) content [%]
E	Compaction energy level (CEL) [kJ/m^3]
E_MP	CEL for modified Proctor (MP) compaction [kJ/m^3]
E_R	Rational CEL (here R = RSP, SP, RMP or MP) [kJ/m^3]
E_RMP	CEL for reduced modified Proctor (RMP) compaction ($E_\text{SP} < E_\text{RMP} < E_\text{MP}$) [kJ/m^3]
E_RSP	CEL for reduced standard Proctor (RSP) compaction ($E_\text{RSP} < E_\text{SP}$) [kJ/m^3]
E_SP	CEL for standard Proctor (SP) compaction [kJ/m^3]
F_F	Soil fines (< 75 μm sized) content [%]
G_s	Specific gravity of soil solids
LL	Liquid limit [%$^\text{MC}$]
MAPE	Mean absolute percentage error [%]
n	Summation index
NAPE	Normalized absolute percentage error [%]
N_C	Number of MDUW:OMC conversions/predictions
NRMSE	Normalized root-mean-squared error [%]
N_T	Number of (compiled/examined) compaction test results
PI	Plasticity index (defined as PI = LL – PL) [%$^\text{MC}$]
PL	Plastic limit [%$^\text{MC}$]
R^2	Coefficient of determination
RMSE	Root-mean-squared error [%$^\text{MC}$ for OMC or kN/m^3 for MDUW]
S	Degree of saturation [%]

S_{opt}	Degree of saturation produced for optimum compaction state [%]
S_{opt}^{R}	S_{opt} for CEL of $E = E_R$ [%]
S_{opt}^{SP}	S_{opt} for CEL of $E = E_{SP}$ [%]
w	Gravimetric moisture content [$\%^{MC}$]
w_{opt}	Optimum moisture content (OMC) for compaction [$\%^{MC}$]
w_{opt}^{R}	OMC for CEL of $E = E_R$ [$\%^{MC}$]
w_{opt}^{RSP}	OMC for CEL of $E = E_{RSP}$ [$\%^{MC}$]
w_{opt}^{SP}	OMC for CEL of $E = E_{SP}$ [$\%^{MC}$]
y_M	Measured OMC [$\%^{MC}$] or MDUW [kN/m^3]
$y_{M(max)}$	Maximum of measured OMC [$\%^{MC}$] or MDUW [kN/m^3] data
$y_{M(min)}$	Minimum of measured OMC [$\%^{MC}$] or MDUW [kN/m^3] data
y_P	Converted/Predicted OMC [$\%^{MC}$] or MDUW [kN/m^3]
γ_d	Dry unit weight [kN/m^3]
γ_{dmax}	Maximum dry unit weight (MDUW) for compaction [kN/m^3]
γ_{dmax}^{R}	MDUW for CEL of $E = E_R$ [kN/m^3]
γ_{dmax}^{RSP}	MDUW for CEL of $E = E_{RSP}$ [kN/m^3]
γ_{dmax}^{SP}	MDUW for CEL of $E = E_{SP}$ [kN/m^3]
γ_w	Unit weight of water [kN/m^3]

1 Introduction

Soil compaction is routinely employed, often in conjunction with supplementary soil stabilization techniques, to satisfy earthworks construction requirements (e.g., Attom 1997; Benson et al. 1999; Sridharan and Gurtug 2004; Zhang et al. 2019; Soltani et al. 2022). The compaction characteristics are commonly measured in accordance with standardized soil laboratory tests. The prevalent methods include the standard and modified Proctor (SP and MP) compaction tests, which measure changes in the soil dry unit weight γ_d with increasing moisture content w for defined compaction energy levels (CELs) of $E = \sim 600$ and ~ 2700 kJ/m^3, respectively (e.g., BS 1377–4, 1990; AS 1289.5.1.1 2017; AS 1289.5.2.1 2017; ASTM D698 2021; ASTM D1557 2021). From the measured γ_d:w plot, the optimum moisture content (OMC or w_{opt}) and corresponding maximum dry unit weight (MDUW or γ_{dmax}) parameters, defined by the peak point of the said plot, can be visually deduced. Because of the time and labor demands associated with conventional/standardized laboratory compaction testing of fine-grained soils, especially for CELs > SP compactive effort, several attempts have been made to either (i) devise alternative compaction testing techniques or (ii) indirectly estimate the optimum compaction parameters through empirical-type correlations. The former mainly involves the application of miniature compaction testing equipment, thereby significantly reducing the amount of soil material needed for, and also alleviating the time and labor demands associated with, quantifying the OMC and MDUW parameters (e.g., Little 1948; Wilson 1950; Singh and Punmia 1965; Sridharan and Sivapullaiah 2005).

In spite of generally promising results, to date, none of these alternate compaction testing methods has been standardized, nor have they gained widespread acceptance among practicing geotechnical engineers, likely owing to their relatively lower accuracy in replicating soil compactability for CELs > SP compactive effort (Santos et al. 2019; Loshelder et al. 2023).

Empirical-type correlations generally aim to eliminate the need for performing compaction testing, instead relying on data-driven mathematical expressions that link the OMC and MDUW parameters to common soil index properties, such as the Atterberg limits and grain-size distribution parameters. For a complete review of these empirical-type correlations, the reader is referred to the papers by Verma and Kumar (2020) and Shimobe et al. (2021). Because of the intricate nature of the compaction process, being governed by several inter-related soil attributes (e.g., gradation, plasticity, clay mineralogy and the soil's chemical composition (Howard et al. 1981; Horpibulsuk et al. 2008)), efforts to establish a universally applicable empirical model capable of consistently producing reliable OMC and MDUW predictions (across rational CELs) based on common soil index properties still remain somewhat inconclusive. In other words, most existing empirical models have been observed to be overly dependent on the specific ranges of soil index properties used for their development, with their predictive performance often diminishing when applied outside of their original calibration domains (Shimobe et al. 2021; Di Matteo and Spagnoli 2023). Moreover, for cases where the MDUW and/or OMC parameters predicted by empirical correlations are overestimated, the theoretically deduced degree of saturation for the γ_{dmax}:w_{opt} prediction often surpasses the physically limiting zero-air-voids (ZAV) saturation line (i.e., implying $S_{opt} \geq 100\%$), thus producing a materially meaningless optimum compaction state (Soltani et al. 2023).

The limitations described above have produced renewed interest in energy conversion (EC) models, defined as a special/separate category of empirical-type correlations that work on the premise of employing measured OMC and MDUW values obtained for a rational CEL (mainly SP) to predict the same for other lower and/or higher compactive efforts (e.g., Hamdani 1987; Blotz et al. 1998; Drew 2005; Horpibulsuk et al. 2008; Vinod et al. 2015; Gurtug et al. 2018; Khalid and Rehman 2018; Shaivan and Sridharan 2020; Shivaprakash and Sridharan 2021; Di Matteo et al. 2022; Di Matteo and Spagnoli 2023; Soltani et al. 2023). In other words, EC-type models eliminate the need to perform additional compaction tests that would otherwise be required to cover a desired/wide CEL range. In general, EC-type models can be classified into two groups, namely (i) aided [energy] conversion (AC) and (ii) direct [energy] conversion (DC) approaches. In addition to the measured OMC and MDUW parameters for a rational CEL, AC-based models also employ soil index properties (mainly the Atterberg limits), as supplementary inputs, to estimate the optimum compaction parameters for other lower and/or higher CELs. On the other hand, DC models postulate that the relationship between the optimum compaction parameters of fine-grained soils across rational CELs is somewhat unique, and hence, reliable conversions of the OMC and MDUW parameters (from one CEL to another) can be directly achieved without the need of acquiring any soil index properties. In fact, the authors' recent findings, originally presented by them in the discussion paper of Di Matteo et al. (2022) and later substantiated in Soltani et al. (2023), have demonstrated that incorporating soil index properties (as inputs) into EC-based models has the general effect of restricting their predictive performance, causing similar limitations as those described above for conventional empirical-type correlations. The latest DC-based EC modeling framework, as proposed by the authors of the present study, can be expressed along these lines (Soltani et al. 2023):

$$w_{opt}^{R} = w_{opt}^{SP}\left(\frac{E_R}{E_{SP}}\right)^{-0.178} \ni \mathrm{R} = \{\mathrm{RSP}, \mathrm{RMP}, \mathrm{MP}\} \tag{1}$$

$$\gamma_{dmax}^{R} = \gamma_{dmax}^{SP}\left(\frac{E_R}{E_{SP}}\right)^{+0.068} \ni \mathrm{R} = \{\mathrm{RSP}, \mathrm{RMP}, \mathrm{MP}\} \tag{2}$$

$$\gamma_{dmax}^{R} = \frac{G_s\gamma_w}{1 + \left(\frac{G_s\gamma_w}{\gamma_{dmax}^{SP}} - 1\right)\left(\frac{E_R}{E_{SP}}\right)^{-0.178}} \ni \mathrm{R} = \{\mathrm{RSP}, \mathrm{RMP}, \mathrm{MP}\} \tag{3}$$

where w_{opt}^{SP} and w_{opt}^{R} are the OMCs for CELs of $E = E_{SP}$ and E_R, respectively; γ_{dmax}^{SP} and γ_{dmax}^{R} are the MDUWs for $E = E_{SP}$ and E_R, respectively; G_s is specific gravity of soil solids; γ_w is unit weight of water (=9.81 kN/m^3); RSP is reduced standard Proctor effort (here defined as $E_{RSP} < E_{SP}$); and RMP is reduced modified Proctor effort (here defined as $E_{SP} < E_{RMP} < E_{MP}$).

Referring to Soltani et al. (2023), the fitting parameters −0.178 and +0.068 in Eqs. 1 and 2, which define the mean rates of decrease in OMC and increase in MDUW with increasing CEL, were deduced based on statistical analyses performed on a large and diverse database of 242 compaction test results obtained for 76 different fine-grained soils. Each of these soils had test results for at least three CELs, collectively covering $E = 225–2708$ kJ/m^3. In examining the predictive performance of Eqs. 1 and 2, Soltani et al. (2023) noticed that in some instances where the converted MDUW and/or OMC parameters were overestimated, the theoretically deduced degree of saturation value associated with the γ_{dmax}^{R}:w_{opt}^{R} conversion exceeded the ZAV line boundary (i.e., giving $S_{opt}^{R} \geq 100\%$). It should be mentioned that this occurrence is not commonplace compared to employing conventional empirical correlations and even AC-based EC models. Nevertheless, towards addressing the inconsistency of $S_{opt}^{R} \geq 100\%$, as well as for improving the accuracy of the MDUW conversions, Soltani et al. (2023) proposed the application of Eq. 3 (in lieu of Eq. 2). Note that Eq. 3 was established using basic soil volume–mass relationships (in conjunction with Eq. 1) and on the premise that the value of S_{opt} for a given fine-grained soil does not change significantly across different CELs (Blotz et al. 1998; Gurtug and Sridharan 2004; Horpibulsuk et al. 2008, 2009; Shimobe et al. 2021; Soltani et al. 2023).

Notwithstanding the promising results observed for the EC-based modeling framework given by Eqs. 1–3, its inherent data-driven nature warrants further validation studies based on additional independent datasets. Such validation studies would foster greater confidence within the geoengineering community in adopting this practical and user-friendly modeling framework in routine practice. To this end, the present study aims to critically examine the asserted accuracy of Eqs. 1–3 in converting the OMC and MDUW parameters of dissimilar fine-grained soils across rational CELs. This is achieved by performing comprehensive statistical analyses on a newly compiled database of 206 compaction test results, entailing 70 very different fine-grained soils and accounting for CELs ranging between 202.0 and 2723.5 kJ/m^3.

2 Compiled Database of Compaction Test Results

A large independent database of 206 compaction test results, compiled from 38 different investigations (see Table 1), was employed to critically examine the validity of the EC-based modeling framework given by Eqs. 1–3. The database included compaction test results for 70 fine-grained soil materials (herein designated as S_1–S_{70}). Inclusive of SP measurements, test results for at least two CELs were available for each database soil, with 31 of them having data for at least three CELs. As is evident from Table 1, the database soils cover broad ranges of soil index properties, with C_F (< 2 μm) = 5–97%, F_F (< 75 μm) = 40–100%, LL = 19–365 %^{MC}, PL = 7.9–50.0 %^{MC}, PI = 5–333 %^{MC} and $A = PI/C_F = 0.28$–6.50 (the notations C_F, F_F, LL, PL, PI, A and MC represent the clay content, fines fraction, liquid limit, plastic limit, plasticity index, soil activity and gravimetric moisture content, respectively). Referring to Fig. 1a, which illustrates the database soils (excluding soils S_{13}, S_{14}, S_{32} and S_{41}–S_{44} from Agus (2005), Tüfekçi et al. (2010) and Bhat et al. (2015) for which the Atterberg limits were not reported) plotted in the British-standard plasticity chart (BS 5930 2015), the database soils comprised of 54 clays and 9 silts, with a detailed classification analysis presented in Fig. 1b.

Figure 2a illustrates the variations of MDUW (i.e., $\gamma_{dmax} = 11.7$–22.3 kN/m^3) against OMC (i.e., $w_{opt} = 6.3$–40.0 %^{MC}) for the compiled database. The optimum compaction parameters are strongly correlated with each other, exhibiting, for instance, the following exponential relationship (with $R^2 = 0.930$ for $N_T = 206$ compaction test results): $\gamma_{dmax} = 23.655\ exp(-0.019\ w_{opt})$, which defines the blue chain curve in Fig. 2a. Note that this relationship, along with the other three correlations outlined in Fig. 2a, conform to the well-established "path of optimums" correlation framework reported in numerous earlier studies (e.g., Gurtug and Sridharan 2004; Sivrikaya et al. 2008; Gurtug et al. 2018; Di Matteo and Spagnoli 2023; Soltani et al. 2023). Regarding the CEL, the compaction data cover RSP, SP, RMP and MP efforts (i.e., CEL range of $E = 202.0$–2723.5 kJ/m^3), comprising 31, 70, 42 and 63 cases, respectively (see Fig. 2b). It should be reiterated that the database assembled in the present investigation is entirely different from the one previously employed by the authors (Soltani et al. 2023) in developing the DC-based EC modeling framework given by Eqs. 1–3.

Table 1 Summary of the assembled database of $N_T = 206$ compaction test results

Reference	Soil ID	N_T	G_s	LL ($\%^{MC}$)	PI ($\%^{MC}$)	F_F (%)	C_F (%)	$A = PI/C_F$	CEL (kJ/m^3)
[1] Bell (1977)	S_1	3	2.68	24.0	5.0	61.0	—	—	284.4, 592.5, 1303.5
[2] Broderick and Daniel (1990)	S_2	2	2.59	58.0	24.0	100.0	—	—	593.7, 2681.3
[3] Kotdawala et al. (1994)	S_3–S_8	12	2.61–2.67	30.0–43.0	9.0–21.0	58.7–93.3	10.2–30.5	0.33–1.38	598.5, 2504.1
[4] Attom (1997)	S_9	10	2.70	81.8	45.4	92.0	74.0	0.61	355.6, 474.2, 592.7, 711.3, 790.3, 987.8, 1185.4, 1637.8, 2155.1, 2693.8
[5] Marinho and Stuermer (2000)	S_{10}	2	2.70	48.0	19.0	—	45.0	0.42	593.7, 2681.3
[6] Al-Amoudi et al. (2002)	S_{11}	3	2.70	31.3	8.9	49.0	16.2	0.55	591.0, 1245.0, 2682.0
[7] Barrera (2002)	S_{12}	2	2.71	32.0	16.0	60.6	16.1	0.99	600.0, 2700.0
[8] Agus (2005)[A]	S_{13}–S_{15}	6	2.65[A]	130.0[A]	97.0[A]	50.0–100.0	20.0–40.0	2.43[A]	600.0, 1000.0
[9] Drew (2005)[B]	S_{16}–S_{22}	34	2.66–2.77	19.0–49.0	6.0–19.0	43.5–98.4	6.5–39.2	0.28–0.92	355.0, 592.0, 987.0, 1643.0, 2693.0
[10] Sridharan and Sivapullaiah (2005)	S_{23}	2	2.60	37.0	20.3	59.0	17.0	1.19	593.7, 2681.3
[11] Browne (2006)	S_{24}, S_{25}	4	2.66, 2.65	29.7, 83.7	21.8, 59.8	57.4, 65.1	9.2, 16.8	1.30, 6.50	592.7, 2693.0
[12] Osinubi and Nwaiwu (2006)	S_{26}	2	—	44.0	20.0	52.0	12.8	1.56	605.9, 1009.2
[13] Osinubi et al. (2009)	S_{27}	3	2.76	42.0	10.0	73.5	—	—	605.9, 1009.2, 2723.5
[14] Vipulanandan et al. (2009)	S_{28}, S_{29}	4	2.69, 2.69	42.0, 48.0	26.0, 31.0	—	—	—	593.7, 2681.3
[15] Chen (2010)	S_{30}, S_{31}	6	2.62, 2.63	39.0, 365.0	15.0, 333.0	50.0, 50.0	—	—	360.0, 600.0, 2700.0
[16] Tüfekçi et al. (2010)	S_{32}	2	—	—	—	100.0	97.4	—	593.7, 2681.3
[17] Viswanadham et al. (2010)	S_{33}	2	2.55	35.0	15.0	80.1	46.4	0.32	593.7, 2681.3
[18] Mehmood et al. (2011)	S_{34}	4	—	64.0	42.0	99.0	14.9	2.82	202.0, 562.0, 1534.0, 2556.0
[19] Bose (2012)	S_{35}	2	2.65	170.0	120.0	81.7	69.1	1.74	592.8, 2693.3
[20] Celauro et al. (2012)	S_{36}	2	—	51.5	24.2	92.3	43.6	0.56	593.7, 2681.3
[21] Askarinejad (2013)	S_{37}	2	2.60	27.0	7.0	40.0	5.0	1.40	600.0, 2700.0
[22] Eberemu (2013)	S_{38}	4	2.61	42.0	24.0	56.7	—	—	336.4, 605.9, 1009.2, 2723.5
[23] Eberemu et al. (2013)	S_{39}	3	2.54	34.4	21.0	56.8	—	—	605.9, 1009.2, 2723.5
[24] Uba (2014)	S_{40}	2	2.59	51.0	25.0	95.0	56.0	0.45	600.0, 2700.0
[25] Bhat et al. (2015)	S_{41}–S_{44}	8	—	—	—	47.0–49.4	—	—	593.7, 2681.3

Table 1 (continued)

Reference	Soil ID	N_T	G_s	LL (%^{MC})	PI (%^{MC})	F_F (%)	C_F (%)	$A = PI/C_F$	CEL (kJ/m^3)
[26] Hassan and Toll (2015)^C	S_{45}	2	2.60	43.3	19.6	72.0^C	37.0^C	0.53^C	593.7, 2681.3
[27] Hezmi et al. (2015)	S_{46}	2	—	54.0	22.0	—	—	—	593.7, 2681.3
[28] Sterpi (2015)	S_{47}	3	2.74	32.0	14.0	88.1	24.4	0.57	237.0, 356.0, 593.0
[29] Amadi and Osinubi (2016)^D	S_{48}	3	2.60	42.2	22.2	57.0^D	17.0^D	1.31^D	593.7, 1009.2, 2681.3
[30] Kodikara et al. (2016)^E	S_{49}, S_{50}	12	2.62, 2.65	60.5, 72.4	32.6, 39.9	94.0, 100.0	50.0, 75.0	0.43, 0.80	224.0, 467.0, 560.0, 934.0, 1669.0, 2548.0
[31] Signes et al. (2016)	S_{51}	2	2.79	52.2	28.1	96.4	47.2	0.60	593.7, 2681.3
[32] Li et al. (2017)	S_{52}	3	2.83	70.5	33.5	88.0	70.0	0.48	356.2, 593.7, 831.2
[33] Mamatha and Dinesh (2017)	S_{53}	2	2.72	71.0	48.0	90.0	54.0	0.89	593.7, 2681.3
[34] Prasanna et al. (2018)^F	S_{54}–S_{60}	27	2.61–2.74	38.5–65.0	19.0–36.0	40.0–100.0	—	—	355.5, 592.5, 1616.0, 2693.3
[35] Sreelekshmypillai and Vinod (2019)	S_{61}, S_{62}	6	—	57.0, 73.0	27.5, 48.2	—	—	—	355.5, 592.5, 2693.3
[36] Umar and Elinwa (2020)	S_{63}	4	2.50	38.0	16.0	62.0	13.9	1.15	331.1, 596.0, 993.3, 2681.8
[37] Arama and Gençdal (2022)	S_{64}–S_{69}	12	—	41.0–81.0	20.0–51.0	67.7–94.5	—	—	593.7, 2681.3
[38] Hamza et al. (2023)	S_{70}	2	2.71	53.2	31.1	97.9	37.7	0.83	593.7, 2681.3

N_T, number of reported compaction test results; G_s, specific gravity of soil solids; LL, liquid limit; PI, plasticity index; F_F and C_F, fines (< 75 μm) and clay (< 2 μm) fractions, respectively; A, soil activity; CEL, compaction energy level

^{A}LL, PI and $A = PI/C_F$ were reported only for one soil sample (out of three examined), and G_s for all three soil samples was 2.65

^{B}One soil sample (out of seven examined) did not include compaction test results for CEL = 1643.0 kJ/m^3

^{C}Grain-size distribution data (i.e., F_F and C_F) for the same soil were reported in Mendes (2011)

^{D}Soil gradation data were taken from Amadi and Eberemu (2013)

^{E}All index properties for the same soils were reported in Islam and Kodikara (2016)

^{F}One soil sample (out of seven examined) did not include compaction test results for CEL = 1616.0 kJ/m^3

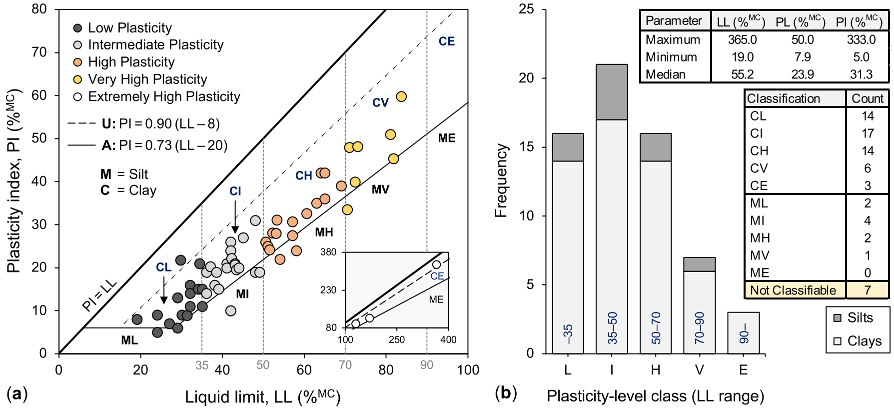

Fig. 1 Plasticity characteristics of the database soils (excluding soils S_{13}, S_{14}, S_{32} and S_{41}–S_{44} for which the Atterberg limits were not reported in the original sources): **a** database soils plotted on the British-standard plasticity chart (BS 5930 2015); and **b** detailed analysis of the soil classification results

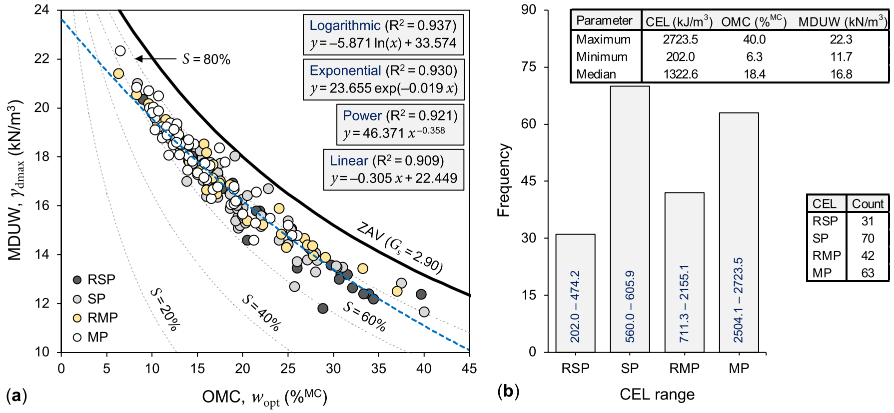

Fig. 2 Compaction characteristics of the database soils: **a** variations of MDUW against OMC; and **b** detailed analysis of the compaction test results. Note: ZAV denotes the zero-air-voids saturation line (or $S = 100\%$) for an assumed specific gravity value of $G_s = 2.90$

3 Results and Discussion

3.1 Converting SP to Other Rational CELs

Figure 3a illustrates the variations of converted against measured OMC values for the compiled database, with the converted RSP, RMP and MP OMCs obtained from the measured SP by Eq. 1. The converted and measured values are strongly

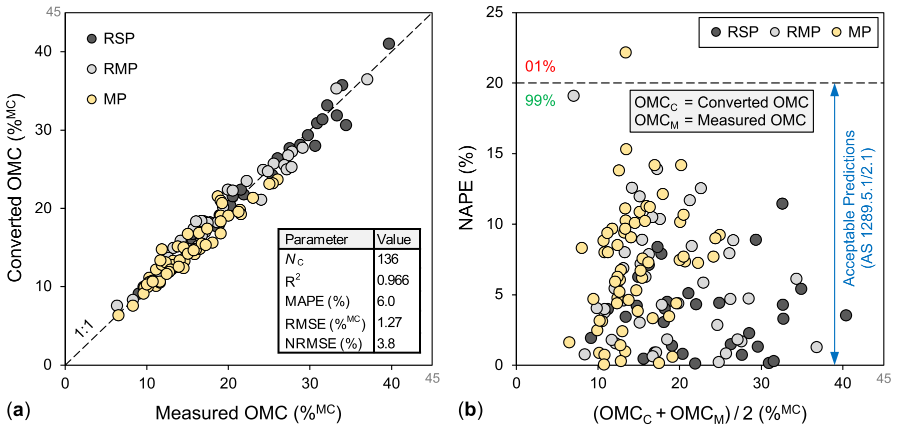

Fig. 3 **a** Correlation and **b** NAPE scatter plots illustrating the level of agreement between the SP-converted (by Eq. 1) and measured OMC values for the investigated RSP, RMP and MP CELs

correlated with each other, exhibiting a very high R^2 value of 0.966 (for $N_C = 136$ conversions). The average prediction error associated with these OMC conversions was quantified using the normalized root-mean-squared error (NRMSE) and mean absolute percentage error (MAPE) parameters, both being dimensionless fit-measure indices, expressed as percentage, through the following relationships (Soltani et al. 2018; Soltani and O'Kelly 2022):

$$\mathrm{NRMSE} = \frac{\mathrm{RMSE}}{y_{\mathrm{M(max)}} - y_{\mathrm{M(min)}}} \times 100\% \ni \mathrm{RMSE} = \sqrt{\frac{1}{N_{\mathrm{C}}} \sum_{n=1}^{N_{\mathrm{C}}} \left[y_{\mathrm{M}(n)} - y_{\mathrm{P}(n)}\right]^2} \quad (4)$$

$$\mathrm{MAPE} = \frac{1}{N_{\mathrm{C}}} \sum_{n=1}^{N_{\mathrm{C}}} \left| \frac{y_{\mathrm{M}(n)} - y_{\mathrm{P}(n)}}{y_{\mathrm{M}(n)}} \right| \times 100\% \quad (5)$$

where RMSE is the root-mean-squared error (in $\%^{\mathrm{MC}}$ for OMC or kN/m^3 for MDUW); y_{M} and y_{P} are the measured and converted (or predicted) OMC or MDUW data, respectively; $y_{\mathrm{M(max)}}$ and $y_{\mathrm{M(min)}}$ are the maximum and minimum of the measured OMC or MDUW data, respectively; N_{C} is the number of compaction conversions (or predictions); and n is the index of summation.

The NRMSE and MAPE parameters for the OMC conversions made by Eq. 1 were calculated as 6.0% and 3.8%, respectively, both being acceptable in view of the typically allowable 5–10% reference threshold. Although the high R^2 and low NRMSE or MAPE values obtained for the OMC conversions depicted in Fig. 3a would normally warrant accepting the predictive capability of Eq. 1, these fit-measure indices alone are generally not able to elucidate the practical implications of employing Eq. 1 in routine practice (Soltani and O'Kelly 2021a). That is, in addition to conventional statistical fit-measure indices (e.g., R^2, RMSE, NRMSE and MAPE), it is also imperative to critically scrutinize the prediction residual values (Bland and Altman 1986; Soltani and O'Kelly 2021b). In the present investigation, this was achieved by the normalized absolute percentage error (NAPE) parameter, which, for a given conversion, can be calculated as follows (AS 1289.5.1.1 2017; AS 1289.5.2.1 2017):

$$\text{NAPE} = \left| \frac{y_{\text{M}} - y_{\text{P}}}{0.5\left(y_{\text{M}} + y_{\text{P}}\right)} \right| \times 100\% \tag{6}$$

where y_{M} and y_{P} are the measured and converted (or predicted) OMC or MDUW data, respectively.

According to the Australian AS 1289.5.1.1 (2017) and AS 1289.5.2.1 (2017) standards for SP and MP laboratory compaction testing, acceptable NAPE values for the OMC and MDUW measurements considering a given fine-grained soil can be as high as 20% and 4%, respectively. Based on laboratory compaction tests performed on a wide variety of soils, AS 1289.5.1.1/AS 1289.5.2.1 advises that "*these [NAPE = 20% and 4%] values, in 95% of cases, should not be exceeded*," while acknowledging that "*in some cases, such as heavy clays, these [NAPE = 20% and 4%] values may be exceeded*." Accordingly, in the present investigation, NAPE values of 20% and 4% were chosen as reference values for cross-examining the prediction residuals associated with the OMC and MDUW conversions (made by Eqs. 1–3), respectively. Figure 3b illustrates the distribution of the NAPE values for the OMC conversions produced by Eq. 1. Compared to the ≥ 95% requirement suggested in AS 1289.5.1.1/AS 1289.5.2.1, this figure shows that 99% (= 135/136 × 100%) of the NAPE values for the OMC conversions were lower than 20%. In other words, it is implied that 99% of the differences between the converted and measured OMC values shown in Fig. 3 are statistically insignificant, being within the acceptable margin of OMC measurement errors permitted by AS 1289.5.1.1 (2017) and AS 1289.5.2.1.5.2.1 (2017). In view of all the above, Eq. 1 can be used with confidence to obtain reliable $E_{\text{SP}} \rightarrow E_{\text{R}}$ (where R = RSP, RMP and MP) conversions of the OMC parameter.

Correlation and NAPE scatter plots illustrating the level of agreement between the SP-converted (by Eq. 2) and measured MDUW values for RSP, RMP and MP CELs are presented in Fig. 4. As depicted in Fig. 4a, the converted versus measured MDUW correlation exhibits excellent fit-measure indices; that is, $R^2 = 0.957$, NRMSE = 2.7% and MAPE = 2.1% for $N_{\text{C}} = 136$. However, compared to the ≥ 95% requirement suggested in AS 1289.5.1.1/AS 1289.5.2.1, only 85% (= 115/136 × 100%) of the NAPEs associated with the conversions produced by Eq. 2 were lower than the allowable 4% limit (see Fig. 4b). This implies that Eq. 2, albeit useful for preliminary estimation purposes, should generally not be used in place of actual compaction test results. This was the driver for Soltani et al. (2023) to introduce Eq. 3 (for use instead of Eq. 2) in obtaining more accurate MDUW conversions. Nevertheless, having already demonstrated that an accurate conversion of the OMC parameter can be established using Eq. 1, one can therefore directly measure its corresponding MDUW value by performing a more practical single-point compaction test.

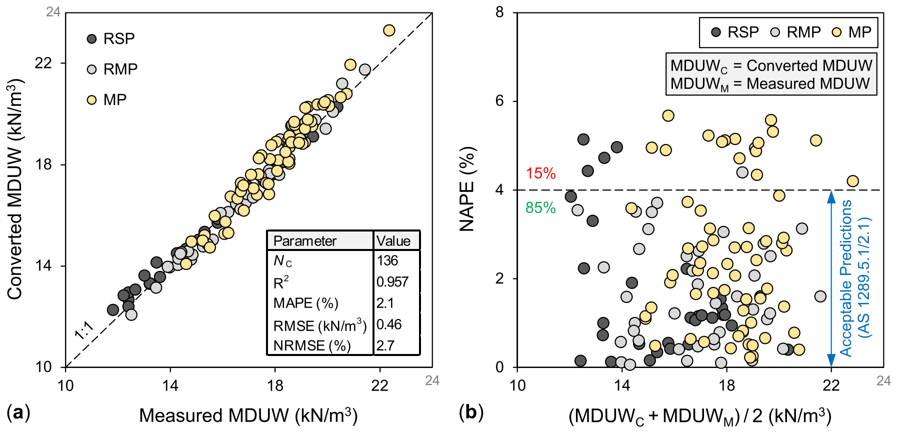

Fig. 4 **a** Correlation and **b** NAPE scatter plots illustrating the level of agreement between the SP-converted (by Eq. 2) and measured MDUW values for the investigated RSP, RMP and MP CELs

Moreover, as described earlier, in some instances where the converted MDUW and/or OMC values are overestimates, the theoretically deduced degree of saturation value for the conversion (calculated using γ^{R}_{dmax} and w^{R}_{opt}) may surpass the physically limiting ZAV line boundary, thereby producing a materially meaningless compaction state (Di Matteo et al. 2022; Soltani et al. 2023). In the present study, a total of 15 conversions (out of $N_C = 136$) obtained based on Eqs. 1 and 2 were found to produce degree of saturation values greater than 100% (i.e., giving S^{R}_{opt} = 101.0–120.0%, with a median value of 102.7%). This inconsistency (of $S^{R}_{opt} \geq 100\%$) was addressed by Soltani et al. (2023) with the introduction of Eq. 3 (in place of Eq. 2), on the premise that the value

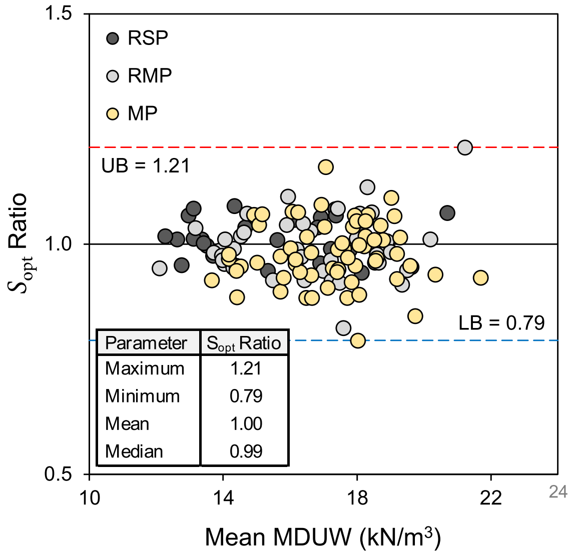

Fig. 5 Variations of the measured S^{SP}_{opt}-to-S^{R}_{opt} ratio against the corresponding $(\gamma^{SP}_{dmax} + \gamma^{R}_{dmax})/2$ value (where R = RSP, RMP and MP) for the compiled database of $N_T = 206$ compaction test results. Note: UB and LB denote the upper and lower S^{SP}_{opt}-to-S^{R}_{opt} ratio, respectively

of S_{opt} for a given fine-grained soil does not change significantly across different CELs. The validity of this assertion, also reported in numerous previous investigations (e.g., Blotz et al. 1998; Gurtug and Sridharan 2004; Horpibulsuk et al. 2008, 2009; Shimobe et al. 2021), is demonstrated in Fig. 5 for the compiled database of $N_T = 206$ compaction test results. As depicted in this figure, the measured S_{opt}^{SP}-to-S_{opt}^{R} ratio (where R = RSP, RMP and MP) ranged between 0.79 and 1.21 (with mean, median and standard deviation values of 1.00, 0.99 and 0.06, respectively), reaffirming the proposition of $S_{opt}^{SP} \approx S_{opt}^{R}$. Note that some of the original sources presented their compaction test results without accompanying specific gravity measurements (see Table 1). For these cases, the highest reported G_s value (of 2.83) in consideration of the entire assembled database was employed in the calculations.

Correlation and NAPE scatter plots depicting the level of agreement between the SP-converted (by Eq. 3) and measured MDUW values for RSP, RMP and MP CELs are presented in Fig. 6. The new MDUW conversions, besides producing physically meaningful values of $S_{opt}^{R} < 100\%$ (when employed in conjunction with Eq. 1), were associated with notably improved R^2, NRMSE and MAPE values of 0.976, 2.0% and 1.6%, respectively (see Fig. 6a). More importantly, 96% (= 131/136 × 100%) of the NAPE values (i.e., satisfying the ≥ 95% requirement suggested in AS 1289.5.1.1/AS 1289.5.2.1) were found to be lower than 4% (see Fig. 6b). In other words, this indicates that 96% of the differences between the converted and measured MDUW values are within the allowable margin of MDUW measurement errors permitted by AS 1289.5.1.1 (2017) and AS 1289.5.2.1 (2017). In view of these results, when the specific gravity value is available (either via direct measurement or based on a reliable assumption), Eq. 3 should be adopted (instead of Eq. 2) for $E_{SP} \rightarrow E_R$ (where R = RSP, RMP and MP) conversions of the MDUW parameter. This is further validation, based on an entirely independent database, of the authors' previous findings and recommendations documented in Soltani et al. (2023).

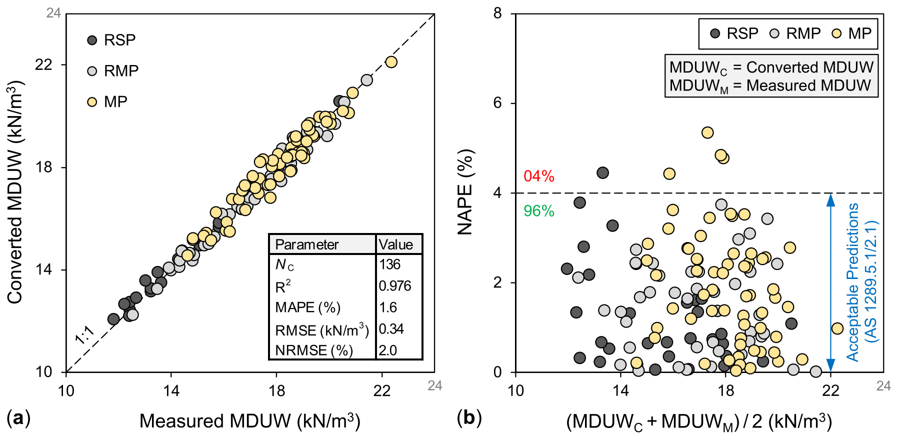

Fig. 6 **a** Correlation and **b** NAPE scatter plots illustrating the level of agreement between the SP-converted (by Eq. 3) and measured MDUW values for the investigated RSP, RMP and MP CELs

3.2 Converting RSP to Other Rational CELs

Given the prevalent usage of the SP compaction test compared to other higher-CEL test methods (e.g., MP and West African RMP), its adoption as an input to estimate the OMC and MDUW parameters for lower and higher CELs is certainly merited. Understandably, however, there may still exist an impetus to reduce the time and labor demands associated with SP compaction testing. To this end, it would be beneficial if one could employ measured compaction data from any user-defined RSP test (with its CEL being, for instance, as low as ~200 kJ/m^3) to produce OMC and MDUW conversions for higher CELs. Rewriting the sub- and super-scripts of "SP" as "RSP" in Eqs. 1–3, the following $E_{RSP} \rightarrow E_R$ relationships are obtained (Soltani et al. 2023):

$$w_{opt}^{R} = w_{opt}^{RSP} \left(\frac{E_R}{E_{RSP}} \right)^{-0.178} \ni E_R > E_{RSP} \tag{7}$$

$$\gamma_{dmax}^{R} = \gamma_{dmax}^{RSP} \left(\frac{E_R}{E_{RSP}} \right)^{+0.068} \ni E_R > E_{RSP} \tag{8}$$

$$\gamma_{dmax}^{R} = \frac{G_s \gamma_w}{1 + \left(\frac{G_s \gamma_w}{\gamma_{dmax}^{RSP}} - 1 \right) \left(\frac{E_R}{E_{RSP}} \right)^{-0.178}} \ni E_R > E_{RSP} \tag{9}$$

where w_{opt}^{RSP} and γ_{dmax}^{RSP} are OMC and MDUW obtained for RSP compaction effort (here defined as any custom compaction test with $E_{RSP} < E_{SP}$), respectively.

Omitting those database soils not including compaction test results for $E_{RSP} < E_{SP}$ (see Table 1), the measured OMC and MDUW values corresponding to the lowest CEL for each remaining soil (all being lower than SP) were applied as w_{opt}^{RSP} and γ_{dmax}^{RSP} in Eqs. 7–9 to produce conversions for higher CELs. The predictive performance metrics for the $E_{RSP} \rightarrow E_R$ (here $E_R > E_{RSP}$) conversions are presented in Table 2. In terms of NAPE distribution, 99% (=88/89×100%) and 93% (=83/89×100%) of the NAPE values (compared to the ≥95% requirement suggested in AS 1289.5.1.1 (2017) and AS 1289.5.2.1 (2017)) obtained for the OMC and MDUW conversions produced by Eqs. 7 and 9 were found to be lower than 20% and 4%, respectively. These favorable results indicate that the differences between the converted and measured optimum compaction parameters are predominantly within the acceptable margins of laboratory measurement errors permitted by the Australian AS 1289.5.1.1 and AS 1289.5.2.1 standards. Note that much like Eqs. 2 and 3 compared earlier in Section 3.1, direct MDUW conversions produced by Eq. 8 were associated with inferior NAPE values compared to those produced by Eq. 9. That is, compared to the ≥95% requirement, only 75% of the MDUW conversions produced by Eq. 8 gave NAPE values lower than the 4% limit. Given the favorable performance metrics obtained for Eqs. 7 and 9 (see Table 2), all being comparable to those noted for Eqs. 1 and 3, it can be concluded that the authors' proposed

Table 2 Summary of the predictive performance metrics for the $E_{SP} \rightarrow E_R$ (Eqs. 1–3) and $E_{RSP} \rightarrow E_R$ (Eqs. 7–9) conversions

Mode of conversion	$E_{SP} \rightarrow E_R$ (i.e., R = RSP, RMP and MP)			$E_{RSP} \rightarrow E_R$ (i.e., $E_{RSP} < E_R \leq E_{MP}$)		
Parameter	OMC (Eq. 1)	MDUW (Eq. 2)	MDUW (Eq. 3)	OMC (Eq. 7)	MDUW (Eq. 8)	MDUW (Eq. 9)
N_C	136	136	136	89	89	89
R^2	0.966	0.957	0.976	0.952	0.931	0.976
RMSE	1.27 $\%^{MC}$	0.46 kN/m^3	0.34 kN/m^3	1.51 $\%^{MC}$	0.60 kN/m^3	0.35 kN/m^3
NRMSE	3.8%	2.7%	2.0%	4.8%	6.2%	3.7%
MAPE	6.0%	2.1%	1.6%	6.6%	2.7%	1.6%
% NAPE < 20%[A]	99%	—	—	99%	—	—
% NAPE < 4.0%[A]	—	85%	96%	—	75%	93%
% NAPE < 4.5%	—	88%	98%	—	80%	96%

N_C, number of compaction conversions (or predictions)

[A]The NAPE limits of 20% and 4% represent the highest allowable measurement error in the OMC and MDUW parameters, respectively, as recommended by the Australian AS 1289.5.1.1 (2017) and AS 1289.5.2.1 (2017) standards for SP and MP laboratory compaction testing

EC-based modeling framework (Soltani et al. 2023) has been further confirmed to achieve reliable $E_{RSP} \rightarrow E_R$ conversions. Overall, therefore, the results presented in this study reaffirm the authors' viewpoints that they originally presented in the discussion paper of Di Matteo et al. (2022) and later substantiated in Soltani et al. (2023), namely:

For fine-grained soils, the optimum compaction parameters (i.e., OMC and MDUW) across rational CELs are somewhat uniquely related.

The effects of fine-grained soil attributes (e.g., gradation, plasticity and clay mineralogy) on soil compactability are adequately captured and explained by the measured OMC and MDUW values that are employed as the conversion inputs/predictors.

4 Summary and Conclusions

Among the many proposals for estimating the optimum compaction properties of fine-grained soils across rational CELs, EC-based models seem to be gaining increased attention. These models work on the premise of employing measured OMC and MDUW values obtained for a rational CEL (usually SP) to predict the same for other lower and/or higher compactive efforts. Towards fostering greater confidence for its adoption in routine geoengineering practice, the present investigation revisited the authors' recently proposed EC-based modeling framework for converting the OMC and MDUW parameters of fine-grained soils across rational CELs (i.e., Eqs. 1–3 and 7–9). Specific aims were to critically examine its asserted accuracy and hence identify its true potentials and/or limitations. These aims were achieved by performing comprehensive statistical analyses on a newly compiled database of 206 compaction test results, entailing 70 different fine-grained soils (covering LL = 19–365 $\%^{MC}$ and PI = 5–333 $\%^{MC}$) investigated for CELs ranging between 202.0 and 2723.5 kJ/m^3.

It was demonstrated that 99% and 96% (compared to the ≥ 95% requirement recommended by the Australian AS 1289.5.1.1 and AS 1289.5.2.1 standards for SP and MP laboratory compaction testing) of the differences between the SP-converted (i.e., obtained by Eqs. 1 and 3) and measured OMC and MDUW values, respectively, fall within the allowable margins of routine moisture content and dry unit weight measurement errors permitted by the said standards. Equally favorable results were also noted for the RSP-based conversions (i.e., employing Eqs. 7 and 9). These findings reaffirm the authors' viewpoint that the optimum compaction parameters across rational CELs are somewhat uniquely related. Furthermore, they demonstrate that the effects of fine-grained soil properties (e.g., gradation, plasticity and clay mineralogical composition) on soil compactability are adequately captured and explained by the measured OMC and MDUW values that are employed as the conversion inputs/predictors. In summary, the examined EC-based modeling framework (i.e., given by Eqs. 1–3 and 7–9) proposes a practical approach to reliably achieve "direct" conversions of the optimum compaction parameters of fine-grained soils across different CELs, only requiring the energy ratio parameter E_R/E_{SP} or E_R/E_{RSP}. In respect of

MDUW, superior prediction accuracy is achieved using Eqs. 3 and 9 over Eqs. 2 and 8, respectively, but with the added input requirement of the specific gravity value (either obtained via direct measurement or based on a reliable assumption). Based on the presented experimental evidence, the user-friendly framework can be adopted with confidence to eliminate the time and labor demands associated with performing additional compaction tests that would otherwise be required to cover a desired/wide CEL range.

Acknowledgements Not applicable.

Author Contribution Amin Soltani: conceptualization, methodology, validation, formal analysis, investigation, data curation, writing—original draft, visualization, project administration; Brendan C. O'Kelly: methodology, writing—review and editing; Suksun Horpibulsuk: validation, writing—review and editing; Abbas Taheri: methodology, validation, writing—review and editing.

Funding Open Access funding enabled and organized by CAUL and its Member Institutions.

Data Availability All data generated or analyzed during this study are included in this published article.

Declarations

Ethics Approval and Consent to Participate Not applicable.

Consent for Publication Not applicable.

Competing Interests The authors declare no competing interests.

References

Agus, S.S.: An experimental study on hydro-mechanical characteristics of compacted bentonite–sand mixtures. Dissertation, Bauhaus University, Weimar (Bauhaus-Universität Weimar), Germany (2005). https://doi.org/10.25643/bauhaus-universitaet.646

Al-Amoudi, O.S.B., Asi, I.M., Al-Abdul Wahhab, H.I., Khan, Z.A.: Clegg hammer–California-bearing ratio correlations. J. Mater. Civ. Eng. **14**(6), 512–523 (2002). https://doi.org/10.1061/(asce)0899-1561(2002)14:6(512)

Amadi, A.A., Osinubi, K.J.: Soil–water characteristic curves for compacted lateritic soil–bentonite mixtures developed for landfill liner applications. In De A, Reddy KR, Yesiller N, Zekkos D and Farid A (Eds), Geo-Chicago 2016 (GSP 271): Sustainable Geoenvironmental Systems, pp 488–497. American Society of Civil Engineers (ASCE): Reston, VA, USA (2016). https://doi.org/10.1061/9780784480144.048

Amadi, A.A., Eberemu, A.O.: Characterization of geotechnical properties of lateritic soil–bentonite mixtures relevant to their use as barrier in engineered waste landfills. Niger. J. Technol. **32**(1), 93–100 (2013)

Arama, Z.A., Gençdal, H.B.: Simple regression models to estimate the standard and modified Proctor characteristics of specific compacted fine-grained soils. In El Naggar H and Barros J (Eds), Proceedings of the 7th World Congress on Civil Structural, and Environmental Engineering (CSEE'22), pp ICGRE 232:1–9. Avestia Publishing: Ottawa, ON, Canada, (2022). https://doi.org/10.11159/icgre22.232

AS 1289.5.1.1: Methods of testing soils for engineering purposes: soil compaction and density tests — determination of the dry density/moisture content relation of a soil using standard compactive effort. Standards Australia (SA): Sydney, NSW, Australia (2017)

AS 1289.5.2.1: Methods of testing soils for engineering purposes: soil compaction and density tests — determination of the dry density/moisture content relation of a soil using modified compactive effort. Standards Australia (SA): Sydney, NSW, Australia (2017)

Askarinejad, A.: Failure mechanisms in unsaturated silty sand slopes triggered by rainfall. Dissertation, ETH Zürich, Switzerland. (2013). https://doi.org/10.3929/ethz-a-010002526

ASTM D1557: Standard test methods for laboratory compaction characteristics of soil using modified effort (56,000 ft-lbf/ft^3 (2,700 kN-m/m^3)) ASTM International: West Conshohocken, PA, USA (2021). https://doi.org/10.1520/d1557-12r21

ASTM D698: Standard test methods for laboratory compaction characteristics of soil using standard effort (12,400 ft-lbf/ft^3 (600 kN-m/m^3)) ASTM International: West Conshohocken, PA, USA (2021). https://doi.org/10.1520/d0698-12r21

Attom, M.F.: The effect of compactive energy level on some soil properties. Appl. Clay Sci. **12**(1–2), 61–72 (1997). https://doi.org/10.1016/s0169-1317(96)00037-3

Barrera, M.: Estudio experimental del comportamiento hidro-mecánico de suelos colapsables. Dissertation, Polytechnic University of Catalonia (Universitat Politècnica de Catalunya), Catalonia, Spain. (2002) https://doi.org/10.5821/dissertation-2117-93532

Bell, J.R.: Compaction energy relationships of cohesive soils. Transp. Res. Rec. **641**, 29–34 (1977)

Benson, C.H., Daniel, D.E., Boutwell, G.P.: Field performance of compacted clay liners. J. Geotech. Geoenviron. Eng. **125**(5), 390–403 (1999). https://doi.org/10.1061/(asce)1090-0241(1999)125:5(390)

Bhat, S.B., Kunroo, D., Ahmad, Z.: Comparison of maximum dry density, optimum moisture content and strength of granular soils using different methods of compaction. Intl. J. Civil Eng. Technol. **6**(2), 1–5 (2015)

Bland, J.M., Altman, D.G.: Statistical methods for assessing agreement between two methods of clinical measurement. Lancet. **327**(8476), 307–310 (1986). https://doi.org/10.1016/s0140-6736(86)90837-8

Blotz, L.R., Benson, C.H., Boutwell, G.P.: Estimating optimum water content and maximum dry unit weight for compacted clays. J. Geotech. Geoenviron. Eng. **124**(9), 907–912 (1998). https://doi.org/10.1061/(asce)1090-0241(1998)124:9(907)

Bose, B.: Geo-engineering properties of expansive soil stabilized with fly ash. Electr. J. Geotech. Eng. **17**, 1339–1353 (2012)

Broderick, G.P., Daniel, D.E.: Stabilizing compacted clay against chemical attack. J. Geotech. Eng. **116**(10), 1549–1567 (1990). https://doi.org/10.1061/(asce)0733-9410(1990)116:10(1549)

Browne, M.J.: Feasibility of using a gyratory compactor to determine compaction characteristics of soil. Dissertation, Montana State University, USA (2006)

BS 1377–4: Methods of test for soils for civil engineering purposes — part 4: compaction-related tests. British Standards Institution (BSI): London, UK (1990)

BS 5930: Code of practice for ground investigations. British Standards Institution (BSI): London, UK (2015)

Celauro, B., Bevilacqua, A., Bosco, D.L., Celauro, C.: Design procedures for soil–lime stabilization for road and railway embankments. Part 2 – experimental validation. Proc. Soc. Behav. Sci. **53**, 568–579 (2012). https://doi.org/10.1016/j.sbspro.2012.09.907

Chen, Y.: An experimental investigation of the behavior of compacted clay/sand mixtures. Dissertation, University of Delaware, USA (2010)

Di Matteo, L., Spagnoli, G.: Predicting compaction properties of soils at different compaction efforts. Proc. Instit. Civil Eng. Geotech. Eng. **176**(2), 146–156 (2023). https://doi.org/10.1680/jgeen.21.00017

Di Matteo, L., Spagnoli, G., Soltani, A., Azimi, M., O'Kelly, B.C.: Discussion: Predicting compaction properties of soils at different compaction efforts. Proc. Instit. Civil Eng. Geotech. Eng. **175**(5), 549–552 (2022). https://doi.org/10.1680/jgeen.22.00053

Drew, I.R.: Influence of compaction energy on soil engineering properties. Dissertation, Iowa State University, USA (2005)

Eberemu, A.O.: Evaluation of bagasse ash treated lateritic soil as a potential barrier material in waste containment application. Acta Geotech. **8**(4), 407–421 (2013). https://doi.org/10.1007/s11440-012-0204-5

Eberemu, A.O., Amadi, A.A., Osinubi, K.J.: The use of compacted tropical clay treated with rice husk ash as a suitable hydraulic barrier material in waste containment applications. Waste. Biomass Valoriz. **4**(2), 309–323 (2013). https://doi.org/10.1007/s12649-012-9161-3

Gurtug, Y., Sridharan, A.: Compaction behaviour and prediction of its characteristics of fine grained soils with particular reference to compaction energy. Soils. Found. **44**(5), 27–36 (2004). https://doi.org/10.3208/sandf.44.5_27

Gurtug, Y., Sridharan, A., İkizler, S.B.: Simplified method to predict compaction curves and characteristics of soils. Iran. J. Sci. Technol. Trans. Civil Eng. **42**(3), 207–216 (2018). https://doi.org/10.1007/s40996-018-0098-z

Hamdani, I.H.: Use of one-point Proctor standard compaction method for computing modified AASHO compaction parameters. In 62nd Annual Proceedings of the Pakistan Engineering Congress, Paper No. 502, pp 183–194. Pakistan Engineering Congress: Lahore, Pakistan (1987)

Hamza, M., Farooq, K., Rehman, Z.U., Mujtaba, H., Khalid, U.: Utilization of eggshell food waste to mitigate geotechnical vulnerabilities of fat clay: a micro–macro-investigation. Environ. Earth Sci. **82**(10), 247 (2023). https://doi.org/10.1007/s12665-023-10921-3

Hassan, A.A., Toll, D.G.: Water content characteristics of mechanically compacted clay soil determined using the electrical resistivity method. In Winter MG, Smith DM, Eldred PJL and Toll DG (Eds), Geotechnical Engineering for Infrastructure and Development: XVI European Conference on Soil Mechanics and Geotechnical Engineering, pp 3395–3400. ICE Publishing: Westminster, London, UK (2015). https://doi.org/10.1680/ecsmge.60678.vol6.532

Hezmi, M.A., Saari, R., Zahari, M.Z., Abdullah, R.A., Yunus, N.Z.M., Rashid, A.S.A.: Soil water characteristic curves of compacted kaolin for various initial moisture content. Jurnal. Teknologi. **76**(2), 39–44 (2015). https://doi.org/10.11113/jt.v76.5431

Horpibulsuk, S., Katkan, W., Apichatvullop, A.: An approach for assessment of compaction curves of fine grained soils at various energies using a one point test. Soils Found. **48**(1), 115–125 (2008). https://doi.org/10.3208/sandf.48.115

Horpibulsuk, S., Katkan, W., Naramitkornburee, A.: Modified Ohio's curves: a rapid estimation of compaction curves for coarse- and fine-grained soils. Geotech. Test. J. **32**(1), 64–75 (2009). https://doi.org/10.1520/gtj101659

Howard, R.F., Singer, M.J., Frantz, G.A.: Effects of soil properties, water content, and compactive effort on the compaction of selected California forest and range soils. Soil Sci. Soc. Am. J. **45**(2), 231–236 (1981). https://doi.org/10.2136/sssaj1981.03615995004500020001x

Islam, T., Kodikara, J.: Interpretation of the loading–wetting behaviour of compacted soils within the "MPK" framework. Part I: Static compaction. Canadi. Geotech. J. **53**(5), 783–805 (2016). https://doi.org/10.1139/cgj-2014-0316

Khalid, U., Rehman, Z.U.: Evaluation of compaction parameters of fine-grained soils using standard and modified efforts. Intl. J. Geo-Eng. **9**(1), 15 (2018). https://doi.org/10.1186/s40703-018-0083-1

Kodikara, J., Islam, T., Rajeev, P.: Interpretation of the loading–wetting behaviour of compacted soils within the "MPK" framework Part II: Dynamic compaction. Canad. Geotech. J. **53**(5), 806–827 (2016). https://doi.org/10.1139/cgj-2014-0317

Kotdawala, S.J.: A study of soil compaction with miniature sheeps-foot rollers and development of a knowledge-based system. Dissertation, Kansas State University, USA (1994)

Li, Z.S., Fleureau, J.M., Tang, L.S.: Aspects of compaction and drying–wetting curves of a subgrade clayey soil. Géotechnique **67**(12), 1120–1126 (2017). https://doi.org/10.1680/jgeot.16.t.010

Little, A.L.: Laboratory compaction technique. In Hanninga N, Oosterholt GA, Geuze ECWA and Koppejan AW (Eds), Proceedings of the 2nd International Conference on Soil Mechanics and Foundation Engineering (Volume 1), pp 224–229. Haarlem: Rotterdam, the Netherlands (1948)

Loshelder, J.I., Errigo Chanis, A.L., Coffman, R.A.: Proctor and Harvard miniature compaction energy comparison. Geotech. Test. J. **46**(2), 364–378 (2023). https://doi.org/10.1520/gtj20210087

Mamatha, K.H., Dinesh, S.V.: Resilient modulus of black cotton soil. Intl. J. Pavem. Res. Technol. **10**(2), 171–184 (2017). https://doi.org/10.1016/j.ijprt.2017.01.008

Marinho, F.A., Stuermer, M.M.: The influence of the compaction energy on the SWCC of a residual soil. In Shackelford CD, Houston SL and Chang NY (Eds), Geo-Denver 2000 (GSP 99): Advances in Unsaturated Geotechnics, pp 125–141. American Society of Civil Engineers (ASCE): Reston, VA, USA (2000) https://doi.org/10.1061/40510(287)8

Mehmood, E., Ilyas, M., Farooq, K.: Effect of initial placement conditions on swelling characteristics of expansive soils. In Han J and Alzamora DE (Eds), Geo-Frontiers 2011: Advances in Geotechnical Engineering, pp 2397–2403.American Society of Civil Engineers (ASCE): Reston, VA, USA (2011) https://doi.org/10.1061/41165(397)245

Mendes. J.: Assessment of the impact of climate change on an instrumented embankment: an unsaturated soil mechanics approach. Dissertation, Durham University, UK (2011)

Osinubi, K.J., Nwaiwu, C.M.: Compaction delay effects on properties of lime-treated soil. J. Mater. Civ. Eng. **18**(2), 250–258 (2006). https://doi.org/10.1061/(asce)0899-1561(2006)18:2(250)

Osinubi, K.J., Eberemu, A.O., Amadi, A.A.: Compacted lateritic soil treated with blast furnace slag as hydraulic barriers in waste containment systems. Intl. J. Risk Assess. Manag. **13**(2), 171–189 (2009). https://doi.org/10.1504/ijram.2009.030328

Prasanna, H.S., Devanur, J., Vishwanath, C.: Compaction behavior of China clay–sand mixtures. Intl. Res. J. Eng. Technol. **5**(12), 950–962 (2018)

Santos, L.A., Lopes, S., Silva, J.: Difficulties of using the Harvard miniature compaction apparatus as a reference test in the study of soil compaction. In Sigursteinsson H, Erlingsson S and Bessason B (Eds), Geotechnical Engineering, Foundation of the Future: XVII European Conference on Soil Mechanics and Geotechnical Engineering, Paper No. 620, pp 1–6. Icelandic Geotechnical Society (IGS): Reykjavík, Iceland (2019). https://doi.org/10.32075/17ecsmge-2019-0620

Shaivan, H.S., Sridharan, A.: Comparison of reduced modified Proctor vs modified Proctor. Geotech. Geol. Eng. **38**(6), 6891–6897 (2020). https://doi.org/10.1007/s10706-020-01405-3

Shimobe, S., Karakan, E., Sezer, A.: Improved dataset for establishing novel relationships between compaction characteristics and physical properties of soils. Bull. Eng. Geol. Env. **80**(11), 8633–8663 (2021). https://doi.org/10.1007/s10064-021-02456-3

Shivaprakash, S.H., Sridharan, A.: Correlation of compaction characteristics of standard and reduced Proctor tests. Proc. Instit. Civil Eng. Geotech. Eng. **174**(2), 170–180 (2021). https://doi.org/10.1680/jgeen.20.00060

Signes, C.H., Garzón-Roca, J., Fernández, P.M., de la Torre, M.E.G., Franco, R.I.: Swelling potential reduction of Spanish argillaceous marlstone Facies Tap soil through the addition of crumb rubber particles from scrap tyres. Appl. Clay Sci. **132–133**, 768–773 (2016). https://doi.org/10.1016/j.clay.2016.07.027

Singh, A., Punmia, B.C.: A new laboratory compaction device and its comparison with the Proctor test. Highway Research News **17**, 37–41 (1965)

Sivrikaya, O., Togrol, E., Kayadelen, C.: Estimating compaction behavior of fine-grained soils based on compaction energy. Can. Geotech. J. **45**(6), 877–887 (2008). https://doi.org/10.1139/t08-022

Soltani, A., O'Kelly, B.C.: Discussion of "The flow index of clays and its relationship with some basic geotechnical properties" by G. Spagnoli, M. Feinendegen, L. Di Matteo, and D. A. Rubinos, published in Geotechnical Testing Journal 42, no. 6 (2019): 1685–1700". Geotechn. Test. J. **44**(1), 216–219 (2021a). https://doi.org/10.1520/gtj20190423

Soltani, A., O'Kelly, B.C.: Reappraisal of the ASTM/AASHTO standard rolling device method for plastic limit determination of fine-grained soils. Geosciences **11**(6), 247 (2021b). https://doi.org/10.3390/geosciences11060247

Soltani, A., O'Kelly, B.C.: Reappraisal of fall-cone flow curve for soil plasticity determinations. Geotech. Test. J. **45**(1), 225–243 (2022). https://doi.org/10.1520/gtj20200312

Soltani, A., Deng, A., Taheri, A., Sridharan, A., Estabragh, A.R.: A framework for interpretation of the compressibility behavior of soils. Geotech. Test. J. **41**(1), 1–16 (2018). https://doi.org/10.1520/gtj20170088

Soltani, A., Deng, A., Taheri, A., O'Kelly, B.C.: Intermittent swelling and shrinkage of a highly expansive soil treated with polyacrylamide. J. Rock Mech. Geotech. Eng. **14**(1), 252–261 (2022). https://doi.org/10.1016/j.jrmge.2021.04.009

Soltani, A., Azimi, M., O'Kelly, B.C., Horpibulsuk, S.: Converting optimum compaction properties of fine-grained soils between rational energy levels. Transport. Geotech. **42**, 101096 (2023). https://doi.org/10.1016/j.trgeo.2023.101096

Sreelekshmypillai, G., Vinod, P.: Prediction of CBR value of fine grained soils at any rational compactive effort. Int. J. Geotech. Eng. **13**(6), 560–565 (2019). https://doi.org/10.1080/19386362.2017.1374495

Sridharan, A., Gurtug, Y.: Swelling behaviour of compacted fine-grained soils. Eng. Geol. **72**(1–2), 9–18 (2004). https://doi.org/10.1016/S0013-7952(03)00161-3

Sridharan, A., Sivapullaiah, P.V.: Mini compaction test apparatus for fine grained soils. Geotech. Test. J. **28**(3), 240–246 (2005). https://doi.org/10.1520/gtj12542

Sterpi, D.: Effect of freeze–thaw cycles on the hydraulic conductivity of a compacted clayey silt and influence of the compaction energy. Soils Found. **55**(5), 1326–1332 (2015). https://doi.org/10.1016/j.sandf.2015.09.030

Tüfekçi, N., Özçoban, M., Yalçin, S., Asci, Y., Akgüner, C.: Adsorption and permeability of clays permeated with ferrous iron and manganese. Fresenius Environ. Bull. **19**(8B), 1703–1714 (2010)

Uba, M.Z.: Geotechnical evaluation of kaolin clay treated with groundnut shell ash as suitable material in municipal solid waste containment systems. Dissertation, Fatih University, Turkey (2014)

Umar, S.Y., Elinwa, A.U.: Evaluation of the hydraulic conductivity of compacted laterite–metakaolin mixtures for solid waste leachate containment. FUOYE J. Eng. Technol. **5**(1), 119–124 (2020). https://doi.org/10.46792/fuoyejet.v5i1.476

Verma, G., Kumar, B.: Prediction of compaction parameters for fine-grained and coarse-grained soils: a review. Int. J. Geotech. Eng. **14**(8), 970–977 (2020). https://doi.org/10.1080/19386362.2019.1595301

Vinod, P.P., Sridharan, A., Soumya, R.J.: Effect of compaction energy on CBR and compaction behaviour. Proc Instit Civil Eng. Ground Improve. **168**(2), 116–121 (2015). https://doi.org/10.1680/grim.13.00059

Vipulanandan, C., Tritico, P., Goldberg, L.: Construction quality: laboratory compaction tests are not the compaction standard. In Proceedings of the Texas Section of the American Society of Civil Engineers (ASCE), pp 1–9. ASCE: Austin, TX, USA (2009)

Viswanadham, B.V.S., Jha, B.K., Pawar, S.N.: Experimental study on flexural testing of compacted soil beams. J. Mater. Civ. Eng. **22**(5), 460–468 (2010). https://doi.org/10.1061/(asce)mt.1943-5533.0000045

Wilson, S.D.: Small soil compaction apparatus duplicates field results closely. Eng. News Rec. **145**(18), 34–36 (1950)

Zhang, J., Soltani, A., Deng, A., Jaksa, M.B.: Mechanical behavior of micaceous clays. J. Rock Mech. Geotech. Eng. **11**(5), 1044–1054 (2019). https://doi.org/10.1016/j.jrmge.2019.04.001

Authors and Affiliations

Amin Soltani[1,2] · Brendan C. O'Kelly[3] · Suksun Horpibulsuk[4,5] · Abbas Taheri[6]

✉ Amin Soltani
a.soltani@federation.edu.au

Brendan C. O'Kelly
bokelly@tcd.ie

Suksun Horpibulsuk
suksun@g.sut.ac.th

Abbas Taheri
abbas.taheri@queensu.ca

1 Institute of Innovation, Science and Sustainability, and Future Regions Research Center, Federation University, Churchill, VIC 3842, Australia

2 Department of Infrastructure Engineering, Faculty of Engineering and Information Technology, The University of Melbourne, Parkville, VIC 3010, Australia

3 Department of Civil, Structural and Environmental Engineering, Trinity College Dublin, Dublin D02 PN40, Ireland

4 School of Civil Engineering, Suranaree University of Technology, Mueang Nakhon Ratchasima District, Nakhon Ratchasima 30000, Thailand

5 Academy of Science, The Royal Society of Thailand, Bangkok 10300, Thailand

6 The Robert M. Buchan Department of Mining, The Stephen J. R. Smith Faculty of Engineering and Applied Science, Queen's University, Kingston, ON K7L 3N6, Canada