



Comparing Several Different Numerical Approaches for Large Deformation Modeling with Application in Soil Dynamic Compaction

Rashid Dastgerdi Hajivand¹ · Mohammad Ahmad Mahmoudi Zamani² · Dariusz Knez³ · Kamran Kazemi⁴

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Abstract

Dynamic compaction (DC) is vastly utilized to improve the strength characteristics of the soils. To predict the soil deformations derived from the DC operations, usually numerical simulation analysis is applied. For the conduction of such simulations, several numerical approaches with different elemental formulations can be used. From the perspective of finite element analysis (FEA), there are four main formulations including the Lagrangian, Arbitrary Lagrangian-Eulerian (ALE), Coupled Lagrangian-Eulerian (CEL), and Smoothed Particle Hydrodynamic (SPH). In this research, a comparative study has been conducted to evaluate the computational efficiency of those four approaches in the prediction of soil large deformations during the DC operations. To do this, for a DC operation executed in a road embankment construction project in China, the real field data was compared to the results obtained from the numerical simulations via the ABAQUS program. The findings demonstrate that of all those approaches, the Lagrangian approach delivers the minimum accuracy of the predicted results, albeit with the least running time. In contrast, the ALE formulation predicted closer estimations of soil deformations although it was found to be less time-efficient. Interestingly, the CEL and SPH approaches predicted the soil deformations with the maximum degree of accuracy whereas they were not as time-efficient as the Lagrangian approach. To address this issue, a hybrid model of Lagrangian and SPH formulations was constituted to satisfy the maximum accuracy with the minimum running time. Such a hybrid model is highly applicable for the accurate prediction of soil large deformations during the DC operations.

Keywords Dynamic numerical analysis · Finite element analysis · ABAQUS · Dynamic compaction · Embankment · Tamper

✉ Mohammad Ahmad Mahmoudi Zamani
zamani@agh.edu.pl

Extended author information available on the last page of the article

1 Introduction

A major proportion of the prevalent engineering phenomena deals with the large collision events. In all of them, dynamic analysis of the collision process is almost commonly complex, and requires three-dimensional numerical simulations to monitor the response of the striking objects during the impact. Some important applications are analysis of meteoroid impact in metrological engineering, car/aircraft crashworthiness in mechanical engineering, and soil dynamic compaction in civil engineering (Sareen et al., 2002; Evans et al., 2016). Regarding the two latter cases, the results obtained from the numerical simulations are effectively utilized for the enhancement of the physical and mechanical properties of cars/aircrafts, and the loose soils.

Soil embankments are implemented in many road, railway, and dam construction projects. Without an adequate degree of soil compaction, the embankment will encounter post-construction settlements together with potential slope instabilities (Zhang et al., 2018; Ji et al., 2018). To address this issue, usually rolling compaction, and dynamic compaction (DC) are deployed to slam the soil, thereby increasing the overall density and bearing capacity of the embankment (Mayne et al., 1984; Shenthana et al., 2004; Menard & Broise, 1975; Van Impe & Bouazza, 1997; Feng et al., 2015). As a result, the porosity, which is a determining factor in the mechanical behavior of the soil (Knez & Zamani, 2021a), declines effectively. In dynamic compaction operations, a tamper (normally the weight is between 5 and 40 tons) is raised to a special height (normally between 15 and 30 m), and then it falls freely. This technique has been frequently reported to be highly efficient in enhancement of the strength properties of the different granular soils, and more than this, in prevention of liquefaction issues related to the silty soils (Moldovan et al., 2017; Bouzelha et al., 2017; Araei et al., 2012). So far, a series of physical models (Oshima et al., 1996; Oshima & Takada, 1998), field tests (Mayne et al., 1984; Feng et al., 2015; Lukas, 1995; Feng et al., 2010; Feng et al., 2011), and numerical analyses (Ghanbari & Hamidi, 2015; Zekkos et al., 2013; Matsui & San, 1992; Gu & Lee, 2002; Lee & Gu, 2004; Zhou et al., 2020; Wang et al., 2017) have been conducted to investigate the effect of dynamic compaction on soil embankments.

For numerical modeling of the dynamic compaction process, different researchers have used the well-known geotechnical programs such as PLAXIS, ABAQUS, FLAC3D, and COMSOL. Each of the mentioned commercial programs have been formulated with a specific numerical method, i.e., Finite Element Analysis (FEA), Finite Difference Analysis (FDA), Discrete Element Analysis (DEA), etc. The FEA-based programs are highly suitable in the simulation of civil engineering problems (Hardik, 2018). Generally, each FEA-based program can provide the geotechnical engineer with one or more numerical approaches. Such numerical approaches encompass Lagrangian, Arbitrary Lagrangian-Eulerian (ALE), Coupled Lagrangian-Eulerian (CEL), and Smoothed Particle Hydrodynamic (SPH). Up to now, several comparative studies have been conducted to assess the capability of such numerical approaches in

the computation of large soil deformations related to different civil engineering problems. In most of them, two or more numerical approaches were compared with the real field tests. In the following paragraph, a concise description related to this state-of-the-art is elaborated.

Bojanowski and Kulak in (Bojanowski & Kulak, 2010) compared the accuracy of the Lagrangian, ALE, and SPH approaches for the calculation of large deformations in soil. To do this, they carried out a set of on-site tests in which a steel pad penetrated into a silty sand layer. The observations were compared with the results achieved from the numerical simulations via the Lagrangian, ALE, and SPH approaches. They reported that the main drawbacks of the Lagrangian and ALE approaches were mesh distortion, and high CPU requirements, respectively. Nevertheless, the SPH approach was referred as a highly reliable approach which gave more accurate predictions contrasted to the ALE and Lagrangian approaches. In another comparative study, (Trajkovski, 2017) assessed the performance of ALE and SPH approaches in the prediction of large blast-induced displacements in some structures. They concluded that the SPH approach provided more accurate results than the ALE formulation. Another attempt to compute the penetration depth of a penetrometer to a soil layer was carried out by (Evans et al., 2016). They utilized an Eulerian-based approach instead of the common Lagrangian-based approaches to avoid the mesh distortion problems. Based on the validation with the experimental tests, they concluded that the Eulerian-based approach is more reliable and accurate than the Lagrangian approach for soil compact analysis. In addition, (Seetamsetti, 2012) compared the Lagrangian, ALE, and SPH approaches in the computation of large deformations in soft soil and water during the aircraft collisions. It was deduced that the results obtained from the ALE and SPH approaches are closer to the experiments results. They also proposed to adopt the CEL approach in the prediction of such large deformations.

As it was mentioned in the previous paragraph, a number of past investigations were conducted to evaluate the capability of those numerical approaches in the prediction of the soil large deformations during the different impact events. The objective of this research is to compare the efficiency of the four aforementioned numerical approaches in the calculation of soil large deformations during the dynamic compaction operations. To do this, the four approaches have undergone a close scrutiny to study their capability in the analysis of the soil dynamic compaction in terms of the maximum accuracy and minimum running time. The pertinent case study is a high embankment executed on Ping-Zang Expressway in Hebei Province, China. The real field data obtained in the dynamic compaction operation are compared with the results achieved from each aforementioned numerical approach. In addition, a hybrid approach which incorporated the Lagrangian and SPH formulations was applied to mitigate the limitations related to the four aforementioned numerical approaches. The performance of the hybrid model was compared with the other four approaches and the real field data. The hybrid model was found to be more accurate and time-efficient in comparison to the four available numerical approaches.

The structure of this article has been arranged as follows: firstly, in section 2, a description related to the different concepts of numerical approaches is presented.

Afterward, section 3.1 elaborates the procedure of the soil dynamic compaction process in civil engineering projects. Then, section 3.2 describes the details related to the field project and embankment soil properties. In section 4, the context is then followed by the presentation of the findings from the conducted numerical simulations via the aforesaid numerical approaches. Afterward, an inclusive discussion about the results is implemented in section 5. And eventually, the paper terminates with a conclusion on the key findings, along with the useful recommendations related to applications of different FEA approaches in numerical modeling of soil dynamic compaction operations. The obtained results in this article can be generalized and utilized to other collision events in different scientific areas.

2 Description of Different Numerical Approaches

In this section, the basic theories relevant to the major numerical approaches, i.e., Lagrangian, Eulerian, ALE, CEL, and SPH are elaborated. The focus is on the description of the fundamental formulations conducted by each approach to calculate the soil large deformation during the dynamic compaction process. In general, all FEA-based numerical approaches have been established on two inclusive formulations: Lagrangian, and Eulerian (Donea et al., 2004; Knez & Khalilidermani, 2021). However, in recent years, creative combinations of those two formulations have led to some modified approaches such as AEL, CEL, and SPH. In what follows, the generic concept of each numerical approach is described.

2.1 Lagrangian Approach

In the purely Lagrangian formulation, the mesh changes as a response to the applied forces while the total mass, momentum, and energy remain consistent within the mesh (Evans et al., 2016). During the motion of the material (soil), the nodes also accompany in the same direction with the same magnitude of displacement. Via the Lagrangian approach, the tracking action of the interface between the two collided materials gets very simple, and hence, the stress-strain history of each node can be conveniently evoked after solving the problem (Donea et al., 2004). Such formulation is very successive in numerical simulation of the small-strain interactions, boundary deformations, sophisticated meshes, and recording the stress-strain history (Evans et al., 2016). However, for soil large deformations, due to the distortion of the mesh, they may predict unrealistic results (Bojanowski & Kulak, 2010; Donea et al., 2004). Similarly, for the high-speed impact problems in which an extensive shock renders to severe deformations within the bodies, i.e., astronomical and meteoroid impacts (Knez & Khalilidermani, 2021; Knez & Zamani, 2021b), the Lagrangian formulation is not effectively applicable. Figure 1 demonstrates the mesh deformation via the Lagrangian formulation applied in impact analysis.

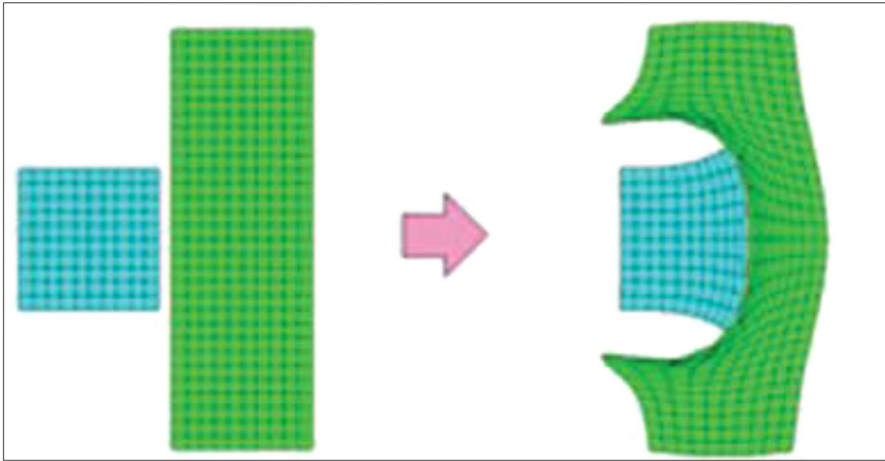


Fig. 1 Mesh deformation in impact analysis (Lagrangian formulation)

2.2 Eulerian-Based Approach

In the purely Eulerian-based formulation, the mesh is constant in the space, and it is independent from the soil (material) deformations (Donea et al., 2004). This key characteristic renders the Eulerian-based formulation superior to the Lagrangian-based one for numerical simulation of large deformations, especially when the collision speed is intensively substantial (Evans et al., 2016; Trajkovski, 2017). Nevertheless, the Eulerian-based formulation has some drawbacks such as the limited capability in the generation of highly complex meshes, recording

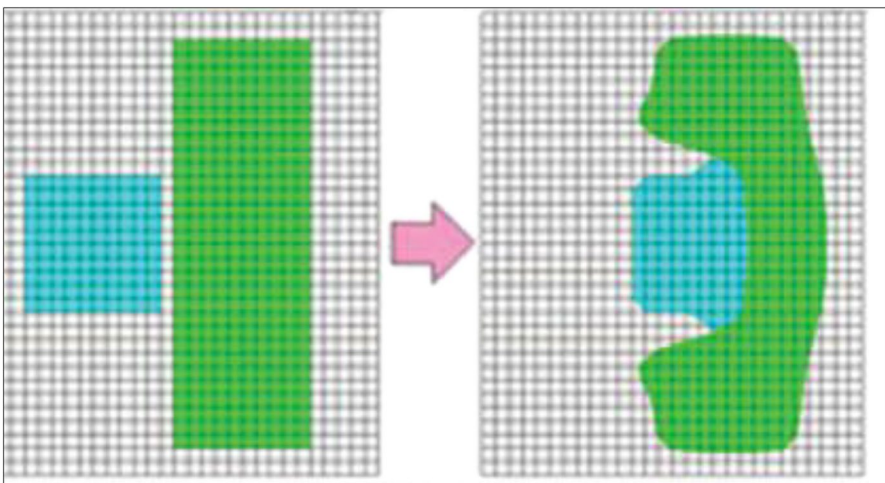


Fig. 2 Mesh deformation in impact analysis (Eulerian formulation)

the stress-strain history, and modeling of the flexible twisty surfaces at the collision interface (Evans et al., 2016; Donea et al., 2004). Figure 2 exhibits the mesh deformation due to the impact process via the Eulerian formulations.

2.3 ALE Approach

As it was previously expressed, the available methods commonly utilize the traditional Lagrangian and Eulerian formulations for the description of the soil (material) motion (Donea et al., 2004). The ALE approach was introduced to capture the advantages of both pure Lagrangian and Eulerian methods, thereby mitigating the problems related to the utilization of those traditional approaches (Seetamsetti, 2012). The main benefit of the ALE approach is that the reference computational domain can move arbitrarily, and autonomous of the deformation of the material (Donea et al., 2004). The motion of such a reference domain is manifested through an arbitrary number of grid points. Consequently, the motion of those grid points can be regarded as the motion of the finite element mesh (Donea et al., 2004). Hence, in contrast to the Lagrangian approach, the grid nodes in the ALE formulation are independent from the motion of the simulated soil (Bojanowski & Kulak, 2010). According to (Trajkovski, 2017), when the ALE-based mesh contains fluids such as water, the problem of “fluid leakage” may occur. Such a problem takes place when the mesh and containing fluid lack parallelism. Figure 3 compares the mesh deformation in ALE formulation with the Lagrangian approach.

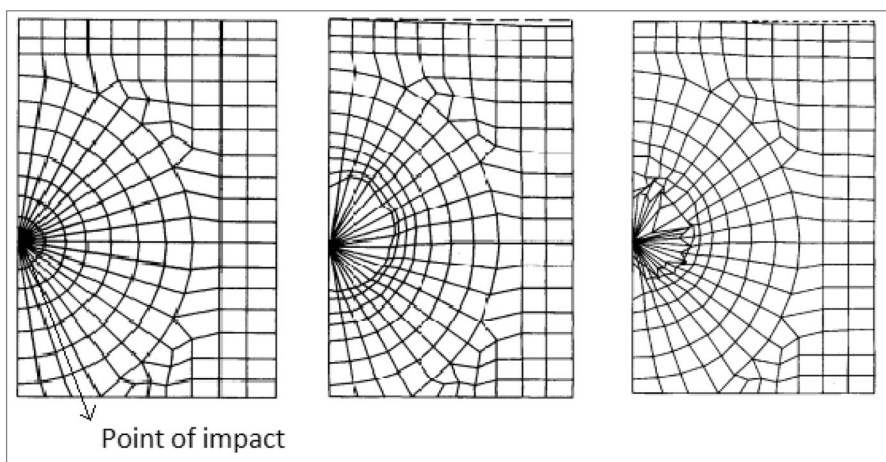


Fig. 3 Mesh deformation in impact analysis. Initial mesh (Left); ALE (middle), and Lagrangian (right). The point of impact lies at the center of the circular part

2.4 CEL Approach

Another way to capture the advantages of both Eulerian and Lagrangian formulations is to use the CEL approach (Chmelnizkij et al., 2019). In this approach, the tamper is modeled by the Lagrangian elements whereas the soil is simulated using the Eulerian elements. Therefore, simulation of soil large deformations is feasible since the Eulerian-based soil elements are capable of flowing through the soil grid nodes (Donea et al., 2004), thereby permitting the simulation to be continued without error in run the process. Figure 4 demonstrates the behavior of the simulated soil in CEL formulation versus the Lagrangian approach. The main merit of the CEL approach is originated from the utilization of the Eulerian volume fraction (EVF) function (Qiu et al., 2011). When the tamper collides with the soil, the CEL approach computes the induced deformations based on the calculation of EVF for each soil element. When the soil is moved through the Eulerian elements, the nodes are consistent, and therefore, the soil is traced through the elements. If an element is entirely filled by the soil, the value of the EVF will be equal to 1. If there is no soil in an element, the corresponding EVF will be equal to 0. In case in which the soil partially fills an element, the value of the corresponding EVF is between 0 and 1 (Qiu et al., 2011).

2.5 SPH Approach

The SPH approach has an entirely Lagrangian formulation, and it does not need to mesh generation. In fact, in this approach, instead of conventional elemental meshes, a series of particles are used to simulate the soil (material) grains. This approach was first utilized by (Mayne et al., 1984) in 1977, to simulate the collision events in extraterrestrial and cosmic environments. Having significant capability in the simulation of large-speed collisions, the SPH approach was subsequently utilized

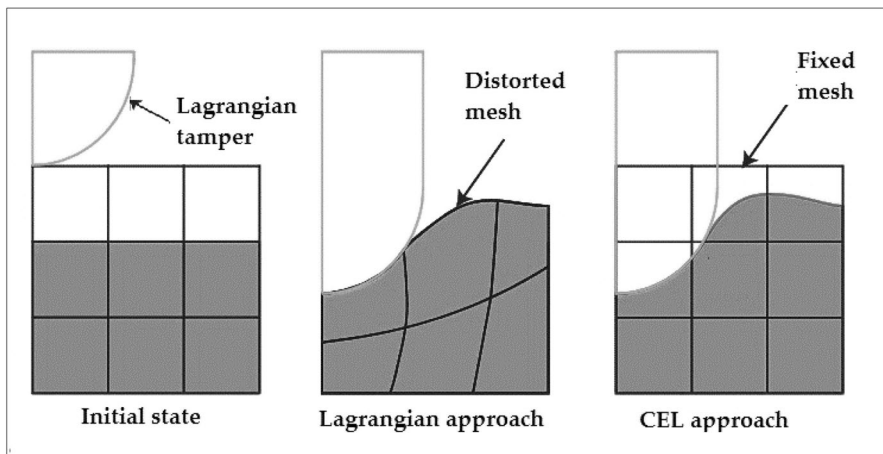


Fig. 4 Mesh deformation in CEL formulation versus the Lagrangian approach

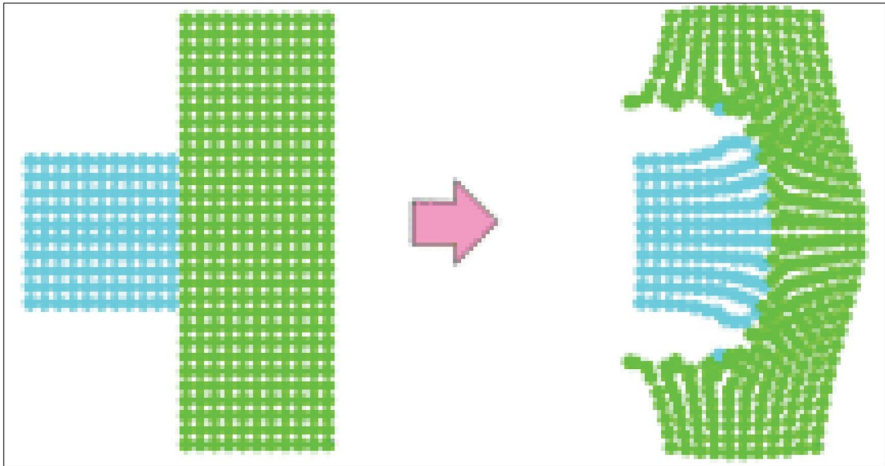


Fig. 5 Impact-induced deformations via the SPH formulation

in large deformation problems on the Earth (Araei et al., 2012). At present, the SPH approach has achieved an ongoing popularity in fluid dynamics, and large strain problems. Figure 5 displays the simulation of the impact-induced deformations via the SPH formulation.

3 Problem Statement

3.1 Dynamic Compaction Process

In nature, the unconsolidated soil and rock grains are deployed in many geo-related engineering applications, e.g., mining engineering, petroleum engineering, and civil engineering (Knez & Calicki, 2018; Knez & Mazur, 2019; Knez et al., 2019). Among the civil engineering applications, dynamic compaction is widely used to condense and strengthen the granular soils, especially in roads, railways, and dam construction projects. This method is categorized among the soil improvement techniques. In this method, a tamper (normally the weight varies from 5 tons to 40 tons) is released from a height (normally the height varies from 15 to 30 m) to collide with the embankment surface at a high speed (Pourjenabi et al., 2013). When the tamper collides with the embankment, the maximum energy is transferred into the soil particles exactly located under the tamper. The more distance between the tamper and the soil grains, the less transferred energy to displace the soil. Furthermore, while the soil grains under the tamper are compacted, a proportion of the soil particles experiences a heave phenomenon as a consequence of the induced vibration. The domain of this heave can also be predicted by the means of numerical simulations.

Figure 6 shows a schematic diagram of wave propagation through the soil dynamic compaction process. When the tamper strikes with the embankment, its speed is calculated as

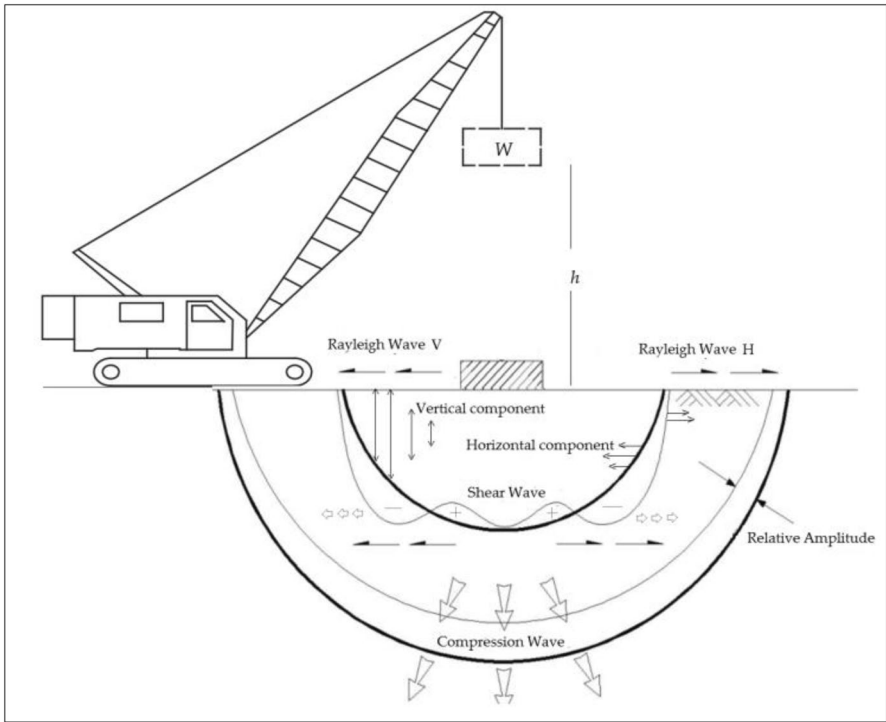


Fig. 6 Schematic diagram of wave propagation in soil dynamic compaction process

$$V_0 = \sqrt{2gh} \tag{1}$$

Where V_0 represents the velocity of the tamper in the first strike, g is the gravitational acceleration, and h indicates the falling height. This value is used in subsequent numerical simulations. During any penetration process, the on-site stress regime shifts, and the new stresses are balanced (Knez, 2014). Theoretically, the maximum impact load of the tamper on the soil can be defined as (Moon et al., 2019):

$$F = \sqrt{\frac{32WhG_d r}{\pi^2 (1 - \nu)}} \tag{2}$$

where F represents the maximum impact force, W indicates the mass of the tamper, h indicates the falling height of the tamper, G_d stands for the dynamic shear modulus of the soil, r represents the radius of the tamper, and ν indicates the Poisson's ratio.

Fig. 7 Dynamic compaction machine in the project



Table 1 Properties of the tamper used in the numerical modeling

Property	Unit	Value
Mass	kg	13,520
Falling height	m	15
Diameter	m	1.6
Height	m	0.84
Elastic modulus	GPa	90
Poisson's ratio	-	0.2
Density	kg/m ³	8009

3.2 Project Description

In this research, to evaluate the accuracy and time-efficiency of the five aforementioned approaches, a high embankment in Ping-Zang Expressway in Hebei Province, China, was selected as the case study. In the field, a rectangular area with the dimension of 12×10 m was chosen to record the real vertical deformations of the soil during the dynamic compaction operation (Zhang et al., 2019). Figure 7 illustrates the dynamic compaction machine utilized in the project. The cylindrical tamper was 1.6 m in diameter, and 0.84 m in height. Furthermore, its mass was 13,520 kg (Zhang et al., 2019). During the compaction operation, it was dropped from the height of 15 m above the embankment surface. The results of the dynamic compaction process at the first strike of the tamper were recorded to validate the accuracy of the aforementioned numerical approaches. Table 1 demonstrates the properties of the tamper used in the numerical simulations.

Furthermore, the geotechnical properties of the filling soil were measured through different tests. Such characteristics have been tabulated in Table 2. For the

Table 2 Properties of the filling soil

Property	Unit	Value
Cohesion	kPa	5
Internal friction angle	degree	33
Elastic modulus	MPa	38
Poisson's ratio	-	0.3
Density	kg/m ³	2500

measurement of the soil cohesion and internal friction angle, a direct shear test was performed. Moreover, the elastic modulus was determined using a standard penetration test. The Poisson's ratio was determined using a uniaxial loading test. The density of the soil was also measured through the conduction of the Sand Cone test.

In addition, based on the Unified Soil Classification System (USCS), the soil was classified as GC-CL (clayey gravel with many fines). The in situ soil included mainly the mafic minerals eroded from the adjacent igneous rocks. The groundwater table was also reported at the depth of 35 m below the embankment toe. Hence, in the numerical simulations, no groundwater effect was applied in the calculations.

For numerical modeling of the soil, the Mohr-Coulomb failure criterion was chosen to represent the behavior of the soil under the compaction process. Therefore, the data presented in Table 2 were imported as the input variables for the Mohr-Coulomb failure criterion. It should be mentioned that this criterion is a reasonable constitutive model to represent the soil/rock behavior under different loading conditions (Zamani & Knez, 2021). Since the objective of this research is to perform a comparison between the different numerical approaches, it is expected that the type of criterion does not affect the numerical results. However, in case of having enough data about the soil properties, other failure criteria such as Mogi-Coulomb, Hoek-Brown, and or Drucker-Prager can also be utilized.

4 Numerical Simulation

Several numerical simulations were conducted to contrast the accuracy and time efficiency of the different numerical approaches with the real field data. The examined numerical approaches included the Lagrangian, ALE, CEL, SPH, and the hybrid Lagrangian-SPH formulations. The latter was an initiative technique to benefit the advantages of both Lagrangian and SPH formulations. The numerical observations including the accuracy and time efficiency of those five approaches were compared together.

In what follows, the numerical results together with a comparative assessment between the mentioned approaches are elaborated. It is noteworthy that in all numerical meshes, the general contact was set as the contact type between the soil and the tamper. Moreover, in analyses that use one of the plasticity constitutive models (Mohr-Coulomb, in this research), a significant amount of energy dissipation occurs during the soil plastic flow. Hence, the selection of damping parameters is

less critical to the outcome of the analysis than if an elastic model were to be used. In this research, no Rayleigh damping effect was specified to the models (undamped condition).

4.1 Lagrangian Approach

For this case, the generated mesh was in the form of a quarter cylinder, which due to the symmetry conditions, a three-dimensional plane of symmetry was used to increase the modeling accuracy. Figure 8 shows the corresponding mesh geometry. The exact dimensions of the mesh as well as the type of the different elements are also illustrated in the figure. Moreover, to prevent wave reflection, the infinite elements were utilized to generate the mesh. The generated mesh contained a number of 36,000 elements. The size of the elements increased incrementally from 0.1 m (for the central elements) to 1 m (for the outside elements).

The modeling steps were as follows: in the first step, the in situ stresses including the vertical stress, maximum horizontal stress, and the minimum horizontal stress

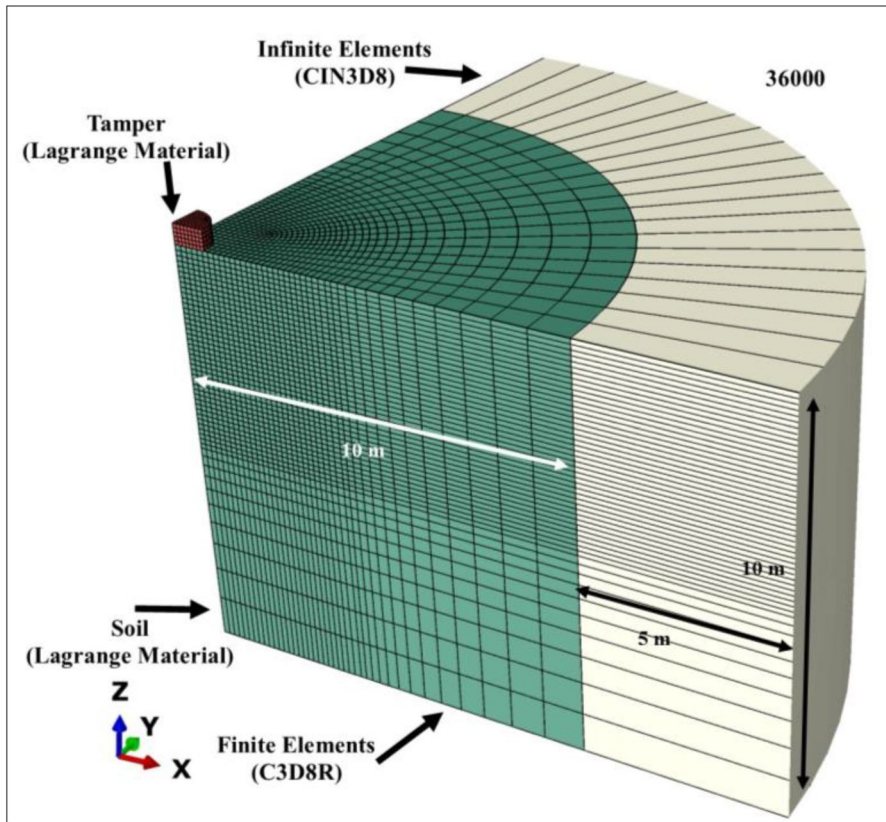


Fig. 8 Three-dimensional finite element mesh for the Lagrangian model

were applied to the model. The ratio of the maximum horizontal stress to the vertical stress, K_0 , was considered equal to 0.42. Then, the Mohr-Coulomb failure criterion was specified to simulate the soil mechanical behavior under the loading conditions. The data summarized in Table 2 were assigned to the soil elements. Afterward, the gravitational acceleration was applied to the model to create a state of equilibrium in the mesh.

In the generated model, semi-infinite boundaries were used to reduce the effect of waves returning from the boundaries. In addition, because there is an axial symmetry in the geometry of the problem, the boundaries were considered as cylinders to reduce the amount of elements, thereby decreasing the problem solving time. The size (horizontal radius and vertical height) and of the elements at the collision interface were set equal to 0.1 m. On the other hand, for the far elements from the collision interface, larger dimensions were applied. Based on Eq. 1, an initial velocity of 17.2 m/s was given to the tamper at the impact moment. The geometry of the tamper was based on Table 1, and it transferred a kinetic energy equivalent to 2000 kN.m to the soil surface.

In this research, only the numerical results of the first impact are compared with the results obtained from the field data. Since the loading conditions, boundary conditions, and soil constitutive behavior was equal for all meshes, only the first blow was considered as an indicator to show the accuracy of the different numerical approaches. In better words, the response of the soil under the first blow is relatively adequate to judge the accuracy of the results calculated by different numerical approaches. In the field, the value of soil vertical deformation under the center of the tamper was measured as 31.1 cm. After the creation of the numerical model and specifying the required data, the model was run to solve the relevant dynamic

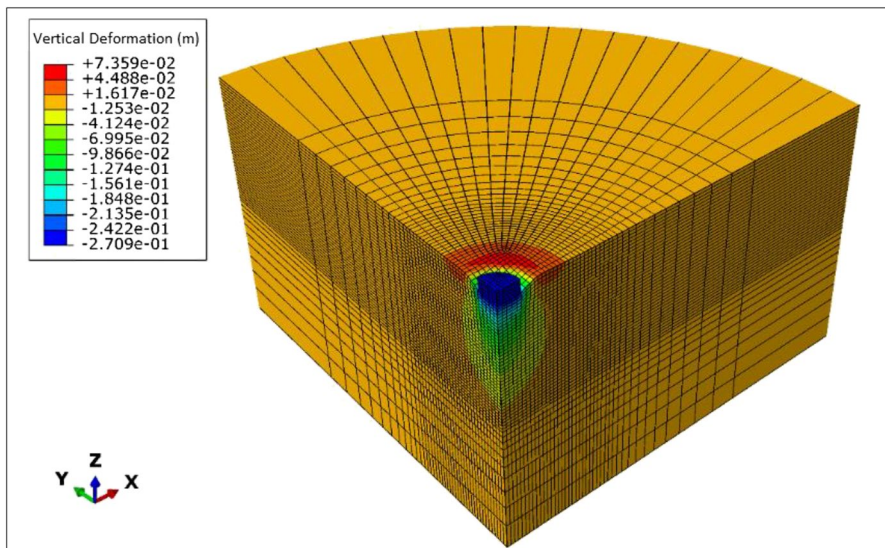


Fig. 9 Contour of soil vertical deformations simulated via Lagrangian approach

compaction problem. Figure 9 shows the vertical displacements for such a Lagrangian model. As it can be seen, the maximum vertical deformation was obtained equal to 27.09 cm while the real field counterpart was 31.1 cm. Hence, it can be observed that the difference between the numerical result and the real data is about 4 cm, which is significant for the typical road embankments. It is again mentioned that such vertical deformation belonged to the soil particles exactly under the tamper center (the soil particles which were situated exactly at the contact of the tamper center and the underlying soil).

4.2 ALE Approach

In this model, the mesh properties are the same as the Lagrangian method; in other words, the mesh, boundaries, and dimensions of the model are the same as those used in the Lagrangian one. The difference is that only the elements which undergo the large deformations were modeled based on the ALE formulation to perform the re-meshing process three times in each increment (Fig. 10); in better words, in each increment that the calculations were performed, the mesh was updated three times so that the large deformations of the elements did not cause inaccuracy in the results. Figure 11 shows the results obtained from this technique. As it can be observed, the maximum vertical displacement has been

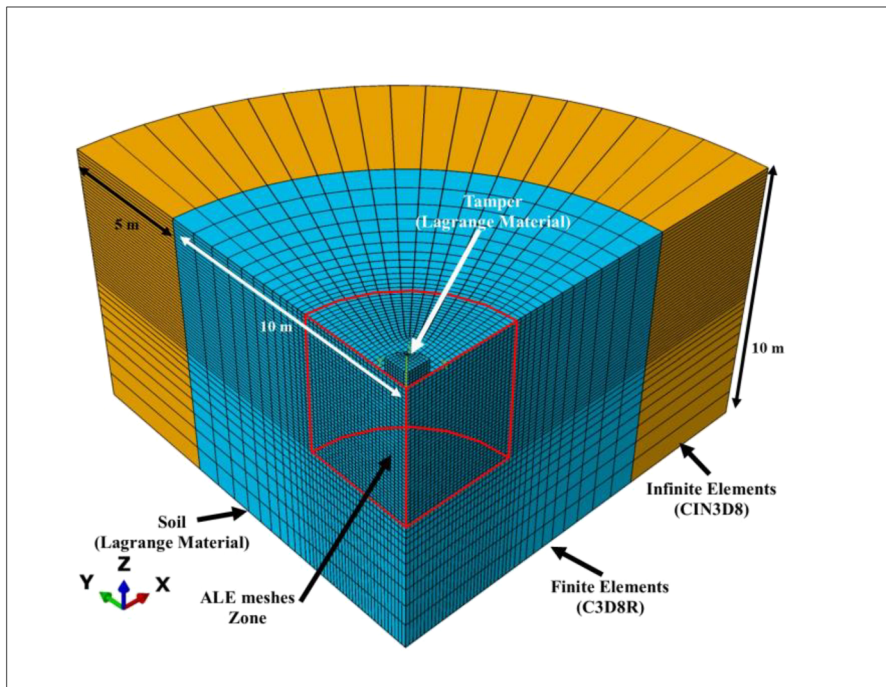


Fig. 10 Three-dimensional finite element mesh for ALE model

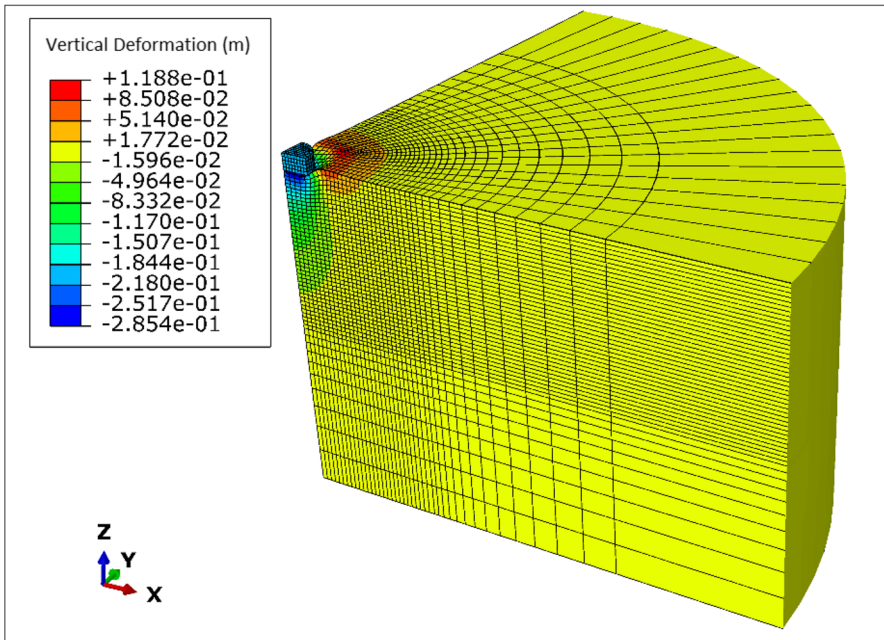


Fig. 11 Contour of soil vertical deformation for ALE model

calculated as 28.54 cm while that of the real field data was 31.1 cm. Therefore, the discrepancy between the numerical result and the real field data is obtained as 2.56 cm which is still significant. It is noteworthy that the ALE approach can be deployed in both two-dimensional and three-dimensional conditions.

4.3 CEL Approach

The theory of this model has already been explained in section 2.4. The geometry of this model has been shown in Fig. 12. The dimensions and the type of elements have also been illustrated in the figure. The mesh contained 36,300 elements. Moreover, the size of the elements increased incrementally from 0.1 m (for the central elements) to 1 m (for the outside elements). The relevant Lagrangian explicit steps were performed, and the Eulerian conditions were taken into account so that soil large deformations could be simulated without error and inaccuracy. It is worth mentioning that the CEL approach can only be used for 3D modeling not 2D modeling. Figure 13 shows the results obtained from this approach. According to this figure, the maximum vertical deformation was computed as 30.51 cm. Thus, the discrepancy between the obtained numerical result and the real field data is about 0.59 cm which is relatively acceptable. Therefore, the CEL approach predicted the vertical deformation of the soil much closer than the Lagrangian and ALE approaches.

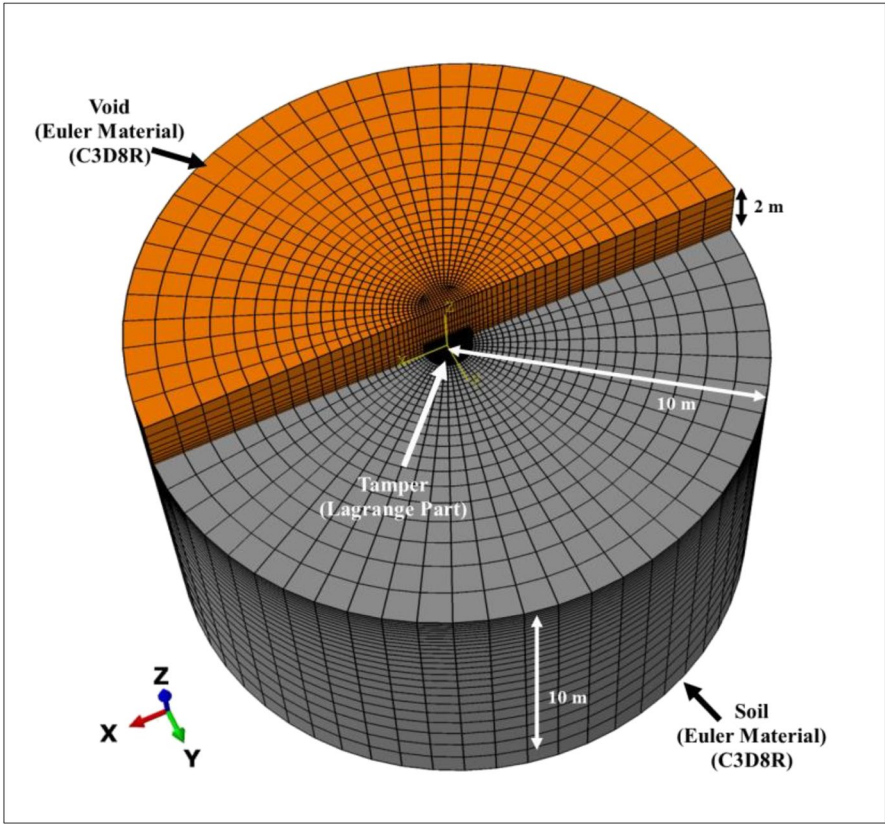


Fig. 12 Three-dimensional finite element mesh for CEL model

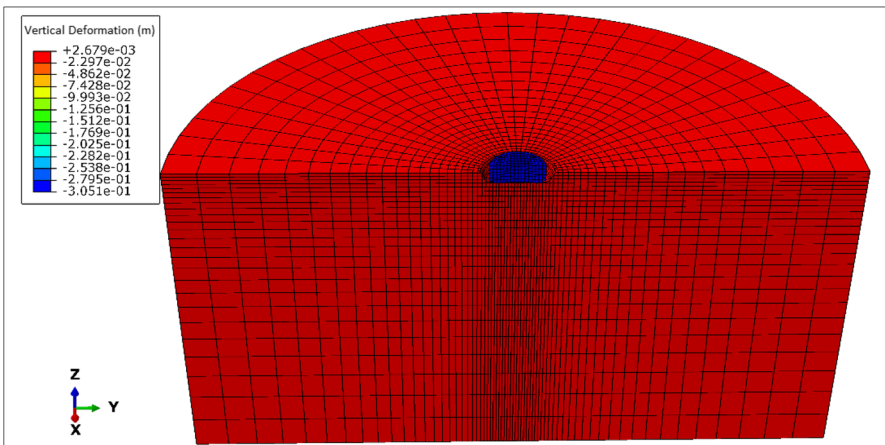


Fig. 13 Contour of soil vertical deformation for CEL model

4.4 SPH Approach

This approach is basically a mesh-less formulation. The modeling method is the same as mentioned for the Lagrangian approach; the only difference between the two formulations is choosing the type of the soil elements. This feature has been seen in ABAQUS software, and there is no need to change the model INP codes. However, the tamper is still considered as a Lagrangian body because it has a high stiffness compared to the soil, and experiences very negligible deformations. Furthermore, the soil was modeled as smooth hydrodynamic particles so that the large deformations proceeded without an “excessive mesh convergence (distortion)” error. The geometry of this model has been shown in Fig. 14. Moreover, the dimensions and the type of elements have been demonstrated in the figure. The mesh is composed of 36,800 elements with a constant size equal to 0.1 m. Due to the hardware limitations, and the fact that this approach has a much longer running time than other methods, at first, the mesh generation process was performed using smaller dimensions to approximate the required running time. Then,

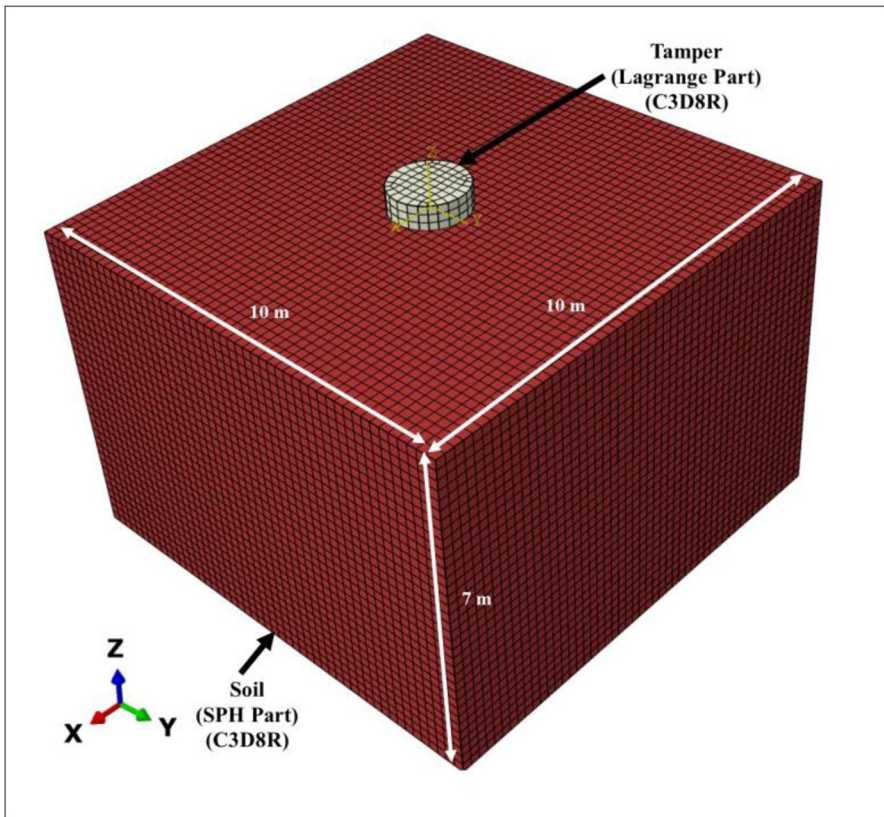


Fig. 14 Three-dimensional finite element mesh for SPH model

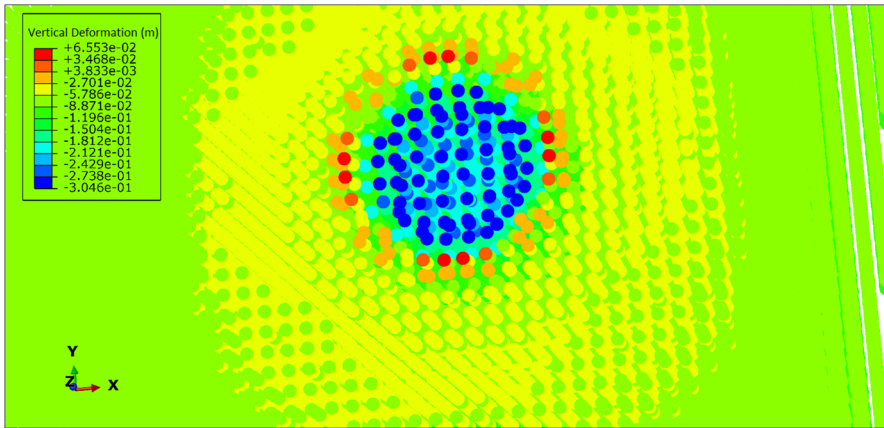


Fig. 15 Contour of soil vertical deformation for SPH model

the exact dimensions were applied to the mesh, and therefore, the main mesh was created.

Figure 15 shows the results obtained from this approach. As it can be observed, the maximum value of vertical deformation was calculated equal to 30.46 cm. Therefore, the difference between the numerical result and the real field data is 0.64 cm which is almost close to the CEL approach's estimation. Similar to the previous approaches, the maximum vertical deformation pertained to the soil particles directly under the tamper center. This approach greatly increases the solving time while it is much more suitable for the large and sudden deformations. In addition, this technique can only be used in three-dimensional modeling.

4.5 Hybrid Lagrangian–SPH Approach

This model was an innovative approach to capture the advantages of both lagrangian and SPH formulations. The main advantage of the Lagrangian approach is its fast running time. In addition, the extraordinary characteristic of the SPH formulation is its high accuracy in computation of the precious soil deformations. To reduce the computation time, only the part of the soil that underwent the large deformations (according to the results of the previous models) was modeled as the SPH particles, and the far parts were modeled as the Lagrangian elements. Moreover, semi-infinite boundaries were considered to increase the quality of the results. Figure 16 illustrates the pertinent mesh geometry. The exact dimensions of the mesh together with the type of the elements have been also illustrated in the figure. Moreover, the generated mesh contained a number of 36,900 elements. The size of the elements increased incrementally from 0.1 m (for the central elements) to 1 m (for the outside elements).

After the creation of the geometry, and assigning the required data, the model was run, and the results were recorded. Figure 17 shows the results obtained from this approach. Based on this figure, it can be expressed that the maximum vertical

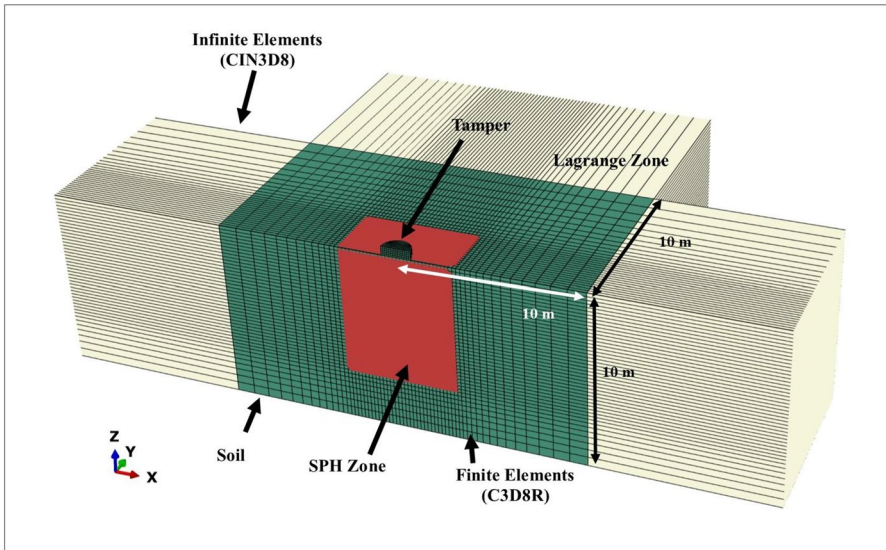


Fig. 16 Three-dimensional finite element mesh for hybrid model

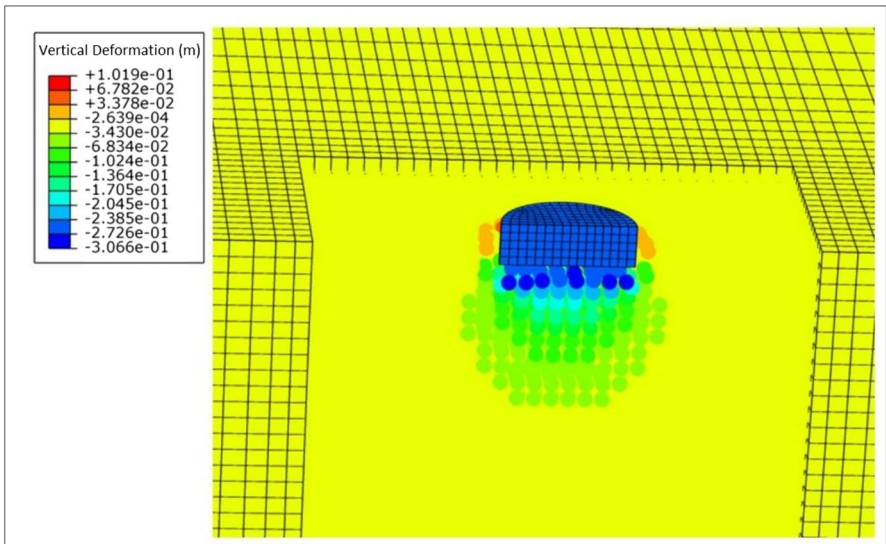


Fig. 17 Contour of soil vertical deformation via the hybrid model

deformation just under the tamper center is equal to 30.66 cm which is 0.44 cm less than the real field data. From this result, it was deduced that the hybrid Lagrangian-SPH approach is an accurate and reliable formulation for the computation of soil large deformations. In fact, the results of this model are closer to the real field data,

and on a more positive note, the problem running time is less than the case in which the whole soil was modeled as the SPH particles. Therefore, it can be concluded that this approach has the potential to be used as a suitable solution for modeling the large deformations and high-velocity collisions. By using this hybrid model, the imperative necessity for computers with high CPUs can be mitigated to a great extent. It is worth mentioning that this approach can be used only in 3D modeling.

5 Results and Discussion

The maximum vertical deformation obtained from the different numerical approaches has been illustrated in Table 3. It should be noted that the maximum vertical deformation belongs to the soil particle which was situated exactly under the tamper center (the soil particle at the contact of the tamper center and the soil). As it can be seen, the maximum and minimum accuracy belong to the hybrid and Lagrangian approaches, respectively. Furthermore, the results obtained from the CEL, SPH, and hybrid approaches are very close to the real field data. In addition, the numerical results obtained by the Lagrangian and ALE approaches are close together.

In Table 3, the row of “error percentage” represents the discrepancy percentage between the maximum vertical deformation obtained from the real field and each numerical formulation. Two fundamental formulations, i.e., the purely Lagrangian and Eulerian approaches, are not adequately computation-efficient since they predicted the soil large deformations with a noticeable error percentage equal to 13%. This is why other researchers in different areas strived to incorporate those two formulations together, or with other approaches to increase the accuracy of the computed results (Evans et al., 2016). In this research, as it was expected, such combined approaches, i.e., ALE, CEL, and SPH formulations enhanced the precision of the numerical results. Therefore, the findings of this research are in concurrence with the previous investigations made by (Bojanowski & Kulak, 2010; Trajkovski, 2017; Seetamsetti, 2012).

The Lagrangian approach predicted less accurate vertical deformations than others. The reason pertains to the limitation of the mesh distortion in the Lagrangian formulation. By contrast, the ALE approach delivered better predictions than the lagrangian approach although the meshes were quite the same. The reason is that the ALE formulation was assigned to the central elements which underwent large deformations. The rest of the mesh contained the Lagrangian-based elements. Since

Table 3 Comparison between the maximum vertical displacements predicted by different approaches vs. real data

Records	Unit	Lagrangian	ALE	CEL	SPH	Hybrid	Real
Maximum vertical deformation	cm	27.1	28.55	30.51	30.46	30.66	31.1
Error percentage	%	13	8	2	2	1.4	-
Running time	minutes	80	110	128	180	95	-

during the running process, the re-meshing operation was performed three times in each increment, such larger deformations were allowed to expand without mesh distortion problem occurrence. In fact, the nodes of the ALE-based elements were dependent from the motion of the simulated soil.

The accuracy of the CEL approach is more significant than the lagrangian and ALE formulations. The reason is that the entire soil elements were modelled as Eulerian elements. This advantage allowed the soil to be deformed without the mesh distortion problem. As it was mentioned in section 2.4, such merit comes from the Eulerian volume fraction (EVF) function defined in the CEL formulation.

The SPH approach also has predicted very close values of soil vertical deformation. In contrast to the Lagrangian, ALE, and CEL formulations, the soil elements were defined by the SPH particles which were capable of performing and recording the soil large deformations without the mesh distortion problem. This exceptional characteristic has led to the utilization of SPH formulation in complex impact problems (Araei et al., 2012).

Concerning the hybrid model, its high accuracy derives from the utilization of the SPH formation for the soil elements which underwent the large deformations. The outer part of the mesh was modelled by Lagrangian formulation, thereby leading to a less running time than the purely SPH formulation.

In Table 3, the duration of running time has also been mentioned for each numerical approach. The minimum running time belongs to the lagrangian approach while the maximum running time belongs to the SPH model. As it can be seen, the running time of the hybrid model is far less than the purely SPH model. Hence, it can be expressed that the accuracy and time efficiency of the hybrid model makes it very potent tool to calculate the soil large deformations during the DC operations. The running time of the ALE and CEL approaches lies in the time domain required for hybrid and SPH formulations.

Figure 18 demonstrates also the maximum vertical deformation of the soil particles under and around the tamper. The green dashed line represents the maximum vertical deformation measured in the field. The red dashed line represents

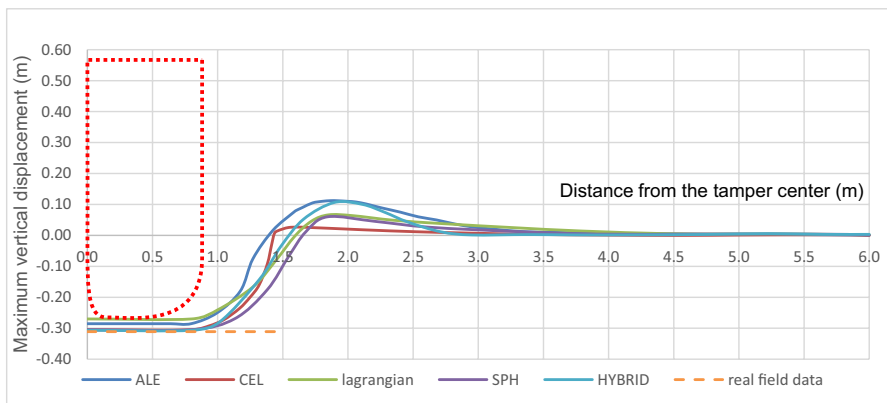


Fig. 18 Maximum vertical displacement for all numerical approaches

the location of the half of the tamper in the real field test. The height of the tamper is 0.84 m, and the radius of the tamper is equal to 0.8 m. Apart from the comparison between the five numerical approaches, some other conclusions can be deduced for application in future numerical simulations. Firstly, it can be seen that due to the tamping vibration effect, a wide area around the tamper experiences a heave of the soil particles. This area spans between 1.6 and 4.5 m far from the tamper center. Therefore, the heave phenomenon starts at a distance equal to $2r$ (where r represents the tamper radius). Furthermore, the tamping process creates a “heave” zone with radius 5 times as the tamper radius. This key point must be regarded specially in mesh generation during numerical modeling. Very large or very small mesh dimensions will negatively influence the time-efficiency and accuracy of the results, respectively. In this study, the least radial dimension of the cylindrical meshes was equal to 6 m which is consequently acceptable.

In this research, all input data were consistent for the tamper and the soil. In future works, the effect of tamper shape (Mehdipour & Hamidi, 2017), soil failure criterion (Pourjenabi et al., 2013), tamping (loading) rate (Bojanowski & Kulak, 2010), temperature (Rajaoalison et al., 2019; Knez & Rajaoalison, 2021), grain size (Khalilidermani et al., 2021), etc. on the accuracy of results computed by different numerical approaches can also be investigated. This may seem to be a null hypothesis although there must be sufficient evidence to reject this claim. To do this, as well as the numerical approaches, other probabilistic techniques, e.g., Monte Carlo simulation, can be adopted to recognize the effect of each parameter on the DC results (Quosay et al., 2020; Quosay & Knez, 2016). Using these techniques, the geotechnical engineer may predict the penetration depth of the tamper even via the number of tamper drops and the machine-specific energy (Khalilidermani & Knez, 2022).

6 Conclusions

For impact problems dealing with the large deformations, an appropriate numerical approach must be selected to achieve the reliable results. In this research, five different numerical approaches including Lagrangian, Arbitrary Lagrangian-Eulerian (ALE), Coupled Eulerian-Lagrangian (CEL), Smoothed particles hydrodynamic (SPH), and a hybrid model of Lagrangian-SPH were weighed up in terms of the computation accuracy and solving time efficiency. For this purpose, those approaches were adopted to simulate the large soil deformations during the dynamic compaction process in a high embankment located in the Ping-Zang Expressway in Hebei Province, China. The numerical results of those five approaches were validated with the real field data recorded during the dynamic compaction operation in the project. Based on the conducted research, the following results were concluded:

1. The Lagrangian approach predicts less vertical displacement than other approaches. The reason pertains to the limitation of the mesh distortion of Lagrangian formulation. Nevertheless, the Lagrangian approach required the lowest running time although its results are not as accurate as other approaches. The measured discrepancy between the maximum vertical displacement obtained from the

- Lagrangian approach and the real field data was equal to 13%. In general, this method seems not to be suitable for high-speed collisions and large deformations.
2. The ALE approach is more accurate than the Lagrangian approach. However, the solution time was found to be several times longer than the Lagrangian counterpart.
 3. The purely SPH approach took much longer running time than other approaches. In fact, computers with high CPU are required for numerical modeling of complex impact problems via the SPH approach. However, it delivers very close results to the real field data.
 4. The CEL approach presents relatively accurate results, and the solution time is also suitable. The solution time lies between the solution times of the SPH and ALE approaches.
 5. The approaches of CEL, SPH and hybrid give close results to the real field data. Furthermore, of all the different approaches, the hybrid model has the maximum accuracy together with the minimum solution time.
 6. For this current dynamic compaction problem, with increase of the collision speed, the Lagrangian approach was not capable of solving the problem, and the ALE approach had a very long computation time. However, the other three approaches suitably simulated the large deformations, and predicted the results close to the real field data.
 7. In simulating the high-velocity collisions or explosion problems, the priority is with the SPH approach, hybrid, and then CEL approach. In low-velocity collisions and relatively low displacements, the Lagrangian approach gives good results with the shortest time of the running process.
 8. In simulating the dynamic soil compaction operation, it is recommended to adopt three approaches of CEL, SPH, and hybrid of Lagrangian-SPH for this purpose.
 9. Moreover, from the different numerical simulations conducted in this research, it was deduced that the tamping vibration creates a heave zone with the length equal to five times as the tamper radius. This upper bound must be regarded when the geometry of the numerical mesh is created for the DC problems. In addition, such an upper bound can also be utilized in determining the influence of tamping on the adjacent structures such as buildings in urban areas (Pourjenabi & Hamidi, 2015).

In this study, only the impact of the first blow was numerically modelled. In addition, the crater depth was used as a comparative index to evaluate the accuracy of each numerical approach. For the future works, it is recommended to consider the multiple number of tamper blows along with the measurement of improvement depth to further assess the effectiveness of the applied numerical approaches.

The results gained through this research are quite applicable to other engineering phenomena dealing with the severe collision events. Such areas can be crashworthiness of the cars, aircrafts, blasting problems, meteoroid impacts, earthquake issues, and different dynamic analyses.

Availability of Data and Material The datasets analyzed during the current study are available from the corresponding author on reasonable request.

Author Contribution All authors contributed to the drafting, editing, and preparation of this manuscript through the following: Conceptualization: Rashid Hajivand Dastgerdi; data curation: Rashid Hajivand Dastgerdi; writing—original draft preparation: Rashid Hajivand Dastgerdi and Mohammad Ahmad Mahmoudi Zamani; writing—review and editing: Rashid Hajivand Dastgerdi and Mohammad Ahmad Mahmoudi Zamani; visualization: Rashid Hajivand Dastgerdi, Mohammad Ahmad Mahmoudi Zamani, and Dariusz Knez; methodology: Rashid Hajivand Dastgerdi and Mohammad Ahmad Mahmoudi Zamani; formal analysis: Kamran Kazemi and Dariusz Knez; software: Rashid Hajivand Dastgerdi; investigation: Kamran Kazemi; resources: Kamran Kazemi; validation: Dariusz Knez; supervision: Dariusz Knez; and project administration: Dariusz Knez. Furthermore, all authors have read and approved the submitted version of the manuscript.

Declarations

Ethics Approval and Consent to Participate Not applicable

Consent for Publication Not applicable

Competing Interests The authors declare no competing interests.

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Authors and Affiliations

Rashid Dastgerdi Hajivand¹  · Mohammad Ahmad Mahmoudi Zamani²  ·
Dariusz Knez³  · Kamran Kazemi⁴ 

Rashid Dastgerdi Hajivand
dastgerd@agh.edu.pl

Dariusz Knez
knez@agh.edu.pl

Kamran Kazemi
kamrankazemi2626@gmail.com

- ¹ Faculty of Civil Engineering and Resource Management, AGH University of Science and Technology, 30-059 Krakow, Poland
- ² Department of Drilling and Geoengineering, AGH University of Science and Technology, 30-059 Krakow, Poland
- ³ Department of Drilling and Geoengineering, AGH University of Science and Technology, 30-059 Krakow, Poland
- ⁴ Department of Civil and Environmental Engineering, Shiraz University of Technology, Shiraz, Iran