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TECHNICAL PAPER

Limit Analysis Optimization of Design Factors for Mechanically Stabilized Earth Wall-Supported Footings

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Abstract Increasingly in the past few decades, mechanically stabilized earth (MSE) walls have been a common choice for earth retention solutions due to their ease of construction, cost-effective design, and flexibility. However, limit state capacity of footings atop a MSE structure, such as a bridge abutment, is a complex and non-intuitive stability problem dependent on factors like reinforcement layout and strength, location of footing, and failure mechanism. In order to evaluate the effects of these factors, a parametric study was performed utilizing a robust tool in the framework of limit analysis of plasticity. It demonstrates the effect of the parameters on bearing capacity and optimized reinforcement efficiency, all in context of an existing MSE wall-supported bridge abutment. It is shown that closely spaced reinforcement layouts allow for more efficient reinforcement behavior and improved bearing capacity nearer to the wall facing, while inverse, detrimental trends were observed for wider reinforcement spacing. The implications of this analysis illustrate that an optimized approach involving footing location, reinforcement strength, and reinforcement spacing allow for reduced bridge deck length requirements. This clearly has economic consequences. The optimization of these design factors is especially applicable to MSE wall-supported bridge abutments since the location of the footing affects expensive elements such as length of girders. Fundamentally, this work adds to the limited existing knowledge of bearing capacity of footings on top of MSE walls.

Keywords Bridge abutment · MSE walls · Spread footings

Background

Mechanically stabilized earth (MSE) walls are an increasingly common means of earth retention for a variety of applications. Their function is simple and efficient—use

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tensile reinforcements and a facing to allow for soil to retain itself as well as surcharge loads. This coupled stability problem, combined with an assumed stress distribution based on earth pressure principles, allows for a simplified design methodology to evaluate the reinforcement tensile strength needed to produce an adequately stable structure. The lateral earth pressure approach has been refined to include a variety of geometries, but little insight exists into the stability and design requirements for MSE structures supporting a footing—an increasingly important function, especially in context of true bridge abutments, structures that support the deck via spread footings.

Literature on the behavior of footings atop of MSE structures is limited to primarily instrumentation and field testing observing the service state, not limit state, performance of MSE wall-supported footings. Wu et al. [1] conducted field testing of surcharge preloading of MSE walls, determining that deformation can be reduced in soil and reinforcement due to the application of a surcharge prior to service. Similarly, Tatsuoka et al. [2] found that deformations of a footing on reinforced soil structures could be greatly reduced with preloading and subsequent tensioning of reinforcements. Helwany et al. [3] performed a finite element analysis of MSE wall abutments and evaluated the deformational behavior of the wall under various surcharges, while evaluating effects of varying reinforcement spacing, stiffness, and backfill soil. Yoo et al. [4] observed the behavior of two-tiered MSE wall under surcharge loading and modeled its performance using three-dimensional analysis. Gotteland et al. [5] compared loading tests with numerical modeling and described failure behavior of abutments under surcharge loading. However, there is little insight into the coupled behavior of footing location, reinforcement spacing, and reinforcement strength on the bearing capacity and global stability of MSE wall abutment structures.

Construction of retaining structures for bridge abutments serves a relevant economical function—shortening the required span of a bridge deck, an expensive component of bridge construction. Retaining walls in lieu of sloping abutments have been used to shorten this span. However, significant economic impact has been achieved by using MSE walls. The cost savings attained from reducing a bridge deck span, even slightly, combined with spread footings can be significant, thus justifying the proper utilization of MSE wall construction, a design objective that has been demonstrated for years. However, it is important to evaluate the true strength limit state of these MSE wall-supported spread footings and their associated collapse mechanisms.

Conventional design of spread footings is dictated by two uncoupled design criteria. One is tolerable settlement (i.e., serviceability) and second is adequate margin of safety against failure (i.e., strength limit state). Each of the two design aspects is commonly assessed using different and unrelated types of analyses. Most of the work reported in the literature addresses the serviceability issue of spread footings over MSE walls. While this performance criterion is important, it does not necessarily imply the existing margin of safety against collapse. This aspect is especially important in MSE walls as stability is hinging upon the long-term strength of the reinforcement. That is, rupture of one reinforcement layer may lead to domino effect where other layers are progressively ruptured, resulting in a catastrophic failure. Equally important is that bearing capacity assessment complements the serviceability check where each approach utilizes a different design methodology. Hence, it provides an implicit indication that there is no major error in settlement assessment. However, evaluating the bearing capacity and global stability of the reinforced soil composite that serves as the foundation for a bridge footing is based on a complex mechanism, dependent on location of the footing, footing loads (live and dead), reinforcement spacing, tensile strength of the reinforcement, facing type, and fill material properties. Additionally, assumed failure mechanisms for such analyses are not straightforward, as failure may manifest as a soil shear surface with tensile rupture of all layers of reinforcements, pullout failure of reinforcements under low confinement, connection failure at the facing, or a combination of all. The complexity and non-intuitive nature of this failure mechanism requires a stability analysis that does not a priori assume a slip surface geometry (i.e., circular, log-spiral, planar surface, etc.) or reinforcement stress distribution at the critical limit state. In order to meet these requirements, a parametric study was performed using robust numerical simulations based on computational limit analysis founded on plasticity principles. This study discusses the effects of footing location and reinforcement design, all in context of impacting the required bridge deck length and/or reinforcement quantities.

Numerical Model

The numerical model geometry is based on the actual design specifications of a wall constructed in Colorado [6] as a true bridge abutment (see Fig. 1). The geometry of this structure includes two distinctive reinforced sections. One consists of a primary reinforcing structure containing 15 layers of type 1 geogrid (see Table 1), surcharged by a footing. The footing supports the live and dead loads of a bridge deck while also retaining the second reinforced soil section behind it with four reinforcements—three type 2 geogrids and one type 3 geogrid (Table 1). Geogrid lengths increased linearly from the facing with increased wall height (Fig. 2), starting with a length of 7.8 m at the base and extending to nearly 11 m at the top. The primary MSE wall was 5.9 m in height. A footing 3.81 m wide and 2 m high supported a working load of 572 kN per unit length. Facing elements were modular concrete blocks with connectors for attaching reinforcements. Both the footing and the individual facing blocks were treated as rigid materials (no internal failure). The reinforced fill was cohesionless select material with unit weight of 20 kN/m^3 and default strength characteristics in accordance to American Association of State Highway and Transportation Officials (AASHTO) requirements, where $\phi'=34°$. The interface friction angle between the soil and reinforcements was modeled as $^2/_3\tan\phi=0.51$. Interface friction between the soil and the facing, or internal, interface friction between the facing blocks was ignored as to primarily focus on the limit state internal stability behavior of the reinforced soil mass and the footing surcharge. Connection of the reinforcement to the facing and friction in between facing blocks was very high to model the internal stability of the structure. The purpose for the high connection interaction was to simulate mechanical connection to the facing blocks, allowing the critical failure mechanism to be tensile rupture, either at the facing or within the backfill. In the case of very weak connections, a localized failure through the upper facing blocks would likely govern stability; however, this analysis assumed strong connections to examine the effects of reinforcement spacing on the internal stability of the entire reinforced mass. The soil in front of the MSE wall was removed from the model as passive earth pressures were ignored in analysis.

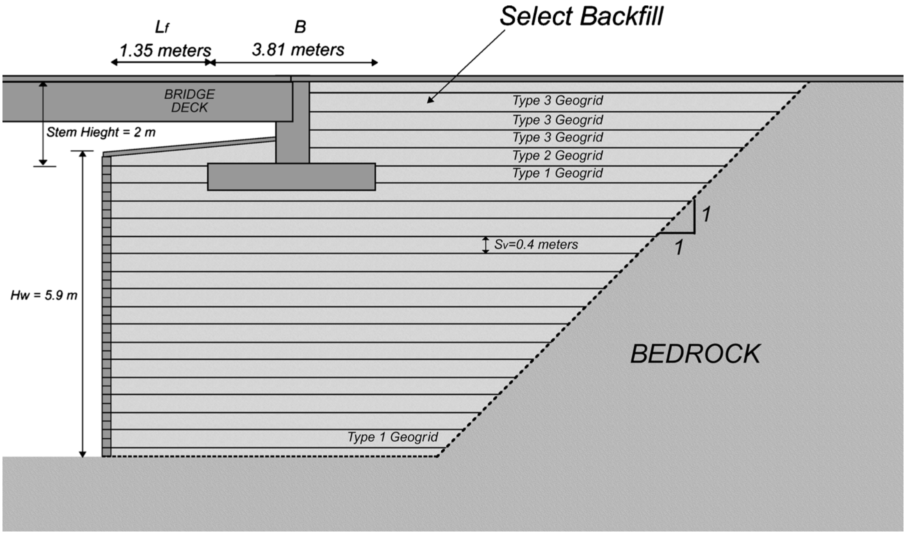

Fig. 1 Geometry of CDOT MSE wall (reproduced from Abu-Hejleh et al. [7])

In order to adequately model the stability of complex MSE structures without an assumed failure surface geometry, numerical modeling was used to determine the ultimate bearing capacity and critical failure mechanism based on limit analysis principles. The utilized limit analysis (LA) tool allows for the use plasticity to perform a robust investigation into the effects of soil shear strength and the reinforcement layout (i.e., spacing and length) on the required tensile strength of reinforcements necessary to avoid failure or limit state. LA modeling treats soil as a material that obeys an associated flow rule and is perfectly plastic [7]. The upper-bound plasticity LA methodology used in this study is governed by the principle that once the rate of work along an admissible failure surface due to external loads exceeds work done by internal stresses, collapse occurs [7, 8]. LA is used as a stability tool with commercially available software, LimitState:GEO [9], to study the necessary model parameters and how they affect stability. This program uses an algorithm called Discontinuity Layout Optimization (DLO) to discretize the geometry of the model for potential slip surfaces. This algorithm functions by assigning nodes, prescribed to a certain density as potential locations for a slip surface to propagate, that is, every slip surface analyzed using LA methodologies must pass through one or more of the specified nodal points [8], as shown in Fig. 3. The use of DLO utilizes linear programming optimization resulting in the failure mechanism without an a priori assumed failure geometry. Despite being an upper-bound limit analysis, the high level of discretization for potential discontinuities shrinks the gap between upper- and lower-bound solutions, facilitating results within

Table 1 Design properties of the reinforcements

Geogrid	Long-term design strength (kN/m)
Type 1 (red)	27
Type 2 (blue)	11
Type 3 (green)	6.8

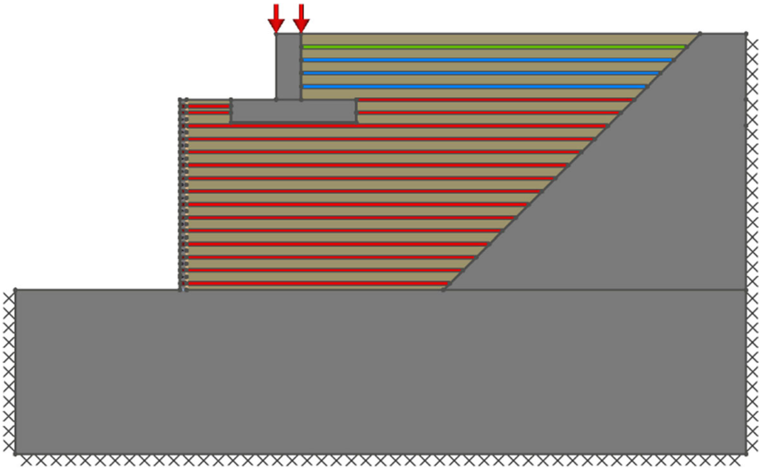

Fig. 2 Design geometry and numerical model of MSE wall with nodal discretization

range of lower-bound solutions. Prior work with LA has shown excellent agreement with rigorous limit equilibrium (LE) methods [10, 11] and robust analysis for MSE structures [12, 13]. Rigorous LA also satisfies equilibrium at a limit state; however, it is free of the assumptions used in LE analysis. Hence, it was deemed an acceptable tool to provide insight into the stability of MSE walls, a soil structure that is highly complex in geometry and behavior. However, since a critical failure surface must pass through a connection between two nodes or any combination of connections, the critical failure mechanism and its associated stability is affected by nodal density—increased nodes in a system lead to more accurate discretization of the solution. Parametric studies were performed to ensure that the strength attained from the analysis did not change significantly with added nodes. It was found that there was no noticeable change in stability when more than 3,800 nodes was used for the most demanding analyses, resulting in 4,000 nodes being used for the geometry of all of the models. An analysis that requires no assumed failure surface is necessary to define the stability of an inherently complex soil structure with a non-intuitive failure mechanism, such as MSE wall-abutment structure.

Three different reinforcement layouts were chosen to demonstrate the effects of vertical reinforcement spacing on bearing capacity: S_{v_dense}=0.2 m (dense), S_{v_design}= 0.4 m (actual design specification), and S_{v_wide}=0.6 m (wide), where S_v is the vertical spacing between each layer of geogrid (see Fig. 4). Within each reinforcement layout, placement of the footing was varied to demonstrate the effect of the bridge deck loading

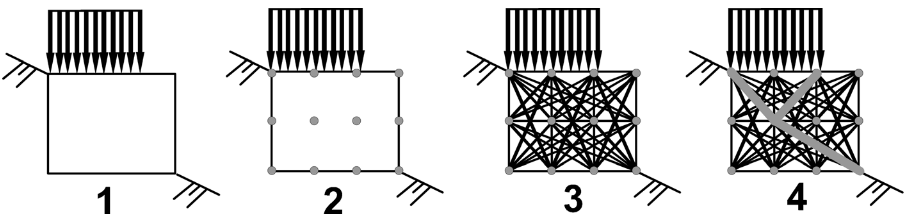

Fig. 3 Schematic of DLO algorithm

location on bearing capacity. This toe of the 3.8-m-wide footing was placed at L_f equal to 0.15, 0.3, 0.6, 1.0, 1.35 (actual design), 2.0, and 3.0 m from the facing to simulate the effects of footing location on the critical failure mechanism and bearing capacity—a concern that has a direct relationship to material costs due to bridge deck length requirements and required reinforcement quantities. Finally, the strength of the reinforcements was varied as a proportion of the design values (Table 1). Specifically, this ratio of model reinforcement strength to the actual design reinforcement strength for all reinforcement types, defined as the *tensile strength ratio* (TSR), was varied from 1 (design values) to 0 (no reinforcement strength). This parameter, applied proportionally to every reinforcement layer, was defined as

$$\mathrm{TSR} = \frac{\text{Model Reinforcement Strength}}{\text{Design Reinforcement Strength}} \tag{1}$$

The model was run by placing an increasing load on the footing, representative of the bridge deck load placed on the footing. Overall, the load on the footing also considered the load exerted by the retained soil on the stem of the footing; this load was an internal load independent of the vertical deck load. The vertical load, $Q_{\text{ult-model}}$ was increased successively until failure, providing an ultimate bearing capacity of the footing—a value that could be compared to the load required by the bridge deck design, Q_{design} (115 kPa), and defined the bearing capacity factor of safety as

$$\mathrm{FS_{BC}} = \frac{Q_{\text{ult-model}}}{Q_{\text{design}}} \tag{2}$$

Typically, the factor of safety for bearing capacity should exceed 3.0 for shallow foundations like strip footings. The factor of safety was evaluated for a variety of reinforcement spacing scenarios, reinforcement strengths, and footing locations. The effect of these parameters coupled with numerical models provides insight into the optimization of design criteria for MSE wall-supported bridge abutments to meet required design specifications.

Results

A series of approximately 170 numerical models was performed to evaluate the effects of reinforcement spacing, tensile strength, and footing location for the given MSE wall-supported bridge abutment. From these results, failure mechanism, bearing capacity, and factor of safety were recorded and compared for the varied parameters. It is shown that smaller reinforcement spacing can allow for shorter bridge deck requirements with

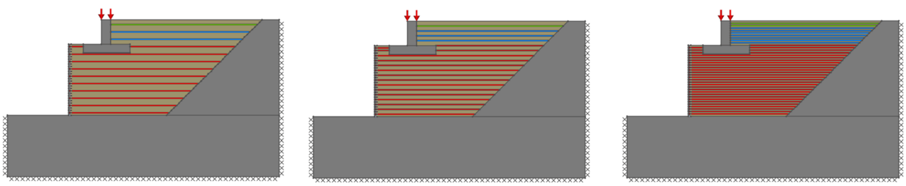

Fig. 4 Models of the MSE wall (*left*—S_{v_wide}; *middle*—S_{v_design}; *right*—S_{v_dense})

greater efficiency than that of wide spacing, potentially offsetting prohibitively expensive costs for bridge deck lengths.

Bearing Capacity and the Effects of Footing Location

The actual design and baseline geometry (S_v=0.4 m) was analyzed by varying tensile strength ratio for the reinforcements (15 type 1, long-term design strength [LTDS]=27 kN/m; 3 type 2, LTDS=11 kN/m; 1 type 3, LTDS=6.8 kN/m) and space between the footing's toe and the back of the wall facing. The model representative of the actual earth structure (TSR=1.0, L_f=1.35 m) was found to have a FS_{BC} of approximately 3.5, meeting the general design criteria for strip footings (greater than 3, see Fig. 5). It was observed that the reinforcement strength could be reduced by approximately 15 % and still meet the required bearing capacity design criteria of FS=3; note that this does not account for reduction factors like creep or construction damage—that is, the reinforcement design strength considered here is its long-term strength. The long-term tensile design strength of the reinforcements could be reduced by up to 64 % and still be critically stable (FS=1), demonstrative of a significant difference between expected loads and design loads at the limit state.

Location of the footing had a significant effect on the bearing capacity of the MSE wall-supported bridge abutment. When full LTDS was applied (TSR=1.0), FS_{BC} reduced to 2.6 when the footing was only 0.15 m from the facing (the AASHTO limit for L_f) and increased to 6.5 when placed 3 m from the facing. This demonstrates a reduction of wall location influence on the bearing capacity as the footing is set back further. Setback of the footing can result in savings in reinforcement material costs as the required TSR values to meet FS_{BC}=3 results in reinforcement strength reduction, TSR, of 0.69 and 0.43 for L_f of 2 and 3 m, respectively (i.e., weaker reinforcement is needed to produce adequate FS). This reduced reinforcement strength as L_f increases, of course, requires more bridge deck materials—a stipulation that may offset the potential savings in reinforcement strength, highlighting a need for optimization of design needs and costs for the abutment structure.

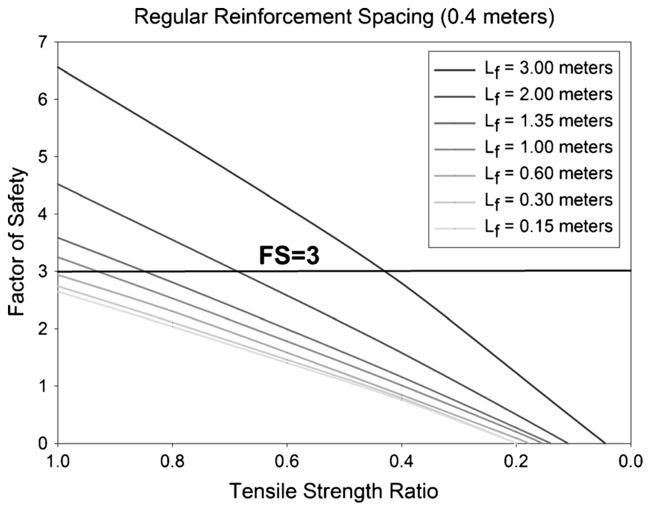

Fig. 5 Factor of safety for footing on MSE wall, S_v=0.4 m

In order to define the effects and potential benefits of using closely spaced reinforcements (i.e., $S_v \leq 0.2$ m), a parametric study, similar to the one for the baseline case of S_v= 0.40 m, was performed varying TSR and footing location to evaluate bearing capacity and the associated collapse state. This new spacing led to an increase from 19 to 38 reinforcement layers (30 type 1, LTDS=27 kN/m; 6 type 2, LTDS=11 kN/m; 2 type 3, LTDS= 6.8 kN/m), essentially doubling the number of each reinforcement type by adding a layer in between existing reinforcements. It was observed that use of closely spaced reinforcements resulted in significantly higher bearing capacities (see Fig. 6). For the baseline design geometry (TSR=1.0, L_f=1.35 m) and closely-spaced reinforcements, the bearing capacity of the footing increased by a factor of 3.3 from FS_{BC}=3.5 to 11.5. This is suggestive that the increased strength attained from doubling the amount of reinforcements is not a linear function. At 50 % of the design reinforcement strength for all of the reinforcement layers (TSR=0.5), the factor of safety for L_f=1.35 m dropped to 5.5, still 57 % larger than that in the baseline case (S_v=0.4 m, TSR=1.0, L_f=1.35 m), where the FS_{BC} was 3.5. For the baseline geometry, TSR values of 0.28 and 0.14 were required to attain a FS_{BC} of 3.0 and 1.0, respectively. For the actual design, TSR values of 0.82 and 0.35 were required for FS_{BC} of 3.0 and 1.0, respectively, indicating a significant reduction in required tensile strength for closely spaced reinforcements. Inclusion of closely spaced reinforcements also allowed for improved stability when the footing was located close to the facing. Consistent with current practice limits for footing setback, at L_f of 0.15 m, only 35 % of the original reinforcement design strength was required to maintain a FS_{BC} of 3.0. Conversely, larger footing setbacks led to increasing conservatism and significantly higher FS values, demonstrative of a local bearing capacity failure as opposed to a compound stability problem related to the MSE wall.

A final parametric study varying TSR and L_f was performed on the same baseline geometry, this time with 13 evenly spaced reinforcement layers (ten type 1, LTDS= 27 kN/m; two type 2, LTDS=11 kN/m; one type 3, LTDS=6.8 kN/m) signifying wide reinforcement spacing (S_v=0.6 m). This wider spacing had a notable, detrimental effect on the stability of the MSE wall-supported footing. When evaluating stability for the baseline scenario (TSR=1.0, L_f=1.35 m), the FS for bearing capacity reduces by

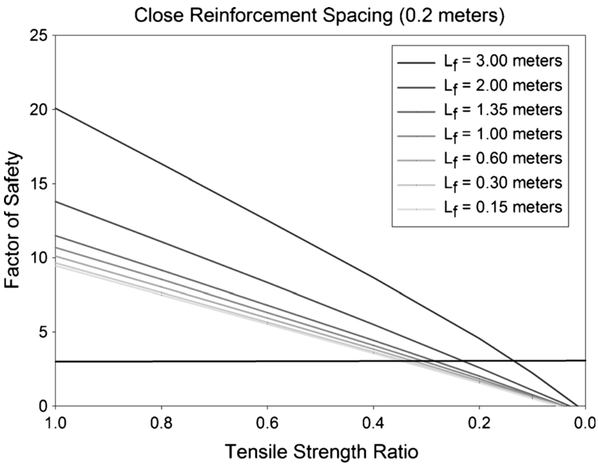

Fig. 6 Factor of safety for footing on MSE wall, S_v=0.2 m (close-spaced reinforcement)

almost 42 % to a factor of safety of 2.06 when compared to the actual design (see Fig. 7). Effectively, the setback of the footing must be greater than 2 m to attain a FS_{BC} greater than the 3.0 required for design. That is, when L_f=2 m and the reinforcement is full strength (TSR=1.0), the factor of safety is just 2.62. It demonstrates that increasing spacing by 50 % does not result in a linear change in structural stability. In fact, increasing the spacing reduced the reinforcing materials in the structure by only 32 % (13 reinforcements/19 reinforcements in the actual design), but the bearing capacity FS_{BC} decreased by 35–65 % for comparable TSR and L_f values. Specifically, the larger bearing capacity reductions occurred for weaker reinforcements and footings located close to the wall facing.

Slip Lines and Collapse Mechanisms

In addition to establishing factor of safety for a variety of footing locations and reinforcement tensile strengths, the slip lines representative of the critical collapse state and failure mechanism were observed. Such mechanism helps making the numerical results less abstract while possibly explaining peculiar behavior. Moreover, it enables one to judge the reasonableness of the results. While subjective judgment could be dangerous, seasoned designers may have "intuition" and experience allowing the associated mechanism to be instructive.

When the footing was close to the wall facing, the collapse state followed a compound punching, curved failure stemming from the heel of the footing to the toe of the MSE wall (see Fig. 8). When reinforcement TSR values were larger (i.e., stronger reinforcement), rupture occurred at the connection (Fig. 8a). When TSR values were lower (i.e., weaker reinforcement), a secondary, planar slip surface occurred at the top tier of reinforcements, behind the footing stem, in addition to the primary collapse mechanism through the toe of the wall (Fig. 8b, d). This manifestation of a curved primary slip surface below the footing and a planar shear plane above the footing was consistent for L_f values of up to 1.35 m. However, a slightly different failure mechanism occurred at higher L_f values coupled with higher TSR values (full or nearly full reinforcement strength). Specifically, a pullout mechanism occurred at the top few

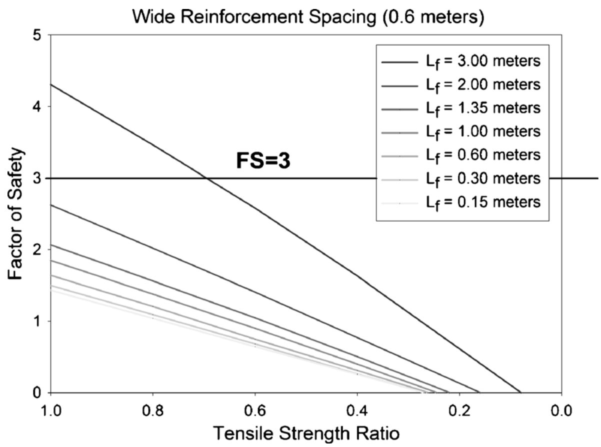

Fig. 7 Factor of safety for footing on MSE wall, S_v=0.6 m (wide-spaced reinforcement)

layers of reinforcement as their strength was high enough to resist rupture and the overburden low enough to allow for a quasi-pullout collapse mechanism of the top few layers in addition to the primary failure surface through the toe. This compound failure mechanism was only present for higher reinforcement tensile strengths and larger footing setbacks (greater than 2 m, Fig. 8c).

The critical failure mechanisms for the closely spaced reinforcement case followed a similar trend to that of the regular spacing (see Fig. 9a, b). That is, a curved failure surface occurred between the heel of the footing and the toe of the MSE wall when the footing was located closer to the facing of the wall. However, as L_f increased (i.e., further setback of the footing), a larger compound failure developed, predominantly occurring as a combined curved failure through the toe of the wall and a pullout of a large mass of soil (Fig. 9c, d) with the failure mass size dependent on the tensile capacity of the reinforcement (i.e., stronger reinforcements imply a larger failed soil zone).

When wide reinforcement spacing is applied, the slip lines of the critical collapse state of the wall scenarios followed a similar trend as shown before, manifesting a compound failure mechanism with a slip surface beneath the footing and pullout of the soil mass behind it when the footing exceeded a L_f of 1.35 m. The only notable difference is that a larger mass of soil seemed to mobilize when reinforcement strengths had lower TSR values. As seen before, this failure mechanism transformed into a simplified curved failure surface as the footing was located closer to the facing (see Fig. 10).

Practical Implications

Reinforcement Strength to Benefit Ratio

It was demonstrated that closely spaced reinforcements significantly increased the bearing capacity of the footing, allowing for shortened bridge deck length and reduced reinforcement tensile strength. It was shown that if the actual design

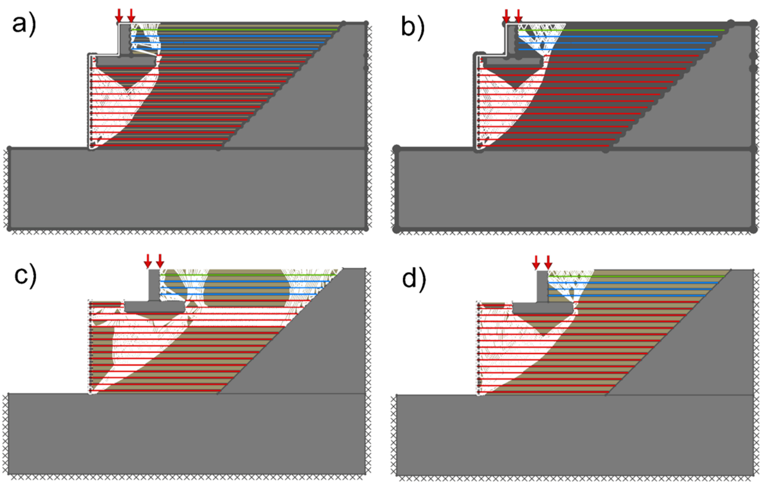

Fig. 8 **a** L_f=0.15 m; TSR=1.0 (type 1=27 kN/m; type 2=11 kN/m; type 3=6.8 kN/m). **b** L_f=0.15 m; TSR= 0.2 (type 1=5.4 kN/m; type 2=2.2 kN/m; type 3=1.36 kN/m). **c** L_f=2.0 m; TSR=1.0. **d** L_f=2.0 m; TSR=0.2

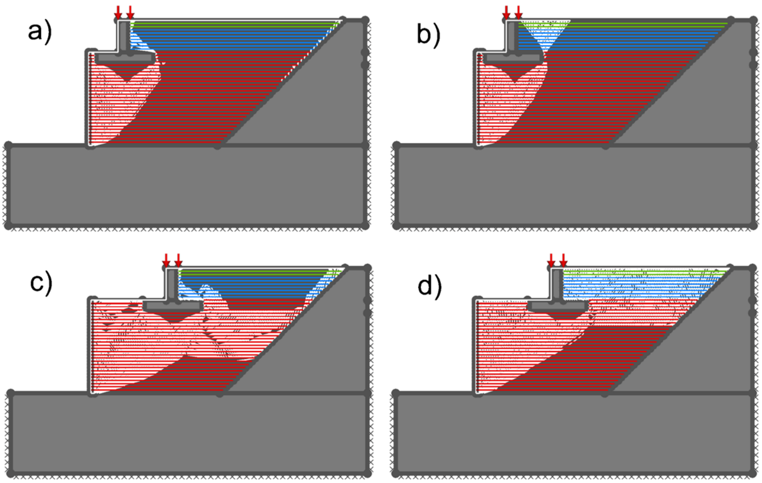

Fig. 9 **a** L_f=0.15 m; TSR=1.0. **b** L_f=0.15 m; TSR=0.1. **c** L_f=3.0 m; TSR=1.0. **d** L_f=3.0 m; TSR=0.1

of the MSE wall analyzed in this paper had utilized close reinforcement spacing (i.e., S_v=0.2 m), the bearing capacity would increase by 230 %, while the same case would only need 28 and 32 % of the reinforcement strength to maintain a FS_{BC} of 3.0 (required minimum bearing capacity FS) and 3.5 (original FS for actual design case), respectively. This is implicative of significantly reduced reinforcement strength. In order to define this relationship of material strength, reinforcement efficiency, and its benefits, the results are presented as ratio of the percentage change in bearing capacity compared to a percentage change in the total sum of reinforcement tensile strengths based on the baseline reinforcement spacing scenario (S_v=0.4 m). This relationship is called the reinforcement strength to benefit ratio (SBR) and defined as

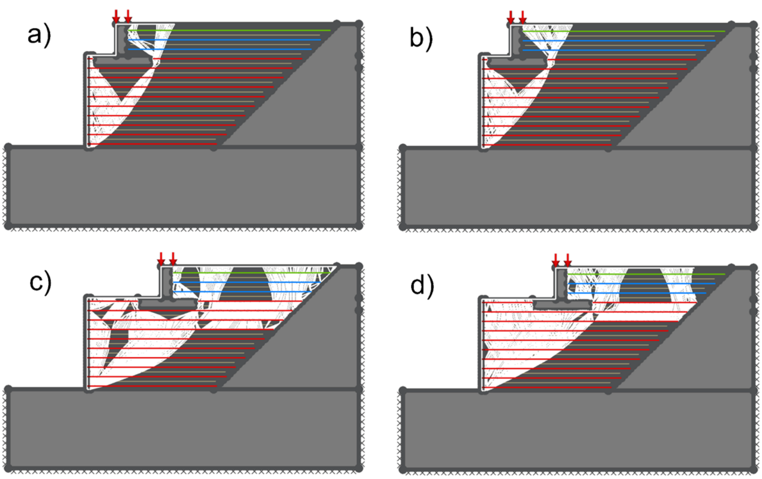

Fig. 10 **a** L_f=0.15 m; TSR=1.0. **b** L_f=0.15 m; TSR=0.4. **c** L_f=3.0 m; TSR=1.0. **d** L_f=3.0 m; TSR=0.4

$$\mathrm{SBR} = \frac{\text{Change in Bearing Capacity } (\%) + 100\%}{\text{Change in Total Sum of Reinforcement Tensile Strengths } (\%) + 100\%} \quad (3)$$

For example, for the baseline case (FS_{BC}=3.5 for TSR=1.0, L_f=1.35 m, S_v=0.4 m), a FS_{BC} of 3.5 is calculated. When the reinforcements are closely spaced for the same geometry and reinforcements, the reinforcement total-summed tensile strength is doubled, implying a 100 % increase in tensile capacity. Concurrently, the FS_{BC} increases by 206 %, so the SBR for this comparison is 1.53. Using the SBR relationship, it is seen that the bearing capacity increases at a higher rate than the reinforcement added, suggesting improved reinforcement efficiency by the SBR value greater than 1. When the SBR is less than 1, it is implicative of less efficient bearing capacity compared to summed reinforcement strengths (i.e., less benefit due to mobilization of reinforcement).

Reinforcement Strength Reduction Ratio

The inverse of this relationship (i.e., 1/SBR) directly represents how much reduction or increase in reinforcement tensile strength (and material if linearly related) is needed to support the same load as the baseline case. This relationship is called the strength reduction ratio (SRR) and is defined as

$$\mathrm{SRR} = \frac{\text{Change in Total Sum of Reinforcement Tensile Strengths } (\%) + 100\%}{\text{Change in Bearing Capacity } (\%) + 100\%} \quad (4)$$

For the same example, in order to maintain the same FS_{BC} for the baseline case (FS_{BC}=3.5 for TSR=1.0, L_f=1.35 m, S_v=0.4 m), the SRR becomes 0.65. This primarily means that only 32.5 % of the reinforcement strength is required. However, since the summed reinforcement strength is doubled for the closely spaced case, it implies that 65 % (32.5 %×2) of the reinforcement strength compared to the regular spacing scenario is required to maintain the equivalent factor of safety for the same footing location—a reduction of 35 % in summed reinforcement strength. A SRR value less than a unit value implies that reinforcement strength can be reduced to maintain a certain FS_{BC}. Inversely, a SRR value greater than a unit implies that stronger reinforcements are needed to maintain the same level of stability for bearing capacity.

Efficiency of Reinforcement Layout

The results of the parametric study imply a significant gain in efficiency for a closely spaced reinforcement layout for a variety of scenarios. That is, the SBR value increased significantly, representative of gain in bearing capacity that was larger in proportion than the added tensile strength of the now increased quantity of reinforcement layers (38). This was true for a range of TSR values, applied uniformly to reinforcement strength in the compared baseline and closely spaced cases. This trend is especially notable when the footing was closer to the facing of the wall, reaching a SBR of 2.35 (see Fig. 11a) when tensile strengths are lower. This is sensible as the effects of the footing on the stability of the wall facing decrease as its location is set back further, mobilizing less required tensile strength for stability and less reinforcement efficiency in maintaining stability, as demonstrated by SBR values of 1.55, 1.60, and 1.79 for L_f of

3.0, 1.35, and 0.15 m, respectively (Fig. 11a). This increased efficiency in reinforcement tensile strength compared to reinforcement spacing translates into potentially shortened bridge deck length requirements to maintain the same level of stability (FS_{BC}), as shown by the SRR (Fig. 11b). The SRR values range from 0.57 to 0.65 (35 to 43 % reduction in material strength) when using full reinforcement strength (TSR=1.0) but begin to demonstrate significant benefits at lower reinforcement strengths, with SRR values ranging from 0.54 to 0.42 for TSR=0.4 (Fig. 11b).

Converse to the results of the densely spaced reinforcement system, a loss in design efficiency was demonstrated by application of the widely spaced reinforcement layout (S_v=0.6 m). When comparing TSR, which affects the tensile strength of all reinforcements uniformly, SBR values decreased as reinforcement strengths decreased, demonstrating lower efficiencies as the footing location neared the wall facing. That is, the bearing capacity when compared to the baseline case reduced more proportionally than the sum of the fewer (13) reinforcement layers. This is intuitively a sensible result since the effect of reinforcements on wall stability becomes less important as the footing is located further from the wall, demonstrated by a SBR near 1.0 (same as baseline, control case) for a L_f of 3.0 within the spectrum of reinforcement strengths (see Fig. 12a). The SRR demonstrated an increase in total tensile strength needs to maintain the stability of each baseline case, reaching nearly double for footings near the wall with lower tensile strengths (Fig. 12b). This low efficiency design can be less economical than that of the densely spaced case. Specifically, comparing SBR of a footing placed 0.15 m from the facing with a TSR of 0.4, one can see that the gain in efficiency of reinforcement material quantities used is nearly fivefold (SBR_{dense}=2.35/SBR_{wide}= 0.5=4.7).

Discussion

Reinforcement Spacing

It was found that there were significant non-linear differences in efficiency for varying reinforcement spacing. This efficiency, described by factors SBR and SRR, demonstrated increased efficiency with denser spacing, suggestive of reduced distance between footing toe and wall facing (less bridge deck length). This comparison shows that

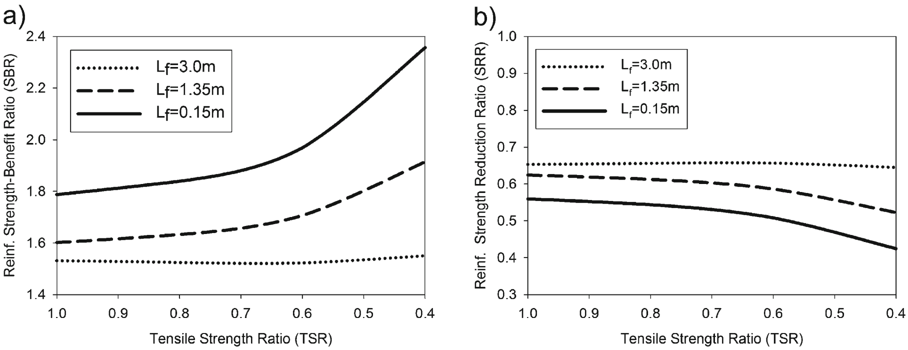

Fig. 11 Dense spacing: **a** SBR (*left*) and **b** SRR (*right*)

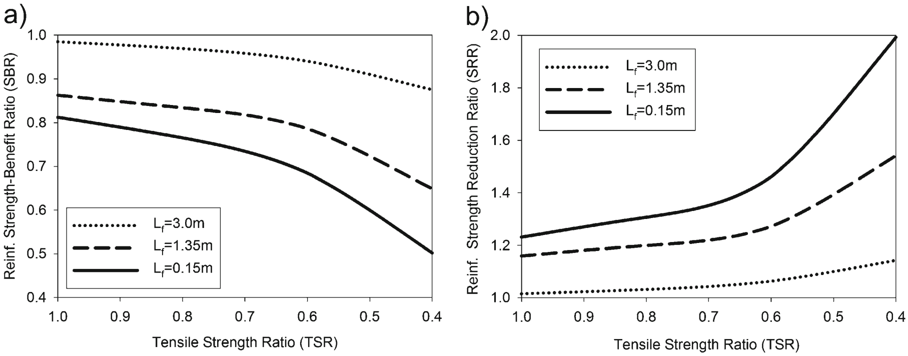

Fig. 12 Wide spacing: **a** SBR (*left*) and **b** SRR (*right*)

the reinforcement efficiency is steadier (less change) as the footing distance from the toe is larger, suggestive of less influence from the wall on bearing capacity. However, the influence of reinforcement strength on efficiency for footings placed close to the facing (L_f=0.15 m) is remarkable. For closely spaced reinforcements, the gain in bearing capacity compared to increased total reinforcement tensile strengths varies from SBR=1.8 to 2.35, demonstrative of gains of over 80 to 135 % in efficiency in comparison to the regularly spaced reinforcement layout (no change in efficiency is SBR=1, see Fig. 11). Inversely related to this significant gain in efficiency is the SRR, representative of the potential reduction in reinforcement material to maintain the same level of stability as the baseline scenario (S_v=0.4 m) under the assumption that reinforcement strength is directly related to reinforcement quantities. Based on this relationship, it was shown that reinforcement-summed total strength requirements for footings placed near the wall facing could be reduced by 44 to 58 %, even considering the increased number of reinforcements. This shows significant savings potential in reinforcement and bridge deck costs by increasing reinforced soil efficiency with smaller reinforcement spacing, implicative of better distribution of tensile strength to the wall backfill due to the denser inclusion of tensile reinforcement elements. For the actual design case geometry, use of closely spaced reinforcements would have been 60 % more efficient, implying that total reinforcement material costs could have been reduced by over 35 %. This is demonstrative of greater efficiency in soil-reinforcement composite behavior as the reinforcements are distributed more throughout the reinforced soil mass, ultimately distributing the geogrid tensile strength more evenly to the soil mass, which has no strength in tension.

Based on these results, it is intuitive that the use of wide reinforcement spacing can have a detrimental effect on the distribution of tensile strength to the reinforced soil mass. This reduced efficiency due to larger spacing (S_v=0.6 m) is demonstrated by the SBR and SRR values shown in Fig. 12. Similar to the dense reinforcement scenario, the role of reinforcement efficiency is more understated as the distance between the wall facing and the footing increases as the influence of the wall is reduced. However, for shorter distances between the footing and the wall, the reduced reinforcement efficiency of the backfill becomes more pronounced. Specifically, the SBR values for larger reinforcement spacing show 80 % efficiency (20 % loss from baseline case) at higher reinforcement strengths, reducing to only 50 % efficiency at lower strengths. This is demonstrative of 25–100 % more reinforcement materials. This significant reduction in

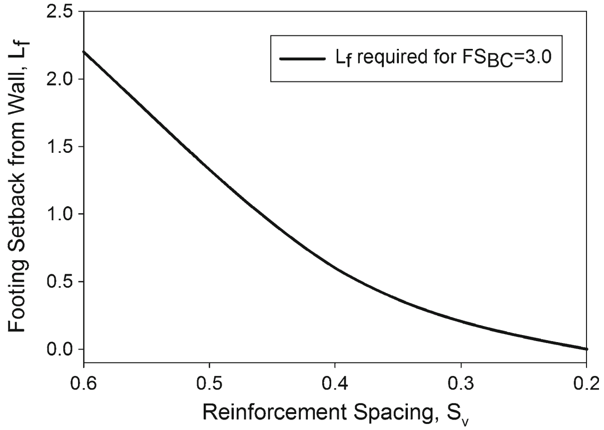

Fig. 13 Footing setback lengths (L_f) required to attain a factor of safety of 3.0 for the bearing capacity of an MSE wall abutment-supported footing

cost benefit is indicative of poor distribution of tensile strength to the wall backfill material, resulting in lower stability for the footing and MSE wall.

Footing Placement

The results of the series of parametric studies are instructive about the effects of reinforcement spacing, reinforcement strength, and surcharge location on the bearing capacity of footings supported by MSE walls. Various combinations of these parameters can allow for an optimized placement of a footing to potentially decrease material costs through reduced bridge deck length. As expected, bearing capacity of the footing increased as its location was further from the wall facing. This added capacity and the subsequent failure mechanism are implicative of a transition from a classical failure mechanism, a slip surface from the heel of the footing to the toe of the wall, to a compound failure mechanism including much of the backfill and reinforced soil zone. Such a transition suggests a shift from a slope/retaining structure-related failure to a

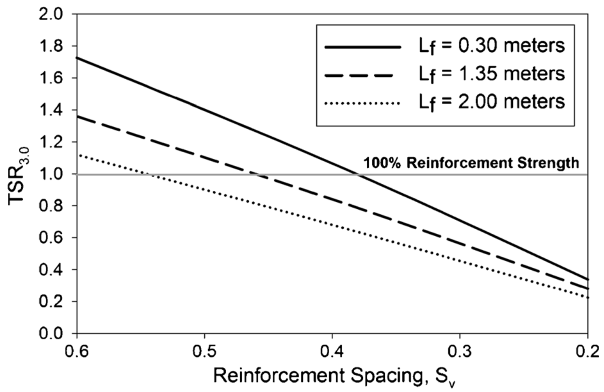

Fig. 14 Proportion of total long-term design strength (TLTDS), classified as $TSR_{3.0}$, required to support a footing with bearing capacity of 3.0 for varying reinforcement spacing

compound bearing capacity failure, congruent with the idea that larger footing setbacks will reduce the influence of the wall on footing bearing capacity. The bearing capacity of the footing was lower as its location approached the facing of the MSE structure but was still shown to be adequate for design when using higher reinforcement strengths. This reduction in required footing setback distance (L_f) suggests potential cost savings by means of shortened bridge deck length, a structural component that can be significantly more expensive than the higher-strength reinforcement needed to compensate for the increased internal stability requirements of the MSE structure.

Bearing Capacity

One alternative means of demonstrating the effects of reinforcement spacing on bearing capacity is to approach the problem from the traditional bearing capacity perspective, that is, establishing a factor of safety of 3.0. In Fig. 13, the effects of reinforcement spacing are obvious: denser reinforcement spacing allows for greatly reduced footing setback to attain a FS_{BC} of 3. Greater setback of the abutment footing translates directly into costs by increased bridge deck length requirements, demonstrated by an increase in L_f of 2.2 m per side when comparing widely spaced reinforcements (S_v=0.6 m) to close spacing (S_v= 0.2 m). Similar observations can be made from a comparison of the tensile strength ratio (a proportion of total long-term design strength, or TLTDS) required to attain FS_{BC}=3.0 compared to a variety of reinforcement spacing layouts (see Fig. 14). This comparison demonstrates that for the original design reinforcement TLTDS values (TSR=1.0), reinforcement spacing of 0.53, 0.46, and 0.38 were required to maintain FS_{BC}=3.0 for footing setbacks of 2.0, 1.35, and 0.3 m from the facing, respectively. This is indicative of a need to optimize footing setback (and bridge deck length) with reinforcement strength and spacing to meet bearing capacity requirements. It is important to note that dense spacing still provides a significant reduction in required reinforcement strength to maintain an adequate factor of safety for bearing capacity. Similarly, as bridge deck length is reduced, reinforcement spacing must be reduced as well, but the economic benefits in reduced requirements for bridge materials can possibly offset the increased amount or strength of reinforcement materials required, demonstrating a potentially viable option for more efficient and cost-effective bridge abutment design.

Conclusions

A series of parametric studies consisting of approximately 170 numerical models using limit analysis was performed, observing the effects of reinforcement strength, footing location, and reinforcement spacing on the bearing capacity of MSE wall-supported footings, specifically in context of bridge abutment design. The results demonstrated several notable trends with economic implications for future bridge abutment design. Specifically:

- Location of a load-bearing footing on the crest of an MSE wall can have a significant impact on bearing capacity and critical failure mechanism of the earthen structure. As the footing location nears the facing of the wall, the bearing capacity reduces and the failure mechanism tends to become a curved shear zone tangent to the toe of the wall and heel of the footing. Greater setback of the footing reduces the influence of the facing of the wall on stability, increasing bearing capacity and manifesting failure as a compound mechanism involving an active slip surface and a general shear bearing capacity failure.

- Spacing between reinforcement layers can have a major impact on the bearing capacity of a MSE wall-supported footing. Denser reinforcement layouts allow for significantly increased stability, allowing for greater bridge load, shortened bridge deck length with a shortened L_f, reduced reinforcement strength, and more efficient mobilization of composite strength by reinforced soil zone. Wider spacing had the opposite effect, with reduced efficiency relative to the total reinforcement strength. This trend suggests that more reinforcement material is needed to maintain a level of stability due to inefficiency in distribution of tensile strength, a likely result of reduced composite behavior of the backfill due to the larger spacing between reinforcement layers.
- The spacing of reinforcement had a notable effect on bearing capacity, but the relationship between increased reinforcement material and increased bearing capacity was not linear. Specifically, significant gain in efficiency was demonstrated from smaller vertical spacing between reinforcement layers, while a loss of efficiency resulted from wider spacing. Doubling the amount of reinforcement from the baseline scenario (S_v=0.4 m) to the closely spaced scenario (S_v=0.2 m) increased the reinforcement material by 100 %; however, the gain in bearing capacity ranged between 155 and 235 %, a notable, non-linear gain in reinforcement efficiency. The higher end of this increased efficiency was notable as the reinforcement neared the facing. This demonstrates that closely spaced reinforcements not only allowed for footing placement closer to the wall facing but also for significantly increased efficiency of reinforcement materials. Widely spaced reinforcements showed a non-linear loss in efficiency compared to the reduction in reinforcement materials.
- Change in efficiency due to varying reinforcement spacing could allow for cost savings through reduced material costs, that is, significant savings in reinforcement material quantities could occur due to increased reinforcement density and improved efficiency in mobilizing reinforcement tensile strength. For the actual design case, it is shown that use of dense reinforcements can allow for the footing to be placed closer to the facing. This demonstrates that use of closely spaced reinforcement layers for MSE wall-supported footings provides efficient design and holds significant potential cost savings by means of bridge deck length requirements.

The combination of the potential benefits highlights two potential tools for bridge abutment design that deserve added consideration for future construction: (1) mechanically stabilized earth walls as support for footings, especially in context of bridge abutments, and (2) use of closely spaced reinforcement systems due to more cost-efficiency in bearing capacity stability.

References

1. Wu, J.T.H., Ketchart, K., Adams, M.: GRS bridge piers and abutments. *Report FHWA-RD-00-038*. FHWA, US Department of Transportation, pg. 136. (2001)
2. Tatsuoka, F., Uchimura, T., Tateyama, M.: Preloaded and prestressed reinforced soil. Soils Found **37**(3), 79–94 (1997)

3. Helwany, S., Wu, J., Kitsabunnarat, A.: Simulating the behavior of GRS bridge abutments. ASCE J Geotech Geoenviron Eng **133**, 129–1240 (2007)
4. Yoo, C., Kim, S.: Performance of a two-tier geosynthetic reinforced segmental retaining wall under a surcharge load: full-scale load test and 3D finite element analysis. Geotext Geomembr **26**(6), 460–472 (2008)
5. Gotteland, P., Gourc, J.P., Villard, P.: Geosynthetics reinforced structures as bridge abutments: full scale experimentation and comparison with modelisations. In: Wu, J.T.H. (ed.) Mechanically stabilized backfill, pp. 25–34. A.A. Balkema Publisher, Rotterdam (1997)
6. Abu-Hejleh, N., Outcalt, W., Wang, T. and J. Zornberg: Performance of geosynthetic reinforced walls supporting the Founders/Meadows Bridge and approaching roadway structures. Colorado Department of Transportation Report No. CDOT-DTD-R-2000-5, Denver, CO. (2000)
7. Chen, W.F.: Limit analysis and soil plasticity. J. Ross Publishing, Ft. Lauderdale, FL (2008)
8. Smith, C.C., Gilbert, M.: Application of discontinuity layout optimization to plane plasticity problems. Proc R Soc A **463**(2086), 2461–2484 (2007)
9. LimitState (2013) LimitState:GEO (version 3.0) [Software].
10. Leshchinsky, B.: Comparison of limit equilibrium and limit analysis for complex slopes. Proceedings of GeoCongress 2013, San Diego. (2013)
11. Yu, H.S., Salgado, R., Sloane, S.W., Kim, J.M.: Limit analysis versus limit equilibrium for slope stability. ASCE J Geotech Geoenviron Eng, ASCE **125**(10), 915–918 (1999)
12. Leshchinsky, B. (2013). Mechanically stabilized earth walls: parametric study of reinforcement tensile loads under limit state. Proceedings of the international symposium on design and practice of geosynthetic-reinforced soil structures, Bologna.
13. Clarke, S.D., Gilbert, M., Smith, C.C.: Modelling discrete soil reinforcement in numerical limit analysis. Can Geotech J **50**(7), 705–715 (2013)