



Particle model from quantum foundations

Leonardo Chiatti · Ignazio Licata

Received: 28 July 2016 / Accepted: 30 October 2016 / Published online: 15 November 2016
© The Author(s) 2016. This article is published with open access at Springerlink.com

Abstract Some ideas are explored concerning the structure of elementary particles (specifically, leptons and hadrons), formulated within the context of a theory of the objective collapse of the wavefunction recently proposed by the authors. In accordance with this hypothesis, to each interaction that induces a discontinuity (quantum jump) in the evolution of the state of an elementary particle, two de Sitter half-spaces are associated, respectively, connected with the outgoing state and its conjugate. It is in these spaces that the structural constituents of the particle lie (quarks in the case of a hadron). The mass of free particles (leptons and hadrons) is given by the energy associated with their time localization in the jump, while the interaction between quarks belonging to a same hadron leads to a chromodynamic coupling constant that ensures both confinement and asymptotic freedom. It is possible to write a toy Hamiltonian in which the organization of quarks in hadrons appears *ab initio*, and which includes terms both for the exchange of quarks and for the creation/annihilation of pairs, thus avoiding the problem of bottom-up hadronization. In this scenario, the genesis of Regge trajectories is briefly discussed and it is argued that their slope can be quantized; finally, a reinterpretation is suggested of the classical Veneziano and Virasoro amplitudes.

Keywords Quantum jump · de Sitter spacetime · Elementary particles · Strong interactions · Regge trajectories · Veneziano and Virasoro amplitudes

1 Introduction

Quantum mechanics (QM) is the universally accepted reference theory for the study of microphenomena—involving a finite number of degrees of freedom—on a molecular, atomic, nuclear and particle scale. In this theory, as is known, there are two distinct coexisting modalities of the time evolution of the states of a system: one is unitary

L. Chiatti (✉)
ASL VT Medical Physics Laboratory, Via Enrico Fermi 15, 01100 Viterbo, Italy
e-mail: leonardo.chiatti@asl.vt.it

I. Licata
Institute for Scientific Methodology, Via Ugo La Malfa 153, 90146 Palermo, Italy
e-mail: ignazio.licata@ejtp.info

I. Licata
School of Advanced International Studies on Applied Theoretical and Non Linear Methodologies of Physics, 70100 Bari, Italy

and deterministic, described by the system Hamiltonian, the other non-unitary and non-deterministic, consisting in the reduction of the system state (wavefunction collapse) that is formally represented by a projection operator. This is the well-known “projection postulate” introduced by Von Neumann as early as in the first formalization of non-relativistic QM [1–3]. While there are detailed rules on the construction of the Hamiltonian of a given quantum system (rules for the replacement of classical quantities with operators, reordering, and so on), the quantum formalism in its current formulation does not provide indications on the collapse mechanism nor on the circumstances of this event. This gap has given rise to a number of conflicting schools of thought, according to which the collapse: (1) is not a physical phenomenon but an epistemic convention; (2) is a physical phenomenon confined to measurement procedures; (3) is a physical phenomenon of more general significance. The reader may refer to the vast specialized literature for a discussion of the different approaches (a foretaste of which can be found in references [4–10]). In this work, we shall focus on a recent proposal of ours [11], exploring its possible developments in the sphere of particle physics, with special reference to hadron structure.

According to this proposal, which subscribes to the third line of thought listed above, the collapse phenomenon and that of the “quantum jump” induced in a quantum microsystem by an interaction are basically identical. The notion of quantum jumps between the discrete levels of a quantized system was introduced by Bohr in his famous work on the hydrogen atom [12], but this concept is usually understood in its more extensive sense of discontinuity in the evolution of the quantum state. In the rest of this work we shall confine our attention to elementary particles. For particles, the quantum jump is a localization *in time*, either accompanied or not by a localization in space; normally, however, the particle remains delocalized in space. For example, the quantum jump of an atomic electron is a localization of this electron in the time domain (outside of this event, the electron remains silent in its stationary state); the electron, however, remains spatially delocalized in accordance with the final orbital of the transition. The jump is induced by the interaction of the electron with the electromagnetic field. Therefore, it is the *interaction* that presents a twofold aspect: one described by its Hamiltonian and connected with the unitary evolution of quantum state (in the example, the time-dependent superposition of the initial and final electron orbitals), the other—not described by the Hamiltonian—related to the jump. The jump is formally represented by the projector $|\psi\rangle\langle\psi|$ on the final orbital ψ .

The essence of the proposal is to take the formalism seriously and to interpret the projector as a physical event that consists in the ordered succession of three steps: (1) the $\langle\psi|$ section of the projector switches off the $|\psi\rangle$ component of the incoming state, halting its run in external time; (2) passage to an “a-spatial” condition in which the de Broglie phase factor $\exp(i\omega t)$ of the $|\psi\rangle$ component ($\omega = mc^2/\hbar$ is here the oscillating frequency of the electron in its proper time t) is recoded as an oscillation of an unobservable background described by a complex time without spatial coordinates, an ancestor of laboratory physical time; (3) inverse decoding of this background oscillation, with the restitution of the original de Broglie factor; the $|\psi\rangle$ section of the projector reappears as an outgoing state. From the perspective of the external time domain we have an instantaneous event of localization of the state $|\psi\rangle$, which appears as incoming to this event as well as outgoing from it, so that no change of physical quantities related to this state really occurs [13].

We note that, though the succession of steps (1)–(3) is time reversal, the a-spatial background exchanges a positive energy with the particle field during creation and a negative one during annihilation. Therefore, the possibility for the background to perform work is confined to the creation process. If the time order of the succession is chosen such that the exchange of negative energy precedes the exchange of positive energy, causality is always respected. The projector can also act on the left, annihilating the $\langle\psi|$ component of the incoming state and creating the outgoing state $\langle\psi|$, a conjugate of the preceding one. In this case, a positive energy exchange will be associated with the annihilation of $\langle\psi|$ (equivalent to the creation of $|\psi\rangle$), while a negative energy exchange will be associated with its creation (equivalent to the annihilation of $|\psi\rangle$); this ensures the absence of advanced effects.

This leads us to think about the concept of “particle” not as a carrier of permanent properties but as a *connection process between events*, in accordance with the idea that a physical theory must be solidly built around the observed events [14].

Whilst the reader can refer to the original articles for the necessary details [11, 15], we shall mention the points that are most significant here. Firstly, a time scale θ_0 is introduced which is associated with the temporal (and possible

spatial) localization of the elementary particles in a quantum jump; such a scale corresponds to the time necessary for light to cross the classical electron radius (approximately 10^{-23} s). The particle mass is the energy involved in the time localization (subsequently corrected by other effects) and its amount is determined by θ_0 . It is also, in natural units, the oscillation frequency of the particle when it is in the a-spatial intermediate virtual state. By interpreting the imaginary part of complex time as an inverse temperature, it is also possible to derive the Hagedorn distribution of hadronic states; specifically, it is possible to obtain the correct Hagedorn temperature value from θ_0 [11].

Two distinct de Sitter spacetimes of maximum radius $c\theta_0$ (actually, as we will see later in this paper, two halves of the same de Sitter spacetime), which are the internal spaces of the particle in the ψ state, turn out to be associated with the state $|\psi\rangle$ and its conjugate $\langle\psi|$. In the case of a hadron, such spaces contain its quarks, in a similar way to bag models explored in 70s and early 80s [16]. When the time localization of a hadron in a quantum jump occurs (the hadron is the physical particle undergoing the jump), it implies the simultaneous localization of all the quarks belonging to it, as its internal centres of interaction (they form a colour singlet). This “holistic” constraint on quarks (a kind of *causa formalis*) can be implemented in compliance with the relativity principle, assuming that the tangent point of de Sitter spaces with external spacetime is the application point of unit vectors whose elongations on the de Sitter horizon define the spatial positions of the quarks at the time of the jump. In such a context, it is possible to introduce the colour degree of freedom in a clear manner. After a general discussion of the problem in Sect. 2, a colour interaction is introduced by hand in Sect. 3 that complies with the rules of quantum chromodynamics (QCD) within the static limit. In Sect. 4, quark coupling is discussed and emphasis is laid on how the experimentally observed dependence of the chromodynamic coupling constant on energy can be explained in the context of present model.

The particular quantum jump consisting of a strong interaction vertex between hadrons is relevant. Since the state $|\psi\rangle$, undergoing the localization in the time domain, consists of a product of asymptotic hadronic states, the state $|\phi\rangle$ on which the operator $|\psi\rangle\langle\psi|$ acts must consist of an entanglement of such products. Each term of this entanglement will represent a possible combination of hadrons outgoing from the interaction vertex and the combination actually selected by the quantum jump $|\psi\rangle\langle\psi|$ will be $|\psi\rangle$. In our opinion, this fact must be interpreted as meaning that each quark has, at every moment of time, to belong to a specific hadron (although two hadrons may appear as “indistinguishable” within the entanglement), because there are no free quarks in terms of $|\phi\rangle$ expansion. Also, since the various hadrons appearing in each single term of $|\phi\rangle$ are free, quarks belonging to different hadrons cannot be coupled by colour interaction. These conclusions, which seem to us the natural consequence of the application of the concept of quantum jump to the hadronic matter, are more stringent than purely “unitary” bottom-up description provided by QCD, which instead allows the “fusion” of hadrons in the interaction vertex and colour coupling between quarks of different hadrons. The latter, in fact, was developed in a historical period in which the direct observation of quantum jumps in specific physical systems had not yet been accomplished and the very existence of the phenomenon was dubious (remember that the first of these observations is dated 1986 [17]). In QCD the projection operators intervene only as an expression of the measurement operations of asymptotic states, with respect to which the quark confinement has been ascertained many years ago. But if we accept the idea that the projection operators intervening in the very definition of the interaction vertex, this latter being a quantum jump, the emergence of constraints on the dynamics of a contact interaction, as is the strong interaction, appears to be a natural consequence. More precisely: since the colour interaction must be limited to quarks within a single hadron (which must be colourless) and each quark must at every moment belong to a hadron, the strong interaction between hadrons must consist of exchanging quarks and the creation/annihilation of really neutral mesons. These processes (who must maintain each involved hadron colourless) are the only ones capable of transforming an entanglement of hadrons in an entanglement of hadrons through a contact interaction.

Thus, the structure of strong interaction appears to be developed on a number of levels: at the intra-hadronic level, we have colour coupling between quarks belonging to the same hadron (colour coupling between quarks from distinct hadrons is not permitted in this approach); at the extra-hadronic level, we have the exchange of quarks between hadrons whose de Sitter spaces fall within the maximum radius $c\theta_0$. This, in a full QFT (quantum field theory) description, is supplemented by the creation and annihilation of quark pairs in the form of really neutral mesons with their associated de Sitter spaces. It is possible to write a toy Hamiltonian that includes all these

contributions and it is presented in Sect. 5. Naturally, the rearrangement of quarks in interacting hadrons must satisfy global constraints and it is therefore a non-local process; it does not, however, entail a hyperluminal exchange of energy or information between quarks that can be detected from the outside, and it therefore satisfies the principle of relativity.

At the “local” level, the coupling of two quarks is constructed by hand according to QCD rules; however, at the global level, the quarks are subjected to the constraint that they belong to a given hadron. Therefore, the deconfinement cannot be identified with the liquidation of this condition of belonging, which actually is permanent; rather, it is to be identified with the establishment of a condition of free exchange of quarks between hadrons. Should this possibility be confirmed by further research, it would indicate a strategy to work around the long-standing problem of describing hadronization. Indeed, the organization of quarks in hadrons would not emerge as a result of their “local” colour interactions by following a bottom-up process, but would be constantly present in the Hamiltonian description. This would do away, amongst other things, with the problem of dynamically describing the breaking of the chiral symmetry, since such a phenomenon would never appear.

Finally, it must be noted [11] that a hadron of mass m can be associated with an “inertia moment” $mc^2\theta_0^2 \approx (\hbar/c^2\theta_0)(c^2\theta_0^2) = \hbar\theta_0$ which must not, of course, be interpreted in the classical terms of a continuous distribution of mass but as a conversion factor between an internal rotational frequency and spin. On this basis, it is possible to derive Regge trajectories with quantized slopes that constitute, from this point of view, an expression of the finite value of θ_0 . This subject is set forth in Sect. 6. In Sect. 7 this view of spin is applied to the reinterpretation of a classical topic: the Veneziano [18] and Virasoro [19] amplitudes.

2 The hidden side of interactions: the particle–vacuum dialogue

Our considerations are based, therefore, on quantum jumps associated with interactions between particles. Elementary particles whose state can appear as incoming to (or outgoing from) an event represented by a projection operator are, by definition, real or “physical” particles. Leptons and hadrons can be real particles, as can gauge quanta; we shall confine our attention to leptons and hadrons. Instead, quarks and gluons are permanently “virtual” entities exchanged in an interaction vertex but never detected as asymptotic states. From our point of view, the quantum jump is the localization event of the state of a real particle in the time domain. As we have said, this event is normally delocalized in accordance with the ingoing (outgoing) particle wavefunction.

At this point, a remarkable observation is that although the quantum jump is instantaneous, the interaction that causes it is not at all so. In fact, a finite energy is exchanged in this interaction, and thus according to the Heisenberg uncertainty principle, its duration in external time must also be finite. In essence, to say that the created/destroyed particle possesses a rest energy mc^2 means that an interaction induces the sharpest particle localization has a duration that does not exceed $\theta = \hbar/mc^2$, a value that depends on the particle. This is the most accurate time localization of that particle. We can thus associate with the particle’s internal processes (at the time of its spatio-temporal localization with maximum accuracy) a maximum duration of θ . Let us consider the initial and final events of one of these processes, whose projections on the external Minkowski spacetime are, respectively, (x_1, y_1, z_1, ct_1) and (x_2, y_2, z_2, ct_2) ; we pose $x = x_2 - x_1$, $y = y_2 - y_1$, $z = z_2 - z_1$, $t = t_2 - t_1$. The duration of this process should not exceed θ . In other words:

$$c^2t^2 - x^2 - y^2 - z^2 \leq c^2\theta^2. \quad (1)$$

The coordinate transformations that leave the (1) unchanged form a de Sitter group; the inequality (1) represents a de Sitter space with a radius of $c\theta$ [see Appendix for details]. We consider this space as the “inner space” of the particle, where its internal processes live.

In order to assure the accordance with experimental data we must pose $c\theta \leq c\theta_0 \approx 10^{-13}$ cm, the well-known scale of elementary particles, common to both leptons (classical electron radius) and hadrons (effective radius of a strong interaction). We assume that $c\theta_0$ is a new fundamental constant of nature. Elsewhere, we have provided

arguments supporting the identification of this constant with the classical electron radius [11,20]. Of course it is not necessary to think $c\theta_0$ as the radius of a sphere, but rather as *effective* quantity. Indeed the introduction of this constant allows us to invert the argument, by associating to each physical particle a given number $z \geq 1$, which is a function of the flavour composition of the real (physical) particle and its spin and parity state. This physical particle is associated with a de Sitter micro-universe with a radius of $c\theta_0/z$, and a “skeleton mass” (really a rest energy) of $\hbar z/\theta_0$. The attribution of this mass (with the sole exceptions of neutrinos and the electron, see [11]) defines the initial core of the particle’s inertia; the physical particle’s effective mass is the sum of the skeleton mass with contributions from the internal degrees of freedom and self-interaction. Determining the number z from algebraic and topological considerations related to abstract graphs that can be connected to any given physical particle (“glyphs”, see [21,22]) is a problem currently under investigation, and will be the subject of a separate paper. In this work we assume z to be a known parameter.

We will now focus on the physical particle’s spatialization process, occurring in an interaction vertex in which the particle spatial position is measured with the maximum accuracy compatible with finiteness of θ . The introduction of a fundamental constant, or “chronon”, θ_0 allows for a definition of a spatial scale $c\theta_0$ (which defines the maximum localization “accuracy”), energy scale (\hbar/θ_0), moment of inertia $\hbar\theta_0$ and a mass scale ($\hbar/c^2\theta_0$). It is remarkable that the Hagedorn temperature, which amounts to about 160 MeV, can be derived from the energy scale $\hbar/\theta_0 = 70$ MeV [11,23,24].

We geometrically interpret the creation/annihilation of a physical particle as a geodetic (or “gnomonic”) projection of an “internal” de Sitter spacetime on the external spacetime, with local preservation of the light cone. This projection produces a Castelnuovo spacetime (Beltrami representation) in which the proper time is limited according to (1), while the contemporaneousness space of the fixed point extends to infinity [25]. However, in the original de Sitter spacetime this space is closed. Note that, in a closed space, field equations that satisfy the Gauss theorem inexorably lead to the conclusion that all charges of that field and present in that space are zero. Thus, moving from a closed space to an open space is equivalent to the creation of charges (or, conversely, to their destruction if the process is viewed from the opposite direction, i.e. as a *back projection*).

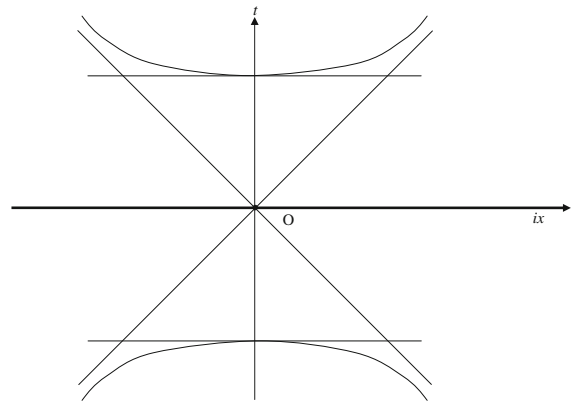
The projection therefore implies the creation of charges, which interact with each other and (due to the finite duration of the interaction and thus the appearance of virtual effects by virtue of the uncertainty principle) with themselves as well. Thus the self-interaction exists only as a concomitant phenomenon of the interaction associated with the projection. The strong interaction radius for the “projected” charges (if they are colour charges) is $c\theta_0$. Out of this radius from the interaction vertex there are no self-interactions at all.

We now need to take a look at the hadron strong processes involving the exchange of quarks (and/or the creation/annihilation of quark–antiquark pairs). The hadron micro-universes entering a vertex of interaction have certain radii, which are generally different from those of hadrons emerging from such a vertex. The reorganization of quark configurations therefore requires an intermediate state, which corresponds to a situation in which individual hadrons are still present with individual de Sitter radii. However, in this condition quark exchanges are possible *between* hadrons within a radius equal to $c\theta_0$, while no colour interaction exists between components of different hadrons; the possibility of a quantum superposition of geometries with different values of z thus arises. The recombination of quarks and genesis of new micro-universes whose radii are the result of such a recombination corresponds to the phenomenon of hadronization. *The hadronization can thus be described as a transition of geometries at the Hagedorn temperature.* This is a well-known scenario which corresponds to the transition from free hadron gas to quark gluon plasma (QGP), or vice versa, occurring at that temperature [24].

3 de Sitter particles

If ψ is the state of a physical particle, two de Sitter micro-universes [11,20] will be associated with the event represented by the projector $|\psi\rangle\langle\psi|$, one for $|\psi\rangle$, the other for $\langle\psi|$. These micro-universes will be de Sitter spaces of identical radii that contain identical physical states. Specifically, we wish to examine the internal structure of leptons and hadrons in this context.

Fig. 1 Castelnuovo spacetime, projection of de Sitter spacetime onto ordinary spacetime. O is the tangent point. The light cone passing through O , the two sheets (past and future) of the Cayley–Klein absolute and the two sheets $t = \pm\theta_0/z$ of the de Sitter horizon of O are represented



To understand how the state vector ψ is associated with a given de Sitter micro-universe, it is necessary to proceed by successive steps. Let us remember that de Sitter spacetime having radius $c\theta_0/z$ is tangent, in the point-event O , to the external spacetime on which it is projected [25, 26]. In any frame of reference in external time, the time coordinate of the point-event $O' \equiv O$ corresponds to the instant of the jump. The projection transforms the de Sitter light cone of O (in de Sitter spacetime) into the light cone of O' in external spacetime. The time axis of O in de Sitter spacetime is projected in a timelike line that passes through O' in external spacetime; we postulate that this line constitutes the time axis of the frame of reference of the relativistic centre of mass of the interaction that induces the jump. The spatial position of O' can be (and normally is) subject to quantum delocalization (superposition of states); in other words, the spatial coordinates of point-event O' in the ordinary spacetime generally remain indefinite, unless the jump is a spatial localization (with maximal precision) of the particle. Putting aside delocalization and assuming that O' is exactly defined, the projection of de Sitter spacetime on external spacetime is a Castelnuovo spacetime [25, 27], such as that shown in Fig. 1. The point O' is, in ordinary QM language, the position of the physical particle.

The spacetime in Fig. 1 is divided into two half-spaces $t > 0$ and $t < 0$, associated with sections $|\psi\rangle$ and $\langle\psi|$, respectively, of the projector $|\psi\rangle\langle\psi|$. If we assume that the projector acts on the right, the half-space $t < 0$ is associated with the destruction of the forward state vector $|\psi\rangle$ which is regenerated in the half-space $t > 0$. If it is assumed that the projector acts on the left, the half-space $t > 0$ is associated with the destruction of the backward state vector $\langle\psi|$ which is regenerated in the half-space $t < 0$. Therefore, the two de Sitter spaces associated with the two sections of the projector are actually the two halves of the same de Sitter space, and from this point on we shall focus only on the half-space $t > 0$, since the physical situation in the corresponding half-space $t < 0$ is its mirror-image.

A further geometrical element that we introduce as a basic ingredient of our description is a set of three timelike versors (that is, unit vectors) R, G, B (or, alternatively, $\bar{R}, \bar{G}, \bar{B}$) applied in O and with a free extreme in the half-space $t > 0$; the meaning of the R, G, B (red, green, blue) labels is that of the three colour eigenvalues of the customary $SU(3)$ colour group of QCD. Each of these versors will have a corresponding opposite versor of identical colour (or anticolour) in the half-space $t < 0$. We shall also assume the possibility that two or three of these versors can be identified, that is coincident in a single versor. The identification of versors R and G will produce a single versor \bar{B} , and so forth, according to the pattern: $RG = \bar{B}$, $GB = \bar{R}$, $BR = \bar{G}$, $\bar{R}\bar{G} = B$, $\bar{G}\bar{B} = R$, $\bar{B}\bar{R} = G$, $GBR = \bar{G}\bar{B}\bar{R} =$ white. The introduction of this set of three versors with these particular properties can be justified starting from topological arguments (Ref. [21] and work in progress), but in this paper we shall limit ourselves to postulating it.

Since the versors are timelike and therefore inside the light cone of O in Fig. 1, the half-line with origin O identified by a versor intersects the de Sitter horizon of O in a specific point. From this point, a perpendicular to the contemporaneousness space of O can be conducted. Naturally, according to the specific identifications assumed for the versors, there will be three distinct possibilities, represented in Figs. 2, 3 and 4, respectively. We assume that the centres of charge inside a physical particle are identified in the segments of the perpendicular lowered from the intersections. There will be three distinct cases; a single white segment (lepton), two segments of complementary

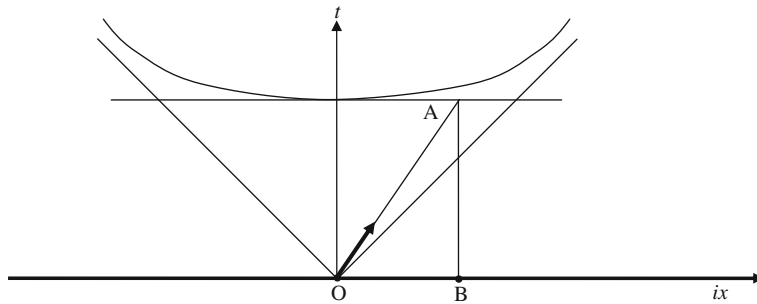


Fig. 2 The three versors are identified in a single white vector. The elongation of this vector meets the de Sitter horizon of O in point A , whose projection on the contemporaneous space of O is B . Segment AB is a lepton, whose position in external spacetime is O . Note that the lepton has a finite (temporal) extension while the point O does not have any dimensions

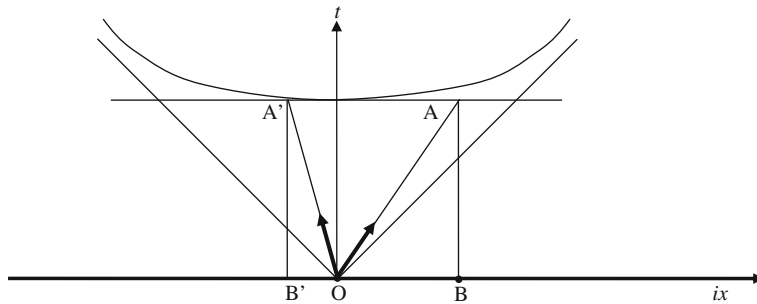


Fig. 3 Two versors are identified in a single vector of a given anticoulour (colour), while the third vector assumes the complementary colour (anticoulour). By repeating the construction of Fig. 1 for the two versors, segments AB and $A'B'$ are obtained. The coloured segment corresponds to a *quark*, the anticoloured one to an *antiquark*. The physical particle is a *meson*, whose spacetime position is O

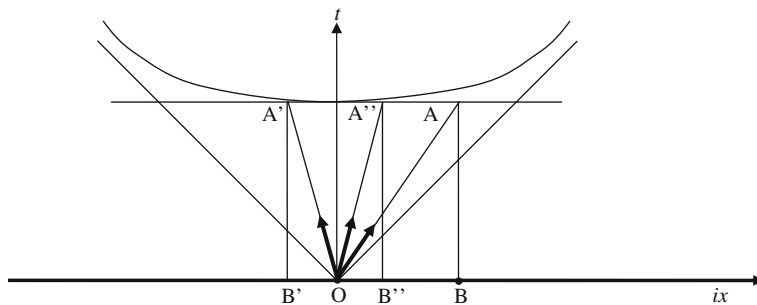


Fig. 4 No mutual identification of the three versors is present. By repeating the construction of Fig. 1 for the three versors, segments AB and $A'B'$ and $A''B''$ are obtained. The three segments are either all coloured, thus corresponding to three *quarks*, or anticoloured, thus corresponding to three *antiquarks*. The physical particle is a *baryon* in the first case, an *antibaryon* in the second case and its spacetime position is O

colours (meson, made up of a quark and an antiquark), three coloured or anticoloured segments (respectively: one baryon formed by three quarks and one antibaryon formed by three antiquarks).

In each case, the physical particle (lepton, antilepton, meson, baryon or antibaryon) will have an overall white colour. If a delocalization of the colour states (quantum superposition) is admitted, the only states allowed are therefore colour singlets. It is evident, from this construction, that the centres of interaction inside the particle are segments of finite time extension (equal to θ_0/z) and lacking spatial extension in the frame of reference of the interaction centre of mass. A spacetime position of the physical particle exists, which is O ; the greatest possible spatial distance of a centre of charge from this position is $c\theta_0/z$, since the versors are timelike. In other words,

both quarks and leptons are confined within the de Sitter micro-universe to which they belong; in actual fact, the physical particle consists of this micro-universe with its centres of charge inside [28, 29].

The temporal extension of the micro-universe (chronological distance of O from its de Sitter horizon) allows the definition of a moment of inertia $mc^2\theta_0^2/z^2 = \hbar\theta_0/z$, where $mc^2 = z\hbar/\theta_0$ is the rest energy of the physical particle [11, 15]. Of course, this moment of inertia cannot be understood in the classical sense of an internal mass distribution. We conjecture it can be a conversion factor between the frequency z/θ_0 (inverse of de Sitter time) and the spin. If $j = 0, 1/2, 1, \dots$ is the eigenvalue of the total spin operator of the physical particle, it is possible to write $j\hbar$ as the product of the moment of inertia $\hbar\theta_0/z$ and the frequency jz/θ_0 , a half-integral multiple of the natural frequency. Should this speculation turn to be true, it would be possible to assign to j the following “classical” meaning: every two θ_0/z periods the physical particle “rotates around itself” $2j$ times. Thus, the existence of chronon could imply the existence of the particular period θ_0/zj which we call hereinafter as “turn”. Turns will be relevant in discussion of Veneziano amplitudes.

4 Masses and couplings

The chronon fixes the time scale for physical particles localization processes in time domain and for processes of interaction between centres of charge within the same physical particle (i.e. quarks belonging to the same hadron) as well. By dividing the elementary quantum of action \hbar for this time scale, an amount of energy is derived which is available to that process, according to the uncertainty principle. The actual energy requested by the process will be a fraction of this available energy, expressed by a suitable coupling constant:

$$\text{energy} = \frac{\text{elementary action}}{\text{process time scale}} \times \text{coupling constant.} \quad (2)$$

For the localization process of a physical particle with $z > 0$ the requested energy is the rest energy associated with the skeleton mass M_{sk} , the time scale coincides with the de Sitter time θ_0/z of the particle micro-universe and all the available energy is requested:

$$M_{sk}c^2 = z \frac{\hbar}{\theta_0}. \quad (3)$$

For the localization process of a physical particle with $z = 0$ (electron and neutrinos, [11]) the available energy is \hbar/θ_0 and the requested energy is the actual rest energy of that particle. The coupling constant is the fine structure constant in case of electron; it is instead a not well-known generation-depending parameter α' for neutrinos:

$$\text{energy} = \frac{\hbar}{\theta_0} \frac{e^2}{\hbar c} = \frac{e^2}{c\theta_0} = mc^2 \quad \text{electron} \quad (4)$$

$$\text{energy} = \frac{\hbar}{\theta_0} \alpha' = mc^2 \quad \text{neutrinos (eigenstates of mass)} \quad (5)$$

As regards the colour coupling between the quarks inside a hadron, it must be considered that the interaction radius between two quarks is not the de Sitter radius of the micro-universe to which they belong, but the universal radius $c\theta_0$. We must therefore consider the two quarks from the perspective of the de Sitter universe with radius $c\theta_0$, with a time axis that coincides with one of these and chosen in such a way that the time 0 coincides with its extreme (Fig. 5). With reference to Fig. 5 we have:

$$(Aq_1)^2 = (q_1q_2)^2 + (Pq_1)^2 = (ix)^2 + (c\theta_0)^2 = c^2\theta_0^2 - x^2, \quad (6)$$

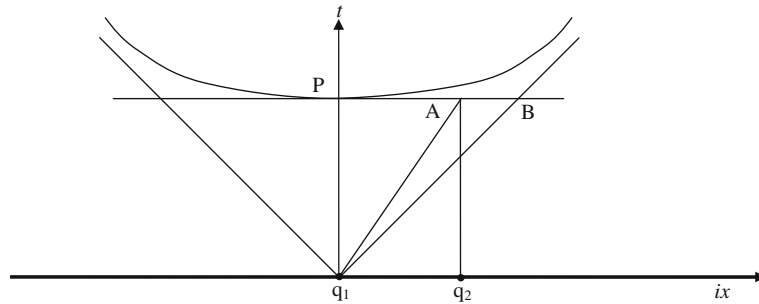


Fig. 5 Pq_1, Aq_2 = quarks; PB = de Sitter horizon of the quark Pq_1 ; Aq_1 = interaction distance between the two quarks

where Pq_1 and Aq_2 are time intervals, whose ends are, respectively, P, q_1 and A, q_2 , representing two quarks separated by a spatial interval $x = q_1q_2$ and Aq_1 is the segment of straight line from A to q_1 . Taking the time scale as Aq_1/c and indicating with $\alpha_{S,0}$ an appropriate strong interaction fine structure constant, one obtains:

$$\text{interaction energy} = \frac{\hbar}{\left(\frac{Aq_1}{c}\right)} \alpha_{S,0} = \frac{\alpha_{S,0}(\hbar/\theta_0)}{\sqrt{1 - \left(\frac{x}{c\theta_0}\right)^2}}. \tag{7}$$

This is practically the same as that in Caldirola’s electron theory [30], in accordance with which the centres of charge—which manifest themselves as time intervals of duration equal to the “chronon”—interact with the external environment only at their extremes. Equation (7) is independent from z , and therefore from the hadron flavour composition. In order to obtain a null coupling energy for $x = 0$, the constant term $\alpha_{S,0}(\hbar/\theta_0)$ must be subtracted. Moreover, the scalar product $\lambda_i \cdot \lambda_k$ of the colour charges of the two quarks i and k (expressed as vectors in the 8-dimensional space of the Gell Mann matrices) must be inserted. Thus the interquark potential becomes:

$$V_{ik} = \left(\frac{\alpha_{S,0} \frac{\hbar}{\theta_0}}{\sqrt{1 - \left(\frac{x}{c\theta_0}\right)^2}} - \alpha_{S,0} \frac{\hbar}{\theta_0} \right) \lambda_i \cdot \lambda_k. \tag{8}$$

This potential explicitly exhibits asymptotic freedom and confinement. Posing $r = c\theta_0$ and:

$$\alpha_S = \alpha_{S,0} \frac{\left(\frac{x}{r}\right)}{\sqrt{1 - \left(\frac{x}{r}\right)^2}}, \tag{9}$$

a coupling constant is thus obtained which is dependent on the distance; it vanishes for $x \rightarrow 0$ and diverges for $x \rightarrow r$. Taking into account the substitution (9), the potential energy (8) assumes the classical Coulomb expression:

$$V_{ik} = \left(\frac{\alpha_S(x)\hbar c}{x} - \alpha_{S,0} \frac{\hbar}{\theta_0} \right) \lambda_i \cdot \lambda_k. \tag{10}$$

This coupling does not depend on the flavour. The SU(N) flavour invariance is however broken by the fixing of z , which assigns to each hadron its own specific de Sitter radius, dependent upon the flavour composition, spin and parity.

We note that the construction in Fig. 5 is reversible; the origin of the axes can be transported in q_2 by carrying out a spatial translation. Following this operation one has the change of coordinates [27,31,32]:

$$x' = \frac{x + T}{1 - Tx/r^2}; \quad t' = \frac{t\sqrt{1 + T/r}}{1 - Tx/r^2}, \tag{11}$$

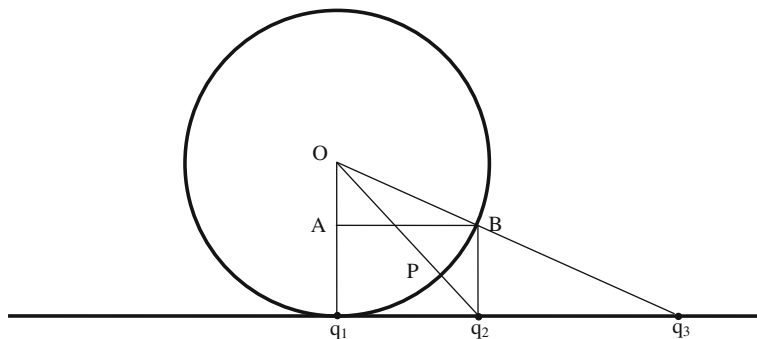


Fig. 6 O centre of projection; q_1, q_2 = positions of two quarks in spacetime

where $T = -x$ is the difference between new and old origin abscissa. One can easily see that for $t = 0$ one has $t' = 0$; in other words, the contemporaneusness space remains unchanged, so that q_1 and q_2 remain “simultaneous”. The lengths undergo the transformation:

$$l = l_0(1 - Tl_0/r^2)/[1 + (T/r)^2] \tag{12}$$

where we define $q_1q_2 = l_0$. And since $T = -l_0$, one has $l = l_0$; in other words, the distance x between the two quarks remains unchanged. By elevating from q_2 the perpendicular to its contemporaneusness space, it will meet the q_2 horizon in a point of the Cayley–Klein absolute [25,27,31]. The horizon tangent to the absolute in this point will not coincide with that passing through P and indicated with PB in Fig. 5. However, on this horizon, a point will exist, located at the spatial distance of $-x$ from q_2 , whose projection on the contemporaneusness space will be q_1 . The distance of this point from q_2 will again be Aq_1 .

When describing the inseparability of both quarks using the language of a “force” mediated by gluons, proper time Aq_1 (Fig. 5) can be considered as the time interval necessary for the exchange of the action $\alpha_{S,0}\hbar$. When the quarks coincide, this interval reaches the maximum value and the coupling is zero; when the point-event A is positioned on the light cone of q_1 , this interval vanishes and the coupling becomes infinitely intense.

The strong fine structure constant (9) is expressed as a function of the distance x between two quarks. The problem arises of how to express it as a function of the energy E exchanged with an external probe, in such a way as to enable comparison of equation (9) with experimental data. For this purpose, let us consider de Sitter spacetime with radius $r = c\theta_0$, tangent to the ordinary spacetime in the position occupied by a quark q_1 , and let q_2 be the spatial position occupied by a second quark (belonging to the same hadron) in the contemporaneusness space of q_1 . The de Sitter space simultaneous to q_1 will be a sphere with radius r (Fig. 6) from whose centre O the second quark, positioned in P , is projected in q_2 . Now if the parallel to Oq_1 is elevated from q_2 , this will intersect the de Sitter space in a point B . The projection of B from O onto the spacetime is q_3 . One has $q_1q_2 = x$, $OB = r$ and we define $q_1q_3 = y$. It is clear from Fig. 6 that:

$$\frac{OA}{r} = \frac{x}{y} \Rightarrow y = \frac{x}{\sqrt{1 - (\frac{x}{r})^2}}; 0 \leq x \leq r; 0 \leq y \leq \infty, \tag{13}$$

while from Eq. (9) one has:

$$\frac{y}{r} = \frac{\alpha_S}{\alpha_{S,0}}. \tag{14}$$

Let us now imagine two quarks, one of which belongs to the hadron, the other to the external probe, positioned at the relative distance x . The energy necessary to resolve this distance is $\hbar c/x$, while the colour interaction between quarks occurs in a time $\theta_0[1 - (x/r)^2]^{1/2}$, so that the action exchanged in the process is:

$$\Delta A = \left(\frac{\hbar c}{x}\right) \theta_0 \sqrt{1 - \left(\frac{x}{r}\right)^2}. \quad (15)$$

In order that this action is exchanged, the two quarks need to belong to a same virtual intermediate hadronic state, because only in this case can they be subject to a mutual colour interaction. We recall that, according to our hypotheses, the colour force only acts between quarks of a same hadron and not between quarks from different hadrons (this restriction does not hold in QCD). The duration of this intermediate state will be τ , and the energy E associated with its formation will be expressed by the uncertainty principle as $E = \hbar/\tau$. The minimum energy scale of hadronic processes is \hbar/θ_0 ; if the action ΔA were developed by a process of constant energy \hbar/θ_0 , this process would take a time τ'' defined through the relation:

$$\frac{\tau''}{\theta_0} = \frac{\Delta A}{\hbar}. \quad (16)$$

We can consider τ'' as the maximum duration required for the occurrence of an interaction where the action ΔA is exchanged. This time is associated, in our model of quantum jump [11, 15], with the imaginary part of the complex time of the (dormant) wavefunction of the particle undergoing the fluctuation of action ΔA ; in our case, this particle is the intermediate hadronic state. According to the model the probability of this fluctuation is [11, 15]:

$$\exp\left(-\frac{\tau''}{\theta_0}\right) = \exp\left(-\frac{\Delta A}{\hbar}\right). \quad (17)$$

Assuming ergodicity, this is the fraction of the maximal de Sitter time θ_0 in which the fluctuation is present. This time fraction will correspond to various time intervals of different duration, each of which can be associated to a certain energy through the uncertainty principle. The minimum energy required will be that corresponding to the case of a single interval having a duration:

$$\tau = \theta_0 \exp\left(-\frac{\Delta A}{\hbar}\right). \quad (18)$$

Thus the minimum required energy is:

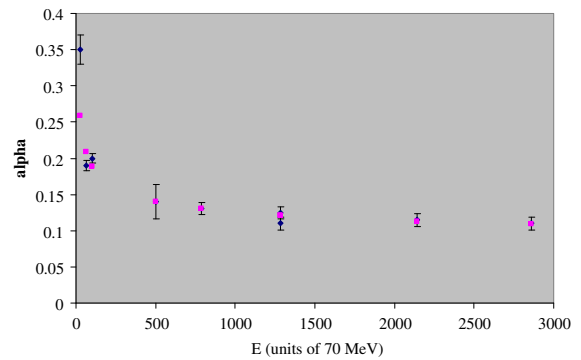
$$E = \frac{\hbar}{\tau} = E_0 \exp\left(\frac{\Delta A}{\hbar}\right), \quad (19)$$

where $E_0 = \hbar/\theta_0$. The energy E is that actually exchanged by the probe, and it is very different from the energy required for the resolution of the x scale, which is $\hbar c/x$. By rewriting $\Delta A/\hbar$ as r/y and bearing in mind Eq. (14), it is possible to rewrite Eq. (19) in the form:

$$\alpha_S(E) = \alpha_S(y) = \frac{\alpha_{S,0}}{\ln\left(\frac{E}{E_0}\right)}. \quad (20)$$

In Fig. 7, the experimental trend of $\alpha_S(E)$ and its best fit through Eq. (20), which corresponds to $\alpha_{S,0} = 0.377$ (with a correlation index R^2 of 0.95), are compared. The good agreement suggests that the polarization of the QCD vacuum is in a certain sense equivalent to the structure of the strong interaction assumed in this paper. We note in particular that Eq. (20) diverges in the conventional limit $\theta_0 \rightarrow 0$.

Fig. 7 Experimental (*blue rhombi*) and theoretical (*magenta squares*) behaviour of α_S as a function of energy. Data from Ref. [33]



5 Toy equations of motion

5.1 Single particle

The position of quarks inside a hadron is encoded in the tangent point O and in the directions of the versors applied in this point. The colour exchanges between quarks are actually colour exchanges between versors and are implemented in O. Certainly, one can equally conceive that these exchanges occur between quarks through the mediation of gluons; but then the gluons are emitted by a quark only under the condition that they are absorbed by a second quark. In other words, the propagation of gluons is confined between quarks; they do not propagate freely. Seen in these terms, the process is non-local, and clearly in the present description a gluonic field in QCD style does not exist. The hadrons are not bound states of quarks determined by the colour interaction; rather, quarks and their colour interaction are an aspect of the global structural unity represented by a hadron. By using the terms *implicate* and *explicate* order introduced by Bohm [34–39], we can say that the hadron represents the explication of a structure implicitly encoded in its set of three versors. The unity of the hadron is thus guaranteed not by an efficient cause (the colour force) but by a formal cause that consists in the explication of that hadron; the colour force is an emergent aspect of this explication.

As a consequence, the equations of motion of the hadrons will not contain couplings with colour fields but only interquark potentials: they will be equations of motion of quarks. The positions of the quarks will be expressed in terms of the coordinates of their projections on ordinary spacetime. The differentiation operators with respect to these coordinates thus represent momenta on Minkowski spacetime. The only, completely indirect, effect of the de Sitter geometry will be apparent in the definition of the masses (z factors) and in the form of the colour coupling between quarks.

For free physical particles (leptons, mesons or baryons) having mass M_{sk} (“skeleton” mass in our model; see Sect. 2 and Refs. [11, 15]) we can postulate the following equation of motion:

$$i\hbar\partial_t\Psi = H\Psi \tag{21}$$

with:

$$H = -i\hbar c \sum_{ic} \alpha_{ic,l}\partial_l + \frac{1}{2}\beta \sum_{ic,kc} V_{ic,kc} + \beta M_{sk}c^2, \tag{22}$$

$$\Psi = \Psi(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_{ic}, \dots, \vec{x}_n; t), \tag{23}$$

where l runs on the values 1,2,3 and $ic, kc = 1, \dots, n$ are indices related to centres of charge. For leptons n is identically 1 because the leptonic micro-universe contains just one centre of charge: the lepton itself. For mesons

n is identically 2 because the mesonic micro-universe contains one quark and one antiquark. For baryons n is identically 3 because the baryonic micro-universe contains three quarks. Different centres are mutually parallel, timelike segments; their beta matrices are thus identified in a single beta matrix even though their Dirac algebras do not overlap.

The appearance of the beta matrix in the second term of (22) ensures the relativistic covariance of Eq. (22), on the assumption that the interquark distances that appear in this term are the spacetime separations of quarks as evaluated in the frame of reference of the interaction centre of mass. In this case, therefore, it does not seem necessary to resort to QFT methods such as the Bethe–Salpeter equation to ensure covariant treatment.

For leptons Eq. (22) is reduced to the Dirac equation. The electroweak interactions can be inserted by substituting the ordinary derivatives with the appropriate covariant derivatives and by expanding the wavefunctions on the appropriate basis states.

In the absence of interactions with external fields, Eq. (22) must admit stationary solutions. For a particle with N centres of charge with a position \mathbf{r}_{ic} and a momentum \mathbf{p}_{ic} we can define the following quantities:

$$\mathbf{R} = \frac{1}{N} \sum_{ic=1}^N \mathbf{r}_{ic}; \mathbf{P} = \sum_{ic=1}^N \mathbf{p}_{ic}; \xi_{ic} = \mathbf{r}_{ic} - \mathbf{R}; \pi_{ic} = \mathbf{p}_{ic} - \frac{\mathbf{P}}{N}. \tag{24}$$

Equation (22) can therefore be rewritten as [40]:

$$H = c \sum_{ic} \alpha_{ic,l} \left(\pi_{ic} + \frac{\mathbf{P}}{N} \right)_l + \frac{1}{2} \beta \sum_{ic,kc} V_{ic,kc} (\|\xi_{ic} - \xi_{kc}\|) + \beta M_{sk} c^2. \tag{25}$$

If a stationary solution Φ exists:

$$H\Phi = \hbar\omega\Phi, \tag{26}$$

then:

$$\begin{aligned} \hbar\omega &= \frac{c}{N} \langle \Phi | \sum_{ic=1}^N \alpha_{ic,l} \mathbf{P}_l | \Phi \rangle + \langle \Phi | c \sum_{ic=1}^N \alpha_{ic,l} \pi_l + \frac{1}{2} \beta \sum_{ic,kc} V_{ic,kc} (\|\xi_{ic} - \xi_{kc}\|) + \beta M_{sk} c^2 | \Phi \rangle \\ &= \langle \Phi | \frac{c}{N} \sum_{ic=1}^N \alpha_{ic,l} \mathbf{P}_l + \beta M^* c^2 | \Phi \rangle + \langle \Phi | c \sum_{ic=1}^N \alpha_{ic,l} \pi_l + \frac{1}{2} \beta \sum_{ic,kc} V_{ic,kc} (\|\xi_{ic} - \xi_{kc}\|) + \beta M_{sk} c^2 - \beta M^* c^2 | \Phi \rangle. \end{aligned} \tag{27}$$

Only the second term contains the relative coordinates. The eigenvalue ω will be independent from this term if this latter is identically null, that is if:

$$M^* c^2 = \frac{\langle \Phi | c \sum_{ic=1}^N \alpha_{ic,l} \pi_l + \frac{1}{2} \beta \sum_{ic,kc} V_{ic,kc} (\|\xi_{ic} - \xi_{kc}\|) + \beta M_{sk} c^2 | \Phi \rangle}{\langle \Phi | \beta | \Phi \rangle}. \tag{28}$$

In this case:

$$\hbar\omega = \langle \Phi | \frac{c}{N} \sum_{ic=1}^N \alpha_{ic,l} \mathbf{P}_l + \beta M^* c^2 | \Phi \rangle \tag{29}$$

and M^* is the actual kinematic mass. Since Eq. (29) must hold for every stationary state, an “external” equation of motion is justified with this mass, having solutions that correspond to arbitrary superpositions of stationary states

having identical kinematic mass. As far as the dependence of Φ on the internal coordinates is concerned, it will have to satisfy the boundary condition $\Phi = 0$ for $\|\xi_{ic} - \xi_{kc}\| \geq c\theta_0/z, \forall ic, kc$.

Consequently, in the particle interaction matrix elements, plane wave factors of the $\exp(ip^\mu x_\mu)$ type will be implicit, where p_μ and x_μ are the four-momentum of a physical particle and its conjugate position variable, respectively; the integration on these factors will guarantee the preservation of the four-momentum with regard to the asymptotic states. In addition, these matrix elements will contain integrations on the coordinates of the centres of charge, which will describe the contact effects; in the following section, we shall only consider these coordinates. Finally, we note that posing the second term in the right-hand side of Eq. (22) equal to βV , if V does not depend on t , the square of this equation can be written in symbols as:

$$\begin{aligned} (i\hbar\partial_t + \beta V)(i\hbar\partial_t - \beta V) &= (i\hbar\partial_t)^2 - V^2 = E^2 - V^2 = P^2 + M^2 \\ \Rightarrow E^2 = P^2 + M^2 + V^2 = \Omega^2 + M^2 &\Rightarrow \frac{P}{\Omega} = \frac{1}{\sqrt{1 + (\frac{V}{P})^2}}. \end{aligned} \tag{30}$$

For $P > 0$ one therefore has $P < \Omega$. In other words, a portion of the total momentum of the hadron is transported by the internal colour field. For leptons, V is identically null and the impulse is transported entirely by the only centre of charge present.

5.2 Interactions

It is possible, in line with the assumptions made in this paper, to describe the strong interactions between hadrons with the customary standard second quantization formalism used, for example, in quantum chemistry. It is necessary, however, to adopt some precautions on the definition of the state of vacuum. One must firstly distinguish a leptonic vacuum $|0\rangle_L$ from a hadronic vacuum $|0\rangle_H$. A generic multi-lepton state will be defined by the relation:

$$\Lambda = \prod_i \lambda_i^+ \Lambda_i^+ |0\rangle_L = \lambda_1^+ |0\rangle_1 \lambda_2^+ |0\rangle_2 \lambda_3^+ |0\rangle_3 \dots |0\rangle_L = |\alpha_1 \alpha_2 \alpha_3 \dots\rangle |0\rangle_L. \tag{31}$$

In this relation, the operators:

$$\Lambda_i^+ = |0\rangle_i |0\rangle_L \langle 0|_L, \quad \Lambda_i = |0\rangle_L \langle 0|_L \langle 0|_i \tag{32}$$

create from (destroy in) the general leptonic vacuum the vacuum that corresponds to the micro-universe of the i th lepton. On this vacuum, the customary second quantization fermionic operators λ_i^+ and λ_i create and destroy, respectively, the centre of charge corresponding to the lepton i . It is assumed that $\langle 0|_H |0\rangle_L = 0$, so that the fermionic sector and the baryonic one are disconnected. The leptonic second quantization Hamiltonian is simply:

$$\hat{H} = \sum_{i,t,s} \langle t, i | h_i | s, i \rangle \lambda_{t,i}^+ \lambda_{s,i}, \tag{33}$$

where the index i runs along the leptons and h_i is the free Hamiltonian given by the first and third terms of Eq. (22), in which ic is fixed.

A multi-hadron state will be defined by the relation:

$$B = \prod_{i,j(i)} a_{ij}^+ B_i^+ |0\rangle_H = \prod_{j(1)} a_{1j}^+ |0\rangle_1 \prod_{j(2)} a_{2j}^+ |0\rangle_2 \dots |0\rangle_H = |\alpha_1 \alpha_2 \alpha_3 \dots\rangle_1 |\alpha_1 \alpha_2 \alpha_3 \dots\rangle_2 \dots |0\rangle_H \tag{34}$$

in which the operators:

$$B_i^+ = |0\rangle_i |0\rangle_H \langle 0|_H, \quad B_i = |0\rangle_H \langle 0|_H \langle 0|_i \tag{35}$$

create from (destroy in) the general hadronic vacuum the vacuum that corresponds to the micro-universe of the i th hadron. On this vacuum, the customary second quantization fermionic operators a_j^+ and a_j create and destroy, respectively, the centres of charge (quarks) that correspond to the hadron i .

The time evolution of the state (34) will be governed by the four-term Hamiltonian:

$$\hat{H} = \hat{H}_{\text{free}} + \hat{H}_{\text{exchange}} + \hat{H}_{\text{pairs}} + \hat{H}_{BM}. \tag{36}$$

The first term is the free evolution term:

$$\hat{H}_{\text{free}} = \sum_{i,t,s} \langle t, i | h_i | s, i \rangle a_{t,i}^+ a_{s,i} + \frac{1}{2} \sum_{i,t,s,t',s'} \langle t', i; s', i | V | t, i; s, i \rangle a_{s,i}^+ a_{t,i}^+ a_{t,i} a_{s,i} \tag{37}$$

in which the index i runs along the hadrons; h_i is given by the first and third terms of Eq. (22), while the second term of Eq. (37) comes from the second term of Eq. (22). To keep the notation from being heavy, from here on we shall write:

$$t = \bar{s} \tag{38}$$

to indicate that the t and s states are characterized by the same colour (anticolour) and flavour (antiflavour). Instead we shall write:

$$t = \bar{s} \tag{39}$$

to indicate that they are characterized by opposite colour and flavour states.

The second term of Eq. (36) describes the exchange of membership of two quarks, that is the fact that one of them, belonging to hadron i , moves to hadron j , and another, belonging to hadron j , moves to hadron i . This term is written:

$$\hat{H}_{\text{exchange}} = \frac{1}{2} \sum_{i,j \neq i,t,s,\bar{t},\bar{s}} \langle \bar{t}, i; \bar{s}, j | | t, j; s, i \rangle a_{s,j}^+ a_{t,i}^+ a_{t,j} a_{s,i}, \tag{40}$$

where brackets indicate the coefficients of sum. The third term describes the creation/annihilation of quark–antiquark pairs in the form of really neutral mesons:

$$\hat{H}_{\text{pairs}} = \sum_{i,j \neq i,t,s,\bar{t},\bar{s}} \langle \bar{t}, i; \bar{s}, i; t, j | | s, j \rangle a_{s,i}^+ a_{t,i}^+ B_i^+ a_{t,j}^+ a_{s,j} + h.c. \tag{41}$$

The fourth and final term describes the conversion of three mesons into a baryon and an antibaryon, as well as the inverse process:

$$\begin{aligned} \hat{H}_{BM} = & \sum_{\substack{i \neq j, j \neq k, i \neq k \\ l, m \\ s, t, u, v, w, z \\ \bar{s}, \bar{t}, \bar{u}, \bar{v}, \bar{w}, \bar{z}}} \langle \bar{s}, m; \bar{u}, m; \bar{w}, m; \bar{t}, l; \bar{v}, l; \bar{z}, l; | | s, i; t, i; u, j; v, j; w, k; z, k \rangle \\ & a_{s,m}^+ a_{u,m}^+ a_{w,m}^+ B_m^+ a_{t,l}^+ a_{v,l}^+ a_{z,l}^+ B_l^+ B_k a_{z,k} a_{w,k} B_j a_{v,j} a_{u,j} B_i a_{t,i} a_{s,i} + h.c. \end{aligned} \tag{42}$$

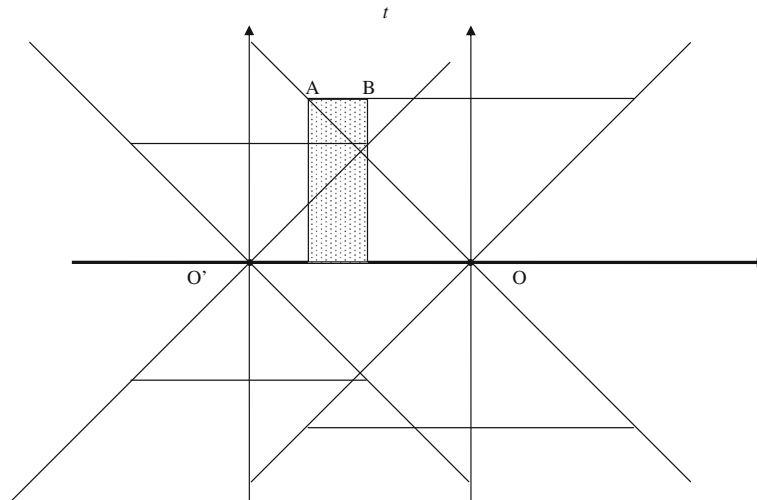


Fig. 8 In the contemporaneity space region of O and O' between the perpendiculars lowered from points A and B , quarks belonging to one of the two hadronic micro-universes can “trespass” into the other micro-universe

in which it is understood that the s, u, w states are coloured, while states t, v, z are anticoloured or vice versa, and the total colour is white in both cases. As can be seen, starting from “well-formed” hadrons ($q\bar{q}, qqq, \bar{q}\bar{q}\bar{q}$) the Hamiltonian (36) produces well-formed hadrons.

As regards the coefficients of sums in Eqs. (40), (41) and (42), we shall make the assumption that they are determined solely by geometric superposition and that no physical factors are involved. For example, as regards Eq. (40):

$$\langle \tilde{t}, i; \tilde{s}, j | | t, j; s, i \rangle = \langle \tilde{t}, i; \tilde{s}, j | t, j; s, i \rangle = \int \int dx_1 dx_2 \psi_{\tilde{s}, j}^*(x_1) \psi_{\tilde{t}, i}^*(x_2) \psi_{t, j}(x_2) \psi_{s, i}(x_1) \quad (43)$$

It must be noted that the states with the same coordinate belong to different micro-universes, so that even in the case that they belong to an orthonormal basis the result of integration is not, as is usual (and as one would have by eliminating the i, j indices), the product of two Kronecker symbols. The integrand is null everywhere except in the spatial region of superposition of the two micro-universes. To clarify this concept, let us consider Fig. 8, in which the light cones and the horizons of the two micro-universes are considered. The tangent points O and O' are assumed to be simultaneous, and the time axes are assumed to be parallel to the time axis of the frame of reference of the interaction centre of mass, exactly as would be the case in a quantum jump. In these conditions, the quarks belonging to the first hadron, having de Sitter radius $c\theta_0/z'$, are time segments extended from the contemporaneity plane to the horizon, with a spatial position that lies between $-c\theta_0/z$ and $+c\theta_0/z$ around O . Similarly, the quarks belonging to the second hadron, having de Sitter radius $c\theta_0/z'$, are time segments extended from the contemporaneity plane to the horizon, with a spatial position that lies between $-c\theta_0/z'$ and $+c\theta_0/z'$ around O' . The region on the contemporaneity space of O and O' that lies between the perpendiculars lowered from points A and B will be the region that contributes to integral (43). The integral will depend on the distance of O and O' , on z and z' . In any case, points O and O' are normally delocalized in accordance with the wavefunction of the two hadrons.

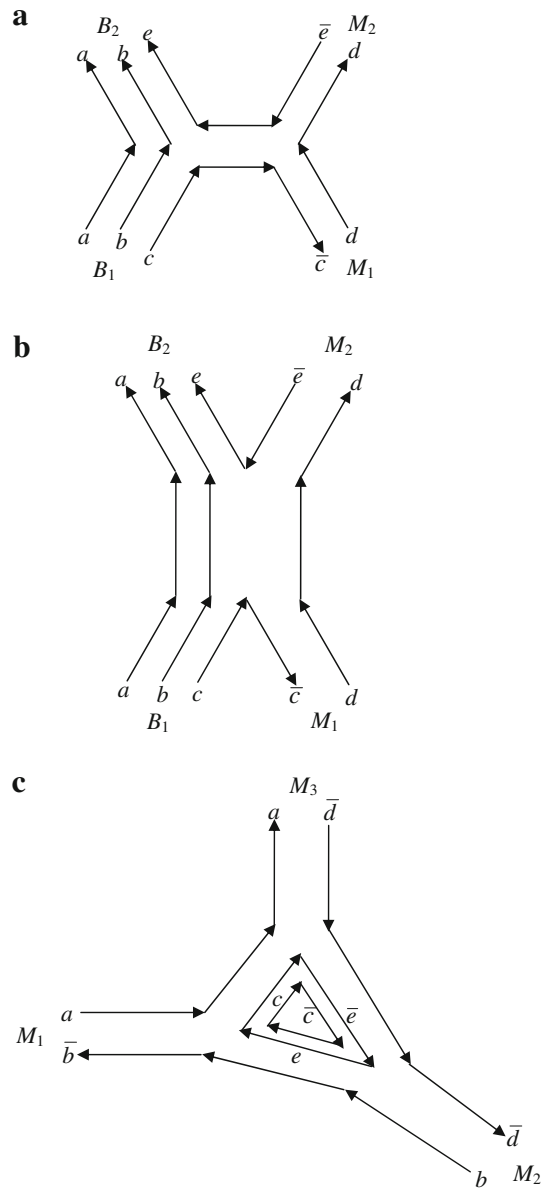
In Fig. 9a–c some examples are described of processes mediated by the exchange of virtual hadrons that should be possible according to Eq. (36). For virtual hadrons, z must be delocalized (in other words, the mass is not defined and one has a quantum superposition of de Sitter geometries). An interaction channel requiring a defined z should therefore tend to be less favoured; this is what happens if the final state is created by Eq. (41) and the initial state is annihilated by Eq. (41), without other concurrent processes (according to the Okubo–Zweig–Iizuka rule).

Needless to say, the equations proposed in this section are nothing more than a description of the concepts set forth in this paper and must be taken as a sort of toy model.

Fig. 9 a Quark-line diagram of the process:
 $B_1(abc) + M_1(\bar{c}d) \rightarrow$ (pair creation) $B_1(abc) + M_1'$
 $(\bar{c}d) + M_2'(e\bar{e}) \rightarrow$ (exchange) $B_1'(abc) + M_3$
 $(\bar{c}e) + M_2(d\bar{e}) \rightarrow$ (exchange) $B_2(abe) + M_5$
 $(\bar{c}c) + M_2(d\bar{e}) \rightarrow$ (pair annihilation) $B_2(abe) + M_2(d\bar{e})$.

b Quark-line diagram of the process: $B_1(abc) + M_1(\bar{c}d) \rightarrow$ (exchange) $B_1'(abd) + M_1'(\bar{c}c) \rightarrow$ (pair annihilation) $B_1'(abd) \rightarrow$ (pair creation) $B_1'(abd) + M_2'(e\bar{e}) \rightarrow$ (exchange) $B_2(abe) + M_2(d\bar{e})$.

c Quark-line diagram of the process:
 $M_1(a\bar{b}) + M_2(b\bar{d}) \rightarrow$ (double pair cr.)
 $M_1(a\bar{b}) + M_2(b\bar{d}) + M_1'(e\bar{e}) + M_2'(\bar{c}c) \rightarrow$ (baryon-meson coupling) $B_1(aec) + \bar{B}_2(\bar{b}\bar{c}\bar{e}) + M_2(b\bar{d}) \rightarrow$ (exchange) $B_1'(e\bar{b}c) + \bar{B}_2(\bar{b}\bar{c}\bar{e}) + M_3(a\bar{d}) \rightarrow$ (baryon-meson coupling) $M_4'(e\bar{e}) + M_5'(b\bar{b}) + M_6(\bar{c}c) + M_3(a\bar{d}) \rightarrow$ (triple pair annihilation) $\rightarrow M_3(a\bar{d})$

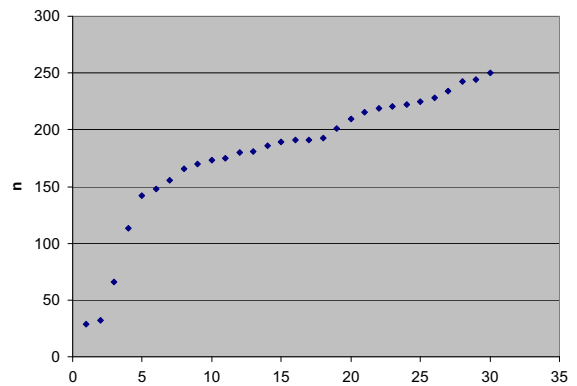


6 Origin of Regge trajectories

Let us now analyse in particular how a Regge trajectory originates. Using qualitative arguments, we will attempt to illustrate the genesis of the trajectories starting from de Sitter micro-universes.

When the hadron exits from the interaction sphere of radius $c\theta_0$ where it has been formed, it has a definite skeleton mass m , where $\theta_0/z = \hbar/mc^2$ (the effective mass will also contain contributions from internal degrees of freedom and perturbative effects). The newly formed mass m is therefore associated with a moment of inertia $mc^2\theta_0^2$ with respect to the centre of the sphere. The natural angular pulsation of the new-born hadron is z/θ_0 , or more generally its integer submultiple $z/(n\theta_0)$, so that the quantity of angular momentum transferred from the interaction vertex to the hadron is given by:

Fig. 10 Orbital Regge trajectory slopes ordered by increasing values of n (Eq. 45). Data from Ref. [42]. The linear piecewise behaviour is apparent, in accordance with a quantized variation of n



$$\Delta J = \Delta \left[(mc^2\theta_0^2) \left(\frac{z}{n\theta_0} \right) \right] = \Delta \left[(mc^2\theta_0^2) \left(\frac{mc^2}{n\hbar} \right) \right] = \Delta \left[(mc^2)^2 \left(\frac{\theta_0^2}{n\hbar} \right) \right]. \tag{44}$$

Bearing in mind that $J = j\hbar$ we obtain, for n constant:

$$(mc^2)^2 = jn \left(\frac{\hbar}{\theta_0} \right)^2 + \sigma, \tag{45}$$

where σ , which can be positive, zero or negative, represents the intercept of the trajectory [41]. Pulsations which are greater than z/θ_0 obviously have no relevance, because the hadron temporal localization makes the higher harmonics lose their physical meaning. Pulsations which are lower than z/θ_0 should be its submultiples, because angular motion is sampled with a maximum frequency given by the inverse of the micro-universe de Sitter time. In other words, z/θ_0 becomes a Nyquist frequency. Equation (45) defines a rectilinear Regge trajectory whose slope is quantized, because n is an integer. The function $z(j)$ fixes n as a quantum number related to flavour composition and parity. For a discussion about experimental evidences of quantization the reader should consult Ref. [42], from which data Fig. 10 is derived.

7 Veneziano and Virasoro amplitudes

We shall now focus on a particular interpretation of the classical four-point Veneziano and Virasoro amplitudes [18, 19]. Currently, these amplitudes (valid at tree level) are deduced using string theory methods and details of this derivation can be found in any textbook. We propose here a possible different interpretation, based on the chronon concept. The central idea in the formulation of these two amplitudes is the exchange of Regge trajectories having the form:

$$\alpha(s) = \alpha(0) + \alpha' s, \tag{46}$$

where $\alpha(0)$ is the intercept and α' is the slope, which is here assumed to be the same for all the trajectories. As we have seen, this position poses problems from an experimental point of view, and we shall take it as a valid approximation for a restricted set of mutually close trajectories. By adopting the $(+, -, -, -)$ signature for the spacetime metric, the Mandelstam variables relating to a collision with two incoming particles (with p_1, q_1 four-momentum) and two outgoing particles (with p_2, q_2 four-momentum) are the following:

$$\begin{aligned} s &= (p_1 + q_1)^2 \\ t &= (p_1 - p_2)^2 \\ u &= (p_1 - q_2)^2 \end{aligned} \quad (47)$$

with the constraint:

$$s + t + u = \sum_{i=1}^4 m_i^2, \quad (48)$$

where m_i is the mass of the i th incoming-outgoing particle. Equations (46) and (48) are consistent with the normalization condition:

$$\alpha(s) + \alpha(t) + \alpha(u) = 2 \quad (49)$$

adopted for the Veneziano amplitude. This is written in the symmetrical form:

$$A(s, t, u) = V(s, t) + V(t, u) + V(s, u) \quad (50)$$

with:

$$V(s, t) = \int_0^1 x^{-\alpha(s)}(1-x)^{-\alpha(t)} dx = B(1-\alpha(s), 1-\alpha(t)) \quad (51)$$

that is, Euler's beta function, or also:

$$V(s, t) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}. \quad (52)$$

By adopting the normalization:

$$\alpha(s) + \alpha(t) + \alpha(u) = -1 \quad (53)$$

Equation (52) becomes:

$$V(s, t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}. \quad (54)$$

For the Virasoro amplitude, on the other hand, the normalization used is:

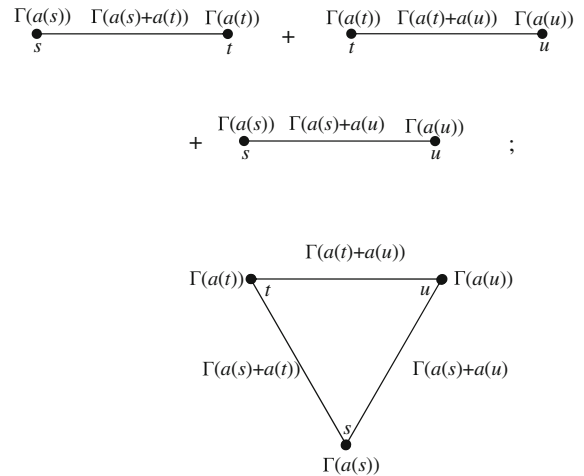
$$\frac{1}{2}[\alpha(s) + \alpha(t) + \alpha(u)] = -1. \quad (55)$$

It takes the form:

$$\bar{A}(s, t, u) = \frac{\Gamma(-\alpha(s)/2)\Gamma(-\alpha(t)/2)\Gamma(-\alpha(u)/2)}{\Gamma(-\alpha(s)/2 - \alpha(t)/2)\Gamma(-\alpha(t)/2 - \alpha(u)/2)\Gamma(-\alpha(s)/2 - \alpha(u)/2)}. \quad (56)$$

Let us now attempt to see how Eq. (51) can be reinterpreted. We firstly observe that the α terms that appear in the exponents are nothing other than continuous generalizations of the total spin eigenvalue j . In each channel (s ,

Fig. 11 Graphs of the Veneziano (above) and Virasoro (below) amplitudes. The symbol a denotes $-\alpha$ in the Veneziano case, $-\alpha/2$ in the Virasoro case



t or u), particles with j spin are exchanged. Let us mention the meaning of this eigenvalue: in the time interval $2\theta_0/z$, where z is proportional here to $s^{1/2}$, $t^{1/2}$, $u^{1/2}$, respectively, the particle “rotates around itself” $2j$ times. We say that the particle is made up of $2j$ turns; this will be the number of turns associated with the complete de Sitter micro-universe (past and future section) in a quantum jump. The number of turns is a relativistic invariant, and particles with different number of turns are exchanged in s , t or u channels, even though the exchanged action is the same.

Equation (51) highlights a distribution of spins, and therefore of “turns”, between two channels: s and t . One can imagine the existence of a box from which events are extracted in a predetermined number. A fraction x of these events forms what we call “a turn on the channel s ”; the remaining fraction $1 - x$ forms what we call “a turn on the channel t ”. We can say that each turn is coupled to the s channel with the amplitude $1/x$, and the t channel with the amplitude $1/(1 - x)$. Each of these amplitudes decreases with the increase of the fraction of events (or “attempts”) required for the appearance of a turn on the corresponding channel. Thus, the amplitude associated with a concatenation of j_1 independent turns on the s channel and a concatenation of j_2 independent turns on the t channel is $x^{-j_1} (1 - x)^{-j_2}$. The number of turns associated with the past (or future) section only of a quantum leap where a particle of spin j is involved is, as we have said, j . Therefore, this amplitude represents the exchange of particles with spin j_1 , j_2 (Regge trajectories), respectively, on s and t channels [we consider only a section of the quantum jump because here we are interested in amplitudes, not in projector]. We thus have the integrand of Eq. (51), in which the exponents j_1 , j_2 are $\alpha(s)$, $\alpha(t)$, respectively.

We have tacitly assumed that the variable x was deterministic. However, x is not fixed by the Mandelstam variables that define the kinematics of the interaction, as it is completely independent from these. Thus, it seems reasonable that x is subject to quantum uncertainty. If this supposition is correct the total amplitude of the process will thus be the sum of the compound amplitudes $x^{-\alpha(s)}(1 - x)^{-\alpha(t)}$ on the different values of x . This integration operation precisely coincides with Eq. (51). There are no statistical weights for the integration variable x because it is uniformly distributed, given the equivalence in principle of the two channels.

Equation (51) is equivalent to Eq. (54) which, when compared with the first graph in Fig 11, provides the function $\Gamma(-\alpha(a))$ as a factor of the vertex a and $1/\Gamma(-\alpha(a) - \alpha(b))$ as a factor of the edge ab , with $a, b = s, t, u$. On this basis, it is possible to deduce the Virasoro amplitude, as an amplitude associated with the second graph in Fig. 11. Naturally, the new normalization (55) must be kept in mind.

The concept of “concatenation of turns” to make a particle derives from that of quantum jump; thus, it cannot be traced to the Hamiltonian developed in Sect. 5. In other words, according to the present interpretation Veneziano amplitudes can be considered as non-Hamiltonian aspects of strong interaction.

8 Conclusions

In this paper, we have proposed a model of elementary particles based on the assumption that the collapse of the wavefunction is a genuine physical phenomenon that can be identified with the quantum jump undergone by the particle in the course of a real interaction with other particles. This process coincides with a *finite* exchange of energy and information [43], governed by a new universal constant, which is the chronon. We have then explored some classical ideas of particle physics (their classification in leptons, mesons and baryons, the confinement of quarks and gluons, Regge trajectories, Veneziano and Virasoro amplitudes) from this point of view. Even though this proposal is similar to models of particle based on de Sitter micro-universes appeared some time ago in the literature [44,45], it differs from them for the role played by a timeless background [46].

Some aspects of the model still pose problems and require further investigation. For example, it is clear that the introduction of the colour degree of freedom in terms of this presentation leads to a description of colour forces that is incompatible with a local gauge theory such as QCD. Nor have detailed calculations been performed as yet of the scattering processes based on the principles of this model; these and other problems need to be developed further and must be left open for the time being. What is interesting, actually, is an approach that places leptons and hadrons on an equal footing, introduces ab initio confinement and seems to lead to a finite mass spectrum (induced by the finite value of the chronon).

In this article we intended to discuss the potential impact, on particle physics, of the assumption that interaction vertices are phenomena of objective reduction of the wavefunction. Phenomena which can be interpreted as the time localization of particles outgoing from vertices, on a time scale defined by the chronon. These quantum leaps thus become non-Hamiltonian aspects of interaction and their presence implies the incompleteness of the Hamiltonian and unitary description offered, for example, from the standard model. At this point the question arises whether it is possible an integration between the two aspects, Hamiltonian and not, within a single coherent descriptive model of interaction. We cannot say to be reached this unification, or if the conflict found with the QCD can be removed by subsequent refinements of the model. However, it is also possible that this conflict could turn itself in a resource. Indeed, the strong interaction is a contact interaction, meaning that quark exchange and their creation/annihilation in a strong process take place within the interaction vertex whose radius is (experimentally) in the order of 10^{-13} cm. Since this radius is about $c\theta_0$ (i.e. c times the temporal localization scale), it is possible in our approach to account for the confinement and at the same time explain the energy scale E_0 in Eq. (20) [considering the experimental uncertainties, the value derived from the model (70 MeV) is consistent with the experimental one for four to five flavours, which is in the order of 180–200 MeV]. QCD, for its part, is currently forced to take this parameter (usually denoted by Λ) from experiment and seems unable to provide demonstrations of confinement from first principles.

Similarly, the predictions of QCD on the existence of glueballs (which is a necessary consequence of the existence of coloured gluons) have not had to date certain experimental confirmation despite 30 years of research. This could be an indication in favour of the non-existence of gluons as individual entities within the hadron, that would support the model proposed here. Another topic addressed in this article was the quantization of the slopes of the Regge trajectories, a phenomenon of which there are currently no ab initio derivations within the non-perturbative QCD.

Leaving open the question of its compatibility with QCD, the model still provides useful additions to the standard model. When $c\theta_0$ is assumed to be the classical electron radius, the energy scale $E_0 = \hbar/\theta_0$ amounts to 70 MeV and this implies the correct Hagedorn temperature of 160 MeV [11]. This connection between leptonic and hadronic worlds mediated by the ubiquitous constant θ_0 seems reinforced by the fact that E_0 is the well-known “quantum of mass” of which the masses of the muon, tau and lighter hadrons (pions, kaons, etc.) are almost exactly half-integer multiples; a circumstance analysed by Nambu as early as in 1952 [47]. These connections do not find a clear explanation in the standard model but become meaningful when it is integrated with the chronon θ_0 . The consequences of introducing the chronon on concepts of inertia and mass [48,49], suggest potential connections with a seemingly distant problem: quantum gravity. In the description proposed here, leptons and hadrons are processes with a minimal temporal extension θ , which does not exceed θ_0 . Naturally, this does not mean that the external fields (as distinct from the gravitational field) cannot “see” point sources such as quarks within a hadron or single lepton. In fact, these fields are coupled to the charges carried by quarks and leptons. From this point of

view, there is no upper limit to the energy transferred in the interaction. Things are very different for gravitation. In general relativity, gravitation coincides locally with inertia, and is thus coupled to the particle inertia. However, as we have seen, the particle inertia is defined only on the time scale θ . On time scales lower than θ , the particle inertia (quantitatively represented by its mass) is a meaningless concept. For instance, no mass for quarks exists in a strict sense [from Eqs. (21), (22)]; moreover, inside micro-universes only a purely inertial, de Sitter geometry holds. Consequently, the particle must decouple from the gravitational field for energy exchanges greater than \hbar/θ . In other words, above Hagedorn's temperature, matter must gradually decouple from gravitation.

Any confirmation of these conjectures would have obvious spin-off effects on widely different research fields, from singularities in black holes (which presumably would not be formed) to the meaning of the Planck scale. Since the latter is nothing but a different guise of the gravitational constant G , it might preserve an informational role in connection with G , though without necessarily being of significance as a "new physics" scale in gravitational interaction processes [46].

Acknowledgements The authors wish to thank an anonymous referee for notes and comments that have improved the quality of this paper.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

Appendix

Let us pose:

$$(c\tau)^2 = c^2t^2 - x^2 - y^2 - z^2, \quad (57)$$

where the squares on the right-hand side are all taken as positive (space coordinates and ct are real). The expression (57) is positive within the "physical" region internal to the light-cone.

Let us consider, in the Minkowskian five-dimensional space $\{ct, x, y, z, c\theta\}$ with signature $[1, -1, -1, -1, -1]$, the four-dimensional manifold:

$$(c\tau)^2 - (c\theta)^2 = -(c\theta_0)^2, \quad (58)$$

where θ_0 is a real and positive constant. The five-dimensional light-cone with origin $(0, 0, 0, 0, 0)$ is:

$$(c\tau)^2 - (c\theta)^2 = 0. \quad (59)$$

We assume the point-event of observation to be placed in $ct = x = y = z = 0$; in coincidence with this point is then $c\tau = 0$. From Eq. (58) the relation $(c\theta)^2 = (c\theta_0)^2$ follows, which is the equation of the four-dimensional space tangent to the manifold (58) in the point-event of observation. When this relation is substituted in Eq. (59), the equation of the intersection of this space and light-cone (59) is derived. This equation is $(c\tau)^2 = (c\theta_0)^2$ that is:

$$\tau = \pm\theta_0, \quad (60)$$

or:

$$c^2t^2 - x^2 - y^2 - z^2 = (c\theta_0)^2. \quad (61)$$

Equations (60) and (61) represent a quadric with two sheets placed at the chronological distance, respectively, $+\theta_0$, $-\theta_0$ from the observer (when measured in its rest frame).

In five-dimensional space the observer $(0, 0, 0, 0, \pm c\theta_0)$ is joined to the origin $(0, 0, 0, 0, 0)$ by a vector of spatial kind of length $c\theta_0$. A coordinate transformation changing a observer point-event in an other must leave unchanged the spatial character and length of this vector. This condition identifies a specific transformation group that is the five-dimensional Lorentz group, or de Sitter group.

The de Sitter group leaves unchanged the entire description represented by Eqs. (58)–(61). de Sitter transformations become singular in correspondence of the observer light-cone $c\tau = 0$ and quadric (61); these manifolds thus form the boundary of the region physically accessible by the observer. Let us remark that posing $(c\theta_0)^2$, instead of $(-c\theta_0)^2$, in Eq. (58) the Eq. (60) takes the form $\tau = \pm i\theta_0$, against the assumed positivity of Eq. (57) within the observer light-cone.

Equation (61), if considered in itself, represents a three-dimensional manifold in a four-dimensional Minkowsky space. It is considered in this context as the three-dimensional boundary of a four-dimensional de Sitter space $c^2t^2 - x^2 - y^2 - z^2 \leq (c\theta_0)^2$. See also Ref. [25].

References

1. Von Neumann, J.: *Mathematische Grundlagen der Quanten Mechanik*. Springer, Berlin (1932). [English translation by Beyer, R.T.: *Mathematical Foundations of Quantum Mechanics*. Princeton University Press, Princeton (1955)]
2. Bub, J.: Von Neumann's projection postulate as a probability conditionalization rule in quantum mechanics. *J. Philos. Logic* **6**(1), 381–390 (1977)
3. Khrennikov, A.: Von Neumann and Luders postulates and quantum information theory. *Int. J. Quantum Inform.* **7**, 1303–1311 (2009)
4. Wigner, E.P.: The problem of measurement. *Am. J. Phys.* **31**, 6–15 (1963)
5. Shimony, A.: Role of the observer in quantum theory. *Am. J. Phys.* **31**, 755–773 (1963)
6. Schrödinger, E.: Die gegenwärtige situation in der quantenmechanik. *Naturwissenschaften* **23**, 807–812, 823–828, 844–849 (1935)
7. Ghirardi, G.C., Rimini, A., Weber, T.: Unified dynamics for microscopic and macroscopic systems. *Phys. Rev. D* **34**, 470–491 (1986)
8. Everett, E.: "Relative state" formulation of quantum mechanics. *Rev. Mod. Phys.* **29**, 454–462 (1957)
9. Zeh, H.D.: On the interpretation of measurement in quantum theory. *Found. Phys.* **1**, 69 (1970)
10. Zurek, W.H.: Pointer basis of quantum apparatus: into what mixture does the wave packet collapse? *Phys. Rev. D* **24**, 1516 (1981)
11. Licata, I., Chiatti, L.: Timeless approach to quantum jumps. *Quanta* **4**, 10–26 (2015)
12. Bohr, N.: On the constitution of atoms and molecules. *Philos. Mag. Ser. 6*(26), 1–25, 476–502, 857–875 (1913)
13. Cramer, J.: The transactional interpretation of quantum mechanics and quantum nonlocality. [arXiv:1503.00039](https://arxiv.org/abs/1503.00039) [quant-ph]
14. Colosi, D., Rovelli, C.: What is a particle? *Class. Quantum Grav.* **26**, 025002 (2009)
15. Chiatti, L.: Is Bohr's challenge still relevant? In: Licata, I. (ed.) *Beyond Peaceful Coexistence. The Emergence of Space, Time and Quantum*, pp. 545–557. Imperial College Press, London (2016)
16. Chodos, A., Jaffe, R.L., Johnson, K., Thorn, C.B., Weisskopf, V.: New extended model of hadrons. *Phys. Rev. D* **9**, 3471–3496 (1974)
17. Bergquist, J.C., Hulet, R.G., Itano, W.M., Wineland, D.J.: Observation of quantum jumps in a single atom. *Phys. Rev. Lett.* **57**(14), 1699–1702 (1986)
18. Veneziano, G.: Construction of a crossing symmetric, Regge-behaved amplitude for linearly rising trajectories. *Il Nuovo Cimento* **57**, 190–197 (1968)
19. Virasoro, M.: Alternative construction of crossing-symmetric amplitudes with Regge behavior. *Phys. Rev.* **177**, 2309–2314 (1969)
20. Chiatti, L.: Choosing the right relativity for QFT. In Licata, I., Sakaji, A. (eds.) *Vision of Oneness*, pp. 365–398. Aracne Editrice, Rome (2011). [arXiv:0902.1293](https://arxiv.org/abs/0902.1293) [physics.gen-ph]
21. Chiatti, L.: Elementary Particles as Signs. *VS* **118**, 105–117 (2014). <http://philsci-archives.pitt.edu/id/eprint/10339>
22. Chiatti, L.: Bootstrapping the QFT, a new road to the elementary particles spectrum. *El. J. Theor. Phys.* **9**, 33–48 (2012)
23. Blanchard, P., Fortunato, S., Satz, H.: The Hagedorn temperature and partition thermodynamics. *Eur. Phys. J. C* **34**, 361–366 (2004)
24. Rafelski, J.: Melting hadrons, boiling quarks. *Eur. Phys. J. A* **51**, 114 (2015)
25. Arcidiacono, G.: The de Sitter–Castelnuovo universe and elementary particle physics. *Gen. Rel. Grav.* **13**, 701–710 (1981)
26. Moschella, U.: The de Sitter and anti-de Sitter sightseeing tour. *Séminaire Poincaré* **1**, 1–12 (2005)
27. Arcidiacono, G.: A new "Projective relativity" based on the De Sitter universe. *Gen. Rel. Grav.* **7**, 885–889 (1976)
28. Recami, E.: Multi-verses, micro-universes and elementary particles (hadrons). [arXiv:physics/0505149](https://arxiv.org/abs/physics/0505149)
29. Recami, E., Castorina, P.: On quark confinement: hadrons as "strong black holes". *Lett. Nuovo Cim.* **15**(10), 347–350 (1976)

30. Caldirola, P.: The introduction of the chronon in the electron theory and a charged-lepton mass formula. *Lett. Nuovo Cim.* **27**(8), 225–228 (1980)
31. Arcidiacono, G.: The de Sitter universe and mechanics. *Gen. Rel. Grav.* **8**, 865–870 (1977)
32. Cacciatori, S., Gorini, V., Kamenshchik, A.: Special relativity in the 21st century. *Ann. Der Physik* **17**, 728–768 (2008)
33. Hinchliffe, L.: Quantum Chromodynamics, Chapter 9; PDG files, updated to Sept. 2005 <http://pdg.lbl.gov/2005/reviews/qcdrpp.pdf>
34. Bohm, D.: *Wholeness and the Implicate Order*. Routledge, London (1980)
35. Frescura, F.A.M., Hiley, B.J.: Algebras, quantum theory and pre-space. *Rev. Bras. Phys.* **14**, 49–86 (1984)
36. Bohm, D., Hiley, B.J., Stuart, A.E.G.: On a new mode of description in physics. *Int. J. Theor. Phys.* **3**, 171–183 (1970)
37. Bohm, D.: Quantum theory as an indication of a new order in physics. B, implicate and explicate order in physical law. *Found. Phys.* **3**, 139–168 (1973)
38. Bohm, D., Davies, P.G., Hiley, B.J.: Algebraic quantum mechanics and pregeometry. [arXiv:quant-ph/0612002](https://arxiv.org/abs/quant-ph/0612002) (2006)
39. Hiley, B.J.: Non-commutative Quantum Geometry: a re-appraisal of the Bohm approach to quantum theory. In: Elitzur, A., Dolev, S., Kolenda, N. (eds.) *Quo Vadis Quantum Mechanics?*, pp. 299–324. Springer, Berlin (2005)
40. Hagelstein, P.L., Chaudhary, I.U.: Kinematic mass of a composite in the many-particle Dirac model. [arXiv:1201.1488](https://arxiv.org/abs/1201.1488) [cond-mat.other]
41. Collins, P.D.B.: *An Introduction to Regge Theory and High Energy Physics*. Cambridge University Press, Cambridge (1977)
42. Chiatti, L.: A simple quantization rule for the slopes of Regge trajectories. *Phys. Ess.* **27**, 143–145 (2014)
43. Licata, I.: Transaction and non locality in quantum field theory. *EPJ Web Conf.* **70**, 00039 (2014)
44. Caldirola, P.: On a relativistic model of the electron. *Nuovo Cim.* **49 A**, 497–611 (1979)
45. Motoyoshi, A.: On the mass spectrum of the extended hadrons. *Lett. Nuovo Cim.* **44**(6), 379–382 (1985)
46. Licata, I.: In and out the screen. On some new considerations about localization and de-localization in Archaic theory. In: Licata, I. (ed.) *Beyond Peaceful Coexistence. The Emergence of Space, Time and Quantum*, pp. 559–577. Imperial College Press, London (2016)
47. Nambu, Y.: An empirical mass spectrum of elementary particles. *Prog. Theor. Phys.* **7**(5), 595–596 (1952)
48. Pavsic, M.: Discrete rest masses resulting from relativistically covariant massless field equations in 5 dimensions. *Lett. N. Cim.* **17**(2), 44–48 (1976)
49. de Sabbata, V., Gasperini, M.: Spontaneous symmetry breaking in a De Sitter metric background. *Lett. N. Cim.* **31**(7), 261–264 (1981)