Economic Theory Bulletin
https://doi.org/10.1007/s40505-024-00263-z

RESEARCH ARTICLE

Financial intermediation and efficient risk sharing in two-period lived OLG models

Paul Ritschel[1] · **Jan Wenzelburger**[1]

Received: 18 November 2023 / Accepted: 26 February 2024
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Abstract
This article investigates a two-period lived overlapping-generations (OLG) model that incorporates financial intermediation. A risk-neutral bank offers loan and deposit contracts that insure risk-averse agents against idiosyncratic income shocks. Agents prefer financial intermediation to capital markets if it provides efficient risk sharing. The analysis demonstrates that in any two-period lived OLG model in which productive capital is increasing in investment levels, financial intermediation, when implemented for the purpose of efficient risk sharing, cannot instigate business cycles or complex dynamics. The resulting dynamics is monotonic and qualitatively indistinguishable from the dynamics of the classical OLG model by Diamond (Am Econ Rev 55(5):1126–1150, 1965). Business cycles may only occur if banks offer inefficient contracts. Efficient contracts will, in general, not induce dynamically efficient growth paths.

Keywords Financial intermediation · Overlapping generations · Risk sharing · Business cycles · Loan contracts

JEL Classification D53 · E32 · E44 · G21 · O41

1 Introduction

Understanding the pace and patterns of economic growth is one of the central topics in macroeconomics. The empirical evidence that a well-functioning financial system is vital for economic development is plentiful, e.g. see Levine (1997) or the excellent

Paul Ritschel and Jan Wenzelburger contributed equally to this work.

✉ Paul Ritschel
paul.ritschel@wiwi.uni-kl.de

Jan Wenzelburger
jan.wenzelburger@rptu.de

[1] Faculty of Business Studies and Economics, University of Kaiserslautern-Landau, Gottlieb-Daimler-Str. 42, 67663 Kaiserslautern, Rhineland-Palatinate, Germany

Published online: 04 April 2024

reviews of the empirical literature by Levine (2005) and Aziakpono (2011). Over the past four decades, relatively few theoretical contributions have incorporated financial intermediation into growth models. These try to link the promotion of economic growth to the fundamental functions that financial intermediaries carry out in an economy, cf. Pagano (1993). The seminal contribution by Greenwood and Jovanovic (1990), for example, highlights how risk sharing and the informational advantage of financial intermediaries encourage high-yield investments and economic growth. Bencivenga and Smith (1991), to mention another important contribution, extends the Diamond and Dybvig (1983) view by demonstrating that liquidity provision induces savings behaviour of agents that enhances capital accumulation.

The literature on business cycles in OLG models with financial intermediation is relatively scarce. Williamson (1987) demonstrates that indivisibilities in investment projects may be a cause for business cycles. Smith (1998) finds that monopolistic financial intermediaries can increase the severity of existing business cycles. Azariadis and Smith (1998) show that bank-loan financed capital investments may generate business cycles if there is an adverse selection problem regarding the ability of the borrowers to honour their debt. Banerji et al. (2004) consider an OLG model in which loan and deposit contracts enable risk-averse agents to completely insure against idiosyncratic income shocks. They argue that risk sharing may expose the economy to endogenous fluctuations in the form of real-sector business cycles and conclude that the promotion of economic growth by financial intermediaries comes at the cost of the full variety of complex dynamics. Finally, the stochastic OLG model with financial intermediation developed in Gersbach and Wenzelburger (2003, 2008, 2012) exhibits persistent business cycles. Macroeconomic productivity shocks trigger the failure of individual production projects. This risk cannot be diversified away so that the model, unlike that in Banerji et al. (2004), has aggregate uncertainty.

This article addresses the extent to which efficient risk sharing can induce endogenous business cycles in two-period lived OLG models, which, in the absence of financial intermediation, are known to admit only monotonic growth. Following on from Banerji et al. (2004) by allowing for the standard class of intertemporal preferences used in that literature, we find that a collective bank can implement the efficient allocation by offering suitable loan and deposit contracts. These contracts provide complete risk sharing and must enlarge the disposable income in order to be accepted by agents. This income effect, which in Banerji et al. (2004) is deemed responsible for causing endogenous fluctuations, is a consequence of a mere incentive problem and it turns out that it does not alter the qualitative dynamics of the economy. The key feature of our model is that productive capital and thus capital income is increasing in investment levels. We demonstrate that financial intermediation which implements the efficient allocation and diversifies away idiosyncratic risk *does not* generate business cycles or even complex dynamics as the dynamics of the economy is always monotonic.

In our framework, agents' incentive compatibility and participation constraints are explicit. Our analysis reveals that even though the bank maximises agents' welfare, incentive problems remain so that the acceptance of an efficient contract is not as straightforward as one would expect. Contrary to Banerji et al. (2004), who argue that financial intermediation may generate a complex *backwards* dynamics, we will focus on the *forward* dynamics of the economy. The reason is that the usefulness of the backwards dynamics for a forward-time interpretation is limited and that the analysis is often restricted to a limited range of model parameters in the neighbourhood of a steady-state solution, e.g. see Grandmont (1989) and Medio and Raines (2006).

The remainder of this article is organised as follows. The next section lays out the basic model and all essential assumptions. In Sect. 3, we formulate the decision problems of both agents and the bank. We then introduce our notion of an efficient contract and establish its existence and uniqueness. Section 4 is dedicated to the dynamics induced by efficient contracts and contains our main results. Section 5 concludes.

2 Model prerequisites

We consider a two-period lived OLG model with discrete time $t = 0, 1, \ldots, \infty$. There is a single perishable good that can be consumed and invested. At the beginning of each period t, a new generation comprising a unit-mass continuum of homogeneous agents is born. Agents are risk-averse and live for two periods. Their intertemporal preferences over consumption are represented by a life-cycle utility function

$$U(c^1, c^2) := u(c^1) + v(c^2),$$

where $c^1, c^2 \geq 0$ denote youthful and old-age consumption, respectively.

Assumption 1 (*Preferences*) The utility functions $u, v : \mathbb{R}_+ \to \mathbb{R}$ are twice continuously differentiable, strictly increasing, strictly concave, and satisfy the Inada conditions.

A young agent may become an entrepreneur by undertaking a risky production project, which may either be successful or fail. The likelihood of success depends on the amount of capital invested and is determined by a *success function* $p : \mathbb{R}_+ \to (0, 1]$ that stipulates the success probability $p(I)$ of the capital investment $I \geq 0$. The uncertainty about the outcome of a project resolves one period after capital has been invested. A project generates a verifiable gross rate of return $\varrho > 0$ if successful and zero if it fails.[1] Invoking the law of large numbers, the productive capital stock of the economy is

$$\Omega(I) := p(I)I.$$

The properties of the success function p are of central importance to our analysis and are stated in terms of properties of Ω.

Assumption 2 (*Productive capital*) The function $\Omega : \mathbb{R}_+ \to \mathbb{R}_+$, defined by $\Omega(I) = p(I)I$, is twice continuously differentiable, strictly increasing, and concave.

[1] As an alternative interpretation, one may think of the entrepreneur as an investor who invests capital into a firm. A failed project is then equivalent to the default of a firm.

The important assumption for our results is that productive capital $\Omega(I)$ is strictly increasing in the investment level I. The concavity of Ω implies that the success probability $p(I)$ is non-increasing in I.[2] Such a choice features the economic intuition that large-scale projects are more likely to fail.

Example 1 The function $\Omega(I) = \frac{\kappa I}{1+I}$, where $0 < \kappa \leq 1$ is some constant, satisfies Assumption 2. The corresponding success probability $p(I) = \frac{\kappa}{1+I}$ is decreasing in I.

The production sector of the economy is perfectly competitive. A neoclassical technology with constant returns to scale transforms labour $N \geq 0$ and real capital $K \geq 0$ into output. Capital depreciates fully during production. We denote by $k := K/N$ the capital-labour ratio and by $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ the production function of the representative firm in intensive form.

Assumption 3 (*Technology*) The production function $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is thrice continuously differentiable, strictly increasing, strictly concave, and satisfies the Inada conditions. Moreover, it holds that

$$\frac{f''(k)k}{f'(k)} > -1 \quad \text{and} \quad \frac{f'''(k)k}{f''(k)} > -2 \quad \text{for all} \quad k \in \mathbb{R}_+.$$

The last two properties imposed on f in Assumption 3 imply that capital income $f'(k)k$ is strictly increasing and strictly concave in the capital-labour ratio k. These properties facilitate the existence and uniqueness of an efficient loan contract and are satisfied by many standard production functions in the literature.[3]

The young generation constitutes the workforce of the economy. Each young agent supplies one unit of labour inelastically to a perfectly competitive labour market. Labour and capital are paid their marginal products. The old generation is retired and receives capital income only. Given a capital investment I, the productive capital stock of the subsequent period is $k = \Omega(I)$ and is paid its marginal product $\varrho = f'(\Omega(I))$. The capital income of the old generation thus becomes

$$g(I) := f'(\Omega(I))\,\Omega(I). \tag{1}$$

The following properties of g are essential for our results.

Lemma 1 (Capital income) *Let Assumptions 2 and 3 be satisfied. Then capital income of the old generation $g : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, defined by (1), is strictly increasing and strictly concave with $g(0) = 0$.*

In the proof of Lemma 1, it is shown that under the hypotheses of Assumption 3, concavity of productive capital Ω is a sufficient condition for strict concavity of capital income g.

[2] Concavity of Ω implies $\frac{\Omega'(I)I}{\Omega(I)} = \frac{p'(I)I}{p(I)} + 1 \leq 1$ so that $p'(I) \leq 0$ for all $I > 0$.

[3] Assumption 3 is fulfilled by the Cobb–Douglas production function and by a wide range of parameterisations of the CES production function.

3 Financial intermediation

To transfer resources to the second period of their lives, young agents may invest part of their wage income into a production project which exposes them to the idiosyncratic risk of an old-age income shock. Agents may form an endogenous coalition in the form of a risk-neutral collective bank as a device to share this risk.[4] The bank offers young agents a loan contract (B_t, I_t, R_t), where $B_t \geq 0$ is the size of the loan, $I_t \geq 0$ is the capital investment into the project, and $R_t \geq 0$ is the gross interest rate on loans. Agents who accept a loan contract are protected by limited liability as they do not have to repay the loan in case their project fails.[5] To finance its loans, the bank raises deposits from young agents by offering a risk-free gross rate $r_t \geq 0$ on deposits.

3.1 Decision problems

Each young agent must decide whether to invest into a project by accepting a loan contract or to undertake the project without funding from the bank instead. Independently of her investment decision, however, a young agent is also allowed to deposit part of her wage income at the bank. The decision problem of a young agent is thus the following. Suppose the bank offers the loan contract (B_t, I_t, R_t) and the deposit rate r_t on savings in period t. Consider first the case in which the agent *accepts* the loan contract. Given her wage income w_t, the agent must decide on how much to consume and how much to save for retirement. By accepting the loan contract, her disposable income becomes $w_t^d := w_t + B_t - I_t$, so that youthful consumption is

$$c^1 = w_t^d - D, \tag{2}$$

where $D \geq 0$ is the amount saved and deposited at the bank. Old-age consumption is $c^{2g} \geq 0$ if the project is successful and $c^{2b} \geq 0$ if the project fails. Since agents have limited liability, the constraint for old-age consumption reads

$$\begin{cases} c^{2g} &= r_t D + \pi(I_t) - R_t B_t \\ c^{2b} &= r_t D \end{cases}, \tag{3}$$

where $r_t D$ are the proceeds from the deposits, $\pi(I_t) := f'(\Omega(I_t))I_t$ is the revenue from a successful project, and $R_t B_t$ is the loan repayment obligation.

The objective of a young agent is to maximise her expected utility of lifetime consumption. Inserting the budget constraints (2) and (3), the agent's objective function

[4] For further details on financial-intermediary coalitions, we refer to the paper by Boyd and Prescott (1986) and, for an overview, to Freixas and Rochet (2008).

[5] We tacitly assume that the bank possesses a monitoring technology that enables it to observe agents' investment behaviour and enforce the contract. A stipulated investment is a particular form of monitoring. For details we refer to the seminal contribution by Holmström and Tirole (1997) on the role of monitoring in settings with limited liability.

becomes

$$\max_{0\le D\le w_t^d} \; u(w_t^d - D) + p(I_t)\, v(r_t D + \pi(I_t) - R_t B_t) + (1 - p(I_t))\, v(r_t D). \tag{4}$$

Given a loan contract (B_t, I_t, R_t), a deposit rate r_t, and a wage rate w_t, a solution to (4) is given by the agent's savings function $S\mathbb{R}_+^5 \to \mathbb{R}_+$, which is defined by

$$\begin{aligned} S(w_t, B_t, I_t, R_t, r_t) \;:=\; & \operatorname*{argmax}_{0\le D\le w_t^d} \; u(w_t^d - D) + p(I_t)\, v(r_t D \\ & + \pi(I_t) - R_t B_t) + (1 - p(I_t))\, v(r_t D). \end{aligned}$$

Inserting S into the objective function in (4) yields the value function for Problem (4), which is denoted by

$$V(w_t, B_t, I_t, R_t, r_t). \tag{5}$$

Consider now the case in which the agent *rejects* the loan contract and undertakes the project without funding from the bank. To do so, she will invest the amount I^{A} into the project and deposit the amount D^{A} at the bank in order to safeguard old-age consumption against the failure of the project. Formally, the corresponding decision problem reads

$$\begin{aligned} & \max_{I^{\mathrm{A}}, D^{\mathrm{A}}} \; u(w_t - D^{\mathrm{A}} - I^{\mathrm{A}}) + p(I^{\mathrm{A}})\, v(r_t D^{\mathrm{A}} + \pi(I^{\mathrm{A}})) + (1 - p(I^{\mathrm{A}}))\, v(r_t D^{\mathrm{A}}) \\ & \text{s.t.} \quad I^{\mathrm{A}}, D^{\mathrm{A}} \ge 0 \;\text{ and }\; I^{\mathrm{A}} + D^{\mathrm{A}} \le w_t. \end{aligned} \tag{6}$$

The value function associated with Problem (6) is well defined and stipulates the agent's reservation utility, which for any given w_t and r_t is denoted by

$$U_{\mathrm{res}}(w_t, r_t).$$

The decision problem of the bank is the following. Since the bank is collectively owned, it offers a loan contract (B_t, I_t, R_t) and a deposit rate r_t so as to maximise the agent's expected utility V given in (5). Using the law of large numbers, the bank correctly anticipates that for any given I_t, the loan default rate is $1 - p(I_t)$. Therefore, the two feasibility constraints of the bank are the *profit constraint*

$$p(I_t) R_t B_t - r_t D_t \ge 0, \tag{PrC}$$

stating that bank profits must be non-negative, and the *resource constraint*

$$D_t \ge B_t, \tag{RC}$$

noting that the bank has no equity. Both the loan and the deposit contract have to be compatible with the young agent's savings behaviour. Since the amount saved is at the discretion of the agent, the bank has to fulfil the *incentive compatibility constraint*

$$D_t = S(w_t, B_t, I_t, R_t, r_t) \tag{IC}$$

in order to obtain the amount of deposits required in (RC). Finally, since an agent may decide to invest without funding from the bank, the loan contract must be designed in such a way that the agent prefers the loan contract to undertaking the project without the bank. Formally, this *participation constraint* reads

$$V(w_t, B_t, I_t, R_t, r_t) \geq U_{\text{res}}(w_t, r_t), \tag{PC}$$

that is, the expected utility of accepting both the loan and the deposit contract is at least as high as the reservation utility.

Given the wage rate w_t, the decision problem of the bank thus takes the form

$$\begin{aligned} &\max_{B,I,R,r\geq 0} \; V(w_t, B, I, R, r) \\ &\text{s.t. } \; p(I)RB - rS(w_t, B, I, R, r) \geq 0, \quad S(w_t, B, I, R, r) \geq B, \\ &\text{and } \; V(w_t, B, I, R, r) \geq U_{\text{res}}(w_t, r). \end{aligned} \tag{7}$$

3.2 Efficient allocations

We next establish the efficient allocation that a myopic social planner would implement. Given the wage rate w_t, the planner's objective in period t is to maximise the welfare of the generation born in t.[6] Applying the law of large numbers, the mass of successful agents in the subsequent period is $p(I)$, while the mass of failed agents is $1 - p(I)$. Capital income in the subsequent period is $g(I)$, independently of the state of nature. The social planner's maximisation problem thus becomes

$$\begin{aligned} &\max_{I,c^1,c^{2g},c^{2b}} \; u(c^1) + p(I)\, v(c^{2g}) + (1 - p(I))\, v(c^{2b}) \\ &\text{s.t. } \; I, c^1, c^{2g}, c^{2b} \geq 0, \quad c^1 + I \leq w_t, \\ &\text{and } \; p(I)\, c^{2g} + (1 - p(I))\, c^{2b} \leq g(I). \end{aligned} \tag{8}$$

The solution $(I_t^\star, c_t^{1\star}, c_{t+1}^{2g\star}, c_{t+1}^{2b\star})$ to Problem (8) will be referred to as *efficient allocation*. It is provided by the following proposition.

Proposition 1 (Efficient allocation) *Let Assumptions 1–3 be satisfied and $w_t > 0$ be given. Then Problem (8) admits a uniquely determined solution $(I_t^\star, c_t^{1\star}, c_{t+1}^{2g\star}, c_{t+1}^{2b\star})$, where the efficient consumption plan is*

$$c_t^{1\star} = w_t - I_t^\star, \qquad c_{t+1}^{2g\star} = c_{t+1}^{2b\star} = g(I_t^\star)$$

[6] Since our research question is not concerned with intergenerational externalities as, for example, in Ennis and Keister (2003), a myopic social planner is justified.

and the efficient investment level $0 < I_t^\star < w_t$ *solves*

$$\max_{0 \le I \le w_t} u(w_t - I) + v(g(I)). \tag{9}$$

Proposition 1 states that if resources are allocated efficiently, the young generation consumes its wage income less the efficient investment level, while the old generation consumes aggregate capital income, which is perfectly smoothed out across both possible states of nature. Observe that the uniqueness of efficient allocations hinges on the concavity of the objective function in (9). This in turn is guaranteed, since u and v are strictly concave by Assumption 1 and g is concave by Lemma 1.

3.3 Efficient contracts

The natural question now is whether financial intermediation that offers loan and deposit contracts in line with Problem (7) can implement the efficient allocation determined in Proposition 1. In situations in which agents' private actions are difficult to control, the arising incentive constraints make it questionable whether an efficient outcome can be achieved, cf. Myerson (1979). To address this problem, we will next define an *efficient contract* as a contract that implements the efficient allocation and is optimal for both agents and the bank.

Definition 1 (*Efficient contract*) Given a wage rate w_t, a loan contract (B_t, I_t, R_t) together with a deposit rate r_t is called an efficient contract (in period t) if the following holds:

(i) The quadruple (B_t, I_t, R_t, r_t) solves the bank's problem (7).
(ii) The allocation induced by (B_t, I_t, R_t, r_t) is efficient in the sense of Proposition 1.

Observe that by definition, any efficient contract must be incentive compatible. With an efficient contract, agents are fully insured and old-age consumption is independent of the success of the project because the bank completely diversifies away idiosyncratic risk. Incentive compatibility of the efficient deposit rate r_t implies in particular that agents decide at their own discretion to save the amount of funds required for complete risk sharing.

Our next proposition establishes the existence of a unique efficient contract.[7]

Proposition 2 (Existence of an efficient contract) *Let Assumptions 1–3 be satisfied and* $w_t > 0$ *be given. Then there exists a uniquely determined efficient contract* (B_t, I_t, R_t, r_t)*, which is given by the following equations:*

(i) *The investment level satisfies* $0 < I_t < w_t$ *and solves*

$$\max_{0 \le I \le w_t} u(w_t - I) + v(g(I)). \tag{10}$$

[7] Note again that the uniqueness of the efficient investment level hinges on the concavity of the objective function in (10) for which the (strict) concavity of u, v, and g is a sufficient condition.

(ii) *The deposit rate is*

$$r_t = g'(I_t). \tag{11}$$

(iii) *The loan interest rate is*

$$R_t = \frac{g'(I_t)}{p(I_t)}. \tag{12}$$

(iv) *The loan size is*

$$B_t = \frac{g(I_t)}{g'(I_t)}. \tag{13}$$

(v) *The profit constraint* (PrC) *and the resource constraint* (RC) *are binding with*

$$D_t = B_t = S(w_t, B_t, I_t, R_t, r_t). \tag{14}$$

An immediate implication of Proposition 2 is that with an efficient contract, the bank extracts no rent as it seizes all the proceeds of the successful projects, $R_t B_t = \pi(I_t)$, and awards the whole surplus to the old consumers.[8] A second implication of Proposition 2 is an *income effect*, already identified in Banerji et al. (2004). Since capital income g is strictly concave by Lemma 1, its elasticity is less than one so that (13) implies

$$B_t - I_t = \left(\frac{g(I_t)}{g'(I_t)I_t} - 1\right) I_t > 0. \tag{15}$$

Hence, the efficient contract enlarges the disposable income of the young agent, $w_t^d = w_t + B_t - I_t > w_t$.[9] By a slight abuse of notation, the savings function corresponding to an *efficient* loan contract in (14) thus takes the standard form

$$S(w_t^d, r_t) := \underset{0 \le D \le w_t^d}{\text{argmax}} \ u(w_t^d - D) + v(r_t D). \tag{16}$$

Since by Assumption 1 both youthful and old-age consumption are normal goods, the young agent will prefer the efficient contract to a pure deposit contract without investing. If g were not concave, then the income effect (15) may be negative in which case the participation constraint (PC) is violated. In the proof of Proposition 2, we show that under Assumptions 1–3, (PC) is satisfied by establishing that the expected utility of an efficient contract is strictly larger than the expected utility of an investment without funding from the bank combined with precautionary savings.

Example 2 For the logarithmic utility functions $u(c^1) = \ln(c^1)$ and $v(c^2) = \beta \ln(c^2)$ with $\beta > 0$, the savings function is independent of the deposit rate because $S(w_t^d) = \frac{\beta}{1+\beta} w_t^d$.

Remark 1 (Tying contracts) If g is non-concave, w_t^d may fall below w_t. In this case, the agent rejects the loan contract because she would be better off saving out of wage income w_t, even without investing into the project. The bank could still implement

[8] Our model may therefore be interpreted as a model of perfect competition among banks.

[9] The efficient contract may be interpreted as the agent selling her project to the bank for the amount $B_t - I_t > 0$ and then saving the amount $S(w_t^d, r_t)$.

the efficient allocation by tying the loan contract (B_t, I_t, R_t) to the deposit contract r_t and offering agents who only want to save a deposit rate $0 < \tilde{r}_t < r_t$ that makes them worse off. The existence of $\tilde{r}_t$ is seen as follows. Strict concavity of v implies

$$U_{\text{res}}(w_t, \tilde{r}_t) < \max\Big\{u(w_t - D^{\text{A}} - I^{\text{A}}) + v(\tilde{r}_t D^{\text{A}} + g(I^{\text{A}})) \,\Big|\, D^{\text{A}} \geq 0,\ I^{\text{A}} \geq 0,\ D^{\text{A}} + I^{\text{A}} \leq w_t\Big\}.$$

For $\tilde{r}_t = 0$, the objective function on the r.h.s. attains its maximum in $(D_t^{\text{A}} = 0, I_t^{\text{A}} = I_t)$, yielding the utility level $u(w_t - I_t) + v(g(I_t))$. This shows that for $\tilde{r}_t = 0$, the agent is strictly better off accepting the tying contract. Since U_{res} is continuous and increasing in r, there exists a largest positive deposit rate $0 < \tilde{r}_t < r_t$, defined by

$$U_{\text{res}}(w_t, \tilde{r}_t) = u(w_t - I_t) + v(g(I_t)),$$

at which the agent is indifferent between the tying contract and investing without funding.

4 Capital accumulation and qualitative dynamics

This section is concerned with the qualitative dynamics induced by efficient contracts. The aggregate capital stock of the economy is determined by the total capital endowment of the successful projects. The law of large numbers implies that on aggregate, the share of successful production projects in period $t + 1$ is $p(I_t)$, where I_t is the efficient investment level defined in (10). It follows from Proposition 2 that I_t is stipulated by the *investment function* $\mathcal{I} : \mathbb{R}_+ \to \mathbb{R}_+$, defined by

$$I_t = \mathcal{I}(w_t) := \operatorname*{argmax}_{0 \leq I \leq w_t} \ u(w_t - I) + v(g(I)). \tag{17}$$

Given the wage rate w_t, the productive capital stock of the subsequent period $t + 1$ thus is

$$k_{t+1} = \Omega(I_t) = \Omega(\mathcal{I}(w_t)).$$

In a perfectly competitive environment, labour is paid its marginal product. Denoting by $w_t = w(k_t) := f(k_t) - f'(k_t)k_t$ the marginal product of labour, it follows that capital accumulation is driven by the time-one map $G : \mathbb{R}_+ \to \mathbb{R}_+$, defined by

$$k_{t+1} = G(k_t) := \Omega\big(\mathcal{I}(w(k_t))\big). \tag{18}$$

The dynamical system (18) governs the forward dynamics of the economy, in the sense that any growth path $\{k_t\}_{t=0}^{\infty}$ of the economy with initial capital $k_0 > 0$ is recursively defined by $k_{t+1} = G(k_t)$, $t = 0, 1, \ldots, \infty$.

We are now in the position to state our main result.

Theorem 3 *(Monotonic dynamics) Let Assumptions 1–3 be satisfied and assume that the bank offers efficient contracts. Then $G' > 0$ so that the dynamics of the economy is monotonic.*

Theorem 3 demonstrates that if financial intermediation implements efficient contracts, then the resulting dynamics is always monotonic. All growth paths $\{k_t\}_{t=0}^{\infty}$ generated by (18) are either monotonically increasing or monotonically decreasing, as in the example portrayed in Fig. 1.

Endogenous fluctuations, including complex dynamics, require the time-one map G to be decreasing at least in some neighbourhood of a steady state $k_\star$ of G. This behaviour is ruled out by Theorem 3. Since $\Omega' > 0$ by Assumption 2 and $w' > 0$ by Assumption 3, the chain rule applied to (18) implies that $G' > 0$ if and only if $\mathcal{I}' > 0$. In the proof of Theorem 3, we establish that the latter condition is ensured by the efficiency of the investment level $\mathcal{I}(w)$ that determines the welfare maximum of agents.

The qualitative dynamics induced by efficient contracts may be classified by the stability properties of the steady states $k_\star = G(k_\star)$.

Proposition 4 (Properties of steady states) *Let Assumptions 1–3 be satisfied and assume that the bank offers efficient contracts. Then the following holds.*

(i) *The origin $k_\star = 0$ is a steady state of G if and only if $w(0) = 0$.*
(ii) *If either $w(0) > 0$ or $\lim_{k\to 0} G'(k) > 1$, then G has at least one positive steady state $k_\star > 0$. The largest one of these steady states is asymptotically stable.*

Proposition 4 implies that the dynamics of our model is qualitatively equivalent to the dynamics of the standard two-period lived OLG model, e.g. see De La Croix and Michel (2002). The following example with a standard parameterisation from the literature on OLG models is insightful.

Example 3 Consider the success function $p(I) = \frac{1}{1+I}$ combined with loglinear utility $u(c^1) = \ln(c^1)$ and $v(c^2) = \beta \ln(c^2)$, where $\beta > 0$, and the Cobb–Douglas production function $f(k) = Ak^\alpha$, where $A > 0$ and $0 < \alpha < 1$. The investment function (17) then takes the form

$$\mathcal{I}(w_t) = \sqrt{\left(\frac{1+\alpha\beta}{2}\right)^2 + \alpha\beta w_t} - \frac{1+\alpha\beta}{2}$$

and the evolution of capital-labour ratios is driven by the map

$$k_{t+1} = G(k_t) = 1 - \left(\frac{1-\alpha\beta}{2} + \sqrt{\left(\frac{1+\alpha\beta}{2}\right)^2 + \alpha\beta(1-\alpha)Ak_t^\alpha}\right)^{-1}.$$

Since $G' > 0$, the dynamics is monotonic and all growth paths with $k_0 > 0$ converge to a unique positive steady state $k_\star$ that is asymptotically stable, cf. Fig. 1.

The result that efficient contracts rule out business cycles and complex dynamics contradicts the findings in Banerji et al. (2004). They argue that efficient risk sharing enabled by financial intermediation may generate endogenous fluctuations in an economy that otherwise would always converge monotonically to a steady state. Our analysis shows that under the premise of efficient risk sharing, the qualitative dynamics of the model is monotonic and that the enlarged disposable income of agents is a mere byproduct of an incentive problem without any impact on the qualitative dynamics.

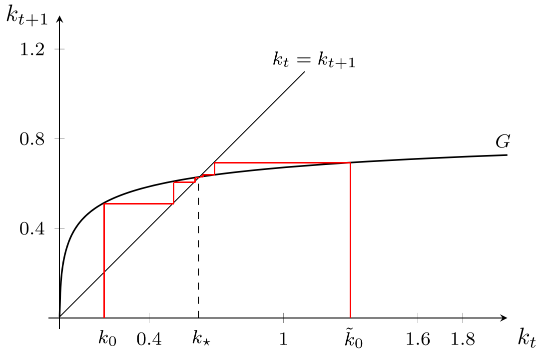

Fig. 1 Monotonic convergence to a unique asymptotically stable steady state $k_\star > 0$ induced by efficient contracts ($A = 30$, $\alpha = 0.6$, $\beta = 1$)

In the discussion-paper version of this article, we analyse an example in which productive capital Ω and thus the objective function in (10) is not concave and show that business cycles can only be triggered by contracts that *do not* maximise agents' welfare and thus *do not* provide efficient risk sharing.[10]

We conclude our analysis by demonstrating that the steady states of the economy will generically be *dynamically inefficient* in the usual sense of macroeconomics. Given a wage rate w_t, youthful consumption in period t is

$$c_t^1 = w_t - I_t, \tag{19}$$

while old-age consumption in period t is

$$c_t^{2b} = c_t^{2g} = g(I_{t-1}) = c_t^2. \tag{20}$$

Adding (19) and (20), total consumption per capita in period t becomes

$$c_t := c_t^1 + c_t^2 = w_t - I_t + g(I_{t-1}) = f(k_t) - I_t. \tag{21}$$

Since $k_{t+1} = \Omega(I_t)$, stationary allocations $(\bar{k}, \bar{c})$, where $\bar{c} = \bar{c}^1 + \bar{c}^2$ denotes stationary total consumption, are given by

$$\bar{c} = \phi(\bar{I}) := f(\Omega(\bar{I})) - \bar{I}.$$

Clearly, stationary total consumption $\bar{c}$ is maximal in a maximum of the function ϕ.

Lemma 2 (Maximal consumption) *Let Assumptions 1–3 be satisfied. Then the map ϕ attains its global maximum at the golden-rule investment level $I_G > 0$, which is uniquely determined by*

$$f'(\Omega(I_G))\,\Omega'(I_G) = 1. \tag{22}$$

Lemma 2 resembles the golden rule of capital accumulation of the standard Solow (1956) growth model. Denote the capital-labour ratio corresponding to I_G by $k_G =$

[10] See Ritschel and Wenzelburger (2023) for a discussion of Banerji et al. (2004).

$\Omega(I_G)$. If capital depreciates fully and the population profile is stationary, as is the case in our model, then Solow's golden-rule capital-labour ratio k_G^{S} is determined by $f'(k_G^{\mathrm{S}}) = 1$. Since f is strictly concave and Assumption 2 implies $0 < \Omega' \leq 1$, it follows from (22) that $k_G \leq k_G^{\mathrm{S}}$, where equality holds if $p'(I_G) = 0$ and $p(I_G) = 1$. Thus, a positive failure rate of the production projects entails a lower golden-rule value in our model.

Observe also that k_G is solely determined by the production function f and the success probability function p. Since any steady state will, by construction, depend on agents' preferences, k_G will generally not be a steady state of the dynamical system (18). Indeed, k_G is a steady state of G if and only if $I_G = \mathcal{I}(w(k_G))$. Stated differently, the golden-rule consumption plan

$$c_G^1 = w(k_G) - I_G, \quad c_G^2 = g(I_G)$$

is a steady-state consumption plan of the dynamical system (18) if and only if I_G solves

$$-u'(w(\Omega(I)) - I) + v'(g(I))\, g'(I) \overset{!}{=} 0.$$

In this case, the economy attains the steady state with the highest possible welfare level. Finally, observe from (15) that

$$c_G^1 + \frac{c_G^2}{r_G} = w(k_G) + \left(\frac{g(I_G)}{g'(I_G)I_G} - 1\right) I_G =: w_G^d,$$

so that in a golden-rule steady state, disposable income is consumed fully. In general, however, a golden-rule steady state will not obtain so that efficient contracts will generically *not induce* dynamically efficient growth paths.

5 Conclusion

This article examined an overlapping-generations model in which financial intermediation arises endogenously as a device to share idiosyncratic risk. Agents' welfare is maximised by efficient contracts that provide complete insurance and perfect consumption smoothing without a premium for the bank. In order to implement efficient allocations, the bank must enlarge agents' disposable income because savings decisions are at their discretion. This implementation is only possible if capital income is concave in the investment level. Otherwise, the implementation of efficient allocations requires tying contracts. Our main contribution to the banking literature is the result that in any two-period lived OLG model with standard preferences and increasing productive capital, financial intermediation that implements efficient risk sharing can neither induce business cycles nor complex dynamics. The resulting qualitative dynamics is always monotonic and thus indistinguishable from the dynamics of the standard two-period lived OLG model. Our analysis reveals that business cycles may only be triggered by contracts which implement a local welfare minimum and would not be accepted by rational agents. To find conditions under which financial intermediation triggers business cycles remains to be an open issue for future research.

Acknowledgements We would like to thank the audience at the 22nd annual SAET conference in Paris and the participants of research seminars at the University of Kaiserslautern-Landau for valuable discussions. We are particularly grateful to Dominique Demougin, Philipp Weinschenk, Fabian Herweg, Tom Rauber, and Jan Münning for helpful comments.

Funding Open Access funding enabled and organized by Projekt DEAL.

Data Availability No data was used for the research in this article.

Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose.

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Appendix A: Proofs

Proof of Lemma 1 Differentiating $g(I) = f'(\Omega(I))\,\Omega(I)$ yields

$$g'(I) = \Omega'(I)\, f'(\Omega(I)) \left[1 + \frac{f''(\Omega(I))\,\Omega(I)}{f'(\Omega(I))}\right].$$

By Assumption 2, $\Omega' > 0$. By Assumption 3, $f' > 0$, $f'' < 0$, and $f''(k)k/f'(k) > -1$ for all $k \in \mathbb{R}_+$. Hence, g is strictly increasing. Since $\lim_{k\to 0}\ f'(k)k = 0$ and $\Omega(0) = 0$, it follows that $g(0) = 0$.[11] Strict concavity of g holds because

$$\begin{aligned} g''(I) = \Omega''(I)\, f'(\Omega(I)) &\left[1 + \frac{f''(\Omega(I))\,\Omega(I)}{f'(\Omega(I))}\right] \\ +\Omega'(I)^2\, f''(\Omega(I)) &\left[2 + \frac{f'''(\Omega(I))\,\Omega(I)}{f''(\Omega(I))}\right] < 0, \end{aligned}$$

noting that $\Omega'' \le 0$ by Assumption 2 and $f'''(k)k/f''(k) > -2$ by Assumption 3. □

Proof of Proposition 1 Let w_t be arbitrary but fixed. By Lemma 1, g is strictly increasing and strictly concave with $g(0) = 0$. Assumption 1 implies that in an optimum,

[11] For a formal proof that Assumption 3 implies $\lim_{k\to 0}\ f'(k)k = 0$, we refer to De La Croix and Michel (2002, p. 308).

$c^1 = w_t - I$. The first-order conditions are therefore

$$-u'(w_t - I) + p'(I)[v(c^{2g}) - v(c^{2b})] + \lambda[g'(I) - p'(I)(c^{2g} - c^{2b})] = 0 \quad \text{(A1)}$$

$$p(I)\, v'(c^{2g}) - \lambda p(I) = 0 \quad \text{(A2)}$$

$$(1 - p(I))\, v'(c^{2b}) - \lambda(1 - p(I)) = 0 \quad \text{(A3)}$$

and the complementary slackness condition is

$$\lambda[g(I) - p(I)\, c^{2g} - (1 - p(I))\, c^{2b}] = 0. \quad \text{(A4)}$$

Assumption 1 implies that in an optimum, the constraint on the consumption plan must hold with equality. Since $g(0) = 0$, the Inada conditions on v imply that $I > 0$ because otherwise $c^{2g} = c^{2b} = 0$ by (A4). Therefore, $0 < p(I) < 1$. Conditions (A2) and (A3) then imply $v'(c^{2g}) = v'(c^{2b}) = \lambda > 0$. Since $v'' < 0$, it follows from (A4) that a social optimum requires

$$c^{2g} = c^{2b} = g(I). \quad \text{(A5)}$$

Moreover, (A1) reduces to

$$-u'(w_t - I) + v'(g(I))\, g'(I) = 0. \quad \text{(A6)}$$

Observe that (A6) is the first-order condition of the maximisation problem

$$\max_{0 \le I \le w_t} \; u(w_t - I) + v(g(I)). \quad \text{(A7)}$$

The objective function in (A7) is either already a continuous function or can be transformed into a continuous function on the compact interval $[0, w_t]$ using the exponential function. Hence, a solution $I_t^\star$ to (A7) exists. It follows that any solution $(I_t^\star, c_{t+1}^{2g\star}, c_{t+1}^{2b\star})$ to the first-order conditions (A1)–(A3) must satisfy (A5) with $I_t^\star$ being a maximiser of Problem (A7). In other words, the social planner's problem (8) reduces to Problem (A7), so that any maximiser $I_t^\star$ of (A7) together with (A5) and $c_t^{1\star} = w_t - I_t^\star$ is a maximiser of (8). The concavity of g implies that the objective function in (A7) is strictly concave so that the social optimum is uniquely determined. □

Proof of Proposition 2 The proof comprises **four steps**.

Step 1 (Relaxed problem). We establish the existence and uniqueness of a solution to Problem (7) without the participation constraint. The Lagrangian of Problem (7) without the participation constraint is

$$\begin{aligned}\mathcal{L}(B, I, R, r, \lambda_1, \lambda_2) := {} & u(w_t + B - I - D) + p(I)\, v(rD + \pi(I) - RB) \\ & + ((1 - p(I))\, v(rD) + \lambda_1(p(I)RB - rD) + \lambda_2(D - B),\end{aligned} \quad \text{(A8)}$$

where $\lambda_1, \lambda_2 \geq 0$ are the Lagrange multipliers and $D = S(w_t, B, I, R, r)$ to simplify notation. The four first-order conditions for a solution (B_t, I_t, R_t, r_t) are:

$$0 = p(I_t)R_t\Big[v'(r_tD_t + \pi(I_t) - R_tB_t) - \lambda_1\Big] + \lambda_2 - u'(w_t + B_t - I_t - D_t) + (\lambda_1 r_t - \lambda_2)\,\frac{\partial S}{\partial B}(w_t, B_t, I_t, R_t, r_t) \tag{A9}$$

$$0 = h'(I_t)\,p(I_t)\,v'(r_tD_t + \pi(I_t) - R_tB_t) + p'(I_t)\Big[v(r_tD_t + \pi(I_t) - R_tB_t) - v(r_tD_t)\Big] + \lambda_1 p'(I_t)R_tB_t - u'(w_t + B_t - I_t - D_t) - (\lambda_1 r_t - \lambda_2)\,\frac{\partial S}{\partial I}(w_t, B_t, I_t, R_t, r_t) \tag{A10}$$

$$0 = \Big[\lambda_1 - v'\big(r_tD_t + \pi(I_t) - R_tB_t\big)\Big]p(I_t)B_t - (\lambda_1 r_t - \lambda_2)\,\frac{\partial S}{\partial R}(w_t, B_t, I_t, R_t, r_t) \tag{A11}$$

$$0 = \Big[p(I_t)\,v'\big(r_tD_t + \pi(I_t) - R_tB_t\big) + [1 - p(I_t)]\,v'(r_tD_t) - \lambda_1\Big]D_t - (\lambda_1 r_t - \lambda_2)\,\frac{\partial S}{\partial r}(w_t, B_t, I_t, R_t, r_t), \tag{A12}$$

where $D_t = S(w_t, B_t, I_t, R_t, r_t)$. The two complementary slackness conditions are:

$$\lambda_1[p(I_t)R_tB_t - r_tD_t] = 0 \tag{A13}$$

$$\lambda_2(D_t - B_t) = 0. \tag{A14}$$

Assume that $\lambda_1 r_t - \lambda_2 = 0$. We will show below with (A34) that in an optimum, this identity must hold. As a consequence, all terms involving derivatives of S in the first-order conditions (A9)–(A12) are zero. Since $p > 0$, (A11) is equivalent to

$$\Big[\lambda_1 - v'(r_tD_t + \pi(I_t) - R_tB_t)\Big]B_t = 0. \tag{A15}$$

By Assumption 1, $v' > 0$ so that two cases can occur in (A15). First,

$$B_t > 0 \quad \text{and} \quad \lambda_1 = v'(r_tD_t + \pi(I_t) - R_tB_t) > 0. \tag{A16}$$

Second, $B_t = 0$.

Case 1. Since $B_t > 0$ and $\lambda_1 > 0$, (A13) requires

$$p(I_t)R_tB_t = r_tD_t, \tag{A17}$$

stating that the profit constraint is binding. Inserting (A16), it follows that (A9) holds with

$$\lambda_2 = u'(w_t + B_t - I_t - D_t) > 0. \tag{A18}$$

Since $\lambda_2 > 0$, (A14) implies that the resource constraint is binding,

$$B_t = D_t > 0. \tag{A19}$$

Using (A19), it follows from (A17) that

$$r_t = p(I_t)R_t. \tag{A20}$$

Inserting (A16) into (A12) yields

$$[1 - p(I_t)]D_t\Big[v'(r_t D_t) - v'(r_t D_t + \pi(I_t) - R_t B_t)\Big] = 0. \tag{A21}$$

(A21) has two possible solutions. First, since $0 < p(I) < 1$ for $I > 0$, $I_t = 0$ is a solution whenever $p(0) = 1$. In this case, (A20) implies $R_t = r_t$ and thus $R_t B_t = r_t D_t$ so that the attained utility level is $u(w_t) + v(0)$. By Assumption 1, this level cannot be optimal. Since $v'' < 0$, the second solution to (A21) is

$$R_t B_t = \pi(I_t). \tag{A22}$$

It follows from (A16) and (A18) that

$$\lambda_1 = v'(r_t D_t) > 0 \quad \text{and} \quad \lambda_2 = u'(w_t - I_t) > 0. \tag{A23}$$

Combining (A20) with (A22) yields

$$B_t = \frac{g(I_t)}{r_t}. \tag{A24}$$

Since $D_t = B_t$, (A24) implies

$$r_t D_t = g(I_t). \tag{A25}$$

Therefore,

$$\lambda_1 = v'(g(I_t)). \tag{A26}$$

Inserting (A19), (A22), (A25), and (A26), Condition (A10) reduces to

$$- u'(w_t - I_t) + v'(g(I_t))\, g'(I_t) = 0. \tag{A27}$$

Condition (A27) determines the optimal investment level I_t. Observe that (A27) is the first-order condition for the maximisation problem

$$\max_{0 \le I \le w_t} \; u(w_t - I) + v(g(I)). \tag{A28}$$

Equations (A22) and (A25) imply that any utility-maximising consumption plan of the relaxed problem (A8) has to satisfy

$$c_t^1 = w_t - I_t \quad \text{and} \quad c_{t+1}^{2g} = c_{t+1}^{2b} = r_t D_t = g(I_t). \tag{A29}$$

Hence, any solution to (A8) is already determined by a solution I_t to Problem (A28). Since Problem (A28) coincides with Problem (10), existence and uniqueness of $0 < I_t < w_t$ obtain from the same arguments as presented in the proof of Proposition 1.

Case 2. If $B_t = 0$, then the profit constraint (PrC) implies that $r_t D_t = 0$. The strict concavity of v yields

$$u(w_t - I - D) + p(I)\, v(\pi(I)) + (1 - p(I))\, v(0) < u(w_t - I) + v(g(I)) \quad \text{(A30)}$$

for all $I, D > 0$ with $I + D \le w_t$. Note that the r.h.s. of (A30) is the objective function of Problem (A28), which assumes its maximum in $0 < I_t < w_t$ with I_t being determined by (A27). Hence, $B_t = 0$ cannot be optimal.

It follows that $B_t > 0$ is optimal and that the optimal solution to the relaxed problem (A8) is uniquely determined by (A27) together with (A29).

Step 2 (Incentive compatibility). In Step 1, the optimal deposit rate r_t has not yet been determined. Given the loan contract (B_t, I_t, R_t) determined in Step 1, the incentive constraint (IC) implies that deposits $D_t = S(w_t, B_t, I_t, R_t, r_t)$ must satisfy the first-order condition

$$u'(w_t + B_t - I_t - D_t) = \Big[p(I_t)\, v'(r_t D_t + \pi(I_t) - R_t B_t) + (1 - p(I_t))\, v'(r_t D_t)\Big] r_t. \quad \text{(A31)}$$

Inserting (A19), (A22), and (A25), Condition (A31) simplifies to

$$-u'(w_t - I_t) + v'(g(I_t))\, r_t = 0. \quad \text{(A32)}$$

A comparison of (A27) with (A32) shows that the optimal deposit rate is

$$r_t = g'(I_t). \quad \text{(A33)}$$

Using (A23), (A26), and (A32), it follows that r_t satisfies

$$\lambda_1 r_t - \lambda_2 = v'(g(I_t))\, r_t - u'(w_t - I_t) = 0, \quad \text{(A34)}$$

thus justifying the assumption made at the outset of the proof. Finally, inserting (A33) into (A24) and (A20) yields

$$B_t = \frac{g(I_t)}{g'(I_t)} \quad \text{and} \quad R_t = \frac{g'(I_t)}{p(I_t)}.$$

Step 3 (Efficiency). To see that the contract (B_t, I_t, R_t, r_t) computed above implements the efficient allocation, recall that the first-order conditions (A6) and (A27) coincide, so that $I_t = I_t^\star$ is the efficient investment level. It follows from (A22) and (A25) that

$$c_{t+1}^{2g} = c_{t+1}^{2b} = r_t S(w_t, I_t, B_t, R_t, r_t) = g(I_t) = g(I_t^\star) = c_{t+1}^{2g\star} = c_{t+1}^{2b\star}.$$

Thus, (B_t, I_t, R_t, r_t) implements the efficient allocation.

Step 4 (Participation constraint). We prove that the relaxed problem without the participation constraint (A8) has the same solution as Problem (7) by showing that agents will accept the efficient contract (B_t, I_t, R_t, r_t) computed above.

An agent who rejects the efficient loan contract may save and invest with idiosyncratic risk, solving Problem (6). By the strict concavity of v, the objective function in (6) satisfies

$$\begin{aligned} & u(w_t - I^{\mathrm{A}} - D^{\mathrm{A}}) + p(I^{\mathrm{A}})\, v(r_t D^{\mathrm{A}} + \pi(I^{\mathrm{A}})) + [1 - p(I^{\mathrm{A}})]\, v(r_t D^{\mathrm{A}}) \\ & \quad \leq u(w_t - I^{\mathrm{A}} - D^{\mathrm{A}}) + v(r_t D^{\mathrm{A}} + g(I^{\mathrm{A}})) \end{aligned} \tag{A35}$$

for all $I^{\mathrm{A}}, D^{\mathrm{A}} \geq 0$ with $I^{\mathrm{A}} + D^{\mathrm{A}} \leq w_t$. Replacing the objective function in Problem (6) with the r.h.s. of Inequality (A35), an auxiliary problem obtains.

We will next establish that the uniquely determined maximiser of this auxiliary problem is $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} = 0)$, where I_t is the efficient investment level, and show that agents will be worse off rejecting the efficient contract.

Observe first that the auxiliary objective function is strictly concave if g is strictly concave. The Inada conditions on u and v imply that any solution $(I_t^{\mathrm{A}}, D_t^{\mathrm{A}})$ to Problem (6) must satisfy $0 < I_t^{\mathrm{A}} + D_t^{\mathrm{A}} < w_t$. Thus, there are three possible solutions to the auxiliary problem.

Case 1: $I_t^{\mathrm{A}} = 0, D_t^{\mathrm{A}} > 0$. The resulting first-order conditions in this case read

$$-u'(w_t - D^{\mathrm{A}}) + v'(r_t D^{\mathrm{A}})\, g'(0) + \lambda_1 = 0 \tag{A36}$$

$$-u'(w_t - D^{\mathrm{A}}) + v'(r_t D^{\mathrm{A}})\, r_t = 0. \tag{A37}$$

The Inada conditions imply that a solution $0 < D_t^{\mathrm{A}} < w_t$ to (A37) exists. Inserting (A37) into (A36), we see that $(I_t^{\mathrm{A}} = 0, D_t^{\mathrm{A}})$ is a possible maximum if

$$\lambda_1 = v'(r_t D_t^{\mathrm{A}})[r_t - g'(0)] \geq 0.$$

However, since $r_t = g'(I_t)$ and $g'' < 0$, it follows that λ_1 must be negative. Hence, $(I_t^{\mathrm{A}} = 0,\ D_t^{\mathrm{A}})$ does not satisfy the first-order conditions.

Case 2: $I_t^{\mathrm{A}} > 0, D_t^{\mathrm{A}} = 0$. The corresponding first-order conditions are

$$-u'(w_t - I^{\mathrm{A}}) + v'(g(I^{\mathrm{A}}))\, g'(I^{\mathrm{A}}) = 0 \tag{A38}$$

$$-u'(w_t - I^{\mathrm{A}}) + v'(g(I^{\mathrm{A}}))\, r_t + \lambda_2 = 0. \tag{A39}$$

As shown in the proof of Proposition 1, the unique solution to (A38) is the efficient investment level $I_t^{\mathrm{A}} = I_t$. Since $r_t = g'(I_t)$, it follows that $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} = 0)$ together with $\lambda_2 = 0$ solves the first-order conditions.

Case 3: $I_t^{\mathrm{A}} > 0, D_t^{\mathrm{A}} > 0$. The resulting first-order conditions are

$$-u'(w_t - I^{\mathrm{A}} - D^{\mathrm{A}}) + v'(r_t D^{\mathrm{A}} + g(I^{\mathrm{A}}))\, g'(I^{\mathrm{A}}) = 0 \tag{A40}$$

$$-u'(w_t - I^{\mathrm{A}} - D^{\mathrm{A}}) + v'(r_t D^{\mathrm{A}} + g(I^{\mathrm{A}}))\, r_t = 0. \tag{A41}$$

A comparison of (A40) with (A41) shows that any solution I_t^{A} requires $r_t = g'(I_t^{\mathrm{A}})$. Since $r_t = g'(I_t)$ and $g'' < 0$, it follows that $I_t^{\mathrm{A}} = I_t$. A comparison with Case 2 shows that $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} = 0)$ solves (A40) and (A41). Since both equations are strictly decreasing in D, no solution with positive savings $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} > 0)$ exists.

These considerations show that the maximum of the auxiliary problem obtains in $(I_t^{\mathrm{A}} = I_t, D_t^{\mathrm{A}} = 0)$ and achieves the utility level $u(w_t - I_t) + v(g(I_t))$. Inequality (A35) implies

$$U_{\mathrm{res}}(w_t, r_t) \leq u(w_t - I_t) + v(g(I_t)) = V(w_t, B_t, I_t, R_t, r_t),$$

showing that agents are indeed willing to accept the efficient contract (B_t, I_t, R_t, r_t). □

Proof of Theorem 3 Endogenous fluctuations are ruled out if $G' > 0$. Differentiating (18) yields

$$G'(k) = \Omega'\big(\mathcal{I}(w(k))\big)\,\mathcal{I}'(w(k))\,w'(k). \tag{A42}$$

By Assumption 2, $\Omega' > 0$. Moreover, $w' > 0$ by Assumption 3. We next show that $\mathcal{I}' > 0$ such that $G' > 0$ holds. The investment function $\mathcal{I}$ is defined by the first-order condition

$$-u'(w - \mathcal{I}(w)) + v'\big(g(\mathcal{I}(w))\big)\,g'(\mathcal{I}(w)) = 0. \tag{A43}$$

Differentiating (A43) yields

$$\mathcal{I}'(w) = \frac{u''(w - \mathcal{I}(w))}{u''(w - \mathcal{I}(w)) + v''\big(g(\mathcal{I}(w))\big)\,g'(\mathcal{I}(w))^2 + v'\big(g(\mathcal{I}(w))\big)\,g''(\mathcal{I}(w))}. \tag{A44}$$

By Assumption 1, $u'' < 0$ such that the numerator in (A44) is strictly negative. Since $0 < \mathcal{I}(w) < w$ is a maximiser, the second-order condition for the objective function in (A28) is satisfied, implying that the denominator in (A44) is strictly negative. Thus, we conclude that (A44) is strictly positive so that $G' > 0$. □

Proof of Proposition 4 (i) Steady states of G are determined by solutions $k_\star \geq 0$ to

$$k \stackrel{!}{=} \Omega\big(\mathcal{I}(w(k))\big). \tag{A45}$$

Note that $\Omega(0) = 0$. If $w(0) = 0$, then $\mathcal{I}(w(0)) = 0$ and, consequently, $k_\star = 0$ solves (A45). On the contrary, if $w(0) > 0$, then the Inada conditions stated in Assumption 1 imply $0 < \mathcal{I}(w(0)) < w(0)$. Since $\Omega' > 0$, it follows that $\Omega\big(\mathcal{I}(w(0))\big) > 0$, showing that $k_\star = 0$ cannot solve (A45). Hence, $k_\star = 0$ solves (A45) if and only if $w(0) = 0$.
(ii) It follows from the definition of Ω and Proposition 2 that

$$0 \leq G(k) = \Omega\big(\mathcal{I}(w(k))\big) \leq f(k) \quad \text{for all} \quad k \geq 0. \tag{A46}$$

If $w(0) > 0$, then $G(0) > 0$, so that in the local neighbourhood of zero, we have $G(k) > k$. On the other hand, if $w(0) = 0$, then $G(0) = 0$. In this case, it follows from the property $\lim_{k \to 0} G'(k) > 1$ that in the local neighbourhood of zero,

$G(k) > k$ holds. Inequality (A46), the strict concavity of f, and the Inada condition $\lim_{k\to\infty} f'(k) = 0$ then imply that there exists at least one $k_\star > 0$ that solves (A45). Of these solutions, the largest one must satisfy $0 < G'(k_\star) < 1$ and thus be asymptotically stable. □

Proof of Lemma 2 Assumptions 2 and 3 imply that $I \mapsto f(\Omega(I))$ is strictly increasing and strictly concave. Hence, a solution I_G to

$$\max_{I\geq 0} f(\Omega(I)) - I \tag{A47}$$

is unique, if it exists. Observe that $f(\Omega(I)) - I \leq f(I) - I$ for all $I > 0$. It follows from Assumption 3 that the function $I \mapsto f(I) - I$ has a unique maximum. Hence, Problem (A47) admits a unique solution $I_G > 0$, determined by the first-order condition (22). □

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